Random location of fuel treatments in wildland community interfaces: a percolation approach

Michael Bevers, Philip N. Omi, and John Hof

Abstract: We explore the use of spatially correlated random treatments to reduce fuels in landscape patterns that appear somewhat natural while forming fully connected fuelbreaks between wildland forests and developed protection zones. From treatment zone maps partitioned into grids of hexagonal forest cells representing potential treatment sites, we selected cells to be treated at random using an algorithm that provides a varying degree of treatment clustering. One thousand or more such maps were used as sample replicates for parameter settings that included landscape size, fraction of area treated, and degree of clustering to test whether continuous fuelbreaks were formed in an acceptable proportion of cases. A shortest path network optimization model was solved for each sample landscape to determine the presence or absence of a continuous fuelbreak and to measure the length of the most direct fuelbreak when one or more were present. By varying the fraction of area treated in a bisection search, we were able to estimate the minimum amount of treatment needed. Results indicated that between 54% and 88% of a forest would need to be treated to form fuelbreaks in 60% or more of the landscapes we modeled.

Résumé : Nous avons étudié l'utilisation de traitements aléatoires corrélés dans l'espace dans le but de réduire les combustibles dans des structures de paysage qui semblent relativement naturelles tout en constituant des coupe-feux complètement reliés entre des forêts non protégées et des zones développées soumises à un régime de protection. À partir de cartes délimitant les zones de traitement et subdivisées en réseaux de cellules hexagonales de forêt représentant les sites potentiels de traitement, nous avons choisi au hasard les cellules à traiter à l'aide d'un algorithme qui produit différents degrés de regroupement des traitements. Un millier ou plus de ces cartes ont été utilisées comme répétitions pour déterminer les paramètres qui incluaient la taille du paysage, la proportion de la superficie traitée et le degré de regroupement pour vérifier si des coupe-feux continus étaient formés dans une proportion acceptable de cas. Un modèle d'optimisation des réseaux par la méthode du plus court chemin a été résolu pour chacun des échantillons de paysage dans le but de déterminer la présence ou l'absence d'un coupe-feu continu et de mesurer la longueur du coupe-feu le plus direct lorsqu'un ou plusieurs coupe-feux étaient présents. En faisant varier la proportion de la superficie traitée dans une recherche du point milieu, nous avons été capable d'estimer la quantité minimum de traitements requis. Les résultats indiquent qu'entre 54 et 88 % d'une forêt a besoin d'être traitée cour constituer des coupe-feux dans 60 % ou plus des paysages que nous avons modélisés.

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Introduction

Interest is currently high in removing forest fuels to reduce threats from fires within and around wildland interface communities in the western United States. Protective measures are typically aimed at meeting at least three fire risk reduction goals: (1) eliminating hazards immediately around homes and other structures (e.g., Cohen 2000), (2) forming fuelbreaks around or adjacent to communities so that suppression forces can more quickly establish firelines or firebreaks (Green 1977), and (3) altering surface and canopy fuels in the surrounding landscape to limit fireline intensities and rates of spread (Byram 1959), to reduce crown fire likelihood (Van Wagner 1977), and to lessen potential fire severity (Ryan and Noste 1985). At the same time, concerns

on the part of local residents and environmental organizations are stimulating research to ameliorate undesirable impacts of widespread forest vegetation and residue removals by using methods such as shaded fuelbreaks (Agee et al. 2000), systematic partial treatment strategies (Finney 2001), and mimicking the effects of natural disturbance processes (Lindenmayer and McCarthy 2002).

In this study, we explore percolation approaches to form fully connected fuelbreaks from spatially correlated random treatments, meeting objective 2 (above). This use of random treatment patterns allows us to mimic the effects of natural disturbances while addressing the landscape-wide fuel treatments identified in objective 3. To date fuelbreak practices have focused primarily on nearly linear treatments as described in Green (1977). Relatively few studies (e.g., Finney

M. Bevers[1] and J. Hof. Rocky Mountain Research Station, 2150A Centre Avenue, Fort Collins, CO 80526, U.S.A.
P.N. Omi. Western Forest Fire Research Center, Department of Forest Sciences, Colorado State University, Fort Collins, CO 80523, U.S.A.

[1]Corresponding author (e-mail: mbevers@fs.fed.us).

2003) have focused on fuel treatments applied randomly across a landscape, although some have investigated the possibility that fire events over an extensive time period might create breaks in fuel that limit the growth of large fires (e.g., Van Wagtendonk 1995; Minnich and Chou 1997).

By correlated random treatment patterns, we mean that from a treatment area map partitioned into a grid of cells representing potential treatment sites, we select the cells to be treated at random using an algorithm that optionally provides a greater than random degree of treatment clustering. Such model landscapes are often used in landscape ecology simulation studies (e.g., Milne 1992; Palmer 1992; With et al. 1997) to provide similarity among random landscape replicates in the number, size, shape, and arrangement of patches and their boundaries (Gardner and O'Neill 1991). The resulting maps are visually similar to maps of real landscapes (Gardner et al. 1987) and seem potentially useful for designing fuel treatment patterns that appear more natural than systematic treatment patterns. Of course, a greater number of randomly treated grid cells would likely be required to form fuelbreaks than if we arranged treatments more strategically. Whether a sufficient number of random treatments is prudent will depend on the costs and benefits associated with specific fuel reductions, which may be perceived differently from one community to another (Mendez et al. 2003).

Percolation theory, described as the quantitative analysis of how connectivity is formed or disrupted in spatially structured systems (Stauffer and Aharony 1994; With 2002), suggests that a very consistent number of randomly treated grid cells would be required to form a continuous fuelbreak across a given landscape. In percolation parlance, connected cells of the same class (either treated or untreated) form clusters. Forming a fully connected fuelbreak from cells treated randomly across a landscape is a matter of treating a large enough fraction of all cells to form a "percolating" or "spanning" cluster of treatments separating the community from the untreated forest. Likewise, this disrupts the connectivity of untreated cells across the landscape, thereby breaking up percolating clusters of untreated forest that would otherwise form an unobstructed fire path to or from the community. The smallest fraction of a landscape grid that must be treated (p_c) for the probability of forming a percolating cluster (PR_c) to meet a predefined standard (e.g., $PR_c \geq 0.99$) is called a critical or percolation threshold.

Percolation theory tells us that the value of this critical threshold will depend on cell geometry (e.g., triangular, square, or hexagonal cells), on landscape dimensions (e.g., the number of rows and columns in a rectangular grid), and on the degree of positive correlation (clustering) among treatments, in addition to the spanning probability chosen as the standard for success. For example, the probability of a spanning cluster occurring across a very large (essentially infinite) number of randomly treated (uncorrelated) sites square in shape drops abruptly from one to zero when fewer than 59.2746% of them are treated (Stauffer and Aharony 1994, p. 17). These relationships are scale independent in the sense that systematic increases or decreases in the size of uniform cells, thereby also increasing or decreasing the size of the landscape, will not change the critical fraction of cells that must be treated. For realistic landscape sizes, however, the small number of cells can substantially affect critical

fraction values, and Monte Carlo simulations are then often used for estimations (see With 2002). Here, we explore how landscape treatment zone depth (in rows) and length (in columns per row), the degree of treatment clustering, and the prespecified standard for treatment spanning probability affect critical threshold values and resulting fuelbreak lengths for rectangular landscapes divided into hexagonal cells (as in Frandsen and Andrews 1979). Hexagonal cells allow us to avoid issues arising from fires spreading across cell vertices as well as edges; while triangular cells also avoid this problem, hexagons appear more natural as shapes for forest treatments.

Methods

To estimate critical treatment thresholds and measure the resulting fuelbreaks, we needed (1) a method to generate random treatment patterns, (2) a method to determine whether a continuous fuelbreak was formed by those treatments and to measure its length when possible, and (3) a search algorithm that would repeat these efforts in a reasonably efficient manner until a critical fraction of the landscape requiring treatment was identified. The three methods that we used are each described in a subsection below.

Generating random landscapes

We begin with a uniformly untreated landscape, and then randomly apply treatments. In a practical problem, fire managers could use irregular landscape polygons from existing maps, accounting for existing canopy gaps and high priority treatment areas by specifying those as "treated" prior to adding random treatments.

Many algorithms can be used to generate random landscapes with controlled levels of clustering. For simplicity, we chose to adapt an algorithm from Fahrig (1997) based on correlated percolation models from Anderson and Family (1988). At the start, a clustering index value between zero and one is specified, where zero implies that treatments will be placed randomly with no added degree of clustering, and one implies that all treated cells except the first chosen must adjoin at least one other cell previously selected for treatment. Candidate cells are then tentatively selected for treatment one by one at random from an initially untreated landscape. If a candidate cell adjoins a cell previously selected for treatment, then that candidate is also treated; otherwise a uniform random number between zero and one is drawn and compared with the clustering index value. If the random number is greater than the clustering index, the candidate cell is treated; otherwise it is not. The algorithm continues in this fashion until the prespecified number of cells to be treated in the landscape has been selected. Thus at each step, one minus the clustering index represents the chance that a candidate cell with no treated neighbors (at that step) will itself be selected for treatment. Figure 1 shows examples of three 20 × 40 landscapes with 62.5% of the cells randomly treated using clustering index values of zero, 0.90, and 0.99, so that the treatment chances were 1.0, 0.1, and 0.01, respectively, for cells with no treated neighbors at the time of selection as candidates.

Identifying and measuring fuelbreaks

To identify and measure the most direct continuous

Fig. 1. Example landscape grids with 20 rows and 40 columns per row of hexagonal cells from which 62.5% were randomly classified as treated using treatment clustering index values of (*a*) 0, (*b*) 0.90, and (*c*) 0.99. Treated cells include those on the fuelbreaks, which are (*a*) 45, (*b*) 55, and (*c*) 43 cells long.
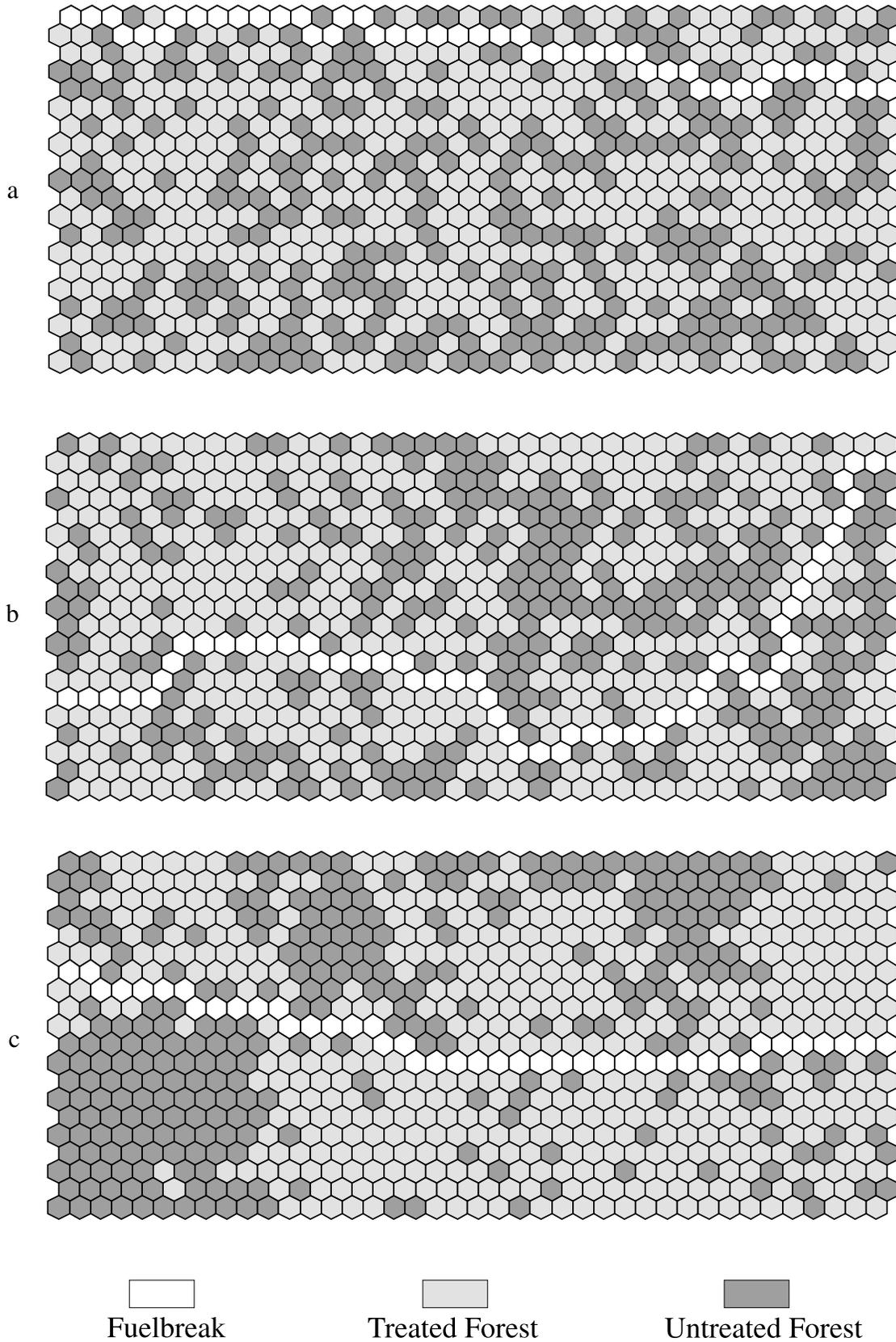
fuelbreak across each randomly treated landscape, we constructed a shortest path network optimization model (Dykstra 1984) formulated as follows:

[1] $\text{Minimize} \sum_{a \in A} x_a$

subject to

[2] $\sum_{a \in S_n} x_a - \sum_{a \in D_n} x_a = \delta_n, \quad \forall n \in N$

[3] $0 \le x_a \le u_a, \quad \forall a \in A$

where n indexes a set N that includes all of the cells (or nodes) in the landscape, plus an auxiliary "supply cell" positioned to the left of the landscape and an auxiliary "demand cell" positioned to the right (as described below), a indexes a set A that includes all directed arcs between all pairs of adjacent cells in the model (also described below), x_a is a variable indicating the amount of flow through directed arc a in the network, δ_n is the "supply" (when positive) from cell n to be moved through the network to meet "demands" (when negative), S_n is the set of directed arcs with flows originating from cell n as a source node, D_n is the set of directed arcs with flows arriving at cell n as a destination node, and u_a is an upper bound on the flow through directed arc a.

Equation 1 minimizes the total flow through the network, treating each cell in the landscape equally. Arcs directed from the auxiliary supply cell, which is given a supply value of plus one ($\delta_n = 1$) in eq. 2, are connected only to forest cells along the left-hand edge of the landscape. Arcs directed to the auxiliary demand cell, which is given a supply value of minus one ($\delta_n = -1$) in eq. 2, are connected only from the forest cells along the right-hand edge of the landscape. All forest cells are given zero supplies ($\delta_n = 0$) and are connected only to adjoining cells. To ensure that only treated cells are connected to form a fuelbreak, we set the upper bounds to zero ($u_a = 0$) in eq. 3 for all arcs with untreated forest cells as either sources or destinations; we set the upper bounds to one ($u_a = 1$) for all other arcs, including those connecting to the auxiliary cells. Thus, the set of balance constraints in eq. 2 forces one supply unit to flow from the auxiliary supply cell, through the network of forest cells, arriving at the auxiliary demand cell. Equation 3 allows that flow only through treated cells, and eq. 1 finds a shortest treated path. When an optimal solution is found, subtracting one from the objective function value (to count cells rather than arcs and to account for the auxiliary cells) gives us the length in forest cells of the shortest complete fuelbreak formed by our random treatments. When a set of random treatments fails to form a complete fuelbreak, an infeasible solution occurs. Because infeasibilities might also arise from computational difficulties, our system then converts the model from a left-to-right to a bottom-to-top percolation orientation and identifies the shortest chain of untreated cells (or "fire path") spanning the treatment area as a check that no continuous fuelbreak exist. Either a fuelbreak or a fire path should always occur exclusively, and did in all of our checks.

Estimating critical treatment fractions

By repeatedly generating landscapes of a given size, fraction treated, and level of spatial correlation, and by solving the resulting network model for each, the fraction of those landscapes with feasible solutions provides an estimate of the probability that a fuelbreak would be constructed. To estimate the critical threshold fraction associated with a given probability standard, we used a midpoint bisection search routine (similar to Gill et al. 1981). In this routine, we first specify an upper and a lower bound on the fraction of the landscape to be treated. We then repeatedly generate randomly treated landscapes with the fraction of cells to be treated set at the midpoint between our lower and upper bounds. If fuelbreaks are formed in an acceptable number of those cases (based on the probability standard), we then reset the upper bound to the current fraction treated and begin again. Alternatively, if the number of cases with fuelbreaks is unacceptable, we reset the lower bound to the current fraction treated and begin again. The algorithm continues in this fashion until the lower and upper bounds are only a single cell apart in the number of cells treated. From this set of results, the smallest fraction of cells treated that meets or exceeds our performance standard is an estimate of the critical treatment threshold p_c for the given set of conditions. Figure 2 shows an example of the output from this procedure.

Results

We solved these optimization models on a 1-GHz Pentium III personal computer using the CPLEX 7.5 callable library of network optimization routines (ILOG 2001). As Fig. 2 shows, some variation in results occurred even when 1000 random landscapes were used to count successes and failures at each treatment level. For example, fuelbreaks were formed in more cases when 1451 cells were treated than when 1469 cells were treated. Consequently, our first experiments were aimed at characterizing variation in the estimated critical threshold treatment levels. At the same time, we found that we had to forego replicating critical threshold estimates for most cases because of the substantial processing times involved; while the example shown in Fig. 2 required 4.8 h to process, a number of the cases we investigated took several days of computing time to produce a single critical threshold estimate.

Table 1 summarizes the results of five repeated critical threshold estimates based on 200 and 1000 random landscapes each for selected sets of parameter values. Results for the first three cases indicated very little difference in variation of observed critical thresholds across different clustering index values, even though we did observe a substantial increase in mean critical threshold for the case with the most clustered treatments. On the other hand, we observed greater sensitivity to changes in PR_c, even with a sample size of 1000 (see the last two cases in Table 1). Based on these results, we decided to use 1000 random landscapes (and more in one case) for subsequent estimates.

Effects of treatment zone depth

To investigate the effect of treatment zone depth into the wildland forest, we estimated the critical treatment fraction and measured the associated fuelbreak lengths for landscapes with 20, 40, and 80 treatable rows. Each landscape had 100 columns (hexagonal cells) per row, and treatments

Fig. 2. Example output from using the bisection search procedure to obtain one estimate of the critical threshold treatment level p_c and measurements of the associated fuelbreak (and fire path) lengths for landscapes with 20 rows, 100 columns per row, a treatment clustering index value of 0.9, an acceptable PR_c value of 0.99, 1000 random landscapes per treatment level, 0.5 as a treatment level lower bound, and 0.75 as a treatment level upper bound. In this case treating 1450 of the hexagonal forest cells appears to be optimal, producing fuelbreaks averaging 109.4 cells in length with a coefficient of variation of about 1.7%.

```
        Start Time:    28-JAN-2003  11:03:12

     Number of treatable nodes =         2000

   P-Value: 0.6250          PRc       Min       Max       Mean        CV
   Nodes Treated:   1250
       Fuelbreak:           0.776   106.000   132.000   117.139      3.166
       Firepath :           0.224    20.000    40.000    22.455     13.401

   P-Value: 0.6875          PRc       Min       Max       Mean        CV
   Nodes Treated:   1375
       Fuelbreak:           0.959   106.000   123.000   111.885      2.063
       Firepath :           0.041    20.000    29.000    22.000     10.378

   P-Value: 0.7188          PRc       Min       Max       Mean        CV
   Nodes Treated:   1438
       Fuelbreak:           0.989   104.000   117.000   109.637      1.635
       Firepath :           0.011    21.000    30.000    24.182     13.752

   P-Value: 0.7344          PRc       Min       Max       Mean        CV
   Nodes Treated:   1469
       Fuelbreak:           0.995   104.000   119.000   108.725      1.567
       Firepath :           0.005    20.000    26.000    22.400     10.758

   P-Value: 0.7266          PRc       Min       Max       Mean        CV
   Nodes Treated:   1453
       Fuelbreak:           0.992   104.000   118.000   109.204      1.583
       Firepath :           0.008    21.000    25.000    22.750      7.270

   P-Value: 0.7227          PRc       Min       Max       Mean        CV
   Nodes Treated:   1445
       Fuelbreak:           0.985   105.000   119.000   109.526      1.664
       Firepath :           0.015    20.000    29.000    22.800     10.993

   P-Value: 0.7246          PRc       Min       Max       Mean        CV
   Nodes Treated:   1449
       Fuelbreak:           0.986   105.000   120.000   109.320      1.591
       Firepath :           0.014    20.000    33.000    22.929     19.137

   P-Value: 0.7256          PRc       Min       Max       Mean        CV
   Nodes Treated:   1451
       Fuelbreak:           0.998   104.000   120.000   109.445      1.604
       Firepath :           0.002    20.000    21.000    20.500      3.626

   P-Value: 0.7251          PRc       Min       Max       Mean        CV
   Nodes Treated:   1450
       Fuelbreak:           0.990   104.000   122.000   109.422      1.686
       Firepath :           0.010    20.000    25.000    22.100      7.222

        End Time:    28-JAN-2003  15:51:22
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were clustered using an index value of 0.7. For perspective we note that if each cell represented 0.25 ha, we would be attempting to protect a boundary about 5 km long by treating zones almost 1 km and more into the forest. PR_c was set at 0.99, so at least 990 of 1000 random landscapes were required to produce continuous fuelbreaks for a treatment level to be acceptable. As Fig. 3 shows, the critical fraction decreases at a decreasing rate from 0.68 to 0.55 as the number of rows increases. This downward trend makes sense when one considers that if a particular treatment level produces continuous fuelbreaks in a 20-row landscape with 0.99

probability, then the probability associated with two 20-row landscapes stacked to form 40 rows will be at least $1 - [(1 - 0.99)(1 - 0.99)] = 0.9999$. This greatly exceeds the performance standard, so a smaller fraction of the area can be treated. The same logic applies as we increase from 40 to 80 rows, but apparently with diminishing effect.

The possibility that a fuelbreak might be formed by treatments in a 40-row landscape even when no fuelbreak is formed in either of the separate 20-row landscapes strengthens the above result and is evidenced by the increases in both mean and maximum fuelbreak lengths shown in Fig. 3

Table 1. Summary results for five repeated critical threshold estimates based on 200 and 1000 random landscapes each for selected parameter values.

Rows	Columns per row	Clustering index	Acceptable PR$_c$	Sample size	Mean p_c (±maximum deviation)
20	100	0.9	0.95	200	0.6823±0.0080
20	100	0.5	0.95	200	0.6410±0.0082
20	100	0.1	0.95	200	0.6429±0.0072
20	100	0.5	0.95	1000	0.6471±0.0018
20	100	0.5	1.00	1000	0.7207±0.0064

Note: p_c, critical or percolation threshold; PR$_c$, the probability of forming a percolating cluster.

Fig. 3. Observed critical thresholds and fuelbreak lengths as the number of treatable rows increases based on 1000 random landscapes per treatment level with 100 columns per row, a treatment clustering index value of 0.7, and an acceptable PR$_c$ value of 0.99.
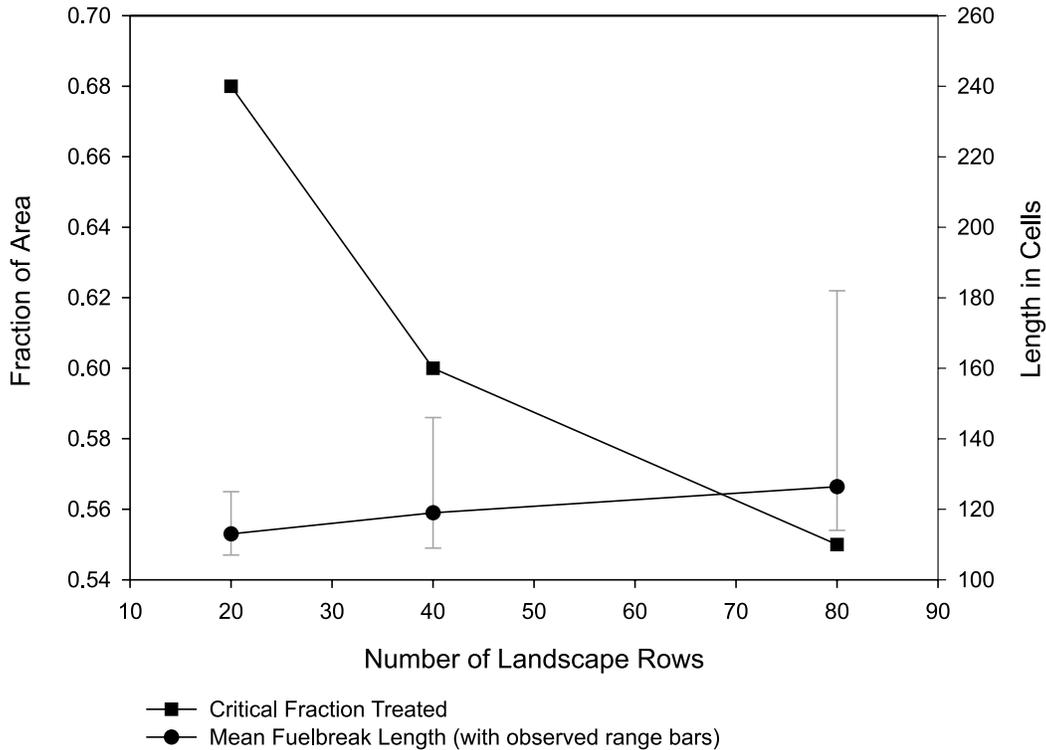
(which are approximately linear in this case). Variation in fuelbreak lengths also increased with the number of rows but this variation was relatively small for all landscape depths: observed coefficients of variation were 1.9%, 3.0%, and 4.9% for landscapes with 20, 40, and 80 rows, respectively.

Effects of treatment zone length

To investigate the effect of protecting a greater area by lengthening the required treatment zone, we estimated the critical fraction and measured the associated fuelbreak lengths for landscapes with 100, 200, and 400 hexagonal cells per treatable row. Each of these landscapes had 20 rows, and treatments were again clustered using an index value of 0.7. As before, at least 990 of 1000 random landscapes were required to produce continuous fuelbreaks for a treatment level to be acceptable. As Fig. 4 shows, the critical fraction increases at a slightly decreasing rate from 0.68 to 0.72 as the number of cells per row increases. This upward

trend also makes sense by logic similar to that used above: if we think of a 200-column landscape as two 100-column landscapes joined end to end, formation of a continuous fuelbreak fails when either of the smaller landscapes fails or when the two partial fuelbreaks fail to join. To overcome the resulting reduction in probability of success, a greater fraction of the total area must be treated. But again, this effect seems to diminish somewhat as the number of treatable landscape columns increases, and in this case mean fuelbreak lengths appear to increase linearly with little variation.

Effects of treatment clustering

To investigate the effect of clustering treated cells, we estimated the critical fraction and measured the associated fuelbreak lengths for landscapes 20 rows by 100 columns in size with treatments clustered using index values of 0.3, 0.5, 0.7, 0.9, and 0.99. At least 990 of 1000 random landscapes were again required to produce continuous fuelbreaks for a treatment level to be acceptable. As Fig. 5 shows, the clus-

Fig. 4. Observed critical thresholds and fuelbreak lengths as the number of treatable columns per row increases based on 1000 random landscapes per treatment level with 20 rows, a treatment clustering index value of 0.7, and an acceptable PR_c value of 0.99.
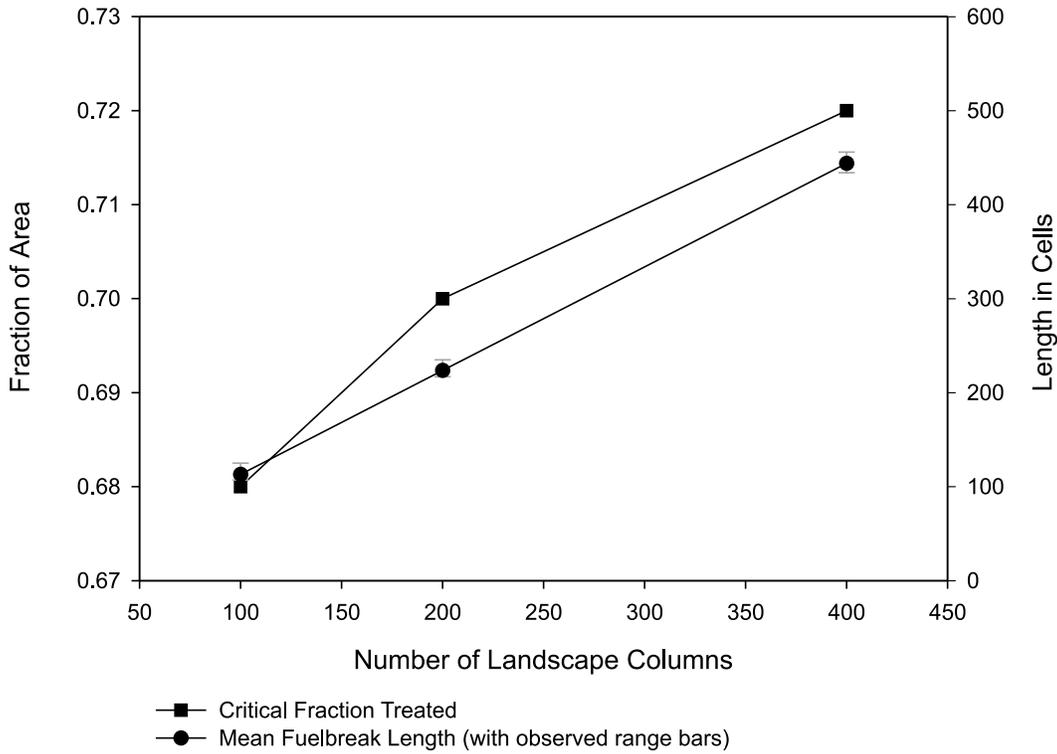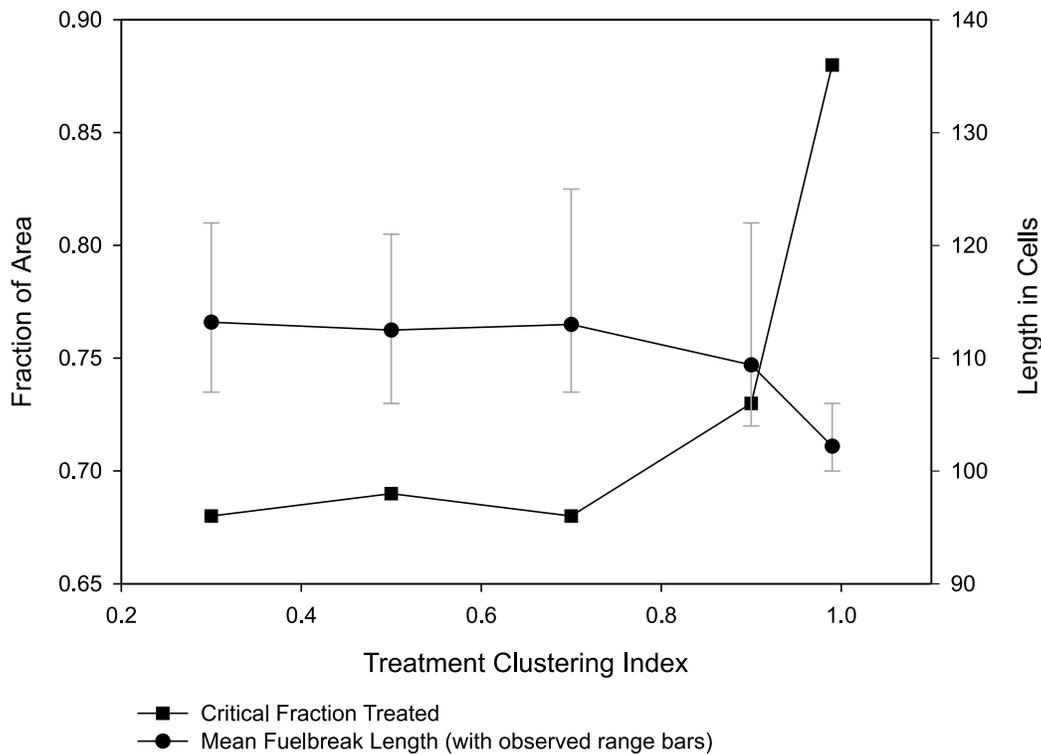

Fig. 5. Observed critical thresholds and fuelbreak lengths as the treatment clustering index varies based on 1000 random landscapes per treatment level with 20 treatable rows, 100 columns per row, and an acceptable PR_c value of 0.99.

tering index has little effect until the value is between 0.7 and 0.9, where the critical fraction begins to rise at a rapidly increasing rate from about 0.68 to a maximum of 0.88. As the critical fraction increases, mean fuelbreak lengths decrease from 113.2 to 102.2 cells, perhaps just indicating that more direct paths tend to occur as more area is treated (when

Fig. 6. Example landscape grid with 20 rows and 40 columns per row of hexagonal cells from which 62.5% were randomly classified as treated using an untreated forest clustering index value of 0.99. The fuelbreak (composed of treated cells) is 45 cells long.

Fuelbreak   Untreated Forest   Treated Forest
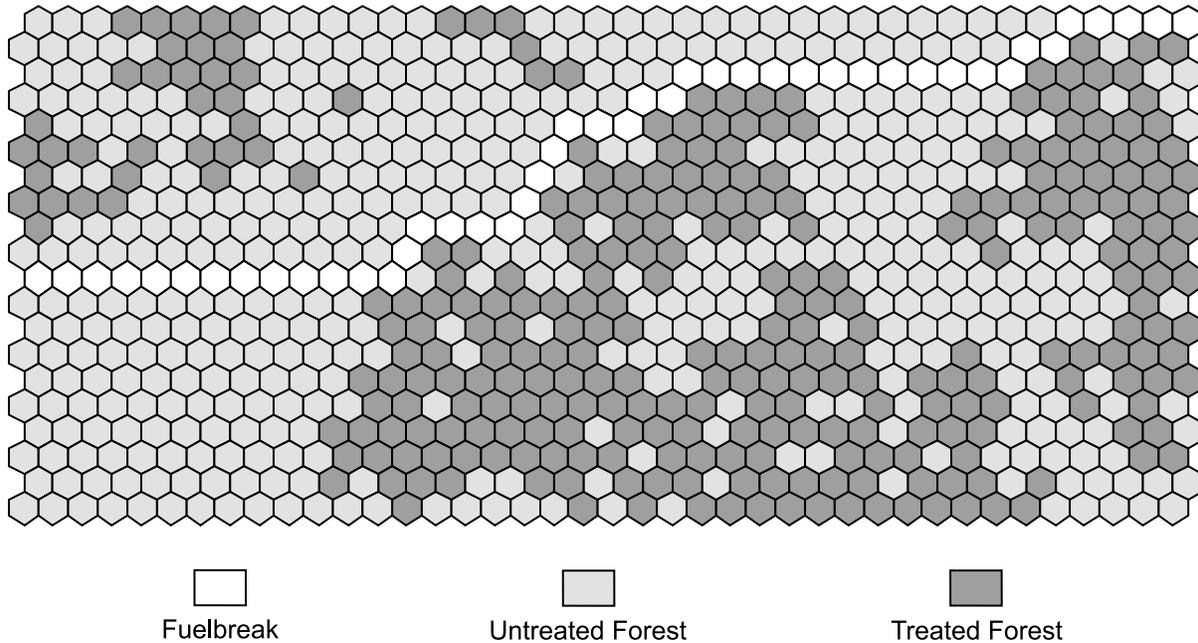
a continuous fuelbreak occurs at all). Observed coefficients of variation in fuelbreak lengths fluctuated between 1.7% and 1.9%, except at the 0.99 clustering index level, where the large critical fraction treated reduced variation to 0.9%.

We also examined the effect of clustering the residual untreated forest cells instead of the treated cells. As Fig. 1*c* shows, clustering treated cells can create a larger number of untreated forest patches (contiguous groups of like cells), occurring to some extent as "perforations" in the treated patches. This effect can be reversed by instead using our clustering algorithm to select which cells will be left untreated. An example comparable to Fig. 1*c* is shown in Fig. 6. Using this approach, we repeated the measurements from Fig. 5 at the 0.99 clustering index value and found no effect on the critical fraction to be treated (still 0.88). Mean fuelbreak length did increase slightly, however, from 102.2 to 103.4 cells with a commensurate increase in the coefficient of variation from 0.9% to 1.7%. These increases likely result from a tendency for treated areas to be somewhat more fragmented when this approach to clustering is used.

Effects of acceptable spanning probability

To investigate the effect of acceptable spanning probability (or proportion as implemented here), we estimated the critical treatment fraction and measured the associated fuelbreak lengths for PR_c values of 0.6, 0.7, 0.8, 0.9, and 1.0, again based on 1000 random landscapes for each treatment level tested. Each landscape was 20 rows by 100 columns, and treatments were clustered using an index value of 0.5; based on the small effect of the change in clustering index from 0.7 to 0.5 in the preceding experiment, we used 0.5 here to speed landscape processing. As Fig. 7 shows, critical fractions increased approximately linearly from 0.59 to 0.61 with PR_c values from 0.6 to 0.8, and then increased more

rapidly up to 0.72. Mean fuelbreak lengths declined from 124.7 to 109.9 cells with increasing PR_c values as the critical fraction increased; at the same time, coefficients of variation declined steadily from 4.4% to 1.4%. In a final examination of PR_c effects, we also obtained results from using a value of 1.0 in a more stringent test with 10 000 random landscapes for each treatment level. The critical fraction increased further to 0.74 and mean fuelbreak length decreased to 108.8 with the coefficient of variation dropping to 1.1%.

Conclusions

The most obvious conclusion from our results is that fuels must be treated throughout most of the area in a forest to create a continuous fuelbreak if treatments are arranged randomly. Also, when random treatments or residual forest areas are intentionally clustered the fraction of area that must be treated tends to increase. These findings are consistent with results from other percolation studies, as is our observation that the rates of change in the critical fraction requiring treatment decline as the number of cells in the landscape increases; as the number of cells approaches infinity, critical fractions are expected to approach constant values (With 2002).

A less obvious conclusion from our results is that fairly direct fuelbreaks tend to be formed through random treatment arrangements near critical fraction values. From the tens of thousands of such landscapes we generated that resulted in continuous fuelbreaks, the fuelbreak with the longest relative length was 1.82 times the length of the treatment area (see Fig. 3), and most were far shorter. Although highly circuitous fuelbreaks were possible given our formulation (e.g., 40 times the length of the treatment area in the case cited above), none were observed. We anticipate that

Fig. 7. Observed critical thresholds and fuelbreak lengths as the acceptable PR_c varies based on 1000 random landscapes per treatment level with 20 treatable rows, 100 columns per row, and a treatment clustering index value of 0.5.
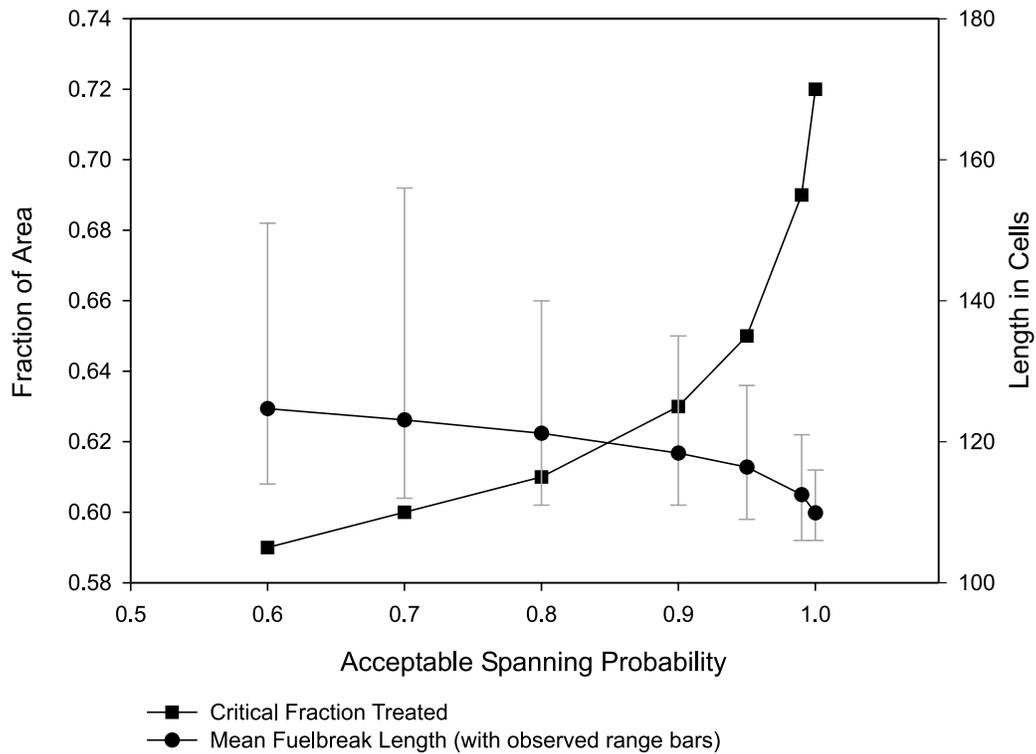
this somewhat surprising result will appeal to fire managers who might have to quickly convert such a fuelbreak into a defensible firebreak.

Observed critical treatment fractions were sensitive to the acceptable spanning probabilities imposed, but the critical fraction was 0.59 even when continuous fuelbreaks were required in only 60% of the random landscapes generated. From a practical point of view, fire managers may not need to be concerned with the exact spanning probability, knowing they could simply generate landscapes until they had a few suitable alternatives to choose from. But our results suggest that they likely would still need to treat a majority of any heavily fueled landscapes to randomly create fuelbreaks with even a modest spanning probability. Because this would also accomplish much additional fuel reduction throughout the wildland–urban interface zone, and because the resulting forest would have a somewhat natural appearance, a random percolation approach might be appealing to some communities. Our study clearly indicates, however, that creating a fuelbreak in this manner will involve a much more extensive treatment effort than the direct creation of a fuelbreak alone.

Acknowledgments

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