PINE

AN UPDATED SITE INDEX EQUATION FOR NATURALLY REGENERATED LONGLEAF PINE STANDS[1]

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Abstract-From 1964 to 1967. the U.S. Forest Service established the Regional Longleaf Growth Study (RLGS) in the Gulf States with the objective of obtaining a database for the development of prediction systems for naturally regenerated, even-aged. longleaf pine stands. The database has been used for numerous quantitative studies. One of these efforts was a site index equation for naturally regenerated longleaf pine stands using data from the first and second m-measurements. The equation performed well except for younger stands less than 20 years in age and was more suited to the East Gulf area than the previously available curves. The sixth remeasurement (30-year) of the RLGS was completed recently and it covers a broader range of longleaf pine stands and longer observation periods. Preliminary results in the development of an updated site index equation using data through the sixth remeasurement are discussed and its performance statistically evaluated.

INTRODUCTION

From 1964 to 1967, Dr. Robert M. Farrar, Jr., and the U.S. Forest Service established the Regional Longleaf Pine Growth Study (RLGS) in the Gulf States (Farrar 1978). The objective was to obtain a database for the development of prediction systems for naturally regenerated, even-aged, longleaf pine stands. During a m-measurement of the RLGS, it was discovered that the two available site index curves did not give comparable estimates of plot site index at the start and end of the five-year growth period (Farrar 1981). Using data from the first and second m-measurement of the RLGS, Farrar developed a site index equation that was published in the Southern Journal of Applied Forestry in 1981. The site index equation (base age 50) is:

$$S=(H)10^{\left[-29.468\left[\frac{1}{50}-\frac{1}{A}\right]-938.97\left[\left(\frac{1}{50}\right)^2-\left(\frac{1}{A}\right)^2\right]-16102\left[\left(\frac{1}{50}\right)^3-\left(\frac{1}{A}\right)^3\right]-88775\left[\left(\frac{1}{50}\right)^4-\left(\frac{1}{A}\right)^4\right]\right]} \quad (1)$$

where

S = height at age 50 (site index) in feet,
H = mean dominant height in feet, and
A = age (ring count at 4 feet plus 7 years).

Farrar's equation is based on a fourth degree polynomial and was weighted by a variable $1/(Age)^2$. The equation predicted minimal changes (< 5 feet) in site index over time for most of the plots. Due to this model form, the equation exhibits some illogical trends at ages below 15 years, producing unrealistic predicted site index values.

Rayamajhi (1996) and other efforts conducted at Auburn University using Farrar's equation for predicting growth and yield of longleaf pine found predicted site index values did not follow the height development pattern in some plots. Because of this and Fat-rat's equation being a fourth degree polynomial, efforts were initiated to produce an updated site index equation.

METHODS

Data came from 1598 observations on 300 permanent 0.1 - or 0.2 - acre plots in even-aged, naturally regenerated longleaf stands. Kush and others (1987) provide details of the study design. Several site index construction techniques were examined. For the purposes of this study, three commonly used models were examined. The 1598 observations of mean dominant height and age were fitted with the following models: 1. Farrar's fourth degree polynomial model fitted to examine the parameters; 2. modified Farrar's model where third and fourth order variables were dropped; and 3. Chapman-Richards non-linear model (Carmean 1972).

The parameters of the models were estimated by using standard linear and non-linear estimation techniques. Using the estimated parameters, site index was estimated for each plot and each m-measurement. The estimated site index for each model was evaluated. Table 1 presents the observations by age and site index based on Farrar's equation (1981).

RESULTS

Farrar Model Updated

Farrar's 1981 model was re-fit with the entire RLGS data set. The site-index equation is:

$$S=(H)10^{\left[14.6755\left[\frac{1}{50}-\frac{1}{A}\right]-1005.4336\left[\left(\frac{1}{50}\right)^2-\left(\frac{1}{A}\right)^2\right]-17194\left[\left(\frac{1}{50}\right)^3-\left(\frac{1}{A}\right)^3\right]-103876\left[\left(\frac{1}{50}\right)^4-\left(\frac{1}{A}\right)^4\right]\right]} \quad (2)$$

The model is still valid with all its parameters significant ($P < .05$). The magnitude of the estimated parameters β_0, β_1, β_2, β_3, and β_4 in equation [2] are very close to the parameters in equation [1] that were estimated by Farrar. However, weighting by $1/(Age)^2$ produced non-significant parameters. This may be due to reduced variability with the addition Of more data, representing a broader range of sites. The estimates of the parameters and its related statistics are given in table 2. The coefficient of determination (R^2) was 76 percent.

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Table 1—Number of observations by age class and site index class for the RLGS data set

Age class	Site index class 50	60	70	80	90	Total
20	23	53	105	85	9	275
30	48	37	86	79	4	254
40	8	34	47	111	23	223
50	17	25	31	74	36	183
60	21	27	37	68	23	176
70	21	23	58	60	17	186
80	15	18	61	33	10	142
90	12		49	8	2	89
100	9	13	35	—	—	57
110	2	7	4	—	—	13
Total	176	267	513	518	124	1,598

Table 2—Parameter estimates and related statistics for the un-weighted Farrar site index model (equation) utilizing the entire RLGS data set

Parameter	Estimate	Standard error	Prob > |T|
β_0	1.8566 1	0.03694	0.0001
β_1	14.67550	5.54674	.0082
β_2	4005.43358	278.92019	.0003
β_3	17194	5653.79557	.0024
β_4	-103876	39588.83331	.0088

The model used is the following:

$$S=(H)10^{\left[\beta_1\left[\frac{1}{50}-\frac{1}{A}\right]+\beta_2\left[\left(\frac{1}{50}\right)^2-\left(\frac{1}{A}\right)^2\right]+\beta_3\left[\left(\frac{1}{50}\right)^3-\left(\frac{1}{A}\right)^3\right]+\beta_4\left[\left(\frac{1}{50}\right)^4-\left(\frac{1}{A}\right)^4\right]\right]}$$

Modified Farrar Model

The parameter estimates using data through the sixth remeasurement of the RLGS did not produce estimates, which were very diierent from those of Farrar (1981). Efforts to produce a model with fewer parameters were undertaken. Farrar's model was reduced to an un-weighted second-degree polynomial. The following site index equation was obtained:

$$S=(H)10^{\left[-6.7463\left[\frac{1}{50}-\frac{1}{A}\right]+35.9519\left[\left(\frac{1}{50}\right)^2-\left(\frac{1}{A}\right)^2\right]\right]} \quad (3)$$

The estimates of the parameters and its related statistics are given in table 3. The R^2 estimated is 78 percent and its parameters were highly significant ($p < 0.0001$).

Non-Linear Model

Several site index models have employed non-linear techniques to estimate parameters. Utilizing the Chapman-Richards function (Carmean 1972), the RLGS data produced the following site index equation:

$$S=H\left[\frac{1-\exp(-0.0568896A_0)}{1-\exp(-0.0568896A)}\right]^{\frac{1}{(1-2.095444)}} \quad (4)$$

where

A_0 is index age.

The estimates of the parameters and its related statistics are given in table 4.

Table 3-Parameter estimates and related statistics for the modified Farrar site index model (equation) utilizing the entire RLGS data set

Parameter	Estimate	Standard error	Prob > |T|
β_0	2.00795	0.00769	0.0001
β_1	-6.74634	55669	.0001
β_2	-35.95195	8.32733	.0001

The model used is the following:

$$S=(H)10^{\left[-\beta_1\left[\frac{1}{50}-\frac{1}{A}\right]+\beta_2\left[\left(\frac{1}{50}\right)^2-\left(\frac{1}{A}\right)^2\right]\right]}$$

parameters were significant when it was refitted without the weight (model above). Farrar's modified model was fit with only one linear and one quadratic term, since it Was flexible enough to account for the variation. In addition, this modified model is more parsimonious and far less complex to use.

Farrar (1981) compared his model to that of Miscellaneous Publication 50 (1976) and Schumacher and Coile (1960) by comparing the standard deviations of difference at the start and end of a 5-year r-e-measurement period for 20-year age classes. For purposes of this study, RLGS plots were divided into IO-year age classes and the standard deviation of site index from the modified model was compared to the updated model.

Table 4—Parameter estimates and related statistics for the non-linear site index model (equation) utilizing the entire RLGS data set

Parameter	Estimate	Asymptotic standard error	Asymptotic 95 percent confidence interval	
			Lower	Upper
β_1	64.7348	0.64156	83.47643	85.99326
$\beta_2$4	2.0.0569	.00272	1.8.05154	2.3.06225

The model used is the following:

$$S=H\left[\frac{1-\exp(-\beta_1 A_0)}{1-\exp(-\beta_1 A)}\right]^{\frac{1}{(1-\beta_2)}}$$

where A_0 is index age.

DISCUSSION

The non-linear model did not perform well in fitting the height development patterns of the RLGS data, especially in the younger and older age classes. The mean square error (MSE) was very high compared to the updated and modified Farrar models. The model seems to contain some specification error producing larger residuals. The specification error in the non-linear model needs to be handled properly. One possibility is considering the effects climate, and possibly changes in climate, may have on the model.

Farrar (1981) used a weighted fourth degree polynomial to fit the re-measured RLGS data (360 observations) available during the study. The data set was much smaller and contained more variability as compared to the current data set. However, the non-parsimonious fourth degree polynomial was flexible enough to fit the data. Farrar's model was reconstructed with the current RLGS data (1598 observations), the third and fourth degree parameters were non-significant with the weight ($1/Age^2$) in the model. The

Using an analysis of variance procedure (ANOVA) and estimating the contrast between the updated and modified model, it was observed that the two models were significantly different at lower (< 40 years old) and higher (> 70 years old) age classes (table 5). Around index age (50 years), the models did not differ statistically. The reason for the similarity around the base age was due to the constraint enforced to pass the site index equation at the base age (50 years). The models were also significantly different in overall comparison ($p < 0.0001$). However, when comparing the models for lo-year age classes, the modified model had a lower standard deviation for 64 percent of the observations. It is also observed that the estimated site index did not change much around the base age (50 years). The number of plots is given by N (table 2) and the absolute difference of site index was higher in higher age class, which might be due to fewer numbers of plots.

A family of site index curves produced by the modified equation is shown in figure 1. The curves have an index age of 50 years.

Table 5—Estimates of absolute differences between the site indices, p-values, and standard errors of estimates by age class comparing Farrar's original site Index model with the modified Farrar site index model

Age class	Estimated difference	Prob. > |T|	Standard error	N
20	2.64	0.0013	0.814	550
30	3.08	.0034	1.048	508
40	.70	.4488	.919	446
50	.05	.9662	1.199	336
60	.65	.6076	I .258	353
70	1.71	.1305	1.131	372
80	2.96	.0134	1.189	284
90	4.08	.0023	1.319	178
100	5.26	.0001	1.308	114
110	6.05	.0252	2.534	26

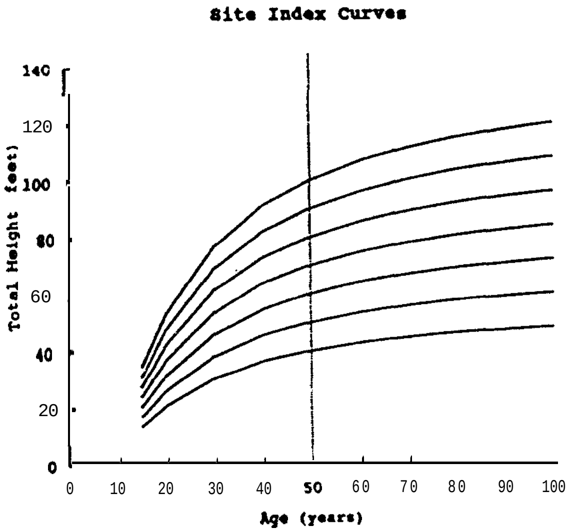

Figure 1—Site-index curves for naturally regenerated longleaf pine in the east Gulf area, index age 50 (from modified Farrar's equation).

SUMMARY

Modified sites index equation for naturally regenerated, even-aged, longleaf stands using Farrar's (1981) site index model was produced by using the current RLGS data set. The modified equation performed as well as the existing equation with only two parameters instead of four. A non-linear model did not fit the observed height development pattern. Another variable, such as climate may be needed in the model that will account for large residuals. Residuals will be examined and efforts will continue for a more precise and flexible model. Site index may be changing (personal communication, Dr. William Boyer, USDA Forest Service, Southern Research Station, Devall Dr., Auburn, AL 36849) and efforts are underway to answer to this question.

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