A COMPARISON OF TWO DOUBLE SAMPLING AUXILIARY VARIABLES WITH INVENTORIES OF VARYING SAMPLE SIZE

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ABSTRACT. —Two auxiliary variables were tested to determine the effects on the precision statement about the mean board foot volume per acre. These comparisons were made with data from three inventories of differing sizes in West Virginia. The sum of the merchantable height on all "in-trees" per point was compared with the more commonly employed auxiliary variable of basal area per acre. The auxiliary variable based on the sum of the merchantable height on "in-trees" resulted in higher correlations with board foot volume per acre than the use of basal area in all three inventories. This increase in correlation did not produce a substantial decrease in the standard error of the mean. A time study was also conducted to investigate the additional time required to estimate the number of logs on "in-trees" per point when compared to just recording basal area per acre. On average, estimating merchantable height on "in-trees" took an additional 13 seconds per point.

Double sampling, also referred to as two-phase sampling, was originally mentioned in forestry literature by Spurr (1952), while the first detailed description of its application was presented by Freese (1962). This sampling technique was developed to take advantage of the relationship between the variable of interest and some easily measured and highly correlated auxiliary variable. The primary objective of double sampling is to reduce total inventory time without sacrificing the precision about the point estimate. This can be accomplished through the use of a carefully selected auxiliary variable. A prerequisite for a good auxiliary variable is one that is easily and thus quickly measured and is also highly correlated with the variable of interest. The most commonly utilized auxiliary variable with double sampling for volume estimates is basal area. This is due to the high correlation between basal area and volume and the fact that basal area can be determined very quickly when using point sampling. Although basal area is an excellent candidate for double sampling, other auxiliary variables can be used. In a New Jersey study by Wiant and Zeide (1979), the sum of the estimated merchantable logs on "in-trees" was initially examined for use as an auxiliary variable. Their results indicated that the sum of the number of logs on "in-trees" provided a higher correlation with board foot volume than basal area per acre. This is a plausible statement since a 20-inch tree with 3-logs and a 20-inch tree with 1-log have the same basal area, but different merchantable heights, and thus different volumes. Another study that utilized merchantable height as an auxiliary variable was reported by Myers (1985) for a simulation study conducted in a 36-year-old even-aged hardwood forest located in northern West Virginia. He concluded that the use of the sum of estimated merchantable logs as the auxiliary variable provided a more precise estimate of board foot volume than basal area using repeated sampling with samples sizes ranging from 20 to 60. Although limited testing of this auxiliary variable appears promising, its use in large-scale forest inventory applications has not been documented. This study was designed to investigate the use of basal area per acre and the sum of the merchantable heights on "in-trees" as auxiliary variables using several inventories that included both large and small sample sizes. In addition to determining the efficiency of these auxiliary variables, field data recorders were used to record point measurement times for the intensively measured points as well as both types of non-intensive points on one of the larger inventories.

Methods

Three separate inventories were utilized as the basis for this analysis. All sample inventories were based on a systematic point sample using a BAF of 20 to inventory sawtimber board foot volume on approximately 70-year-old hardwood forests in West Virginia. Actual inventory area ranged from 11,995 acres to 138 acres (Table 1).

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Table 1.—Tract size, sample size and intensive to non-intensive sampling ratios for three hardwood forest inventories in West Virginia.

Inventory	First Phase (n')	Second phase (n)	Actual ratio	Forest Size (acres)
Coopers Rock	3,124	1,041	1:3	11,995
Kumbrabow	530	216	1:2.45	8,683
Compartment 14	52	18	1:2.89	138

The Coopers Rock inventory, consisting of 11,995 acres, was inventoried during 2000-2001. This inventory includes both the West Virginia University Research Forest (7,594 acres) and a subset of the Coopers Rock State Forest (4,401 acres) located in northern West Virginia. The Kumbrabow inventory was conducted during the summer of 2002 and represents a majority of the area (8,683 acres) that comprises Kumbrabow State Forest located in Randolph County, West Virginia. The third inventory was included to examine the effects of inventory size on the precision statement and is based on a compartment level inventory (Compartment 14) conducted on the West Virginia University Research Forest in 2003. The Kumbrabow inventory was initially designed to test the efficiency of these two auxiliary variables while the Coopers Rock and Compartment 14 inventories were initially comprised of all intensively measured points that were converted to a double sample inventory with an approximate 1 to 3 intensive to non-intensive ratio. The actual plot ratios for each inventory are displayed in Table 1. In each sample inventory both auxiliary variables are known on every point. For the Kumbrabow and Coopers Rock inventories, board foot volume was based on equations by Scott (1979) which are a function of merchantable height in feet. The Compartment 14 inventory utilized form class 78 equations developed by Wiant and Castaneda (1977) where merchantable height was recorded in logs and half logs. All board foot volumes are based on the International ¼-inch log rule.

Results

The relationship between each auxiliary variable and board foot volume per acre was examined on the second phase samples. A ratio estimator was selected since the data appeared to cross at the origin and exhibited increasing variance with increasing values of the auxiliary variable (Figures 1 through 3). There was an increase in the correlation coefficient in all three inventories due to utilization of sum of merchantable heights as the auxiliary variable (Table 2). The estimates of mean board foot volume for the two auxiliary variables produced similar mean values (Table 3). Since the Coopers Rock and the Compartment 14 inventories were originally based on all intensively measured points, the actual mean volume per acre is known to be 10,686 and 12,657 board feet per acre, respectively. In both cases, the sum of the merchantable heights on "in-trees" as the auxiliary variable provided less biased estimates of the mean volume per acre. There was not a substantial increase in precision from utilizing merchantable height as an auxiliary variable (Table 3). The largest increase in precision occurred in the Compartment 14 inventory, which was 1.1 percent lower than that obtained using basal area as the auxiliary variable (Table 3). An evaluation of the mean time to measure each non-intensive plot type indicated that estimating the merchantable heights on "in-trees" took only an additional 13 seconds per point.

Conclusions

Based on a 1:3 ratio of intensive to non-intensive sample points, the use of merchantable height as an auxiliary variable increased the correlation coefficient with board foot volume per acre by 3.8 to 9.8 percent in all three inventories. However, this increase in correlation with volume was not large enough to substantially reduce the standard error of the mean. Although the standard error was reduced in each of the three inventories, reduction only ranged from 0.02 to 1.12 percentage points. Based on a comparison of those inventories where complete enumeration of all points was available, the use of the sum of merchantable heights on "in-trees" as the auxiliary variable resulted in slightly less biased estimates of the mean board foot volume per acre. Additional investigation of these auxiliary variables utilizing intensive to non-intensive ratios with fewer intensive points and their effects on the estimated mean and standard error is recommended.

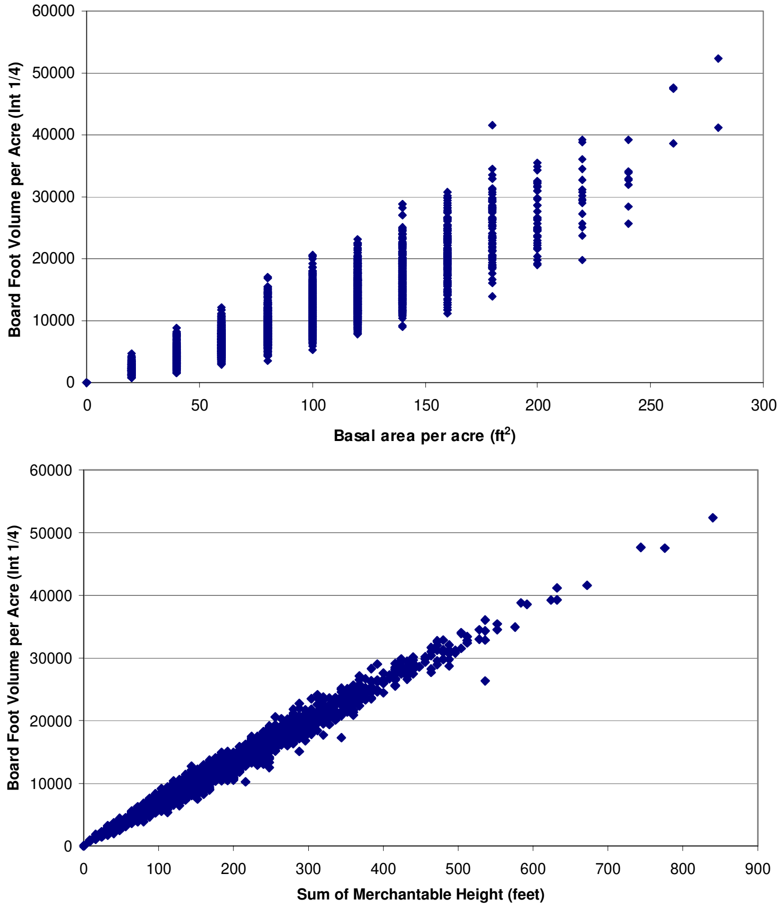

Figure 1.—The relationship between the sum of the merchantable heights on "in-trees" and basal area per acre with board foot volume per acre for the Coopers Rock inventory.

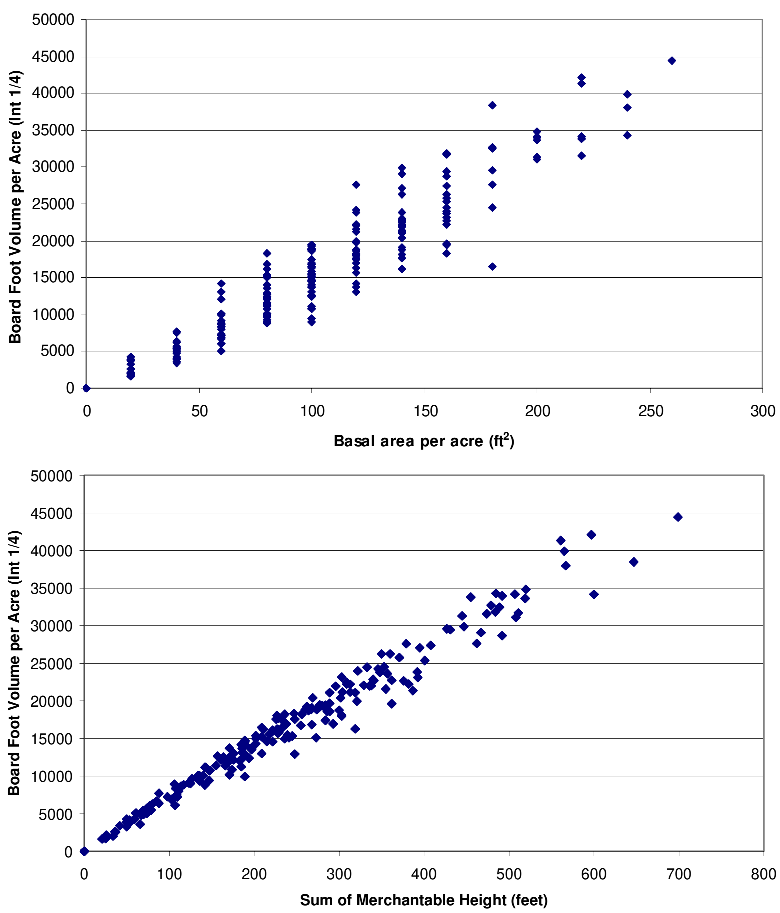

Figure 2.—The relationship between the sum of the merchantable heights on "in-trees" and basal area per acre with board foot volume per acre for the Kumbrabow inventory.

Table 2.—Correlation coefficients by auxiliary variable and inventory.

Auxiliary Variable	Coopers Rock	Kumbrabow	Compartment 14
Basal Area (BA)	0.910	0.952	0.908
Merchantable Height (MHT)	0.991	0.988	0.997

Table 3.—Mean estimates and statistics by auxiliary variable and inventory.

Statistic	Coopers Rock	Kumbrabow	Compartment 14
Mean with BA	10,826	14,445	12,345
Mean with MHT	10,693	14,260	12,504
Standard Error with BA	149	435	1,079
Standard Error with MHT	130	427	953
Standard Error BA (%) [1]	1.38	3.01	8.74
Standard Error MHT (%) [1]	1.22	2.99	7.62

[1]Standard error as a percent of the mean

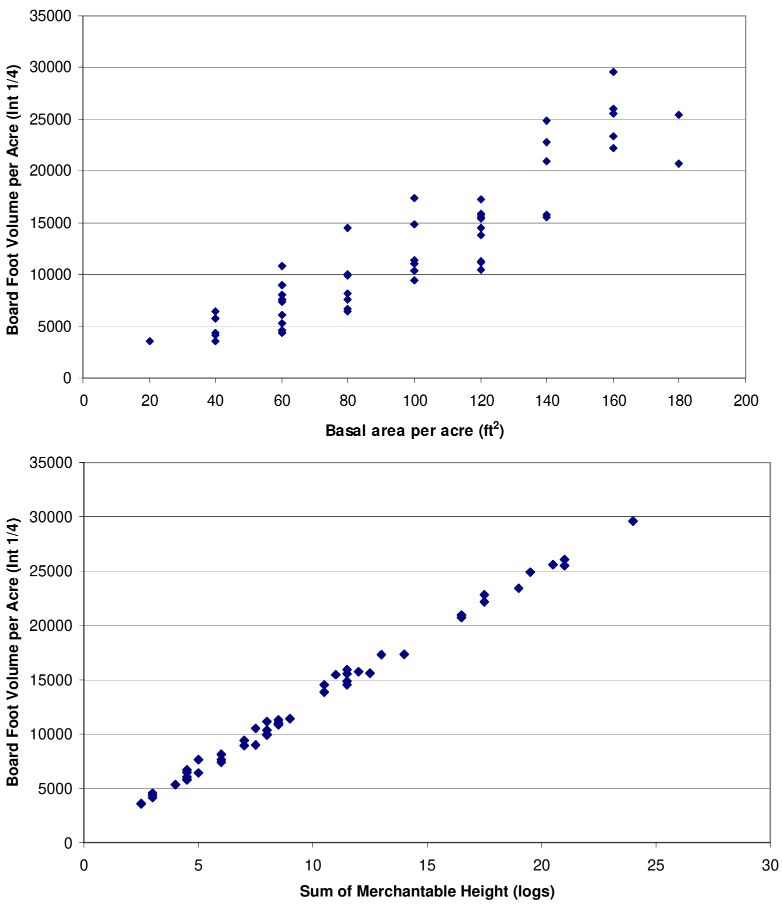

Figure 3.—The relationship between the sum of the merchantable heights on "in-trees" and basal area per acre with board foot volume per acre for the Compartment 14 inventory.

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