PREDICTING THE PAST: A SIMPLE REVERSE STAND TABLE PROJECTION METHOD

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Abstract—A stand table gives number of trees in each diameter class. Future stand tables can be predicted from current stand tables using a stand table projection method. In the simplest form of this method, a future stand table can be expressed as the product of a matrix of transitional proportions (based on diameter growth rates) and a vector of the current stand table. There are cases where the reverse information is needed, i.e. predicting the past instead of the future. Examples of these scenarios include estimating timber damages and retroactively establishing the tax basis of timber that was earlier inherited or purchased. This study focused on procedures used to predict past stand tables from current stand tables and past diameter growth rates. The reverse stand table projection method can be an effective approach to predict the past when not much information is available. Its main drawback is that it has low tolerance for poor estimates of past diameter growth rates, which can result in prediction of negative numbers of trees for some diameter classes.

INTRODUCTION

Forest managers often have to project a stand into the future so that they can evaluate various management alternatives. In some cases, however, there is a need to project a stand backward in time, i.e. to predict the past instead of the future. Examples of these scenarios include estimating timber damages and retroactively establishing the tax basis of timber that was earlier inherited or purchased. In the former scenario, the stand is projected backward to a point in time before the damage and then grown forward to the present using “regular” diameter growth rates obtained by sampling nearby unaffected stands. The difference between observed and predicted current volumes in this case is an estimate of the timber damage that the stand sustained.

Stand tables give number of trees for each diameter class. Although complicated stand table projection algorithms have been developed (Cao and Baldwin 1999a, 1999b; Nepal and Somers 1992; Pienaar and Harrison 1988), the simplest form of stand table projection requires only a stand table and information on diameter growth rates (which can be obtained from increment cores sampled throughout the stand). The objective of this study was to develop procedures for predicting past stand tables from current stand tables and past diameter growth rates.

STAND TABLE PROJECTION

Table 1 shows an example of applying the simple stand table projection method to a hypothetical forest stand. The growth-index ratio (Avery and Burkhart 2002) or movement ratio (Husch and others 2003), which is defined as the ratio of diameter growth and diameter class interval, controls the movement of trees during the growth period. For example, trees in the 6 inch class grew an average of 2.4 inches, resulting in a growth-index of 1.2 (table 1). Therefore, 20 percent of these trees moved up 2 diameter classes, whereas 80 percent of them moved up 1 class. The growth-index ratio of the 10 inch class was 0.9, denoting that 90 percent of trees in this diameter class moved up 1 class, and the rest stayed in that class. Current number of trees in each diameter class is obtained by summing up values along the path indicated by the arrows (Husch and others 2003).

Results from table 1 can be obtained via matrix manipulations. The growth-index ratios from table 1 are used to form matrix A [equation (1)], which is a 5x4 matrix of transitional proportions. The first column of A shows what happened to trees that were in the 6 inch diameter class: no trees remained in the 6 inch class, 80 percent moved up to the 8 inch class, 20 percent moved up to the 10 inch class, and no trees moved up to the 12 inch and 14 inch classes. Likewise,

Table 1—Simple stand table projection of a hypothetical stand

DBH class	10-yr DBH growth	Growth-index ratio	Past stand table	Current stand table	Number trees moving up: No change	1 class	2 classes
- - - - - *inches* - - - - -			- - - *number* - - -				
6	2.4	1.2	313	0	0	250	63
8	2.2	1.1	229	250	0	206	23
10	1.8	0.9	134	282	13	121	0
12	1.6	0.8	70	158	14	56	0
14				56	0		

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the next three columns deal with trees originally from the 8, 10, and 12 inch classes, respectively.

$$A = \begin{array}{cc} & \textit{From} \\ & \begin{matrix} 6'' & 8'' & 10'' & 12'' \end{matrix} \\ \textit{To}\ \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \\ 14'' \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0.2 & 0.9 & 0.1 & 0 \\ 0 & 0.1 & 0.9 & 0.2 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \end{array} \quad (1)$$

If $\underline{x}$ = a 4 x 1 column vector of past number of trees and $\underline{y}$ = a 5 x 1 column vector of current number of trees, then A, $\underline{x}$, and $\underline{y}$ are related as follows:

$$A\underline{x} = \underline{y} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0.2 & 0.9 & 0.1 & 0 \\ 0 & 0.1 & 0.9 & 0.2 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \begin{bmatrix} 313 \\ 229 \\ 134 \\ 70 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \end{matrix} = \begin{bmatrix} 0 \\ 250 \\ 282 \\ 158 \\ 56 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \\ 14'' \end{matrix} \quad (2)$$

REVERSE STAND TABLE PROJECTION

Principle

In the reverse stand table projection problem, the objective is to find the vector of the past stand table ($\underline{x}$) given the current stand table ($\underline{y}$) and the matrix of transitional proportions (A). Assuming that the stand progressed through time following (2), then the vector $\underline{x}$ is solved as follows:

$$A^T A\, \underline{x} = A^T \underline{y} \quad (3)$$

$$\underline{x} = (A^T A)^{-1} A^T \underline{y} \quad (4)$$

or

$$\underline{x} = \left(\begin{bmatrix} 0 & 0.8 & 0.2 & 0 & 0 \\ 0 & 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0.9 & 0 \\ 0 & 0 & 0 & 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0.2 & 0.9 & 0.1 & 0 \\ 0 & 0.1 & 0.9 & 0.2 \\ 0 & 0 & 0 & 0.8 \end{bmatrix} \right)^{-1} \quad (5)$$

$$\begin{bmatrix} 0 & 0.8 & 0.2 & 0 & 0 \\ 0 & 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0.9 & 0 \\ 0 & 0 & 0 & 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0 \\ 250 \\ 282 \\ 158 \\ 56 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \\ 14'' \end{matrix} = \begin{bmatrix} 313 \\ 229 \\ 134 \\ 70 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \end{matrix}$$

Equation (5) demonstrates that the past stand table can be obtained from the current stand table. The problem is that the matrix of transitional proportions, A, is derived from the true average diameter growth rates of the stand (population) and is therefore unknown. It is necessary to obtain a sample to estimate the average diameter growth rate for each diameter class. These growth rates are used to produce growth-index ratios and consequently matrix B, which is an estimate of A. As an example, the following matrix of transitional proportions was derived from sampled diameter growth rates:

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.95 & 0 & 0 & 0 \\ 0.05 & 0.85 & 0.05 & 0 \\ 0 & 0.15 & 0.95 & 0.15 \\ 0 & 0 & 0 & 0.85 \end{bmatrix} \quad (6)$$

Equation (4) becomes

$$\hat{\underline{x}} = (B^T B)^{-1} B^T \underline{y} \quad (7)$$

or

$$\hat{\underline{x}} = \left(\begin{bmatrix} 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 0.85 & 0.15 & 0 \\ 0 & 0 & 0.05 & 0.95 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.95 & 0 & 0 & 0 \\ 0.05 & 0.85 & 0.05 & 0 \\ 0 & 0.15 & 0.95 & 0.15 \\ 0 & 0 & 0 & 0.85 \end{bmatrix} \right)^{-1} \quad (8)$$

$$\begin{bmatrix} 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 0.85 & 0.15 & 0 \\ 0 & 0 & 0.05 & 0.95 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \end{bmatrix} \begin{bmatrix} 0 \\ 250 \\ 282 \\ 158 \\ 56 \end{bmatrix} = \begin{bmatrix} 263 \\ 310 \\ 107 \\ 66 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \end{matrix}$$

where

$\hat{\underline{x}}$ contains predicted values of the past stand table. The magnitude of the difference between the observed value of $\underline{x}$ (equation 5) and the predicted value (equation 8) depends on how well the sample-based transitional matrix (B) estimates the true transitional matrix (A).

To make sure that the result from equation (8) is correct, we need to project $\hat{\underline{x}}$ to the present:

$$\hat{\underline{y}} = B\hat{\underline{x}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.95 & 0 & 0 & 0 \\ 0.05 & 0.85 & 0.05 & 0 \\ 0 & 0.15 & 0.95 & 0.15 \\ 0 & 0 & 0 & 0.85 \end{bmatrix} \begin{bmatrix} 263 \\ 310 \\ 107 \\ 66 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \end{matrix} = \begin{bmatrix} 0 \\ 250 \\ 282 \\ 158 \\ 56 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \\ 14'' \end{matrix} \quad (9)$$

where

$\hat{\underline{y}}$ = the current stand table predicted from $\hat{\underline{x}}$. Both equations (2) and (9) yield identical values for the current stand table.

Other Cases

Some values of the growth-index ratios ($\underline{m}$) may require fine tunings to match observed and predicted current stand tables. We will next consider two of these scenarios.

Case 1: $\underline{m}$ = [2.2   2.15   2.2   2.1]

Equation (10) shows the projection from past to current stand table:

$$A\underline{x} = \underline{y} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0 \\ 0.2 & 0.85 & 0 & 0 \\ 0 & 0.15 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.9 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} 313 \\ 229 \\ 134 \\ 70 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \end{matrix} = \begin{bmatrix} 0 \\ 250 \\ 257 \\ 142 \\ 90 \\ 7 \end{bmatrix} \begin{matrix} 6'' \\ 8'' \\ 10'' \\ 12'' \\ 14'' \\ 16'' \end{matrix} \quad (10)$$

Suppose the following matrix B is the estimate of the transitional matrix (A).

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.75 & 0 & 0 & 0 \\ 0.25 & 0.65 & 0 & 0 \\ 0 & 0.35 & 0.9 & 0 \\ 0 & 0 & 0.1 & 0.8 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \quad (11)$$

The past stand table ($\hat{\underline{x}}$) is predicted from

$$\hat{\underline{x}} = (B^T B)^{-1} B^T \underline{y} = \begin{array}{l} \begin{bmatrix} 333 \\ 267 \\ 54 \\ 102 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} \end{array} \tag{12}$$

The current stand table is then projected from $\hat{\underline{x}}$ and B.

$$B\hat{\underline{x}} = \hat{\underline{y}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.75 & 0 & 0 & 0 \\ 0.25 & 0.65 & 0 & 0 \\ 0 & 0.35 & 0.9 & 0 \\ 0 & 0 & 0.1 & 0.8 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 333 \\ 267 \\ 54 \\ 102 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} = \begin{bmatrix} 0 \\ 250 \\ 257 \\ 142 \\ 87 \\ 20 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \\ 14" \\ 16" \end{array} \tag{13}$$

Note that the result from (13) matches the observed current stand table from (10) except for the largest two diameter classes. The growth-index ratio for the largest diameter class needs to be changed to fix this problem, using the trial-and-error method. This leads to new matrices *B* and *C*, and new solution $\hat{\underline{x}}$. Projection of $\hat{\underline{x}}$ is carried out again as follows:

$$B\hat{\underline{x}} = \hat{\underline{y}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.75 & 0 & 0 & 0 \\ 0.25 & 0.65 & 0 & 0 \\ 0 & 0.35 & 0.9 & 0 \\ 0 & 0 & 0.1 & 0.92 \\ 0 & 0 & 0 & 0.08 \end{bmatrix} \begin{bmatrix} 333 \\ 267 \\ 54 \\ 92 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} = \begin{bmatrix} 0 \\ 250 \\ 257 \\ 142 \\ 90 \\ 7 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \\ 14" \\ 16" \end{array} \tag{14}$$

Now the predicted current stand table matches the observed stand table (10) perfectly.

Case 2: $\underline{m}$ = [0.9   1.1   1.15   1.2]

The current stand table ($\underline{y}$) is projected from the past stand table ($\underline{x}$) and the matrix of transitional proportions (*A*) as follows.

$$A\underline{x} = \underline{y} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0.1 & 0.85 & 0 \\ 0 & 0 & 0.15 & 0.8 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 313 \\ 229 \\ 134 \\ 70 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} = \begin{bmatrix} 31 \\ 282 \\ 206 \\ 137 \\ 76 \\ 14 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \\ 14" \\ 16" \end{array} \tag{15}$$

Let *B* = $\begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 \\ 0 & 0.15 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.75 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$ be an estimate of the transitional matrix (*A*). The past stand table is predicted from *B* and the current stand table ($\underline{y}$):

$$\hat{\underline{x}} = (B^T B)^{-1} B^T \underline{y} = \begin{bmatrix} 298 \\ 242 \\ 126 \\ 67 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} \tag{16}$$

To double check this solution, the current stand table is then predicted from $\hat{\underline{x}}$ and *B*.

$$B\hat{\underline{x}} = \hat{\underline{y}} = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 \\ 0 & 0.15 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.75 \\ 0 & 0 & 0 & 0.25 \end{bmatrix} \begin{bmatrix} 298 \\ 242 \\ 126 \\ 67 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} = \begin{bmatrix} 15 \\ 283 \\ 206 \\ 137 \\ 75 \\ 17 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \\ 14" \\ 16" \end{array} \tag{17}$$

The result from (17) matches the observed current stand table from (15), except for the first two and last two values. The adjustment of the growth-index ratios is carried out in two steps. First, the growth-index ratio for the smallest diameter class is changed using the trial-and-error method. This leads to a new matrix *B* and a new solution $\hat{\underline{x}}$. Projection of $\hat{\underline{x}}$ is carried out again as follows:

$$B\hat{\underline{x}} = \underline{y} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 \\ 0 & 0.15 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.75 \\ 0 & 0 & 0 & 0.25 \end{bmatrix} \begin{bmatrix} 313 \\ 242 \\ 126 \\ 67 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} = \begin{bmatrix} 31 \\ 282 \\ 206 \\ 137 \\ 75 \\ 17 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \\ 14" \\ 16" \end{array} \tag{18}$$

Next, the growth-index ratio for the largest diameter class is adjusted, resulting in a different matrix *B* and its corresponding solution $\hat{\underline{x}}$.

$$B\hat{\underline{x}} = \hat{\underline{y}} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 \\ 0 & 0.15 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.78 \\ 0 & 0 & 0 & 0.22 \end{bmatrix} \begin{bmatrix} 313 \\ 242 \\ 126 \\ 65 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \end{array} = \begin{bmatrix} 31 \\ 282 \\ 206 \\ 137 \\ 76 \\ 14 \end{bmatrix} \begin{array}{l} 6" \\ 8" \\ 10" \\ 12" \\ 14" \\ 16" \end{array} \tag{19}$$

After these two adjustments, the predicted current stand table finally matches the observed stand table (15).

DISCUSSION

In this paper, a simple stand table projection method is shown to be equivalent to the result of multiplying a matrix of transitional proportions (which is based on growth-index ratios) and a vector of past stand table. This system allows the reverse calculation of the past stand table from the current stand table. The reverse stand table projection procedure also adheres to the same assumptions imposed upon the simple stand table projection method as follows:

1. The stand did not lose trees due to mortality. If considerable mortality is suspected, an estimate of mortality for each diameter class should be added to the past stand table predicted from the reverse projection procedure.
2. No ingrowth information is available. The amount of ingrowth, if available, should be deducted from the current stand table before proceeding with the reverse projection procedure.
3. Trees in each diameter class follow a uniform distribution.
4. All trees in each diameter class grew in diameter at the same rate.

5. Estimates of diameter growth rates are reasonably good. This assumption is especially important for the reverse stand table projection method, which is not a robust method. Deviation from the true diameter growth rates translates to an inaccurate matrix of transition proportions and might lead to negative numbers of trees in some diameter classes. Consider the example described in equations (6) and (8). The following estimated matrix of transitional proportions (B),

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.95 & 0 & 0 & 0 \\ 0.05 & 0.6 & 0.05 & 0 \\ 0 & 0.4 & 0.95 & 0.15 \\ 0 & 0 & 0 & 0.85 \end{bmatrix} \tag{20}$$

predicted a past stand table that contains a negative number of trees:

$$\underline{\hat{x}} = (B^T B)^{-1} B^T \underline{y} = \begin{array}{l} \begin{bmatrix} 263 \\ 451 \\ -34 \\ 66 \end{bmatrix} \end{array} \begin{array}{l} 6'' \\ 8'' \\ 10'' \\ 12'' \end{array} \tag{21}$$

Note that the elements of B in (20) are identical to those in (6), except for the second column (containing the growth-index ratio for the 8 inch class). Changing the 8 inch growth-index ratio from 1.15 to 1.4 while keeping the other ratios the same is enough to produce a negative number of trees in the 8 inch class. This example demonstrates that the result of the reverse stand table projection can be extremely sensitive to the estimates of the growth-index ratios.

In summary, the reverse stand table projection procedure can be an effective method to predict the past when not much information is available. Its main drawback is that it has low tolerance for poor estimates of past diameter growth rates, which can result in negative numbers of trees for some diameter classes.

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