Mechanical Grading of Round Timber Beams

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Abstract: Current procedures used to sort round timber beams into structural grades rely on visual grading methods and property assignments based on modification of clear wood properties. This study provides the technical basis for mechanical grading of 228 mm (9 in.) diameter round timbers. Test results on 225 round Engelmann spruce–alpine fir–lodgepole pine beams demonstrate the conservative nature of current timber strength assignments. A good correlation was obtained between static bending strength and static modulus of elasticity (MOE), and between bending properties and dynamic MOE determined in transverse vibration. Relationships were also developed between compressive strength parallel to grain, bending strength, and MOE by transverse vibration. The property relationships developed in this study were used to propose grading criteria consistent with procedures used with mechanically graded softwood dimension lumber. Estimated grade yields obtained using the proposed criteria show that significant increases in yield might be obtained for a specified set of allowable properties than is currently possible by visual grading alone.

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Introduction

Logs used as structural members (round and sawn round timber beams, such as ridge beams, floor joists, and purlins) in log homes are currently graded by visual examination with properties assigned by procedures given in *ASTM D 3957* (ASTM 1990; Hope 1993). This standard provides procedures for grading "wall logs" (which form a load-bearing, solid-wood wall of a building) and "round" and "sawn round" timber beams. Sawn round timbers are shaved or sawn along one side to facilitate their use as a support for floor joists. Allowable properties for round and sawn round timber beams are derived using strength ratios for limiting defects as specified in grading rules and clear wood strength values of *ASTM D 2555* (ASTM 1998). This "clear wood" approach is therefore very similar to the *ASTM D 245/D 2555* (ASTM 1998, 2000c) procedure that was once used to derive allowable properties for most dimension lumber in the United States and is still used for rectangular timbers.

Dimension lumber is often mechanically graded using a device for identifying and appraising one or more physical or mechanical characteristics as well as a visual quality evaluation to sort lumber into categories for assignment of design properties (Smulski 1997). Mechanical grading alternatives improve wood utilization by offering more precise property assignments than are available by traditional visual grading methods. At this time, however, mechanical grading for use by the log home industry is not available for round timbers.

It is likely that the current visual grading and property assignment system does not adequately assess the potential quality of logs for structural use. The conservative nature of the *ASTM D 3957* (ASTM 1990) procedure probably results in the specification of a log larger in diameter than would be required if a more precise grading system were available. Establishment of mechanical grading procedures for structural logs will likely allow the use of smaller diameter logs for a given span and load pattern, thus improving the efficient use of this material.

The work reported here is the second phase of a research program to remove the technical barriers for mechanical grading of logs. The objective of Phase I (Green et al. 2006) was to obtain mechanical properties of 228 mm (9 in.) diameter round timbers in bending and compression parallel to grain that could be used as a basis for proposing a mechanical grading system. The objective of Phase II is to evaluate the proposed mechanical grading system for round timbers and provide estimates of grade yield for potential mechanical grades.

Background

Visual Grading of Round Timbers

In addition to wall logs, two types of structural round timbers are used in log home construction: unsawn and sawn round timbers. The only grade of unsawn round timbers is named "unsawn," and it is primarily intended for bending or truss members. Sawn round timbers have a flattened surface that is sawn or shaved along one side, and they are also primarily intended for use as bending members. The sawn surface is limited to a penetration of no more than 0.30 of the radius of the round log. This limits the reduction in the cross section to less than 10%. Timber Products Inspection, Inc. (TP 1987) has developed structural grading rules for three

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Table 1. Limits on Knot Size and Slope of Grain and Allowable Properties for 228 mm (9 in.) Diameter Cut Round ES–AF–LP Timbers by Visual Grade[a]

			Allowable properties			
Grade[b]	Knot size	Slope of grain	F_b	MOE	F_t	F_c
Unsawn	1/2 diameter	1:15	1,350	1.1	725	625
No. 1	1/3 diameter	1:14	1,100	1.1	600	500
No. 2	1/2 diameter	1:10	900	1.1	500	425
No. 3	3/4 diameter	1:6	525	0.9	275	250

[a]TP (1987). ES–AF–LP is a commercial grouping of Engelmann spruce, alpine fir, and lodgepole pine.

[b]Other limits on grade characteristics are given in TP (1987).

grades of sawn round timbers: No. 1, 2, and 3. Logs of all grades may also be used as compression members; the logs for this usage do not usually have a flattened surface. As with the visual grades of dimension lumber or structural timbers, the grade description of structural logs is a combination of limits on characteristics that affect strength and characteristics that might affect serviceability for the intended application, but not necessarily strength. Table 1 summarizes the limits on knot size and slope of grain for four TP grades. In addition, there are limits on potential decay associated with knots, shake, splits, and compression wood. Examples of serviceability factors include limits on lack of "roundness" and excessive warp and wane.

Allowable properties for sawn round timbers are derived from clear wood data, modified by strength ratios set forth in *ASTM D 3957* (ASTM 1990). This clear wood approach is therefore very similar to the *ASTM D 245/D 2555* (ASTM 1998, 2000c) procedures once used to derive allowable properties for all dimension lumber in the United States and still used for structural timbers. Allowable properties in bending (bending strength, F_b; modulus of elasticity, MOE) and compression parallel to grain (F_c) for the Engelmann spruce–alpine fir–lodgepole pine (ES–AF–LP) species group are given in Table 1. Allowable properties of ES–AF–LP are also established for tensile strength parallel to grain (0.55 of bending strength), strength in compression perpendicular to grain 2.2 MPa (320 lb/in.^{2}), and shear parallel to grain 655 kPa (95 lb/in.^{2}). Additional information on the development of visual grades for log structures is given in Burke (2004).

Mechanical Grading of Round Timbers, Phase I

In Phase I of this project, data were obtained on the mechanical properties of 228 mm (9 in.) diameter round timbers in bending and compression parallel to grain that could be used as a basis for proposing a mechanical grading system (Green et al. 2004). Timbers (n=125) were selected from the inventory of Rocky Mountain Log Homes, Inc. (Hamilton, Mont.). The logs had been cut from standing dead trees and were a mixture of alpine fir (*Abies lasiocarpa*) and lodgepole pine (*Pinus contorta*). All logs were machined to a constant diameter of approximately 228 mm (9 in.) and were approximately 5.0 m (16.5 ft) long. Log moisture content at time of manufacture was below the fiber saturation point. Using a combination of visual characteristics and MOE measurements, the logs were divided into two groups of approximately equal quality. One group was shipped to the University of Idaho for testing in third-point bending (ASTM 1994) and the other to the Forest Products Laboratory for testing as short columns (ASTM 1994). Prior to breaking the specimens, MOE of each log

Table 2. Regression Relationships for 228 mm (9 in.) Diameter Sawn Round Timbers from Phase I (Green et al. 2004)

			Property=$A+BX$			
Property	X	N	A	B	R^2	RMSE[b]
Combined bending and compression data						
E_{tv}	E_{dead}[a]	118	0.644 (0.093)	0.991	0.95	0.234 (0.034)
E_{sw}	E_{dead}[a]	118	1.227 (0.178)	1.033	0.63	0.917 (0.133)
Bending data						
MOE	E_{tv}	60	−0.952(−0.138)	1.071	0.88	0.352 (0.051)
	E_{sw}	56[b]	2.710 (0.393)	0.543	0.67	0.483 (0.070)
	E_{dead}	58[a]	−0.924(−0.134)	1.063	0.94	0.241 (0.035)
MOR	MOE	60	−13.30(−1.930)	5.902	0.61	4.799 (0.696)
	E_{dead}	58[a]	−19.637(−2.848)	6.357	0.56	5.144 (0.746)
	E_{tv}	60	−17.693(−2.566)	6.174	0.51	5.364 (0.778)
	E_{sw}	60	8.343 (1.210)	2.576	0.25	6.633 (0.962)
Compression data						
E_{comp}	E_{tv}	57	0.558 (0.081)	0.903	0.72	0.717 (0.104)
	E_{sw}	57	0.579 (0.084)	0.799	0.76	0.662 (0.096)
	E_{dead}	57	1.020 (0.148)	0.829	0.70	0.738 (0.107)
UCS	E_{comp}	57	4.571 (0.663)	2.090	0.49	3.041 (0.441)
	E_{tv}	57	1.234 (0.179)	2.422	0.60	2.537 (0.368)
	E_{sw}	57	3.399 (0.493)	1.921	0.50	2.813 (0.408)
	E_{dead}	57	2.572 (0.373)	2.212	0.57	2.606 (0.378)

Note: MOR and UCS values are MPa ($\times 10^3$ lb/in^2); MOE values are GPa ($\times 10^6$ lb/in^2); root mean square error (RMSE) values are in the same units as property values.

[a]Two E_{dead} readings lost as a result of LVDT error.

[b]Four "outlying" pieces were dropped as a result of obviously erroneous readings.

was determined by transverse vibration (E_{tv}), by longitudinal stress wave techniques (E_{sw}), and by placing a dead load at the center of the span (E_{dead}). The values of E_{tv} and E_{sw} were determined as possible alternatives for rapid measurement in a mill. Information on species, log grade, knots, twist (slope of grain), moisture content, and specific gravity was also obtained. Additional information on sampling and testing procedures was given in the Phase I report.

The relationships between mechanical properties for the Phase I data are presented in Table 2. At the time of test, average moisture content of bending samples was 15.5% and that of compression samples was 12.3%. There was an excellent correlation between E_{tv} and E_{dead} (R^2=0.95), and between MOE in one-third point bending and E_{dead}(R^2=0.94). Of the two rapid methods of measuring MOE, the correlation was better for transverse vibration (R^2=0.88) than for longitudinal stress wave (R^2=0.67). Wang et al. (2001) also concluded that E_{tv} is better correlated with static MOE than is E_{sw} and noted that E_{tv} measurements are less sensitive than E_{sw} values to geometrical imperfections in the logs.

The relationship between modulus of rupture (MOR) and MOE, both determined by static testing, is shown in Fig. 1. The correlation between MOR and MOE (R^2=0.61) was about the same as that expected for dimension lumber (Doyle and Markwardt 1966) (Table 2). As expected, MOE values determined in bending modes (MOE, E_{tv}, and E_{dead}) correlated better with MOR than did MOE determined in an axial direction (E_{sw}).

All three measures of E correlated equally well with MOE in compression parallel to grain (E_{comp}). Thus, E_{sw} was just as good

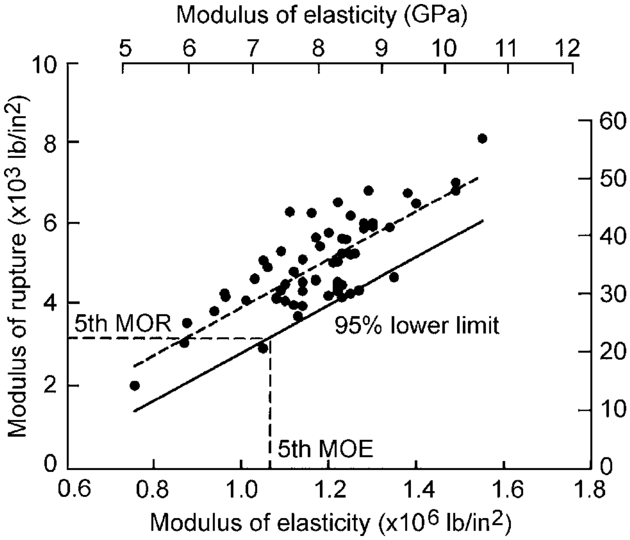

Fig. 1. Relationship between static MOE in bending (one-third point) and MOR for dry 228 mm (9 in.) diameter ES–LP–AF logs from Phases I and II

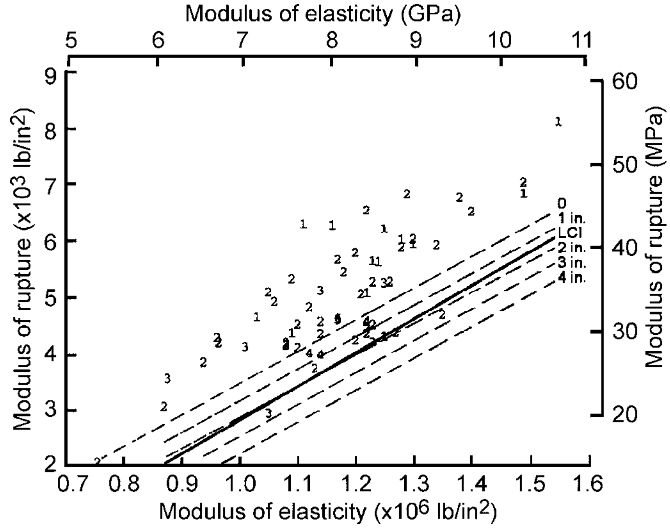

Fig. 2. Effect of knot size on MOR–MOE relationship in one-third point bending (Green et al. 2004)

a predictor of E_{comp} as were the two bending E values. Surprisingly, E_{comp} was not the best predictor of ultimate compressive strength (UCS). With a root mean square error (RMSE) of 2.5 MPa (368 lb/in.^{2}), E_{tv} was the best predictor of MOE, E_{dead} was virtually identical [RMSE=2.6 MPa (378 lb/in.^{2})], and E_{sw} was almost as good [RMSE=2.8 MPa (408 lb/in.^{2})]. Since the difference in RMSE between the predictions of UCS using E_{comp} and E_{tv} is only 503 kPa (73 lb/in.^{2}), perhaps this is just an anomalous result. In any case, current practice for machine stress rated (MSR) lumber is to predict UCS from its relationship with MOR, not from its relationship with MOE (ASTM 2000a).

The relationship between MOR and MOE forms the basis for the MSR grading system, used commercially in the United States since the 1960s for grading dimension lumber (Smulski 1997; Galligan and McDonald 2000). For the MSR grading system, an approximate lower 90% confidence interval (CI) on the predicted MOR values provides the equivalent of the fifth percentile used to determine allowable strength values for visual grades. The equation for the 90% lower CI (LCI) of the logs tested in Phase I (Fig. 1) is

$$\mathrm{MOR}_{0.95\mathrm{LCI}} = 5.902 \times \mathrm{MOE} - 3.084 \qquad (1a)$$

for MOE (10^6 lb/in.^{2}) and MOR (10^3 lb/in.^{2}), and

$$\mathrm{MOR}_{0.95\mathrm{LCI}} = 5.902 \times \mathrm{MOE} - 21.264 \qquad (1b)$$

for MOE (GPa) and MOR (MPa).

If MOE were the only grading criterion, the near minimum (i.e., fifth percentile) MOE would first be determined. For MSR lumber, this has traditionally been set as equal to 82% of the target average MOE value (i.e., $0.82 \times$ average grade MOE), which limits the variability of the lower half of the MOE distribution of the grade to a coefficient of variation (COV) of 11%. For a grade having an allowable E value of 13.8 GPa (2.0×10^6 lb/in.^{2}), the fifth percentile MOE would therefore be estimated as 11.3 GPa (1.64×10^6 lb/in.^{2}). The allowable MOR value, F_b, would be obtained by determining the MOR value predicted from Eq. (1) with an MOE of 11.3 GPa (1.64×10^6 lb/in.^{2}), divided by the general adjustment factor of 2.1 (ASTM 2000c). In this example, the MOR value is thus 6.595×10^3 lb/in.^{2} and the F_b value 3.141×10^3 lb/in.^{2}, which rounds to 3.10×10^3 lb/in.^{2}(21.4 MPa).

To find other variables that might be useful in developing grading criteria, a stepwise regression was run to see what other variables might have predictive capability for MOR. In addition to MOE, the variables included in the regression were the average of the largest knots on each log face, largest knot anywhere in the log, minimum of largest knots on each side, slope of grain, density, specific gravity, and log grade. Modulus of elasticity was by far the most important variable, explaining 61% of the variability and being a significant predictor of MOR ($p=0.0001$). Next in importance was any of the knot measures. All methods of classifying knots were essentially the same in importance. Because of its simplicity as a grading criterion, the maximum knot size was used (Green et al. 2004). This variable was also significant at the 0.0001 level and when added to MOE increased the R^2 value from 61 to 67%. The inclusion of maximum knot size also slightly reduced the variability of a predicted MOR value. The RMSE used in such a prediction decreased from 4.8 MPa (696 lb/in.^{2}) based on MOE alone to 4.5 MPa (647 lb/in.^{2}) using MOE and knot size.

Fig. 2 shows a 95% lower confidence limit of predicted MOR values for the combined MOE–knot size predictive equation for maximum knots ranging from 0 to 102 mm (from 0 to 4 in.). Fig. 2 shows that the limit for the 51 mm (2 in.) knot is very close to the 95% confidence limit for the original MOR–MOE curve, Eq. (1) (indicated as LCI on Fig. 2). This is due in part to the limited number of knots larger than 51 mm (2 in.), as may be seen from the data points on Fig. 2. With more samples of larger knots, the confidence limit for the MOR–MOE curve would be expected to be lower than that for this sample.

The results of Phase I of the study were favorable. However, it is necessary to verify that the proposed procedures can be used to sort mechanical grades of round timbers successfully and to provide initial estimates of log yields by individual mechanical grades. Phase II was designed to provide this information.

Materials and Methods

Selection and Processing

Logs were selected from the inventory of a cooperating log home manufacturer in the Bitterroot Valley of Montana. The material came from dead-standing, typically fire- and beetle-killed trees of mixed softwood species. Because rough logs were processed by the manufacturer into uniform 229 mm (9 in.) diameter cylinders in 4.3 and 4.9 m (14 and 16 ft) lengths, we selected the first 110 processed logs as test specimens, rejecting only those logs with obvious decay or significant shake. The specimens were identified by species and graded as Sawn Round Timbers by a TP Quality Supervisor (TP 1987) and shipped to the University of Idaho. All specimens were then sawn on one side to create an approximated 51 mm (2 in.) wide flattened surface. These sawn round timbers represent a typical structural log profile for bending members, in which the flattened surface would be the load bearing area for a floor or roof member.

An additional 10 logs were selected for testing in compression parallel to grain. These logs were intended to provide lower compression strength values to supplement the values obtained in Phase I and thus provide a better estimate of the UCS–MOR relationship. The logs were selected on the basis of having low MOE values. The logs were shipped to the Forest Products Laboratory in Madison, Wis., for testing as short columns. They were equilibrated for several months at room temperature and 65% relative humidity prior to test.

Flexural Testing

At the test facility, each log was weighed and measured, and the slope of grain was recorded. Two nondestructive evaluation measurements were taken on each log while it was simply supported at the ends. First, the MOE caused by placing a dead load in the middle of the span, E_{dead}, was determined. This was done by first placing a 21 kg (47 lb) preload on the log and then measuring the incremental deformation 60 s after an additional 64 kg (139 lb) dead load was applied. Deflections at midspan were measured with a linear variable displacement transducer (LVDT) to the nearest 0.0025 mm (0.0001 in.). Second, the transverse vibration frequency of each log was determined by striking each log with a rubber-headed hammer at midspan to induce vibration; the output was recorded from load cells located at each support (Fig. 3) (Murphy 2000).

The logs were then placed in a universal testing machine and tested in third-point bending according to *ASTM D 198* (ASTM 1994). For this test, the logs were oriented with the round side down and the load was applied to the flattened surface, as would typically occur in use. The rate of the crosshead motion was 25.4 mm (1.0 in.) per minute, which resulted in an average time to failure of about 3 min. Load–deflection plots were obtained for each specimen for calculation of MOR and static MOE. After testing, a 25 mm (1 in.) thick section of each specimen was removed from an area close to the location of failure to determine moisture content and specific gravity according to *ASTM D 2395* (ASTM 1992b) and *ASTM D 4442* (ASTM 1992a).

Compression Testing

Log selection, grading, measurement of physical characteristics, and nondestructive testing of E_{dead} for Phase I were conducted as described for the bending samples (Green et al. 2004). Stress wave MOE and MOE in compression parallel to grain were not measured on the 10 additional logs. Testing followed procedures given in *ASTM D 198* (ASTM 1994), with lateral supports provided at 305 mm (12 in.) from the ends of the log and at 610 mm (24 in.) spacing in the middle of the log. To facilitate handling, each full-length log was cut in half and each half tested. The average length of the logs at test was 2.5 m (98 in.). The logs were loaded at a rate of crosshead movement of approximately 2.54 mm (0.1 in.), for a time to failure of 5–12 min. Following testing, a 25.4 mm (1 in.) thick cross-section was cut from near failure of each log for determining oven-dry moisture content and specific gravity (ASTM 1992a,b).

Fig. 3. Determination of MOE by transverse vibration

MSR Simulation

In the production of MSR grades for standard 38 mm (nominal 2 in.) thick dimension lumber, two MSR grades are normally produced at one time. Obviously, when producing two or more grades simultaneously, the grade yield of one grade is affected by the higher grades also being produced. In this paper, we assume that only one MSR grade is being produced at a time. This helps provide a more basic understanding of the proposed grading system and would seem a prudent choice when initiating mechanical grading for a new type of structural product (i.e., round timbers).

Individual pieces in the simulation of MSR grades were required to meet three criteria: fifth percentile (minimum) MOE, fifth percentile (minimum) MOR, and grade average MOE. Thus, for grade 1.3*E*, the minimum MOE would be $0.82\times 8.9635=7.350$ GPa $(0.82\times 1.3=1.066\times 10^6$ lb/in.$^2)$ (Fig. 1). Grade names for mechanically graded lumber are specified in terms of the allowable fiber stress in bending (F_b) and the average MOE value assigned to the grade. For example, a grade name of $1650F-1.5E$ indicates an F_b value of 1,650 lb/in.^{2} (11.38 MPa) and an MOE of 1.5×10^6 lb/in.^{2} (10.3 GPa). Because grade names for mechanically graded lumber are legally defined, it is improper to give grade names in SI units. Thus, the assigned allowable properties given in SI units are only approximations of

Table 3. Description of Logs Selected for Phase II

Species	n	Number in grade: Number 1	Number 2	Number 3	Cull
Alpine fir	43	25	8	9	1
Lodgepole pine	40	24	5	12	0
Engelmann spruce	23	13	5	5	0
Western white pine	2	2	0	0	0
Grand fir	1	1	0	0	0

the legally defined values. The minimum MOR value would be estimated from a lower 95% confidence interval on the MOE–MOR regression. For grade 1.3E, placing the value of 1.07 into Eq. (1) yields 22.28 MPa (3.2×10^3 lb/in.^{2}) (Fig. 1). The average MOE of the pieces that pass the first two criteria must equal or exceed the grade average MOE of 8.96 GPa (1.3×10^6 lb/in.^{2}).

Results and Discussion

Test specimens were predominately of three species: Engelmann spruce (*Picea engelmannii*), alpine fir (*Abies lasiocarpa*), and lodgepole pine (*Pinus contorta*). Specimens also included a few pieces of western white pine (*Pinus monticola*) and grand fir (*Abies grandis*). The number of specimens for each species and grade distribution within each species are shown in Table 3. The ES–AF–LP species combination includes the primary species used in log home construction in Montana (Keegan et al. 2000). For the 65 logs graded as No. 1, no defect was grade limiting. In the other grades, slope of grain was the grade-limiting defect for 15 logs, insect holes for 11 logs, red heart for 6 logs, shake or checks for 3 logs each, and wane or white speck for 2 logs each. Knots were listed as the grade-limiting defect for only two logs. Average log moisture content at the time of flexural testing was 15.9%, with a COV of 21.6%. Moisture content ranged from 9 to 24%.

Confirmation of Potential MSR Grades Using Phase II Data

As discussed, the Phase II study provided the opportunity to estimate grade yield using data that are independent of the data used to establish the MOR–MOE relationship. Using the minimum MOE grade criterion, we sorted data from the 109 logs sampled in Phase II into several potential grades. As can be seen for the $1.0E-825F_b$ grade (Table 4), all 109 logs met the 0.82E criteria and 2.1F_b requirement; average MOE of these logs equaled or exceeded the required average grade MOE of 6.895 GPa (1.0×10^6 lb/in.^{2}). Likewise, for the 1.1E and 1.2E grades, the pieces that made the 0.82E criteria also made the other two criteria. Results for grades equal to or greater than 1.3E are slightly different from these results. For grade 1.3E, 94 pieces met the 0.82E and 2.1F_b criteria, but these pieces did not have an average MOE of 8.9 GPa (1.3×10^6 lb/in.^{2}). Thus, six additional pieces, those with the six lowest individual MOE values, were discarded from the grade to raise the average MOE to at least 8.9 GPa (1.3×10^6 lb/in.^{2}). Although 86% of the pieces qualified based on the first two criteria (0.82E and 2.1F_b), only 82% qualified by all three criteria. With higher grades, the problem of meeting the first two criteria but having too low an average MOE for the intended grade worsened. For a proposed $1.7E-2250F_b$ grade, 21% of the 109 pieces met the first two criteria but only 2% made all three criteria. In general, we have not encountered this problem in studies on softwood dimension lumber (Erickson et al. 2000; Green et al. 2000), hardwood dimension lumber [Green et al. (1994), private communication, 2005], or structural timbers (Kretschmann and Green 1999). It is possible that round logs behave differently than the lumber tested in the previous studies. However, this anomalous finding may also be a result of the limited number of specimens (109) in the Phase II study. Further work will be needed to answer this question.

In addition, note that bending strength, indicated by the 2.1F_b criterion, never controlled the assigned grade (Table 4). Thus, lumber that made the 0.82E criterion always had adequate strength. Therefore, the use of Eq. (2), which predicts MOR from both MOE and knot size, would not provide a better estimate of MOR or a more accurate assignment of grade. Two cautions about this conclusion: (1) it is applicable only to the species group sampled in this study and might not be true for other species; and (2) all lumber in Phase II made at least visual grade No. 3. Thus, for all logs, the maximum knot size was less than 75% of the log diameter.

Table 4. Estimated Yield of Mechanical Grades in Phase II Using MOE Criteria from Phase I [Text Eq. (1)]

Grade[a]	Number of pieces making criterion: 0.82E	2.1F_b	Mean E	Percent yield by criterion: Columns 2 and 3	Columns 2, 3, and 4
$1.0E-825F_b$	109	109	109	100	100
$1.1E-1050F_b$	107	107	107	98	98
$1.2E-1300F_b$	100	100	100	92	92
$1.3E-1525F_b$	94	94	89	86	82
$1.4E-1750F_b$	72	72	56	66	51
$1.5E-2000F_b$	51	51	30	47	28
$1.6E-2200F_b$	38	38	14	35	13
$1.7E-2550F_b$	23	23	2	21	2

[a]Assumes production of only one MSR grade at a time.

Improved Estimates Using Combined Phase I and Phase II Data

Visual Grades

Table 5 summarizes the data on bending properties for all logs tested in this study and Table 6 the data for the logs tested in short column compression parallel to grain. Determining the properties of visually graded logs was not an objective of this study, so sample sizes varied considerably by grade. However, because there are little, if any, data available on the properties of logs for these grades, it is instructive to compare trends between grades. For mean MOE, little change in MOE occurred with grade; the difference between MOE of Nos. 1 and 3 logs was only approximately 3%. For mean MOR, the difference between Nos. 1 and 3 logs was only approximately 5%. At the fifth percentile level, a 23% difference in MOR occurred, but the value for No. 2 logs was higher than that for No. 1 logs. However, the small sample size for the Nos. 2 and 3 grades may have increased the variability of fifth percentile estimates for MOR. Except for No. 1 grade logs, there were insufficient data to adequately characterize UCS. Therefore, no comparison of UCS trends across grades is possible.

Table 7 compares the results of our study with the properties assigned by *ASTM D 3957* [see Table 1 and *ASTM D 3957* Appendices (ASTM 1990)]. For MOE, the experimentally deter-

Table 5. Flexural Properties of Visually Graded ES–AF–LP[a] 228 mm Diameter Logs for Combined Phases I and II Data Sets

Property	Grade	Sample size	MC (%)	Mean	Standard deviation	Percentile level 5th	25th	50th	75th
SI units									
E_{dead} (GPa)	No. 1	92	15.5	9.30	3.31	5.76	7.69	8.71	10.34
	No. 2	33	15.4	8.66	1.41	5.91	7.96	8.63	9.27
	No. 3	30	17.2	10.57	7.96	6.30	7.31	8.38	9.63
	Economy	13	17.2	7.99	0.90	NA	NA	7.88	NA
E_{tv} (GPa)	No. 1	92	15.5	8.64	1.25	6.78	7.79	8.43	9.38
	No. 2	33	15.4	8.75	1.08	6.61	8.17	8.77	9.45
	No. 3	30	17.2	8.65	1.40	6.76	7.71	8.47	9.27
	Economy	13	17.2	8.05	0.84	NA	NA	8.00	NA
MOE (GPa)	No. 1	92	15.5	8.56	1.32	6.64	7.61	8.27	9.32
	No. 2	33	15.4	8.54	1.06	6.48	7.92	8.48	9.27
	No. 3	30	17.2	8.31	1.35	6.01	7.42	8.24	9.13
	Economy	13	17.2	7.56	1.02	NA	NA	7.72	NA
MOR (MPa)	No. 1	92	15.5	38.70	1.12	25.45	31.19	38.70	46.24
	No. 2	33	15.4	38.53	6.87	27.76	32.60	37.73	45.04
	No. 3	30	17.2	36.77	7.69	19.69	30.64	35.73	41.02
	Economy	13	17.2	33.19	8.67	NA	NA	30.10	NA
Inch-Pound units									
E_{dead} (10^6 lb/in.^{2})	No. 1	92	15.5	1.349	0.480	0.836	1.115	1.263	1.499
	No. 2	33	15.4	1.256	0.204	0.857	1.154	1.252	1.345
	No. 3	30	17.2	1.533	1.154	0.914	1.060	1.216	1.397
	Economy	13	17.2	1.159	0.130	NA	NA	1.143	NA
E_{tv} (10^6 lb/in.^{2})	No. 1	92	15.5	1.253	0.182	0.983	1.130	1.222	1.361
	No. 2	33	15.4	1.269	0.157	0.959	1.185	1.272	1.371
	No. 3	30	17.2	1.254	0.203	0.980	1.118	1.229	1.345
	Economy	13	17.2	1.167	0.122	NA	NA	1.160	NA
MOE (10^6 lb/in.^{2})	No. 1	92	15.5	1.241	0.192	0.963	1.103	1.200	1.351
	No. 2	33	15.4	1.239	0.154	0.940	1.149	1.230	1.345
	No. 3	30	17.2	1.205	0.196	0.872	1.076	1.195	1.324
	Economy	13	17.2	1.096	0.148	NA	NA	1.120	NA
MOR (10^3 lb/in.^{2})	No. 1	92	15.5	5.613	1.263	3.691	4.523	5.613	6.707
	No. 2	33	15.4	5.588	0.997	4.026	4.728	5.472	6.532
	No. 3	30	17.2	5.333	1.416	2.855	4.444	5.182	5.949
	Economy	13	17.2	4.814	1.258	NA	NA	4.366	NA

[a]ES–AF–LP is Engelman spruce, alpine fir, and lodgepole pine.

mined mean values for each grade were rounded according to the procedures of *ASTM D 245* (ASTM 2000c). The allowable E values in *ASTM D 3957* (ASTM 1990) were already adjusted from the clear wood mean value determined using a center-point load and a span-to-depth ratio of 14:2 to the MOE for a uniform load at a span-to-depth ratio of 21:1. Further adjustment of the experimental MOE values from a third-point load to a uniformly distributed load would be insignificant (ASTM 2002). The experimentally measured bending strength, F_b, was determined by dividing the fifth percentile value from Table 5 by the general adjustment factor for bending of 2.1 and rounding by *ASTM D 245* (ASTM 2000c) procedures. For determining allowable compressive strength, the fifth percentile UCS value from Table 6 was divided by 1.9, multiplied by a "tip factor" of 0.88, and rounded.

The small difference between grades for the experimentally determined MOE values would result in an MOE value of 8.3 GPa (1.2×10^6 lb/in.^{2}) for all three grades if the values were rounded by the procedures of *ASTM D 245* (ASTM 2000c). These values are higher than the assigned values, especially for No. 3 grade logs (Table 7). For F_b, the experimental values are much higher than those assigned by the *ASTM D 3957/D 245* (ASTM 1990, 2000c) process, especially for No. 3 grade. For F_c, only the No. 1 grade had sufficient samples to estimate a fifth percentile. As with F_b, the experimentally determined F_c values for No. 1 are also much higher than is currently assigned.

Property Relationships

Table 8 gives regression relationships based on the combined Phases I and II data sets. Note that Table 8 includes one more piece of bending data than Table 5; one log was graded as a "wall log 40" but its grade as a "cut round timber" was inadvertently not recorded. The inclusion of 109 additional pieces further refined the relationship between MOE and E_{tv} determined in Phase I, but changed the basic relationship only slightly from that given in Table 2 and Fig. 4. Overall, the observed relationship has an excellent coefficient of determination (R^2) of 0.83. The inclusion of additional data slightly improved the correlation between MOE and MOR; the R^2 value increased from 0.61 for Phase I data alone

Table 6. Properties of Visually Graded ES–AF–LP Logs in Compression Parallel to Grain for Combined Phases I and II Data

Property	Grade	Sample size	MC (%)	Specific gravity	Mean	Standard deviation	Percentile level 5th	25th	50th	75th
SI units										
E_{comp} (GPa)	No. 1	43	12.3	0.36	8.14	1.24	6.48	7.31	7.93	8.62
	No. 2	4	13.2	0.37	7.65	1.93	NA	NA	7.72	NA
	No. 3	10	12.0	0.35	8.62	1.38	NA	NA	8.14	NA
UCS (MPa)	No. 1	43	12.3	0.36	20.48	3.65	15.31	7.79	20.96	23.44
	No. 2	4	13.2	0.37	22.75	2.48	NA	8.17	21.65	NA
	No. 3	10	12.0	0.35	8.65	12.27	NA	7.71	20.48	NA
Inch-pound units										
E_{comp} (10^6 lb/in.^{2})	No. 1	43	12.3	0.36	1.18	0.18	0.94	1.06	1.15	1.25
	No. 2	4	13.2	0.37	1.11	0.27	NA	NA	1.12	NA
	No. 3	10	12.0	0.35	1.25	0.20	NA	NA	1.18	NA
UCS (10^3 lb/in.^{2})	No. 1	43	12.3	0.36	3.11	0.53	2.22	2.81	3.04	3.40
	No. 2	4	13.2	0.37	2.97	0.36	NA	NA	3.14	NA
	No. 3	10	12.0	0.35	3.30	1.78	NA	NA	2.97	NA

to 0.68 for the combined data set (Fig. 4). As might be expected, the relationship appears to be independent of species.

Fig. 5 shows the relationship between UCS and MOR for the combined data sets. The MOR data are based on 169 pieces tested in both Phases I and II, and the UCS data include the extra 10 logs tested in Phase II. The relationship is a little more ragged than might be expected and is probably a result of the small number of logs (67) that were tested in compression. A smooth curve was fit to the UCS–MOR data, with the ratio held constant for MOR values above the lowest UCS/MOR ratio on the curve fit. The smoothed relationship is

$$\text{UCS/MOR} = 0.0293R^2 - 0.338R + 1.51 \tag{2a}$$

for MOR $\leqslant 5.78 \times 10^3$ lb/in.^{2}, and

$$\text{UCS/MOR} = 0.535 \tag{2b}$$

for MOR $> 5.78 \times 10^3$ lb/in.^{2}. [For MOR $\leqslant 39.85$ MPa, UCS/MOR $=(6.16\times10^{-4})R^2 - 0.049R + 1.51$; for MOR $\geqslant 39.85$ MPa, UCS/MOR $= 0.535$.]

Table 7. Comparison of Assigned Allowable ES–AF–LP Properties and Experimental Values

Property[a]	Basis	Grade Number 1	Number 2	Number 3
MOE [10^6 lb/in.^{2} (GPa)]	Assigned	1.1 (7.585)	1.1 (7.585)	0.9 (6.206)
	Test data	1.2 (8.274)	1.2 (8.274)	1.2 (8.274)
F_b [10^3 lb/in.^{2} (MPa)]	Assigned	1.100 (7.584)	900 (6.206)	525 (3.620)
	Test data	1.750 (12.1)	1.900 (13.1)	1.350 (9.3)
F_c [10^3 lb/in.^{2} (MPa)]	Assigned	500 (3.448)	425 (2.930)	250 (1.724)
	Test data	1.150 (7.929)	NA	NA

Note: Values derived from data given in Tables 5 and 6. NA (not applicable) indicates that sample size is insufficient to calculate fifth percentile strength values.

[a]Mean MOE values and fifth percentile MOR and UCS values were used to calculate; F_b=MOR/2.1; and F_c=UCS/1.9.

The shape of the UCS/MOR relationship is not unique to logs. It was first noted for standard 38 mm (nominal 2 in.) thick softwood dimension lumber (Green and Kretschmann 1991) and later found to hold for hardwood dimension lumber (Green et al. 1994) and standard 140 by 140 mm (nominal 6 by 6) Southern Pine rectangular timbers (Green and Kretschmann 1997) (Fig. 6). The UCS/MOR relationship for dimension lumber is relatively independent of species, but it decreases slightly with decreasing size (Fig. 7) (Green and Kretschmann 1991). Mean trends in this relationship for standard 38 by 89 mm (nominal 2 by 4) lumber are standardized for mechanically graded dimension lumber in *ASTM D 6570* (ASTM 2000a), and a conservative relationship is provided for standard 38 by 190 mm (nominal 2 by 8) lumber in *ASTM D 1990* (ASTM 2000b). While the relationship for logs would be expected to be numerically different than that for dimension lumber, the form would be expected to be the same. Thus, the relationship for logs would also be expected to decrease with decreasing log size and increasing moisture content, and to be relatively independent of species. Research in progress on small-diameter Douglas-fir and ponderosa pine logs will provide additional information for judging the validity of these assumptions for logs.

It would also be possible to calculate an allowable compression strength using the relationship between UCS and E_{tv}. In this approach, a lower 95% confidence limit would be placed on the relationship, as was shown for bending strength in Eq. (1). For compression strength, the relationship for the combined Phases I and II data is

Table 8. Regression Relationships for Dry Cut Round ES–AF–LP Timbers

Y	X	N	$Y=A+BX$ A	B	R^2	RMSE
MOE	E_{tv}	169	0.0038	0.980	0.83	0.076
E_{tv}	MOE	169	0.2660	0.805	0.70	0.097
MOR	MOE	169	−1.340	5.586	0.68	0.709
	E_{tv}	169	−1.230	5.395	0.53	0.860
UCS	E_{tv}	67	0.798	1.957	0.50	0.395

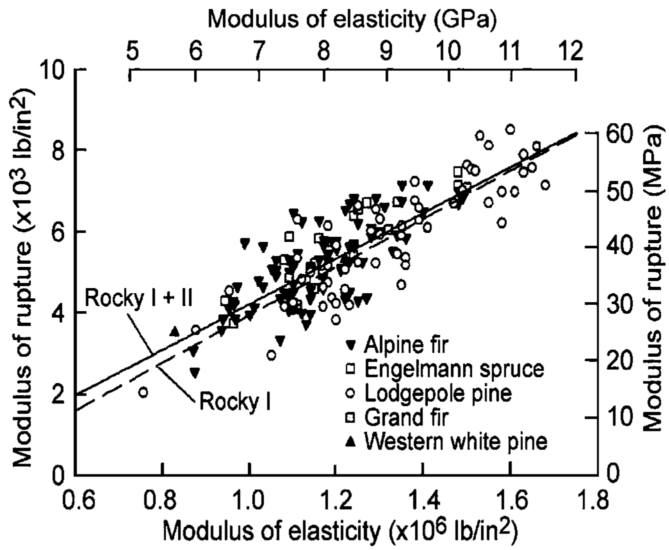

Fig. 4. Bending strength–stiffness relationship for dry ES–LP–AF logs

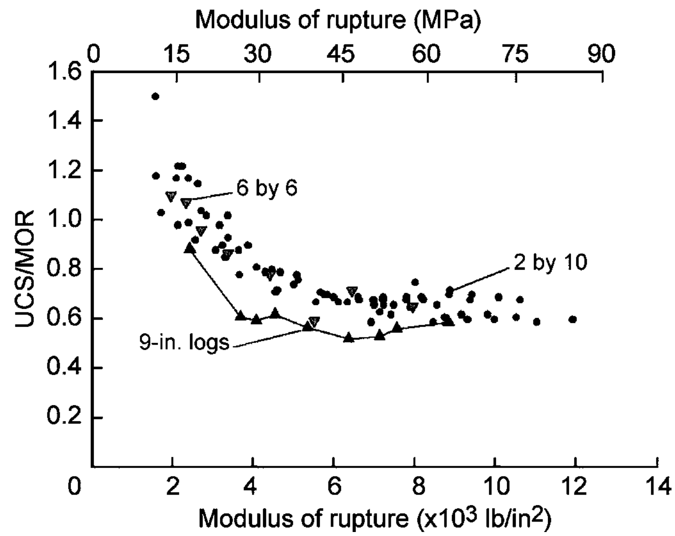

Fig. 6. Compression–bending strength relationships for dry wood products

$$UCS_{0.95LCL} = 1.957E_{tv} + 0.143 \tag{3}$$

Prediction of F_c values using Eq. (3) would be about the same, on average, compared to values determined from the UCS/MOR relationship of Eq. (2). However, since considerably fewer pieces were tested in compression parallel to grain than in bending, the relationship given in Eq. (2) is preferred.

Revision of Potential Mechanical Grading Criteria and Grade Yield Estimates

Fitting a 90% confidence interval to the data shown in Fig. 4 provides a 95% lower confidence bound for estimating F_b, given the fifth percentile MOE estimate ($0.82 \times$ grade average MOE). For the combined data set, this relationship is

$$MOR_{0.95LCI} = 5.586 \times MOE - 2.509 \tag{4}$$

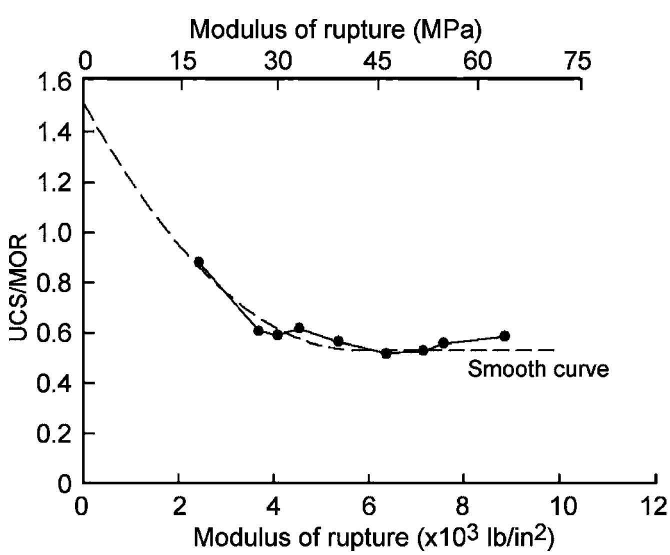

Fig. 5. Relationship between compression and bending strength for dry ES–LP–AF logs

As Table 9 shows, the addition of the 59 logs tested in bending from Phase I to the 109 logs from Phase II did not change the yields significantly from those shown for Phase II alone (Table 4). With the exception of some cull logs that were included in the Phase I data, MOR never limited the yield once the lumber had qualified for the grade based on the $0.82E$ criteria. Meeting the overall required average MOE for the grade as the MSR grade level increases is still problematic and some additional research is needed. Because MOR does not control grade, it would seem possible to test 400–600 logs for MOE alone so as to determine if failure to meet the average MOE requirement is an inherent characteristic of logs or (as seems likely) a result of the limited number of samples available in our study.

Table 10 compares the yield obtained for the visually graded logs in Phases I and II to that for mechanically graded logs having the same MOE. For example, the MOE assigned to visually graded No. 3 ES–LP–AF logs is 6.206 GPa (0.900×10^6 lb/in.^{2}) and the assigned F_b is 3.6 MPa (525 lb/in.^{2}). The yield for No. 3 visual grade logs is 17.9%

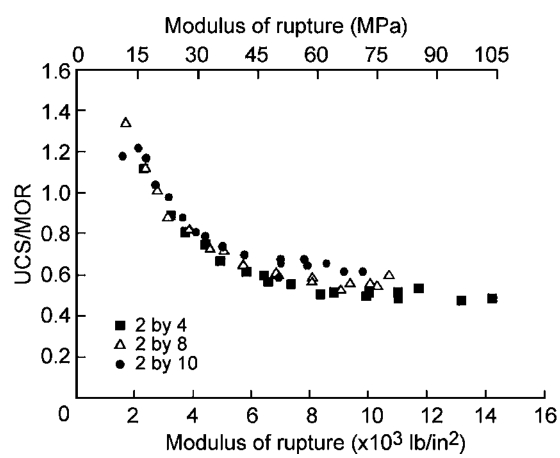

Fig. 7. Effect of lumber size on relationship between compression and bending strength for standard 38 mm (nominal 2 in.) thick Hem–Fir dimension lumber (adapted from Green and Kretschmann 1991)

Table 9. Estimated Yield of Mechanical Grades for ES–AF–LP Logs Using Combined Phases I and II Data Sets[a]

Grade[b]	Pieces making criteria: $0.82E$	$2.1F_b$	Mean E	Percent yield by criteria: Columns 2 and 3	Columns 2, 3, and 4
$1.0E-1000F_b$	168	168	168	99	99
$1.1E-1200F_b$	164	164	164	97	97
$1.2E-1400F_b$	154	154	154	91	91
$1.3E-1650F_b$	143	143	119	85	70
$1.4E-1850F_b$	107	107	62	63	37
$1.5E-2100F_b$	73	73[c]	33	43	19
$1.6E-2300F_b$	45	44	15	26	9
$1.7E-2500F_b$	27	27	2	16	1

[a]Assumes production of only one MSR grade at a time.

[b]F_b values are different than those given in Table 2 because F_b is estimated from Eq. (5) instead of Eq. (1).

[c]Two additional "cull" logs included in Phase I failed F_b but would not have normally been part of the mechanical grading system.

(Table 5). Of course, if the producer were willing to sell the higher value Nos. 1 and 2 grade logs for the price of No. 3 grade, the yield of No. 3 and better would be 92.3% (Table 10). Mechanically graded logs with 6.2 GPa (0.900×10^6 lb/in.^{2}) MOE would have a F_b of 5.34 MPa (775 lb/in.^{2}), and 100% of the logs would make this mechanical grade using all three grading criteria. Slightly over 54% of the logs made No. 1 visual grade, but 97% of the logs would make a mechanical grade having equivalent MOE and the F_b value would be slightly higher. As discussed previously, the MSR simulation assumes that only one MSR grade is being produced at a time. In Table 10, we are not assuming that the $0.9E$ and $1.1E$ grades are pulled simultaneously. This ability to get higher yields for equivalent properties is consistent with results previously found for other mechanically graded products (Green et al. 1994; Kretschmann and Green 1999).

In addition to establishing relationships between MOR and MOE, MOE and E_{tv}, and UCS and MOR, other criteria are needed to specify mechanical grades. Such decisions are the responsibility of the applicable grading agency. However, the results of our study suggest criteria for consideration. As was shown in Phase I (Green et al. 2004), once MOE is determined, slope of grain does not improve the estimation of MOR. In general, a No. 3 grade log was about the lowest grade sampled in this study. Thus, we suggest that for mechanically grading all ES–AF–LP logs, slope of grain should be limited to the maximum amount permitted in a No. 3 visual grade; i.e., a slope <1 in 6 for a 228 mm (9 in.) diameter log. Likewise, in Phase II we found that knots did not control mechanical grade if the logs met the MOE criteria. Because most logs tested in our study were No. 3 grade or better, the maximum knot size specified for No. 3 visual grade logs should also be the maximum recommended for mechanically graded logs. Likewise, the log characteristics that relate to serviceability for No. 3 logs in log home applications should be the maximum allowed in any mechanical grades. Thus, knots should be restricted to $\leqslant 3/4$ of the log diameter. Decay and obvious compression wood are not allowed in the grade. Examples of serviceability factors that should also be considered in the grading criteria include crook, reverse sweep, wane, shake, splits, out of roundness, and holes.

Table 10. Comparison of Estimated Yields for Visually Graded ES–AF–LP Logs with Potential Mechanical Grades with Similar Properties[a]

Required MOE	Allowable bending strength, $F_b(10^3$ lb/in$^2)$: Number 1	Number 2	Number 3	MSR	Percent yield for given MOE: Number 1	Number 2	Number 3	MSR
$0.9E$	—	—	525	775	—	—	17.9	100
$1.1E$	1,100	900	—	1,200	54.4	19.5	—	97
$1.3E$[b]	—	—	—	1,650	—	—	—	70

[a]Assumes production of only one MSR grade at a time.

[b]Higher MOE (10^6 lb/in^2) than available through visual grading for this species group.

Conclusions

From the information presented in this paper on dry 228 mm (9 in.) diameter round timbers of Englemann spruce, alpine fir, and lodgepole pine, we conclude the following.

- Property assignment procedures for visually graded logs using *ASTM D 3957/D 245* (ASTM 1990, 2005c) produce slightly conservative assignment of MOE values and quite conservative assignment of allowable bending and compression strengths parallel to grain.
- A good correlation exists between static MOE in third-point bending and MOE determined by transverse vibration and between static bending strength, static MOE, and MOE by transverse vibration.
- Procedures traditionally used to assign allowable compressive strength parallel to grain to mechanically graded lumber also appear applicable to round timbers.
- There are apparently no technical barriers to developing a mechanical grading system for 228 mm (9 in.) diameter round timber beams.
- Additional research is needed to clarify anticipated grade yields at higher mechanical grades and the applicability of the methods proposed here to logs of other sizes and species. Some of this research is in progress.
- Mechanical grading offers an opportunity to improve the precision of the property assignment for round timber beams and significantly increase grade yield for a given property specification.

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Notation

The following symbols are used in this paper:

E = modulus of elasticity (MOE) in bending;
E_{comp} = MOE in compression parallel to grain;

E_{dead} = MOE determined by center-point dead load;
E_{sw} = MOE determined by longitudinal stress wave techniques;
E_{tv} = MOE determined by transverse vibration;
F_b = allowable bending strength; and
F_c = allowable compressive strength parallel to grain.

References

ASTM. (1990). "Standard practice for establishing stress grades for structural members used in log homes." *D 3957-90*, West Conshohocken, Pa.

ASTM. (1992a). "Standard test methods for direct moisture content measurement of wood and wood-based materials." *D4442-92*, West Conshohocken, Pa.

ASTM. (1992b). "Standard test methods for specific gravity of wood and wood-based materials." *D 2395-92*, West Conshohocken, Pa.

ASTM. (1994). "Standard methods of static tests of lumber in structural sizes." *D 198-94*, West Conshohocken, Pa.

ASTM. (1998). "Standard test methods for establishing clear wood strength values." *D 2555-98*, West Conshohocken, Pa.

ASTM. (2000a). "Standard practice for assigning allowable properties for mechanically graded lumber." *D6570-00*, West Conshohocken, Pa.

ASTM. (2000b). "Standard practice for establishing allowable properties for visually graded dimension lumber from in-grade tests of full-size specimens." *D 1990-00*, West Conshohocken, Pa.

ASTM. (2000c). "Standard practices for establishing structural grades and related allowable properties for visually graded lumber." *D 245-00*, West Conshohocken, Pa.

ASTM. (2002). "Standard practice for evaluating allowable properties for grades of structural lumber." *D 2915-02*, West Conshohocken, Pa.

Burke, E. J. (2004). "Visual stress grading of wall logs and sawn round timbers used in log structures." *Wood Des. Focus*, 14(1),14–20.

Doyle, D. V., and Markwardt, L. J. (1966). "Properties of southern pine in relation to strength grading of dimension lumber." *Research Paper No. FPL–RP–64*, USDA Forest Service, Forest Products Laboratory, Madison, Wis.

Erickson, R. G., Gorman, T. M., Green, D. W., and Graham, D. (2000). "Mechanical grading of lumber sawn from small-diameter lodgepole pine, ponderosa pine, and grand fir trees." *For. Prod. J.*, 50(7/8), 59–65.

Galligan, W., and McDonald, K. A. (2000). "Machine grading: Practical concerns for lumber producers." *General Technical Rep. No. FPL–GTR–7* (revised), USDA Forest Service, Forest Products Laboratory, Madison, Wis. ⟨http://www.fpl.fs.fed.us/documnts/fplgtr/fplgtr7.pdf⟩.

Green, D. W., Gorman, T. E., Evans, J. W., and Murphy, J. F. (2004) "Improved grading system for structural logs for use in log homes." *For. Prod. J.*, 54(9), 52–62.

Green, D. W., and Kretschmann, D. E. (1991). "Lumber property relationships for engineering design standards." *Wood Fiber Sci.*, 23(3), 436–456.

Green, D. W., and Kretschmann, D. E. (1997). "Properties and grading of southern pine timbers." *For. Prod. J.*, 47(9), 78–85.

Green, D. W., McDonald, K. A., Dramm, J., and Kilborn, K. (2000). "Grading options for western hemlock pulpwood logs from southeastern Alaska." *Research Paper No. FPL–RP–583*, USDA Forest Service, Forest Products Laboratory, Madison, Wis., ⟨http://www.fpl.fs.fed.us/documnts/fplrp/fplrp583.pdf⟩.

Green, D. W., Ross, R. J., and McDonald, K. A. (1994.) "Production of hardwood machine stress rated lumber." *Proc., 9th Int. Symp. on Nondestructive Testing of Wood*, September 22–24, 1993, Forest Prod. Soc., Madison, Wis., 141–150.

Hope, M. (1993). "Log grading: development and practical applications." *Proc., Log Home Design, Engineering, and Marketing Conf.*, T. Gorman, ed., WUD Publication No. 93-1, College of Forestry, Wildlife and Range Sciences, Univ. of Idaho, Moscow, Id.

Keegan, C. E., III, Chase, A. L., Shook, S., and Van Hooser, D. D. (2000). "Montana's log home industry: Developments over the last three decades." *Montana Business Q.*, 38(4), 2–9.

Kretschmann, D. E., and Green, D. W. (1999.) "Mechanical grading of oak timbers." *J. Mater. Civ. Eng.*, 11(2), 91–97.

Murphy, J. F. (2000). "Transverse vibration of a simply supported frustum of a right circular cone." *J. Test. Eval.*, 28(5), 415–419.

Smulski, S. G., ed. (1997). "Issue on machine-graded-lumber." *Wood Des. Focus*, 8(2), 1–24.

Timber Products Inspection, Inc. (TP). (1987). *Log home grading rules* (including supplements 1–5), Conyers, Ga.

Wang, X., Ross, R. J., Mattson, J. A., Erickson, J. R., Forsman, J. W., Geske, E. A., and Wehr, M. A. (2001). "Several nondestructive evaluation techniques for assessing stiffness and MOE of small-diameter logs." *Research Paper No. FPL–RP–600*, USDA Forest Service, Forest Products Laboratory, Madison, Wis.