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A PROBABILITY APPROACH TO SAWTIMBER TREE-VALUE PROJECTIONS

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ABSTRACT

The authors present a method for projecting hardwood sawtimber tree values, using tree-development probabilities based on continuous forest inventory (CFI) data and describe some ways to use the resulting value projections to assemble management-planning information.

TIMBER VALUATION is an important part of a forestry business. A timber owner who knows what his trees are worth right now and has a pretty good idea of what they could be worth at the end of some time period can plan his forest management during that period to best meet his financial objectives.

Assigning current values to trees or stands is usually relatively easy. Projecting tree values is a lot harder, especially for hardwood sawtimber, because each sawtimber tree's value is related to its size and quality. And the tree's projected value depends on how it changes in both size and quality.

For any given tree, a wide range of possibilities exists, in both size and quality change and resultant tree value. Probability projection will allow us to cover the entire range of possible tree changes that might be expected, rather than just the average changes. And it will allow us to estimate the likelihood of attaining projected values.

We describe here a method for projecting sawtimber tree values, using tree-development probabilities based on continuous forest inventory (CFI) data for a specific forest. We also describe some ways to use the resulting value projections to assemble management-planning information for a forest. Our aim is to project future values for existing sawtimber trees, and not to project stand tables or yield tables. Thus we will simplify the description of our method by dismissing ingrowth and mortality.

Neither the projections nor their applications are intended to be general guides. Our purpose is to give forest owners a method that they can use to develop comparable management information for their own tracts.

PROBABILITY PRINCIPLES

We can observe an occurrence a large number of times and find the relative frequency with which the various outcomes occur. For example, suppose we measured the diameters of 200 16-inch black oak trees on a series of randomly selected sample plots, then remeasured the same trees 5 years later. Suppose, further, that we found that 120 trees grew 0.5 inch during the 5-year period, 60 trees grew 0.8 inch, and 20 grew 1.0 inch. The relative frequencies of the various growth levels would be:

Diameter change (inch)	*Frequency (number of trees)*	*Relative frequency of occurrence*
0.5	120	120/200 = .60
.8	60	60/200 = .30
1.0	20	20/200 = .10
	200	1.00

We could use these relative frequencies to estimate growth probabilities for the rest of the 16-inch black oaks in our forest.

We could use the same approach to estimate the probabilities for changes in merchantable height and tree quality. Then, by combining the probabilities for all three variables, we could estimate the probabilities of trees developing in a combination of ways.

OBSERVED PROBABILITIES

The data used in this study were obtained from more than 400 randomly selected plots, maintained by the Muskingum Watershed Conservancy District in Ohio. The plots were established in 1955 and remeasured twice—in 1960 and in 1965. The data used came from the 1955 and 1965 remeasurements. We chose the 10-year period because 10 years is commonly used in management planning. It allows for annual fluctuations in precipitation and length of growing season. Thus the changes in diameter, merchantable height, and tree quality are based on 10 years of tree development.

Diameter Changes

Diameters breast high (d.b.h.) had been measured to the nearest 1/10 inch with a diameter tape. The actual diameter changes were found by subtracting a tree's 1955 diameter from its 1965 diameter. But, for simplicity in illustrating our method, we combined the diameter changes into 1-inch classes.

Finding a lot of variation in growth rates from one species to another, we grouped the species into fast, medium, and slow growers. The fast growers included yellow-poplar, red oak, black oak, and soft maple. Medium growers included white ash, hard maple, and American elm. Slow growers included white oak, beech, black cherry, and hickory.

We then sorted the trees in each group into 2-inch diameter classes and, in each diameter class, found the percentage of trees changing by 1-inch growth classes. These percentages, or relative frequencies, indicate the probabilities of attaining given levels of 10-year diameter growth (table 1). For example, we can expect 46 percent of the 14-inch fast growers on the forest to grow 2 inches in 10 years.

We ignored the possible implications of

Table 1.—*Probabilities of trees attaining given levels of diameter growth, by diameter class*

[In percent]

Diameter class (inches)	Number of trees	10-year diameter growth in inches: <1	1	2	3	3+
FAST GROWING TREES *Yellow-poplar, red oak, black oak, and soft maple*						
12	42	5	31	35	22	7
14	26	4	27	46	12	11
16	22	4	19	27	32	18
18	14	—	7	57	36	—
20+	30	7	10	57	23	3
MEDIUM GROWING TREES *White ash, hard maple, and American elm*						
12	38	3	34	42	16	5
14	22	9	36	37	13	5
16	17	12	35	29	18	6
18	5	—	80	20	—	—
20+	6	—	—	66	17	17
SLOW GROWING TREES *White oak, beech, black cherry, and hickory*						
12	92	17	42	34	7	—
14	54	15	40	30	13	2
16	47	6	43	45	6	—
18	12	17	25	50	8	—
20+	26	4	38	43	15	—

stocking level or stand density on individual tree diameter growth. For simplicity, we have assumed constant stand competition. However, the effect of changing stand density should be considered in practical applications.

Merchantable-Height Changes

In each of the CFI tallies every sawtimber tree was assigned a merchantable height for sawlogs. These were estimated to the nearest 2 feet, and the estimates were spot-checked with a clinometer.

(A sawlog is a log that is at least 8 feet long, has a minimum top diameter of 8 inches inside bark, and meets minimum timber-grading specifications for hardwood factory-lumber logs or construction logs, as defined in USDA Forest Products Laboratory studies.)

We found the 10-year height change for each tree. But, because the height changes were not significantly different from one species to another, we did not stratify the trees into species groups. Instead, we sorted all the trees into four height-change classes: those that did not change in height; those that increased less than a half log; those that increased at least a half log but less than one 16-foot log; and those that increased by one or more 16-foot logs.

The relative frequencies of height changes (table 2) show that more than half of the trees failed to increase in height at all and that we can expect less than 10 percent of them to add a log or more in 10 years (less than 1 percent increased a log and half).

Table 2.—*Probabilities of changes in sawlog height, by diameter class*

[In percent]

Diameter class (inches)	Number of trees	10-year change in sawlog height (16-ft. logs)			
		No change	Less than ½ log	½ log to 1 log	1 log or more
12	172	52	27	17	4
14	102	66	12	17	5
16	86	57	19	17	7
18	31	70	18	9	3
20+	62	66	20	11	3

Tree-Quality Changes

Five quality classes were tallied for sawtimber trees on the plots: log grades 1, 2, and 3, tie and structural grade, and cull. These classes were based primarily on standards developed by the USDA Forest Service (*Ostrander 1965*). They were assigned according to butt-log quality. Trees less than 50 percent sound were classed as cull.

We used a slightly different method for estimating probabilities of changes in tree quality. First we sorted the trees in each diameter class into initial (1955) quality classes. Next we sorted the trees in each initial quality class into ending (1965) quality classes. Then we found the proportion of the initial class that the trees in each ending class represented (table 3).

Table 3.—*Calculating the probability of quality changes in 14-inch grade-3 trees*

Tree-quality class		Number of trees	Percent
Initial	Ending		
Grade 3	Cull	1	3
	Tie & structural (T&S)	1	3
	Grade 3	25	65
	Grade 2	8	21
	Grade 1	3	8
	Total	38	100

Probability trees provide a convenient way to display tree-quality changes (fig. 1 and fig. 2). These show, for each diameter class, the number of trees by initial quality class, and the percentage of those trees in each ending quality class.

As we expected, few trees improved in quality during the 10-year period. Quality improvement is a relatively slow process. On the other hand, we did find some trees that dropped in quality class, because of weather damage or increased defect. Such changes, though infrequent, usually occur quickly.

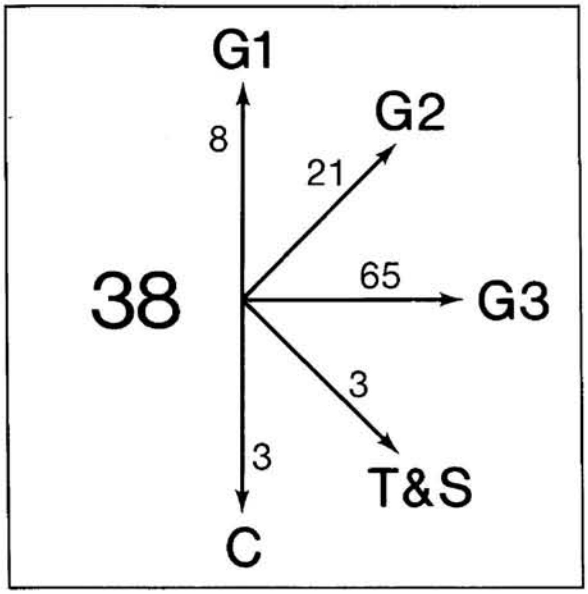

Figure 1.—A sample probability tree, for quality changes in 14-inch grade-3 trees.

The large figure at the left shows number of trees. The figures on the arrows show the percentage of trees moving into the final quality classes.

Probability trees for all five grades and diameter classes are shown in Figure 2.

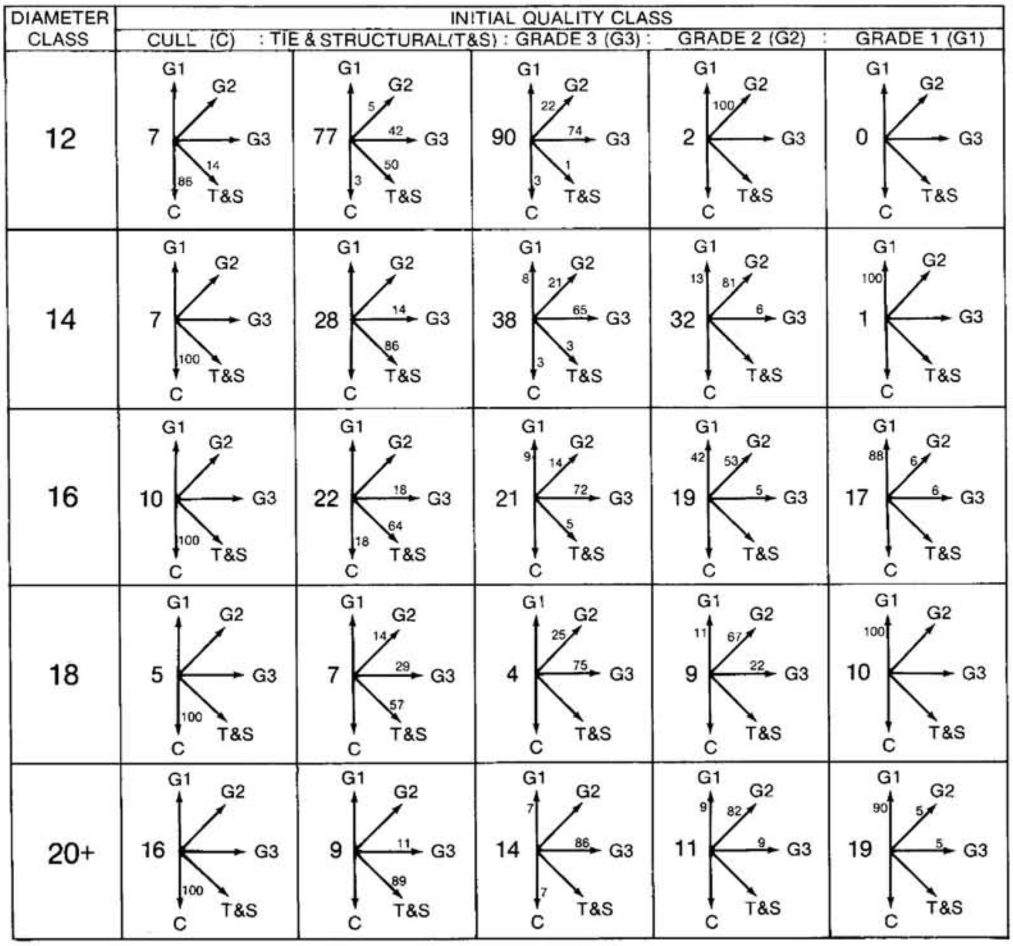

Figure 2.—Probabilities of 10-year quality changes, for five diameter classes and five quality classes.

COMBINED PROBABILITIES

We can combine the probabilities of changes in diameter, merchantable height, and tree quality by multiplying them. In doing so we would be treating the changes as independent occurrences. (Chi-square tests failed to demonstrate any real association between diameter growth and merchantable height growth, except for 12-inch trees. Similar tests failed to demonstrate any real association between diameter growth and tree quality changes, regardless of tree size.) These combined probabilities indicate the likelihood with which a given tree can develop in a combination of ways.

Take an 18-inch 2-log red oak with a butt-log grade of 2, for example. What is the probability of its growing 1, 2, or 3 inches in diameter? Of increasing in sawlog height by another 16-foot log? Or of changing in quality? With what probabilities can we expect various combinations of these changes to occur?

Table 4 has been prepared to show how value projections can be made for an 18-inch, 2-log, grade-2 red oak. Columns 4, 5, and 6 show the results of the changes—all the possible combinations of resultant trees. The probabilities of these changes are shown in columns 7, 8, and 9.

The combined probabilities for each set of possible changes—or for the tree that would result from those changes—are shown in column 10. The probability of each combination is the product of the probabilities of changes in diameter, height, and quality that would result in that combination. For example, the probability of our 18-inch tree growing 2 inches is 0.57; the probability of its merchantable height remaining at two logs is 0.97; the probability of its staying grade 2 is 0.67. Thus the combined probability of our 18-inch, 2-log, grade-2 tree becoming a 20-inch, 2-log, grade-2 tree is 0.57 x 0.97 x 0.67 or 0.370.

We should mention at this point that our method is a transition matrix approach to tree-development estimation. Many excellent texts discuss transition probability theory (*Miller and Starr 1960; Howard 1960*). But, again for simplicity, we chose to avoid direct use of matrix manipulations and mathematical notations.

However, we must point out that in theory a matrix of transition probabilities is assumed to remain constant through time. In reality, transition probabilities might not remain constant. They might change as surrounding conditions in the real world change (*Burford 1966*). This is likely to be the case with most forest land, as the stands progress toward

Table 4.—Value projections for an 18-inch, 2-log, grade-2 red oak

Probability table											Evaluation table						
Initial tree variables			Ending tree variables			Probability of attaining ending tree variables			Probability of attaining all variables,	Trees w/ending characteristics,	Initial value of trees ($)		Ending value of trees ($)		Value change ($)		Rate of value increase per tree,
D.b.h., *in.*	Ht., *ft.*	Quality	D.b.h., *in.*	Ht., *ft.*	Quality	D.b.h., *pct.*	Ht., *pct.*	Quality	*pct.*	*No.*	Per tree	Total	Per tree	Total	Per tree	Total	*pct.*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
18.0	32	2	19.0	32	3	0.07	0.97	0.22	0.015	6	2.82	16.92	–2.35	–14.10	–5.17	–31.02	0
					2	.07	.97	.67	.045	18		50.76	4.21	75.78	1.39	25.02	4.1
					1	.07	.97	.11	.007	3		8.46	10.22	30.66	7.40	22.20	13.7
				48	3	.07	.03	.22	.000	0		—	—	—	—	—	—
					2	.07	.03	.67	.001	0		—	—	—	—	—	—
					1	.07	.03	.11	.000	0		—	—	—	—	—	—
			20.0	32	3	.57	.97	.22	.122	49		138.18	–1.86	–91.14	–4.68	–229.32	0
					2	.57	.97	.67	.370	148		417.36	5.89	871.72	3.07	454.36	7.6
					1	.57	.97	.11	.061	24		67.68	13.13	315.12	10.31	247.44	16.6
				48	3	.57	.03	.22	.004	2		5.64	–3.41	–6.82	–6.23	–12.46	0
					2	.57	.03	.67	.012	5		14.10	5.87	29.35	3.05	15.25	7.6
					1	.57	.03	.11	.002	1		2.82	14.00	14.00	11.18	11.18	17.4
			21.0	32	3	.36	.97	.22	.077	31		87.42	–1.21	–37.51	–4.03	–124.93	0
					2	.36	.97	.67	.234	94		265.08	7.84	736.96	5.02	471.88	10.8
					1	.36	.97	.11	.038	15		42.30	16.41	246.15	13.59	203.85	19.2
				48	3	.36	.03	.22	.002	1		2.82	–2.61	–2.61	–5.43	–5.43	0
					2	.36	.03	.67	.007	3		8.46	8.26	24.78	5.44	16.32	11.3
					1	.36	.03	.11	.001	0		—	—	—	—	—	—
												1,128.00		2,192.34		1,064.34	6.9

maturity. So we must caution that our method for projecting sawtimber tree values should not be used for long-range projections. Rather, it should be used for more immediate planning periods, such as we have done for a 10-year management period in the following discussion.

APPLYING COMBINED PROBABILITIES

Projected Future Trees

From the combined probabilities, we can estimate the likelihood of each of the ways we might expect our initial tree to develop. We can also use them to project the future trees that would result from changes.

Say our sample has an expansion factor of 400, and our initial tree represents 400 similar trees on the forest property. We can proportion those 400 trees according to our combined probabilities and estimate the future number of trees with each combination of ending characteristics (table 4, column 11). This gives us a future tree distribution based on expected changes in the 400 trees represented by our initial tree.

Projected-Value Tables

We can now assign values to our initial and future trees. The tree values used here represent a tree's net worth when converted to 4/4 inch lumber, based on average lumber prices for 1962-67 and including conversion costs (*Trimble and Mendel 1969*). The original 400 trees are valued at $2.82 each (table 4, column 12). But the future tree values vary considerably, from −$3.41 to $16.41. The average is $5.31 (table 4, column 14). The trees with negative values are those that would cost more to convert into lumber than the lumber would be worth.

To compute initial and future values for our 400 trees, we need only to multiply individual tree values (column 12 and 14) by the future tree distribution (column 11). The results of these computations (columns 13 and 15) show that we can expect the original trees to nearly double in value in 10 years.

The changes in value (columns 16 and 17) are simply the differences between future and initial values. Note the wide range in value changes for individual trees. This emphasizes the variation in tree earning power and reflects the many different returns that are possible from trees developing in different ways. The total value change (column 17) indicates the overall accrual in net value that we can expect from the 18-inch, 2-log, grade-2 red oaks on our forest property.

The rate-of-value-increase figures (column 18) were computed by solving for p in the compound-interest formula

$$V_n = V_o (1 + p)^n$$

where V_n = future tree value, V_o = initial tree value, and n = 10 years. Here, too, we find a big spread resulting from the various tree-development possibilities.

MANAGEMENT PLANNING INFORMATION

Up to this point we have generated a good deal of biological and economic data—but all for one tree. The thought of repeating the operation for the rest of the trees in your forest may seem appalling. With the widespread availability of computer services, it need not be. Even hand computations, such as we have made, are really not that laborious when considered in terms of the management-planning information they can be used to develop.

Our 18-inch example tree can be used to point out some types of planning information that we can derive after projecting values for our entire stand. First, we would know the present value of our trees and have a pretty good idea of their future value. In this case we could expect our grade-2, 18-inch red oaks to nearly double in value. In doing so they would be increasing in value at an annual rate of 6.9 percent.

After generating comparable data for the rest of our forest we could aggregate them to find the present and future timber value, the expected change in timber value, and the rate of that change for the entire forest. Information of this sort would give the forest owner a set of value measures that he could use to compare his forest's development against his short-term financial objectives. Such information also would provide him with sideboards for deciding on the levels of management he should consider. It also would give him a yardstick for comparing the benefits from the various management alternatives he might be considering.

We could also aggregate the tree-value measures by species or species groups. We could then compare the economic performance of one species against another. Or we could compare species performance against the owner's economic objectives. Such comparisons could serve as a basis for earmarking desirable or undesirable species and could be worked into silvicultural guides.

We could also use projected tree values to analyze tree development. For an example look closer at the value projections for our 18-inch red oak, and observe some of its development patterns.

Diameter Growth

Most of our value is in trees that we expect will grow 2 inches, but most of our value increase will be in those trees that grow 3 inches in diameter:

Diameter change (inches)	*Trees (no.)*	*Initial value*	*Future value*	*Value change*	*Average value change*	*Rate of value increase (percent)*
1	27	$76.14	$92.34	$16.20	$0.60	1.9
2	229	645.78	1,132.23	486.45	2.12	5.8
3	144	406.08	967.77	561.69	3.90	9.1

The average value increase for a tree that grows 3 inches is nearly double the increase of one that grows only 2 inches. The differences in the rates of value increase are equally striking. We see, too, that our 1-inch growers are not adding much to our forest business. But fortunately we have few of them.

Application of this information depends, of course, on the forest owner's economic objectives. Given those objectives, we could use our value projections to develop management policies based on diameter growth expectations. These, in turn, could be related to tree-vigor classifications that reflect a tree's diameter growth potential and could be incorporated into general cutting guides.

If, for example, the forest owner wanted to build up his capital in growing timber, he would generally favor the 2- and 3-inch growers and eliminate the 1-inch growers, giving more growing space to his faster growing, faster earning trees. The same policy would also satisfy an objective of eliminating those trees whose rate of value increase averages less than 2 percent. But only the fastest growing 18-inch red oaks will, on the average, meet an objective of earning at the rate of 8 percent or higher.

Basing value projections on tree-development probabilities tells us more than which kinds of trees we can expect to meet or fail to meet various economic criteria. It tells us how many. This added information is often useful in scheduling cutting budgets for a forest holding.

Thus the forest owner might well be satisfied with incorporating diameter growth expectations into a general cutting-guide policy. However, he might want to build an exception or two into his general rule. After all, some of the slow growers may meet his financial expectations while some of the fast growers may not. We can look for possible exceptions in the height-change and quality-change patterns of our 18-inch red oak.

Height Change

No luck here. Comparing trees that do not change in height with those that do, we see little difference in value increment:

Height change (feet)	*Trees (no.)*	*Initial value*	*Future value*	*Value change*	*Average value change*	*Rate of value increase (percent)*
0	388	$1,094.16	$2,133.64	$1,039.48	$2.68	6.9
16	12	33.84	58.70	24.86	2.07	5.7

Quality and time play an important part. Upper-stem logs that are added by grade-2 trees are usually poor-quality logs. Their conversion costs often exceed their 4/4 lumber value. They take time to develop quality and become more valuable. So, as long as the owner is interested in value changes over only a 10-year period, he might well instruct his timber-markers to avoid concerning themselves with upper-stem development possibilities. There is not much chance of an 18-inch red oak adding another log, anyhow.

Tree-Quality Changes

But, on the other hand, he should instruct his markers to always consider the possibility of quality changes. For, comparing quality measures, we see some marked and important differences:

Change in butt-log grade	*Trees (no.)*	*Initial value*	*Future value*	*Value change*	*Average value change*	*Rate of value increase (percent)*
From 2 to 3	89	$250.98	$-152.18	$-403.16	$-4.53	(*)
No change	268	755.76	1,738.59	982.83	3.67	8.7
From 2 to 1	43	121.26	605.93	484.67	11.27	17.4

*Negative value.

Note the trees that improve in grade. They add, on the average, more than three times the value that unchanging trees add. Their rate of value increase is twice that of trees that do not improve in grade. These are the trees that are most likely to meet or exceed the owner's financial objectives. They include trees that might grow in diameter as little as 1 inch in 10 years. And the forest owner can anticipate 1 out of 11 18-inch, grade-2 red oaks to improve to grade 1 in 10 years.

The question is, which one? Though there is no sure answer, a woods-wise forester can often make a confident estimate based on an individual tree's unique external appearance. The difference in value increment may well be worth the cost involved with making the estimate.

Even the trees that remain grade 2 can be expected to increase in value at a respectable rate—8.7 percent on the average. The forest owner might happily note that he can expect two-thirds of his 18-inch grade-2 trees to do just that.

As for the trees that drop in grade: such changes can rarely be foreseen. But care can be taken in timber-marking to avoid setting up conditions that might prompt such changes: opening up a stand too much, allowing improper spacing, or leaving a quality tree in a large opening. Feathering—epicormic budding or branching—which often occurs under such conditions, can cause degrade. Being aware of the drastic losses in value that accompany degrade, the forest owner is apt to insist on a policy of proper density and spacing for a given site.

The following statements summarize our analysis of the development patterns in the 18-inch grade-2 red oaks in our forest.

1. Those trees with the potential of improving to grade 1 are potentially big earners.

2. Those that we expect to remain grade 2 are likely to increase in value at a rate that the forest owner deems adequate. Such trees make up the bulk of his 18-inch grade-2 red oaks. The faster these grow, the larger their value increment will be.

3. Those trees that drop in grade will become negative in value. That is, it would cost more to convert them into 4/4 lumber than the lumber would be worth.

4. Not many trees of this size should be expected to increase in merchantable height. Those that do usually have smaller value changes than trees whose sawlog height stays the same.

CONCLUSIONS

Using tree-development probabilities to project sawtimber tree values has certain advantages. It allows us to:

- Project tree quality, which is often expressed in terms of discrete qualitative variables such as butt-log grade.
- Estimate the likelihood with which we can expect a tree to change in any one or any combination of ways.
- Make projections for a full range of tree-development possibilities, which does not limit us to average conditions.
- Use the resulting tree values and their probabilities to develop management-planning information pertinent to the timber owner's economic objectives.

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