
A Qualitative Comparison of Fire Spread Models Incorporating Wind and Slope Effects

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ABSTRACT. Wind velocity and slope are two critical variables that affect wildland fire rate of spread. The effects of these variables on rate of spread are often combined in rate-of-spread models using vector addition. The various methods used to combine wind and slope effects have seldom been validated or compared due to differences in the models or to lack of data. In this study, rate-of-spread predictions from the Canadian Fire Behavior Prediction (FBP) system, McArthur's Mark V forest fire danger meter, the Rothermel empirical model, and the Pagni and Peterson physical model were compared with spread rates observed in a designed laboratory experiment in which wind velocity and slope were varied. Methods of combining wind and slope developed by Albini, McAlpine, and Rothermel were applied to two forms of Rothermel's model. Rothermel's model (original and modified) coupled with Albini's method and Pagni's model predictions closely reproduced the observed shape of the rate-of-spread response to wind and slope. Rothermel's method and McAlpine's method worked well in all cases except in upslope spread with opposed flow. However, Rothermel's model failed to predict a nonzero rate of spread in fuel beds with moisture contents of 35%. Possible causes of overprediction by McArthur's model and the FBP model included: (1) application of equations derived from full-scale fires to laboratory-scale fires, (2) improper selection of fuel type to represent artificial fuel beds, and (3) inaccurate estimation of wind velocity 10 m above the vegetation surface using a logarithmic profile. *FOR. SCI.* 43(2):170-180.

Additional Key Words: Fire behavior, rate of spread.

TO PROTECT RESOURCES FROM FIRE DAMAGE, the causative factors underlying combustion and fire spread in wildland fuels have been studied since the 1920s. Rate of spread, flame length, and flame angle have all been studied to varying degrees, with rate of spread being the most widely studied. We know of no studies in which the combined effects of wind velocity and slope on wildland fire behavior have been examined in an experimental setting. Nor do we know of any instances in which predictions from existing models that contain wind velocity and slope have been simultaneously compared with experimental data in which wind velocity and slope were simultaneously varied.

Efforts to study the physical factors governing wildland fire spread and other measures of fire behavior were organized in the USDA Forest Service's California Forest Experiment Station in the late 1930s and early 1940s. This work culminated in a

mathematical model that included heat transfer terms for conduction, convection, and radiation (Fons 1946). The Forest Service resumed this approach in the late 1950s by establishing fire laboratories in Macon, Georgia; Missoula, Montana; and Riverside, California.

Empirical and statistical rate of spread models have also been developed. An empirical research effort in Canada began in the 1920s. Wildland fire behavior has been measured at field scale in several different fuel types. Nonlinear equations relating spread rate to easily measured variables such as wind speed, ground slope (vertical rise/horizontal distance), temperature and relative humidity have been incorporated into an operational system that is used for fire danger and fire behavior prediction [Forestry Canada Fire Danger Group (FCFDG) 1992]. A similar approach was used in Australia to predict fire danger and rate of spread in grasslands and forests (McArthur 1966, 1967).

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Rothermel (1972) developed parameters from experimental data for a physical model formulated by Frandsen (1971). This model (referred to as the Rothermel model) presently forms the core of the fire danger and fire behavior systems that are used in the United States (Deeming et al. 1977, Albini 1976). Additional research examining various assumptions or components of the Rothermel model has been completed (Albini 1976, Wilson 1990); however, not all proposed changes have been incorporated into the model. Out of all of the physically based rate of spread models that have been developed, only the Rothermel model (1972) has been implemented operationally. As a result of the many research efforts, numerous rate of spread models have been formulated (Catchpole and de Mestre 1986, Weber 1991, Pitts 1991).

Individual Influences of Wind Velocity and Slope

In general, a fire burning upslope aided by wind shows the highest rates of spread, greatest potential for damage, and greatest difficulty to control. Most models were formulated to predict spread for fires aided by wind and slope. Rate of spread for fires spreading either downslope without wind or into ambient wind are often assumed to have equal spread rates (Beaufait 1965, Byram 1959, Byram et al. 1966, McArthur 1967, Prahla and T'ien 1973, Kashiwagi and Newman 1976, Rothermel 1983, Van Wagner 1988, Weber and de Mestre 1990). Typical spread rates for back (windopposed) and downslope laboratory fires ranged from 0.002 to 0.02 m/s. We did not find a physical model for backfire in the literature, probably because backfire is viewed as innocuous. However, backfire is a widely used technique for fire fighting as well as for prescribed fire.

The spread rate of fire spreading with wind (headfire) has often been modeled as a power or exponential function of wind velocity (Fons 1946, Anderson and Rothermel 1965, Byram and others 1966, Rothermel and Anderson 1966, McArthur 1966, 1967, Rothermel 1972, Pagni and Peterson 1973, Nelson and Adkins 1988, FCFDG 1992). In most cases, the exponent for the power function is between 1 and 2. In Pagni and Peterson's formulation, spread rate is a function of $U^{0.8}$ where U is wind velocity. Similarly, the effect of positive slope on rate of spread has been found to be a curvilinear function (Curry and Fons 1938, Byram et al. 1966, McArthur 1967, Rothermel 1972, Hwang and Xie 1984, Weber and de Mestre 1990, FCFDG 1992).

All of these findings are based on studies in which the effects of wind and slope were examined in isolation. Only two studies examined the concurrent effects of wind velocity and slope on wildland fire behavior. Forts (1935) reported the effects of wind velocity and positive slope on the length to width ratio of small elliptical fires burning in ponderosa pine (*Pinus ponderosa* Dougl. ex Laws) needle fuels in the open. A function containing wind speed and slope was developed to predict the ratio of headfire spread to backfire spread.

A similar study conducted by Murphy (1963) used woodflour as a fuel. Fire spread through the fuel but no flaming combustion occurred. Downslope backfire and upslope headfire spread rates

were measured. The ratio between backfire and headfire spread rates was found to vary curvilinearly. Wind and slope effects at the head of the fires were not completely additive.

Of the many models that have been formulated to predict a wildfire's rate of spread, only four were identified in the present study as containing both wind and slope as independent variables. Three of the four models have been implemented as fire behavior prediction systems. The four models are the Rothermel (1972) model, McArthur fire danger meters (Noble et al. 1980), Canadian Fire Behavior Prediction (FBP) system [Forestry Canada Fire Danger Group (FCFDG)1992], and the physical model formulated by Pagni and Peterson (1973).

Combined Influences of Wind Velocity and Slope

Physically Based Methods

As the above discussion indicates, theory and experimental data describing the combined effects of wind and slope on fire spread are scarce (Nelson 1992). Wind is generally viewed as affecting heat transfer from a flame to unburned fuel downwind of the flame primarily by changing the angle of the flame relative to the fuel; changing this angle affects the radiant heat transfer process. Wind increases radiant heat transfer for headfires because the flame is tilted toward the unburned fuel and decreases radiant heat transfer for backfires because the flame is tilted away from the unburned fuel. Wind might affect convective heating ahead of the flame, but this has not yet been experimentally verified.

Early fire modeling efforts treated slope as an added component to wind because slope was considered to have the same effect as wind on rate of spread (Fons 1946). For fire spreading upslope, the angle between the flame and unburned fuel is decreased, thus affecting radiative heat transfer from the flame to unburned fuel in advance of the flame. Similarly, the angle increases for a fire spreading downslope and radiant heat transfer decreases. Pagni and Peterson (1973) used this logic in model formulation.

Other Methods

Rate of spread in the Rothermel model may be viewed as a ratio between the heat received by unburned fuel from the flame to the heat required to ignite the unburned fuel (Rothermel 1972). This is very similar to the approach taken by Pagni and Peterson; however, the heat energy is not partitioned into convection, radiation, and conduction as in the Pagni and Peterson model. In the Rothermel model, a function of wind velocity (ϕ_w) and of slope (ϕ_s) are added and then multiplied by rate of spread for a no wind, no slope fire (R_0). This formulation, equivalent to upslope headfire spread in Equation (1), assumed that the fire will spread under no wind, no slope conditions if all other conditions are equal. The ϕ_w term may be considered to represent forced convection and radiant heat transfer terms while 0, may represent natural convection and radiant heat transfer terms as well. In the Pagni and Peterson model, the flame tilt angle caused by

wind and the slope angle are added; the combined angle is then used to determine radiant heat transfer (see Appendix 2).

Manipulation of the equations (Noble et al. 1980) produced from the McArthur forest fire danger meter yields an equation in which wind and slope angle are added. This is due to the nature of the equations, not due to physical reasoning. In the Canadian FBP, the slope effect is converted to an "effective wind speed" and combined with actual wind velocity using vector addition; this may be viewed as implicitly assuming that wind velocity and slope similarly affect rate of spread.

Albini (1976) extended Rothermel's original formula for the case of wind blowing across a slope at any angle. For upslope headfire spread, Albini's method yielded Rothermel's original model. The special cases from this method in which wind flow and fire spread are parallel to slope include [Equation (1)]:

$$\begin{aligned} \text{Upslope headfire} \quad R_f &= (1 + \phi_w + \phi_s)R_0 \\ \text{Downslope headfire} \quad R_f &= (1 + \max(0, \phi_w - \phi_s))R_0 \\ \text{Upslope backfire} \quad R_f &= (1 + \max(0, \phi_s - \phi_w))R_0 \\ \text{Downslope backfire} \quad R_f &= R_0 \end{aligned} \quad (1)$$

in which R_f is rate of spread (see Appendix 2 for formulas to estimate R_0 , ϕ_w , and ϕ_s). Albini's method was not used in the operational implementation of the Rothermel model.

A simpler, vector-based technique to combine wind and slope was recommended for use with BEHAVE, an operational implementation of the Rothermel model (Rothermel 1983). For wind flow and fire spread parallel to slope, Rothermel's method simplifies to Equation 2, which is similar to Albini's formulation [Equation (1)]. Rate of spread is predicted for wind (R_w) and slope (R_s) separately, and a resultant vector is calculated.

$$\begin{aligned} \text{Upslope headfire} \quad R_f &= (1 + \phi_w + \phi_s)R_0 \\ \text{Downslope headfire} \quad R_f &= (1 + \phi_w)R_0 \\ \text{Upslope backfire} \quad R_f &= |R_s - R_w| = R_0|\phi_s - \phi_w| \\ \text{Downslope backfire} \quad R_f &= R_0 \end{aligned} \quad (2)$$

McAlpine et al. (1991) identified a potential weakness of Rothermel's vector approach when wind and slope vectors were equal in magnitude ($\phi_s = \phi_w$) but opposite in direction. For this scenario, upslope backfire rate of spread is 0 [Equation (2)]. The Canadian FBP currently incorporates the vector method developed by McAlpine et al. (1991) [Equation (3)] for wind flow and fire spread parallel to slope:

$$\begin{aligned} \text{Upslope headfire} \quad R_f &= f(g(U) + h(\theta_s)) \\ \text{Downslope headfire} \quad R_f &= f(g(U) - h(\theta_s)) \\ \text{Upslope backfire} \quad R_f &= f(h(\theta_s) - g(U)) \\ \text{Downslope backfire} \quad R_f &= f(-(g(\theta_s) + h(U))) \end{aligned} \quad (3)$$

in which $f()$, $g()$, $h()$ denote an arbitrary function, U is wind velocity, and θ_s is slope angle.

The fuel moisture content model is also another limiting factor in the Rothermel model. Increased moisture content reduces the rate of spread of the no wind, no slope fire and rate of spread = 0 for moisture contents in excess of a predetermined, fuel-dependent level (extinction moisture content). Extinction moisture content for the standard fuel models used with the Rothermel model does not exceed 40%. However, finite rates of spread were observed in an experiment examining fire spread in live fuels (Martin and Sapsis 1987). Fuel beds of live French broom (*Cytisus monspessulanus* L.) with moisture contents ranging from approximately 30% to 100% were burned on slope angles ranging from 0° to 90°. No ambient wind flow was imposed. Wilson (1985, 1990) addressed extinction moisture content at length, proposed a different formulation of the moisture function, and suggested a wind factor as a possible component for a fuel moisture term in the Rothermel model.

Prior Model Comparison

The Rothermel model has been tested in field settings with varying success (i.e., Andrews 1980, Sneeuwjagt and Frandsen 1977, Gould 1991, Lindenmuth and Davis 1973). The FBP and McArthur models are empirical in nature and were developed in part from rates of spread observed in field settings (FCFDG 1992, McArthur 1966, 1967). The Pagni and Peterson model (1973), a physical model based solely on heat transfer, was only tested using published data from laboratory fires (Anderson and Rothermel 1965). This model requires that many physical properties be known. Flame length, an unknown quantity, is needed to predict another unknown quantity—rate of spread. Unless a model is used to predict flame length, and the predicted flame length is then used to predict rate of spread, Pagni and Peterson's model must be iteratively solved for rate of spread.

Few comparisons of wildland fire models have been reported. Catchpole and de Mestre (1986) qualitatively evaluated several physical models. The Rothermel model, while physical in formulation, was not included in the evaluation because of its empirical component. McAlpine and Xanthopoulos (1990) compared the Rothermel and FBP models using wind tunnel fires burning in ponderosa pine (*Pinus ponderosa* Dougl. ex Laws) needle fuel beds. The FBP model overpredicted rate of spread while the Rothermel model underpredicted it. Cheney (1991) found that the McArthur Forest Fire Danger Meter and Western Australia fire behavior tables produced comparable rate-of-spread predictions for a "standard" jarrah forest. We know of no other direct comparisons between rate of spread models. The comparisons of these various models did not test the wind and slope components of the models together.

This paper discusses results of a qualitative comparison of model predictions from four rate of spread models containing wind and slope with observed rates of spread measured in a laboratory based, designed fire spread experiment in which wind and slope were varied. Of the models examined, the Rothermel model was originally parameterized using laboratory and wind

tunnel data from fires in fuels similar to those used in this study—relatively uniform, machined wooden sticks, dowels, or shavings. The Pagni and Peterson model was validated using some of the same laboratory data (Anderson and Rothermel 1965). The FBP system and McArthur fire danger meters were statistically developed using data collected primarily on field-scale fires burning in natural fuels. Thus comparison of these latter two models results in comparison across scale and between controlled and uncontrolled conditions. Comparison of the models under identical conditions should identify potential weaknesses in modeling the combined effects of wind velocity and slope even though the predicted spread rates may not be correct for the FBP system and McArthur fire danger meters. Due to the limited nature of the data, a positive result does not indicate that the method of combining wind velocity and slope is correct; however, a negative result should stimulate further examination of the interactive effects of wind velocity and slope on fire spread.

Methods

An experiment to examine the interaction between wind velocity and slope angle was conducted during the summer of 1992 using a tilting wind tunnel (Weise 1993). Five nominal wind velocities—two heading (0.4 and 1.1 ms⁻¹), two backing (0.4 and 1.1 ms⁻¹) and no wind—were combined with five slope angles—two upslope (15% and 30%), two downslope (-15% and -30%), and no slope (Table 1). At the USDA Forest Service Southern Forest Laboratory in Macon, Georgia, 65 experimental fires were burned in a portable tilting wind tunnel: two blocks of 30 fires each and an additional 5 fires (Table 1). A block of 30 fires consisted of all 25 wind and slope combinations (5 wind velocities, 5 slopes) and a partial replication consisting of the 4 treatment extremes and the no wind, no slope treatment. The additional 5 fires consisted of a partial replication of treatments in fuel beds with mean stick fuel moisture content of 35% and were used to determine if wind velocity, slope, and fuel moisture content interacted. Stick fuel moisture contents of the 60 fires were not controlled but were fairly constant (11–12% oven-dry weight).

Fuel beds were composed of vertical paper birch (*Betula papyrifera*) sticks and a shallow layer (< 0.5 cm) of evenly distributed coarse aspen (*Populus tremuloides*) excelsior (see Appendix 1 for additional fuel bed parameters). The excelsior provided horizontal continuity and assured successful fire spread for the no wind, no slope treatment—a requirement of the Rothermel model. Rate of spread was estimated using 3 mm chromel-alumel (type K) thermocouples (Weise 1993, Weise and Biging 1994). Flame length was estimated using a video camera and the Fire Image Analysis System (Adkins 1987, Adkins et al. 1994).

Rate of spread was predicted by a total of nine variants of the four basic models. The model variants included are as follows: Canadian Fire Behavior Prediction (FBP) system using the C-7 fuel type; original Rothermel (ORTH) model using vector addition [Equation (2) (Rothermel 1983)]; modified Rothermel (MRTH) model (Wilson 1990) using vector addition; ORTHA is ORTH with Albini's (1976) method [Equation (1)]; MRTHA is

Table 1. Mean observed rate of spread of experimental fires in paper birch stick fuel beds burned under varying wind velocity and slope configurations.

Treatment ¹	Wind velocity (ms ⁻¹)	Slope (%)	Rate of spread ² (x10 ⁻² ms ⁻¹)	
High backing (HB)	1.11	-30	1.58	(0.22)
	1.04	-15	1.00	(0.10)
	1.14	0	1.45	(0.65)
	1.15	15	1.25	(0.25)
	1.11	30	0.65	(0.43)
Low backing (LB)	0.34	-30	1.40	(0.30)
	0.34	-15	1.65	(0.05)
	0.42	0	2.30	(0.30)
	0.35	15	3.35	(1.65)
	0.42	30	11.10	³
No wind	0.00	-30	1.55	(0.25)
	0.00	-15	3.30	(0.60)
	0.00	0	2.68	(0.53)
	0.00	15	3.70	(0.30)
	0.00	30	15.60	(2.10)
Low heading (LH)	0.34	-30	5.00	(0.70)
	0.34	-15	4.30	(0.00)
	0.42	0	7.75	(0.55)
	0.35	15	11.18	(0.60)
	0.42	30	14.15	(1.45)
High heading (HH)	1.11	-30	22.43	(0.41)
	1.04	-15	20.25	(1.25)
	1.14	0	30.70	(2.60)
	1.15	15	46.50	(1.10)
	1.11	30	60.68	(5.57)

¹ Backing indicates that fire is spreading into the wind, heading indicates that direction of fire spread is with the wind.

² Values are mean and standard error.

³ Only one fire successfully spread length of test bed so standard error can't be estimated.

MRTH with Albini's method [Equation (1)]; ORTHF is ORTH with Canadian FBP method (McAlpine et al. 1991) [Equation (3)]; MRTHF is MRTH with Canadian FBP; McArthur's Mark V (MCART) forest fire danger model (Noble et al. 1980); and Pagni and Peterson's model PAGNI (1973) (see Appendix 2). The modified Rothermel model listed above incorporates proposed modifications to the moisture content model (Wilson 1990), but the model for wind and slope interaction is unchanged.

Pagni and Peterson's model contains many physical variables including flame length as input variables. Tabular values were assumed for the input variables when actual values were not measured (Pagni and Peterson 1973, Musselman and Hocker 1981, White 1988). The FBP and McArthur models require wind velocities measured at 10 m above vegetation. The logarithmic wind profile (Albini and Baughman 1979) was inverted to predict 10 m wind velocity from the measured wind tunnel wind velocity. The Rothermel model requires wind velocity at midflame height. Mean wind velocity in the wind tunnel was assumed equal to midflame wind velocity.

The Rothermel and Pagni models use average physical descriptions of the fuel particles. The FBP and McArthur systems use stylized descriptions of fuels that are based on general characteristics of the vegetation. To compare the four models, stylized fuel models

had to be selected for the FBP and MCART models as in McAlpine and Xanthopoulos (1990). The forest equations were used for the MCART model. Fuel type C-7 was used for the FBP. The C-7 fuel type is described as containing light, scattered woody surface fuel with the forest floor dominated by perennial grasses and scattered shrubs (FCFDG 1992). Both the assumed wind velocity profiles and the fuel type assumptions affect the outcome of this study (McAlpine and Xanthopoulos 1990).

Mean rate of spread was estimated for each model by wind speed and slope classes and plotted versus wind speed and slope. Plots of predicted rate of spread were visually compared with observed rate of spread. Because the various assumptions in this study affected the predicted rate of spread for each model, model predictions were not compared statistically with actual rate of spread to assess accuracy. Graphs of the model predictions as influenced by wind velocity and slope were compared to a graph of the observed rate of spread response. The small data set containing rate of spread for five fires in identical fuel beds with higher fuel moisture content was also used for model

comparison. As noted previously, these models were compared across scale and between controlled and uncontrolled conditions. However, the comparison should provide some insight into the various methods that have been devised to combine wind and slope effects.

Results and Discussion

Mean observed rate of spread ranged from 0.007 to 0.06 ms^{-1} (Table 1, Figure 1). Predicted rates of spread for all models were within this range with the exception of FBP and MCART for upslope headfires. Mean observed rate of spread for headfires spreading downslope exceeded rate of spread for backfires (Figure 1). Performance of the models fell within two classes: those that fit the observed data and those that did not. Predictions from models incorporating Albini's method [Equation (1); ORTHA, MRTHA] and the PAGNI model agreed with the observed data (Figure 1a, 1d, 1e). Predictions from models incorporating Rothermel's [Equation (2); ORTH, MRTH]

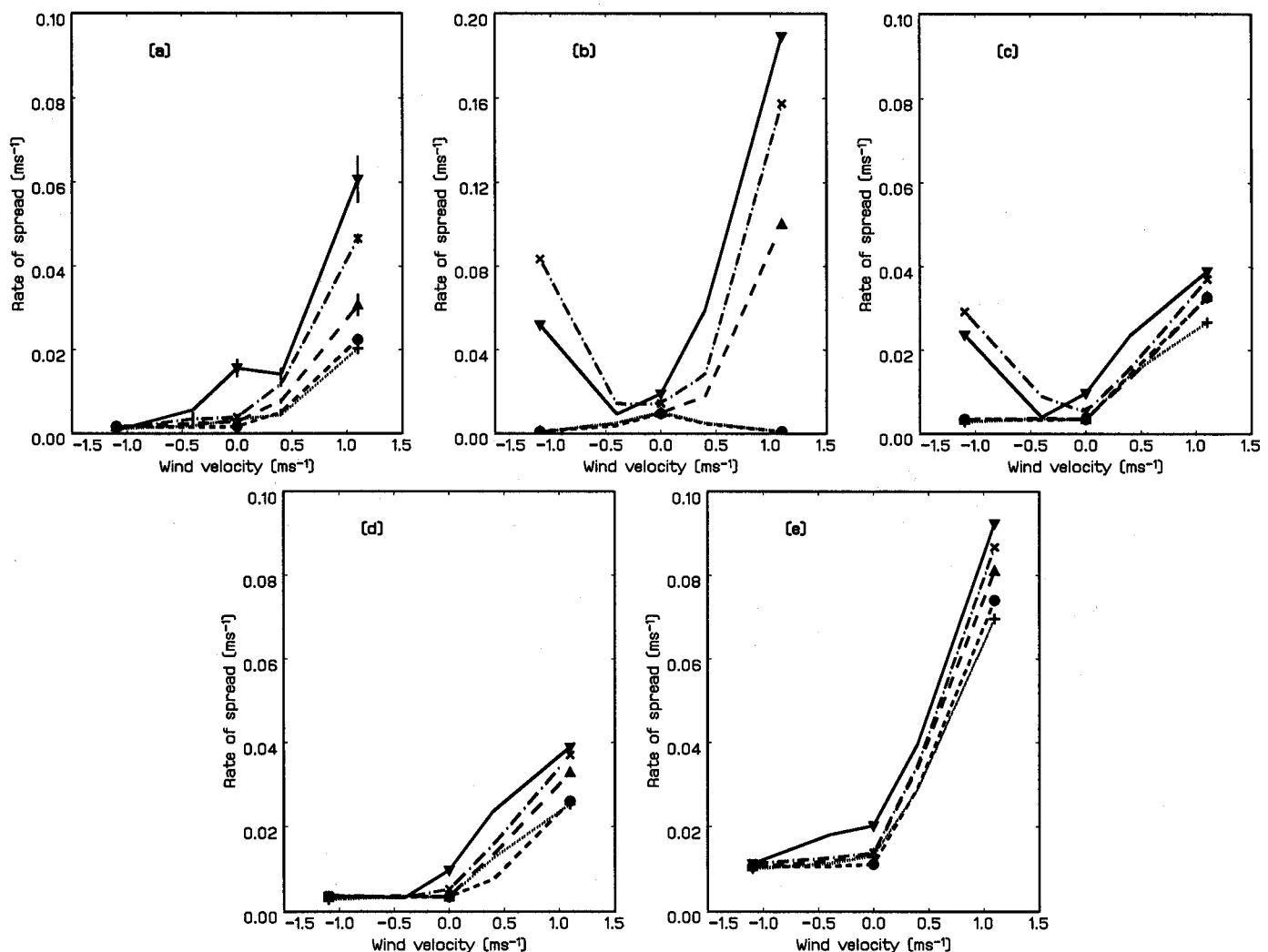


Figure 1. Mean observed rate of spread (a) as a function of wind velocity (ms^{-1}) and slope percent compared with predicted mean rate of spread from (b) Canadian FBP model, (c) original Rothermel model (ORTH), (d) Rothermel model with Albini method (ORTHA), and (e) Pagni and Peterson model (PAGNI). Negative wind velocity indicates fire spreading into wind (backfire). ∇ = 30% (upslope), $\times$ = 15% (upslope), $\triangle$ = 0% slope, $\bullet$ = -15% (downslope), $+$ = -30% (downslope). Vertical bars indicate standard error for observed rate of spread only. Lines connect all points for a particular slope percentage together only and should not be used to interpolate.

and McAlpine's method [Equation (3); FBP, ORTHF, MRTHF] agreed with observed data for all but upslope backfires (Figure 1b, 1c). This behavior was noted in the MCART model also. The reason for the lack of fit of FBP, ORTH, MRTH, ORTHF, and MRTHF with observed rate of spread for the upslope backfires can be attributed primarily to the way in which wind and slope factors were combined and to the assumptions underlying the models.

McAlpine's Vector Method (FBP, ORTHF, MRTHF).

The wind velocity and slope components were combined to produce a combined wind speed for the FBP model and the ORTHF and MRTHF models (Table 2). Effective wind speeds for level and downslope orientations in the FBP model were on the order of $10^{-6} \text{ kmhr}^{-1}$ and did not contribute to the combined wind speed. Addition of the wind and slope components using Equation (3) for the upslope backfire case

Table 2. Selected values of wind and slope components used by various rate of spread models for several wind velocity and slope combinations.

		Slope percent				
Wind ¹		-30	-15	0	15	30
Canadian FBP						
HB	W_c^2	-26.69	-27.65	-27.41	-25.00	-26.69
	S_c	0.00	0.00	0.00	3.89	8.84
LB	W_c	-10.10	-8.41	-10.10	-8.17	-8.17
	S_c	0.00	0.00	0.00	3.84	8.92
Rothermel (ORTH, ORTHA, MRTH, MRTHA)						
HB	ϕ_w	0.00	0.00	0.00	8.42	8.91
	ϕ_s	0.00	0.00	0.00	0.51	2.02
LB	ϕ_w	0.00	0.00	0.00	3.21	3.21
	ϕ_s	0.00	0.00	0.00	0.51	2.02
0	ϕ_w	0.00	0.00	0.00	0.00	0.00
	ϕ_s	0.00	0.00	0.00	0.51	2.02
LH	ϕ_w	3.21	3.21	3.85	3.29	3.85
	ϕ_s	2.02	0.51	0.00	0.51	2.02
HH	ϕ_w	8.91	8.42	9.12	9.19	8.91
	ϕ_s	2.02	0.51	0.00	0.51	2.02
McArthur						
HB	W_A	0.00	0.00	0.00	0.00	0.00
	S_A	0.31	0.55	1.00	1.81	3.18
LB	W_A	0.00	0.00	0.00	0.00	0.00
	S_A	0.31	0.55	1.00	1.81	3.18
Pagni						
LH	Fr	15.1	17.2	16.7	11.9	14.0
HH	Fr	27.0	25.2	24.7	23.7	20.3

¹ Wind velocity and fire spread direction. HH = 1.1 ms^{-1} (high velocity) heading, LH = 0.4 ms^{-1} (low velocity) heading, 0 = no wind, no slope, LB = 0.4 ms^{-1} backing, HB = 1.1 ms^{-1} backing.

² W_c and S_c are wind and slope components of the Canadian FBP system; ϕ_w and ϕ_s are wind and slope factors of the Rothermel spread model (defined in Appendix 2); W_A and S_A are wind and slope term of the McArthur fire danger meter; and Fr is the Froude number ($U/(gL)^{0.5}$) where U is wind velocity, g is gravitational acceleration, and L is flame length.

yielded a finite number (28.89). When the magnitude of the resultant vector was computed, a positive, nonzero combined wind speed resulted leading to model overprediction.

Rate of spread for the downslope backfires was computed using the same combined wind speed. However, the backfire model developed for the FBP was used resulting in model agreement with observed spread rates. The assumption that $\phi_s = 0$ and $\phi_w = 0$ for downslope backfires for ORTHF and MRTHF resulted in close agreement with observed rate of spread.

Rothermel's Vector Method (ORTH, MRTH)

One of the assumptions underlying the Rothermel model is that rate of spread of backfires ($\phi_w = 0$) and fires spreading downslope without wind ($\phi_s = 0$) is equal to rate of spread of a fire spreading without the influence of wind or slope (Table 2). For upslope backfire rate of spread, the combination of ϕ_s and ϕ_w , using Equation (2) resulted in positive, nonzero values. However, ϕ_w , in all four cases was larger than ϕ_s . Per Rothermel (1983), spread would occur in the downslope direction. Rate of spread upslope was not explicitly addressed for this situation. If rate of spread was assumed to equal backfire rate of spread, ORTH and MRTH predictions would have fit the observed data.

Albini's Vector Method (ORTHA, MRTHA)

Albini's method [Equation (1)] differs from Rothermel's method [Equation (2)] in that ϕ_s , does not equal 0 for the downslope headfires as in Rothermel's method. Albini's method was formulated so that the problem identified above for McAlpine's method [Equation (3)] did not arise. As a result, agreement was found between observed and predicted rate of spread as influenced by wind and slope.

McArthur's Fire Danger Meter (MCART)

Agreement between predicted and observed rate of spread for the MCART model was poor. Ignoring the scale effect, the general shape of MCART predictions as a function of wind and slope was similar to FBP, ORTH, MRTH, ORTHF and MRTHF.

Rate of spread for all backfire wind speeds differed appreciably. This difference is due to a slope effect. Rate of spread for upslope backfires was greatly overestimated by MCART. No mechanism to combine wind and slope components for other than upslope headfire spread was found in the literature for the MCART model. Wind was assumed equal to 0 for all backfires. The slope factor for 30% upslope is an order of magnitude greater than the slope factor for 30% downslope (Table 2).

Pagni and Peterson's Physical Model (PAGNI)

Predictions by the PAGNI model for backfires were an order of magnitude larger than observed values. However, the relative insensitivity to change in wind velocity or slope angle for backfires was similar to that observed. For headfires, predicted rate of spread was two to five times observed. While the PAGNI model generally overpredicted rate of spread, the overall shape of the predicted response to wind and slope was similar to the observed response (Figure 1e).

Unlike most of the preceding models, wind and slope interactions were not determined by adding wind and slope components together based on the direction of the slope vector with regard to the wind vector. In PAGNI, slope only appears in one term in the model—the flame radiation term Π_{RF} (see Appendix 2). Wind is present in the wind term Π_w , a dimensionless group known as the Froude number, and in the interior convection term Π_{CI} . For no wind velocity, $\Pi_w = 0$ and the interior and surface convective terms do not affect rate of spread. Similarly, for backfires, interior and surface heating by convection does not occur so those terms are 0 (Pagni and Peterson 1973).

As mentioned previously, wind and slope are combined to estimate flame angle in this model and the wind component of flame angle is $\Pi_w^{0.8}$ (Table 2). The slope component of the flame angle is the arctangent of the slope percentage expressed as a proportion. Flame angle can be computed by adding the wind and slope values directly. Thus for a fire spreading with the high wind speed down a 15% slope, flame angle (from the fuel bed normal) is predicted to be 16.7° ($25.2 + \tan^{-1}(-0.15)$). The wind component for all no wind and opposed flow fires is zero.

Fuel Moisture

Headfires spread successfully in fuel beds with fuel moisture of approximately 35% (Table 3). The intent of the five additional experimental fires was to determine if wind and slope interacted in the same fashion at a moisture content of 35% compared to fuel moisture of 12% and they serve to illustrate some key points in model formulation.

The original Rothermel model formulation assumed steady state spread without wind or slope influences. Wind and slope were simply multipliers of this baseline rate of spread. With this formulation, if a fire did not spread without wind or slope, rate of spread with wind or slope = 0. Wilson (1985, 1990) addressed this issue at length and suggested a wind factor as a possible component for a fuel moisture term.

Of the five experimental fires, only the two headfires successfully spread the length of the fuel bed (Table 3). Neither backfire nor still air fires spread successfully. Virtually all predictions for ORTH, ORTHF, and ORTHA were negative. This

is due in large part to the moisture damping function in the original Rothermel model (see Appendix). The moisture of extinction, the fuel moisture above which a fire will not spread, was estimated to be 17% using the fuel modelling subsystem of BEHAVE (Burgan and Rothermel 1984). Fuel moistures of 35% produced a negative moisture damping value. MRTH, MRTHF, MRTHA and PAGNI all predicted nonzero spread. MRTH, MRTHF, and MRTHA contain Wilson's (1990) modifications eliminating moisture of extinction.

Conclusions

Wind and slope are two critical variables affecting wildland fire spread, but few rate-of-spread models include both variables. Four such models were examined—the Canadian Fire Behavior Prediction system, McArthur's Mark V fire danger meters, the Rothermel model, and the Pagni and Peterson physical model. Various sets of rules have been developed to combine wind and slope effects for the FBP, McArthur, and Rothermel models. Three such sets have been proposed for the Rothermel model. Heat transfer terms associated with convective heating disappear in the Pagni physical model when wind velocity is 0 or in the direction opposite fire spread.

Rothermel's model (original and modified) coupled with Albini's method for combining wind and slope and Pagni's model predictions closely mimicked the shape of the rate of spread response to various combinations of wind and slope. Rothermel's method and McAlpine's method of combining wind and slope worked well in all cases except upslope backfire spread. Albini's method was more flexible than the other vector methods, which resulted in better prediction of upslope backfire spread. The differences between the methods of Albini, Rothermel, and McAlpine occur because of the handling of negative values of $\phi_s - \phi_w$. Albini's method appears to be the most appropriate based on this study.

Overprediction by McArthur's model and the FBP model potentially occurred because of: (1) application of equations derived from full-scale fires to laboratory-scale experimental fires, (2) improper selection of a fuel type to represent the

Table 3. Rate of spread predictions for several models for fuel beds with average fuel moisture content of 35%.

Model	Wind ¹ Slope ²	HH -30	0 0	HB -30	HB 30	HH 30
Actual		0.01433	0.0000	0.0000	0.0000	0.0339
FBP		0.0000	0.0003	0.0000	0.0021	0.0088
ORTH		-0.3896	-0.0396	-0.0291	0.2380	-0.5849
MRTH		0.0121	0.0012	0.0015	0.0092	0.0128
ORTHF		-0.3102	-0.0396	-0.0291	-0.0346	-0.5849
MRTHF		0.0097	0.0012	0.0015	0.0013	0.0128
ORTHA		-0.3102	-0.0396	-0.0291	-0.2726	-0.5849
MRTHA		0.0097	0.0012	0.0015	0.0105	0.0128
MCART		0.0089	0.0154	0.0058	0.0566	0.1093
PAGNI		0.0460	0.0077	0.0073	0.0065	0.0490

¹ Wind velocity and fire spread direction. HH=1.1ms⁻¹ (high velocity) heading, LH=0.4ms⁻¹ (low velocity) heading, 0 = no wind, no slope, LB=0.4ms⁻¹ backing, HB=1.1ms⁻¹ backing.

² Slope percent, negative indicates downslope, positive indicates upslope.

³ Rate of spread in ms⁻¹.

artificial fuel beds, and (3) inaccurate estimation of winds 10 m above the vegetation surface using a logarithmic profile.

The performance of Pagni and Peterson's physical model is promising as wind and slope are combined using physical reasoning and the shape of the predictions was similar to the observed laboratory data, but the model contains flame length as an input. The model also overpredicted spread rates for some wind and slope combinations. Use of a model that accurately describes the physical processes involved in fire spread can theoretically be applied to fire spread in any wildland fuel type. As fire spread models become more accurate due to increased knowledge, the inclusion of wind and slope factors as related to the physical process of fire spread should improve the usefulness of these predictive models.

Albini's and Pagni and Peterson's methods of combining wind velocity and slope effects on wildland fire spread should be considered in the development of improved fire spread models. These two methods were qualitatively the best because the shape of the model predictions was similar to the shape of the laboratory data. However, this study also demonstrated the paucity of rate of spread data (laboratory and field) for situations where wind velocity and slope are affecting fire spread concurrently. Statistical validation of these methods using additional laboratory and field-scale data is needed to identify which method is more reliable and accurate.

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APPENDIX 1: Summary of variables, symbols, and measured or assumed physical quantities.

Variable	Units	Symbol	M/A	Value
Absorptivity, fuel		a	A	0.6
Pyrolysis preexponential		a_p	A	5.3×10^7
Heat capacity, fuel	$\text{kJ}(\text{kg}^\circ\text{C})^{-1}$	c_{pf}	A	2.5
air at 810°C		c_{pg}	A	1.154
Curing index		C	A	100
Diffusivity, mass	m^2s^{-1}	D	A	1.86×10^{-6}
Gravitational acceleration	ms^{-2}	g	A	9.8
Low heat content	kJkg^{-1}	h	A	18608
Heat of vaporization	kJkg^{-1}	h_{vap}, Q_w	A	
of pyrolysis		h_{pyr}, Q_f	A	711
of combustion, volatile gases		h_y	A	12306
Relative humidity	%	H	M	
Conductivity (air at 810°C) $\text{kW}(\text{m}^\circ\text{C})^{-1}$		k_g	A	7.07×10^{-5}
Flame length	m	L	M	
Moisture content	%	m_f	M	
Extinction moisture	%	m_x	A	0.17
Prandtl number		Pr	A	0.704
Reynolds number for fuel		Re_c	M	
Mineral content		s_e	M	0.015
Temperature, ambient	$^\circ\text{C}$	T	M	
ignition		T_{ig}	A	288
flame, ember		T_{fl}, T_b	A	810
Wind velocity	ms^{-1}	U, V	M	
Fuel loading	kgm^{-2}	w_n, w_o	M	0.5617
Slope	$^\circ$	W_s, q	M	
Fuel depth	m	d, l_f	M	0.1143
Flame standoff distance	m	d_g	A	
Emissivity, flame		ε_{fl}	A	0.23
ember		ε_b	A	0.23
Density, fuel	kgm^{-3}	ρ_p, ρ_f	A	609
air at 810°C		ρ_g	A	0.3294
Kinematic viscosity (air at 810)		ν_g	A	1.34×10^{-6}
Stefan-Boltzmann constant	$\text{Wm}^{-2}\text{K}^{-4}$	σ	A	5.67×10^{-8}
Surface area/vol.	cm^{-1}	σ	M	22.75
Fuel bed porosity, packing ratio		ϕ, β	M	0.008

¹ M = measured, A = assumed.

APPENDIX 2: Rate of Spread Models

Pagni and Peterson Model (PAGNI)

The dimensionless rate of spread model was formulated as follows by Pagni and Peterson (1973) in English units.

$$\Pi_v = \Pi_C [\Pi_{CT} + \Pi_W \Pi_{CI} + \Pi_W^{0.8} \Pi_{CS}] + \Pi_{RF} + \Pi_{RB} + \Pi_K / (1 + \Pi_M + \Pi_{PYR})$$

where

$$\Pi_V = \rho_f c_{pf} R \phi (T_{ig} - T) / (\varepsilon_f \sigma T_{fl}^4), \text{ flame spread velocity}$$

$$\Pi_c = \rho_g c_{pg} (T_{fl} - T) (gL)^{0.5} / (\varepsilon_f \sigma T_{fl}^4), \text{ convective coefficient}$$

$$\Pi_{CT} = k_c D / (l (gL)^{0.5}), \text{ turbulent diffusion. Assumed } \Pi_{CT} = 0.01 \text{ (Pagni and Peterson, 1973).}$$

$$\Pi_W = U_w / (gL)^{0.5}, \text{ wind velocity}$$

$$\Pi_{CI} = [1 + 2.4 Pr Re_c^{-0.33}]^{-1}, \text{ interior fuel bed convection}$$

$$\Pi_{CS} = 0.2 v_g^{0.2} L^{0.7} / (g^{0.1} l), \text{ surface convection}$$

$$\Pi_{RF} = 0.5 a (L/l) (1 + \sin(\Omega_s + \tan^{-1}(1.4 \Pi_W))), \text{ flame radiation}$$

$$\Pi_{RB} = (\varepsilon_b \sigma T_b^4) / (\varepsilon_f \sigma T_{fl}^4), \text{ ember radiation}$$

$$\Pi_K = kg(T_{fl} - T_{ig}) / (\delta_g \varepsilon_f \sigma T_{fl}^4), \text{ gas conduction. Assumed } \Pi_K = 1.5 \text{ (Pagni and Peterson 1973).}$$

$$\Pi_M = h_{vap} M_w / (c_{pf}(T_{ig} - T)), \text{ evaporation energy}$$

$$\Pi_{PYR} = h_{pyr} M_{pf} / (c_{pf}(T_{ig} - T)), \text{ pyrolysis energy}$$

$$D = k_g / \rho_g c_{pg}, \text{ gas mass diffusivity}$$

Original Rothermel Model (ORTH) (Rothermel 1972)

The equations listed below reflect changes by Albini (1976) and the metric formulation by Wilson (1980).

$$R = I_R \delta (1 + \phi_w + \phi_s) / (\rho_b \varepsilon Q_{ig})$$

where

$$I_R = \Gamma' w_n h \eta_m \eta_s, \text{ reaction intensity}$$

$$\xi = (192 + 7.9095 \sigma_v)^{-1} \exp \{ (0.792 + 3.7597 \sigma_v 0.5)(\beta + 0.1) \}, \text{ propagating flux ratio}$$

$$\phi_w = C(3.281 U)^B (\beta / \beta_{op})^{-E}, \text{ wind factor}$$

$$\phi_s = 5.275 (\beta^{-0.3} (\tan \Omega_s)^2), \text{ slope factor}$$

$$\rho_b = w_n / \delta, \text{ oven-dry bulk density}$$

$$\varepsilon = \exp(-4.528 / \sigma_v), \text{ effective heating number}$$

$$Q_{ig} = 581 + 2594(M_w), \text{ heat of preignition}$$

$$\Gamma' = \{(\beta / (\beta_{op}) \exp(1 - \beta / \beta_{op}))\}^A / (0.0591 + 2.9266 \sigma_v^{-1.5}), \text{ optimum reaction velocity}$$

$$\eta_M = 1 - 2.59(M_w/M_x) + 5.11(M_w/M_x)^2 - 3.52(M_w/M_x)^3, \text{ moisture damping coefficient}$$

$$\eta_s = 0.174 s_e^{-0.19} \text{ mineral damping coefficient}$$

$$\beta_{op} = 0.20395 \sigma_v^{-0.8189} \text{ optimum packing ratio}$$

$$A = 8.9033 \sigma_v^{-0.7913}$$

$$B = 0.15988 \sigma_v^{0.54}$$

$$C = 7.47 \exp(-0.8711 \sigma_v^{0.55})$$

$$E = 0.715 \exp(-0.01094 \sigma_v)$$

$$\beta = \rho_b / \rho_f$$

Modified Rothermel Model (MRTH) (Wilson 1990)

$$R = (\xi h_v w_o \Gamma \eta_m) (1 + \phi_w + \phi_s) / (\rho_b \varepsilon (Q_f + m_f Q_w))$$

where

$$\xi = 1 - \exp(-0.17 \sigma_v \beta), \text{ propagating flux ratio}$$

$$\phi_w = C(3.281 U)^B (\beta / \beta_{op})^{-E}, \text{ wind factor}$$

$$\phi_s = 5.275 \beta^{-0.3} (\tan \Omega_s)^2, \text{ slope factor}$$

$$\rho_b = w_n / \delta, \text{ oven-dry bulk density}$$

$$\varepsilon = \exp(-4.528 / \sigma_v), \text{ effective heating number}$$

$$Q_w = 4.18(100 - T + 540), \text{ heat of vaporization}$$

$$\Gamma = 0.34 \sigma_v (\sigma_v \beta \delta)^{-0.5} \exp(-\sigma_v \beta / 3) P_f(n_x), \text{ reaction velocity}$$

$$P_f(n_x) = \ln(\sigma_v \beta \delta h_v / Q_w) / (m_f + Q_f / Q_w), \text{ extinction index}$$

$$\eta_m = \exp(-m_f / m_c), \text{ moisture damping coefficient}$$

$$\beta_{op} = 0.20395 \sigma_v^{-0.8189}, \text{ optimum packing ratio}$$

$$A = 8.9033 \sigma_v^{-0.7913}$$

$$B = 0.15988 \sigma_v^{0.54}$$

$$C = 7.47 \exp(-0.8711 \sigma_v^{0.55})$$

$$E = 0.715 \exp(-0.01094 \sigma_v)$$

$$\beta = \rho_b / \rho_f$$

McArthur (MCART) Model

The McArthur fire danger meters were formulated for grassland and forest fuels in Australia (McArthur 1966, 1967). Noble et al. (1980) developed the equations below from the meters. Only the later version of the meter (Mark 5) is presented here.

$$R_q = 0.0012Fw_o \exp(0.0690)$$

where

$$F = 3.35w_o \exp(-0.0897M_w + 0.0403I) \text{ for } M_w < 18.8\%, \text{ fire danger index}$$

$$F = 0.299w_o \exp(-1.686 + 0.0403I)(30 - M_w) \text{ for } 18.8\% \leq MW < 30\%$$

Canadian Fire Behavior Prediction (FBP) Model

The FBP model is empirically derived from a large database of field scale fires. The initial spread index (RSI) is modified for slope by calculating an "effective wind speed" for a given slope and then combining this wind speed with actual wind speed (FCFDG 1992). For fuel type C-7, $a = 45$, $b = 0.0305$, $c = 2.0$.

$$RSI = a[1 - \exp(-b * ISI)]^c$$

where

$$ISI = 0.208e^{0.05039U} \{ 91.9e^{-0.1386M_w} [1 + M_w^{5.31} (4.93 * 10^{-7})] \}$$

a, b, c = fuel type specific parameters.