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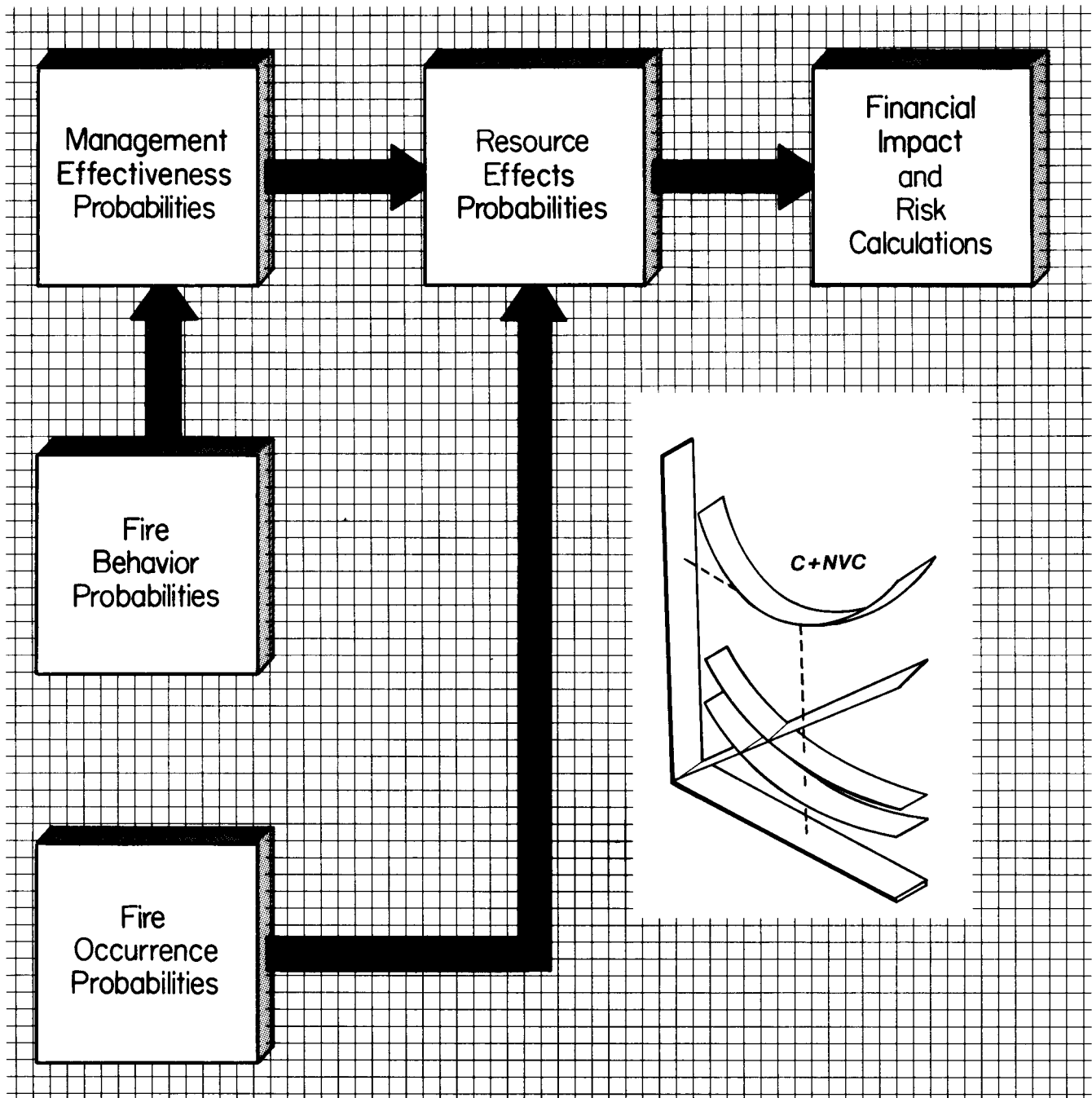
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Probability Model for Analyzing Fire Management Alternatives: theory and structure

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GLOSSARY

A_x	Program for fire management activity type x.	h	Fire type number designating cause and access class.
a	Fire size class (general).	IAA	Initial attack analysis.
a_i	Fire size class after initial attack.	IPC	Inferred parameter cell.
a_E	Burned area vector after escaped fire analysis, with a component for each affected resource.	m_f	Fuel moisture vector for dead fuel particle size classes and live fuels.
B	Set of fire behavior variables.	N	Number of fires.
b	Vector of values for variables in B.	P	Probability function.
b_i	b following initial attack calculations.	PL	Program level (presuppression budget).
b_E	b in escaped fire analysis with a vector for each affected resource.	q_t	Resource change vector per unit burned area in year t.
C	Fixed yearly cost of fire management programs.	Q_T	Total resource change vector per unit burned area through year T
c	Variable suppression costs.	R	Resource management emphasis indicator.
D	Present value cost plus net value change.	r	(1) Fire rate of spread. (2) Resource type.
d	Fire area at detection.	S	Set of fire management situation parameters.
E_α	Set of inferred parameters for fire management activity, α .	s	Value vector for S.
e_α	Value vector for set E_α .	T	(1) Topography descriptor (FMS parameter). (2) Years of fire effects accumulation.
e	Value vector for union of all E_α .	t	Time-years.
EFA	Escape fire analysis (also large fire analysis).	t_d	Time of day class.
F	Vector function relating resource value changes to physical changes (inverse of f, (2)).	t_i	Suppression time in IAA.
F_B	Fire behavior function.	t_y	Time of year class.
FEES	Fire Economics Evaluation System.	u	Slope class for terrain.
FMS	Fire management situation.	$\hat{u}$	Slope-aspect-elevation class.
FMM	Fire management mix.	V	(1) Vegetation class (FMS parameter). (2) Resource value change vector.
f	(1) Fuel model. (2) Vector function relating physical resource changes to dollar values.	v	Vegetative cover type.
f_i	Fuel type used on fire report forms.	w	Windspeed.
G	Set of physical fire effects variables for resources.	Z	(1) Climate zone (FMS parameter). (2) Years of fire occurrence for effects accumulation.
g	Vector of values for G.	Γ	Array of prefire resource characteristics needed to calculate fire effects.
		Ω^t	Postfire resource management program through year t after a fire.
		Φ^t	Set of given parameters and programs through year t.

Increasingly, wildland fire management programs are being judged by their economic efficiency. The use of that criterion to evaluate programs is relatively new, but the application of economic theory to fire management dates from the 1920's (Sparhawk 1925).

The variable nature of wildfires has stimulated planners to recognize that a probabilistic approach to fire management analysis and planning is needed. Only recently, however, have probability concepts been explicitly incorporated into fire analysis and planning systems--and then only in a limited way (Hirsch and others 1979; U.S. Dep. Agric., Forest Serv., 1981b).

At the Pacific Southwest Forest and Range Experiment Station, a multidisciplinary team at Riverside, California, is developing an analytical and planning approach called the Fire Economics Evaluation System (FEES) (Mills and Bratten 1982). Framework for the system is provided by a theoretical probability model designed to analyze program alternatives in wildland fire management. This model includes submodels or modules for predicting probabilities in fire behavior, fire detection, initial attack and large fires, fire effects, resource changes, fire occurrence, and resource values. Generally using state-of-the-art technology and available data, these modules are designed to be modified or replaced in the future as new research dictates.

This report describes the mathematical structure of the probability model and the logic for assembling the modules into an overall system for analyzing wildland fire management.

DESIGN OF THE MODEL

Early in the design phase, the research team decided to follow an approach that would not be site-specific. The system uses two kinds of parameters: *area*, to identify planning situations and *point*, to identify factors that affect fire behavior, occurrence, and effects.

Fire Management Situations

Area parameters serve to identify *fire management situations* (FMS)¹--generalized physical locations with characteristics similar to actual planning areas. A FMS is shown

¹The glossary lists symbols, and their definitions, used in this publication.

in *fig. 1* with an indefinite boundary to indicate the nonspecific site concept. The probability model currently uses these area parameters:

- Climate zone (Z): one of 14 fire weather regions in the United States (Schroeder and others 1964).
- Topography class (T): general topography, such as flat, moderate, or steep land.
- Vegetation class (V): classes derived from standard classification schemes.
- Resource management emphasis (R): relative management importance of various land resources in the area.
- Occurrence types (H): the most important occurrence types defined by combinations of cause and access classes.
- Occurrence levels (N): average number of fires per year per million acres for occurrence types (H).

For each FMS, the planner determines a fire management mix (FMM). Such a mix is a combination of well-defined mixes of fire suppression units and activities, budget or program levels (PL), program emphasis, and treatment.

Inferred Parameter Cells

Point parameters describe the kind of fire environment that could be expected in the fire management situation being analyzed (*fig. 1*). Within a FMS are many kinds of fire sites. The point parameters that describe these sites are necessary for analyzing fire behavior, occurrence, management, and effects. A set of values for these parameters, and hence for a particular kind of fire site, is called an *inferred parameter cell* (IPC).

To predict fire behavior in the model, values for parameters of fuel, weather, and slope of the terrain are used. These and other parameters that determine them comprise the inferred parameter subset E_B . Similarly, E_D is the subset needed to calculate fire detection probability; E_S for calculating fire suppression; and E_R for calculating fire effects on resources.

The entire set of inferred parameters, i.e., the union of subsets E_B , E_D , E_S , and E_R is called E . Some of the parameters in E represent quantities that physically have continuous value ranges. However, for purposes of analysis, each of these ranges is partitioned into a small number of class intervals. The class intervals are numbered for each parameter, and these reference numbers become the range of the parameter. Other parameters, for example, cover type, are basically categorical, and these categories are assigned reference numbers. A vector of reference number values for set E is designated e , or e_α for sets E_α ($\alpha = B, D, S$, or R), and such a vector defines an IPC.

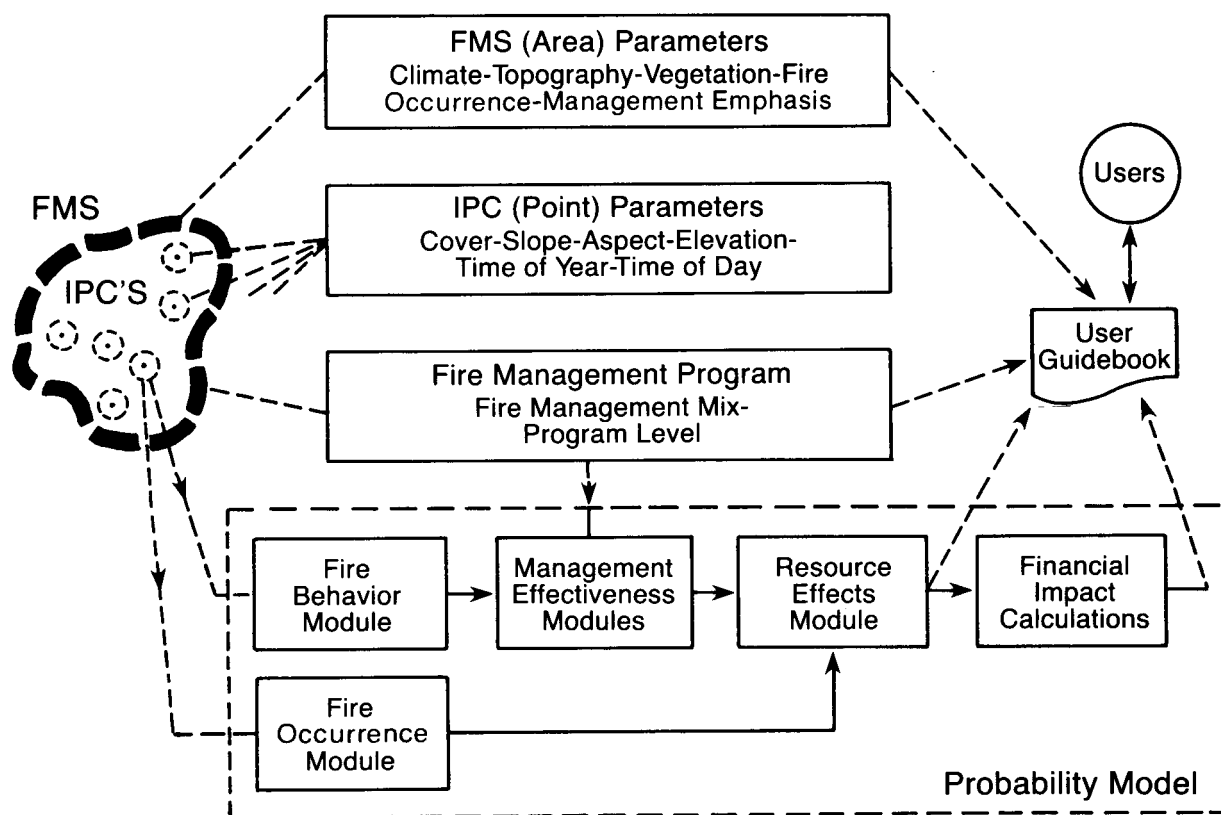


Figure 1--The probability model provides the mathematical framework for the Fire Economics Evaluation System now under development by the Forest Service. Fire management situations

(FMS) are generalized physical location; inferred parameter cells (IPC) are particular kinds of fire location or site.

The IPC's in set E are basic elements of the analysis space. In each IPC, the analysis process will yield probabilities for fire effects on land resources and for variable costs of the specified fire management activities. The effects and costs for any IPC will be realized only if a simulated fire occurs in that cell. This fire occurrence is a random event in the real world and in the model. In any year or fire season, a fire chosen at random will fall in a given cell. This process can be interpreted as a random occurrence of an IPC, or more specifically, of the parameter values defining the IPC. In this sense, e and $e_{\alpha,s}$ for the modules are random vectors.

Fire Management Program Options

A range of fire management programs will be analyzed for each FMS. These programs will be described by well-defined fire management mixes (FMM) of kinds of units and activities, and a range of budget or program levels (PL). The PL represents the total annual budget for the FMS. An FMM is a set of rules for allocation of the PL dollars, first between the various kinds of inputs (fuel treatment, initial attack, etc.) and then between various kinds of expenditures within the input categories.

Outputs

For any combination of FMS, FMM, and PL, the FEES probability model will calculate two basic kinds of outputs. The first is a joint probability distribution for the fire

induced changes in several kinds of forest resource outputs, measured in resource quantities (e.g., timber volume) per million acres per year over a 10 year planning period. These changes can be either positive or negative, including both beneficial and detrimental effects of fires.

The second output will be a probability distribution for the economic efficiency [sic] measure, *cost plus net value change* ($C + NVC$). Here the resource changes over time will be given dollar values which, along with the time stream of fire and resource management costs, will be discounted to present values and summed. Expected values of this "bottom line" output will provide an economic risk-neutral basis for choosing between FM M's and PL's for a given FMS. The probability distributions will provide opportunities for choices based on other kinds of risk preference and consideration of inputs other than FEES to the decision making process.

Model Structure

A more detailed diagram of the probability model is shown in *figure 2*. Information flows between the modules generally from left to right. Logically, the first module to consider is fire behavior. Fire behavior variables include rate-of-spread, intensity, and other quantities derived primarily from these two. Probabilities for fire behavior depend explicitly on the IPC parameters and implicitly on weather data used to estimate the probabilities.

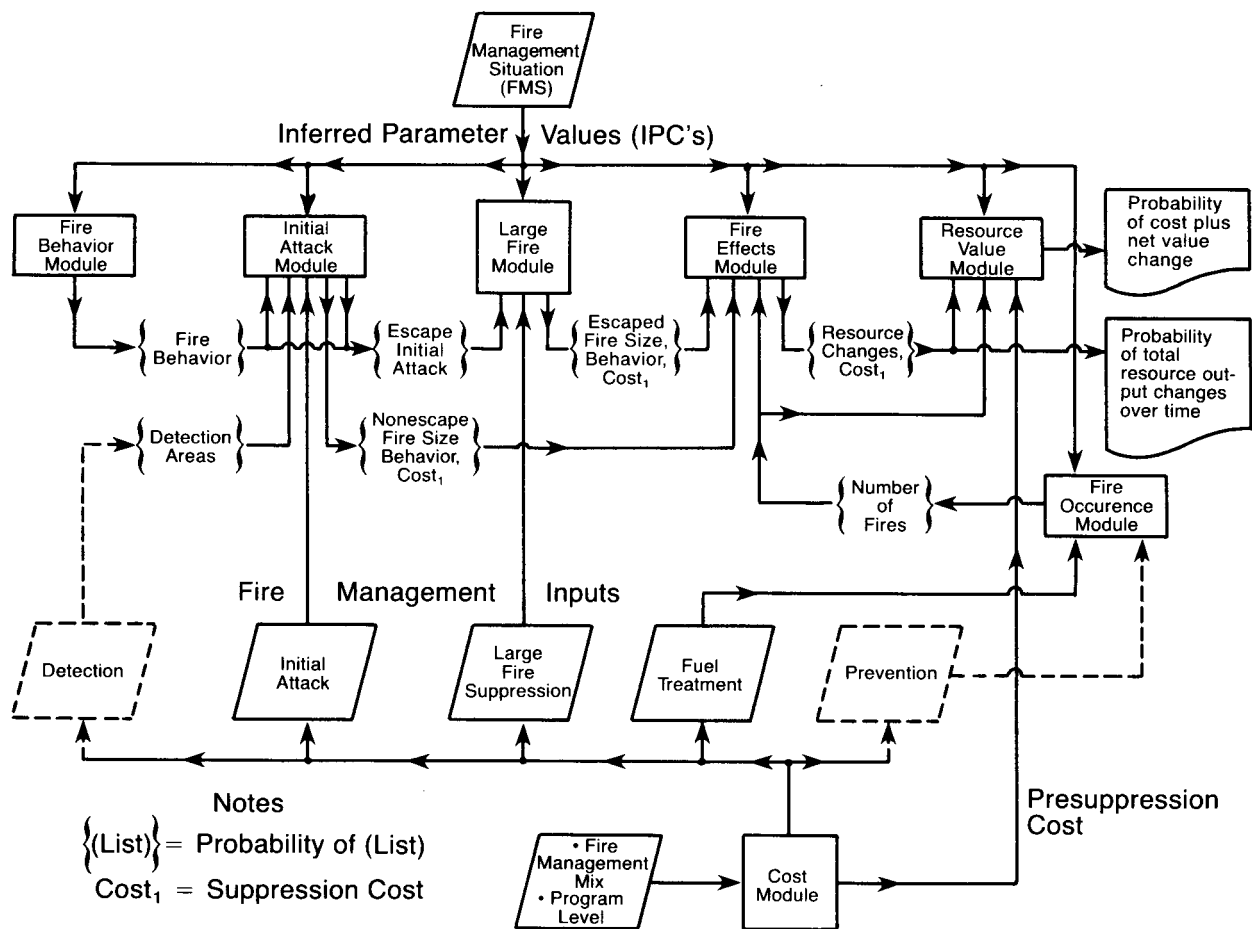


Figure 2--Modules or submodels form the theoretical probability model. They deal with various aspects of the fire management analysis procedure. Information flows between modules generally

Cost-effectiveness models for fire prevention and detection are not presently available, so these inputs are shown as dashed line boxes in *figure 2*. In initial FEES development, detection and prevention activities are assumed to be those historically used in an FMS. The historical dollar expenditures in these areas are subtracted from the PL before using the FMM rules to allocate the remaining dollars to the other inputs. Consistent with these assumptions, probabilities [sic] for detection areas and numbers of fires are estimated from historical fire data and used as inputs to the model.

Given detection area and fire behavior probabilities along with the initial attack inputs or available attack units, the *initial attack module* calculates probabilities for initial attack outcomes for each IPC. These outcomes include a probability for "escape," which is defined as any result from the initial attack or fire behavior calculations in which fire size or intensity exceed values beyond which the initial attack model is invalid. For escape conditions, a *large fire module* combines subjective, and computer techniques to estimate probabilities for final fire sizes, behaviors, and costs.

Given results from these two modules, the *fire effects module* first calculates joint probabilities per fire for sup-

pression costs and resource changes over time in each IPC. Then costs and resource changes and their joint probabilities are combined over all IPC's in the FMS to, in effect, get probabilities for time streams of effects and costs per fire for any fire in the FMS. Probabilities for annual numbers of fires in the FMS are then used to estimate probabilities for total annual resource change and suppression costs in the FMS. The probabilities for total resource output changes over time which are available at this point in the calculations form one part of the model output for use in the "guidebook" of results.

Dollar values are assigned to each single IPC time stream of resource changes in the resource value module. These values and the suppression costs are appropriately discounted, and their probabilities are combined to provide probabilities for present values of resource changes and suppression costs for a single IPC. These probabilities for present values are combined over all IPC's, and probabilities of numbers of fires are used to get probabilities for total suppression costs and net value changes in the FMS. Presuppression costs from the cost module are added to arrive at the "bottom-line" output, for the "guidebook," of probabilities for cost plus net value change.

THE PROBABILITY MODEL

The probability model description uses functional and informal set notation. Symbols used and their definitions are listed in the Glossary. Submodels for estimating fire effects, fire behavior, and other model functions are represented here only as conditional probability distributions in generalized functional notation. Independent and dependent variables or sets are included along with the independent parameters. Random variables and parameters comprising inputs and outputs of the various modules are assumed to be discrete valued. The use of class intervals for defining ranges of the FMS and inferred parameters was described earlier. Similar partitioning is assumed for random variables that are internal to the model.

The following functional notation conventions are used to distinguish between random variables and deterministic parameters, and for other purposes:

$P(x)$ is the probability that discrete random variable X has value x . The convenient procedure of simply writing $P(x)$ with no further explanation will be followed. The domain of the function will be given explicitly only when this is not obvious.

$P(x;a)$ is the probability of x as described above, given deterministic independent parameter a . Similarly, $P(x;a,b)$ has given parameters a , b , and so forth.

$P(x|z)$ is the conditional probability for values of discrete random variable X , given value z for discrete random variable Z . Then, $P(x|z;a)$ is the conditional probability for x , given z and parameter a , and so forth.

$P_n(x)$ uses subscripts on P , as $P_1(x)$ to denote a particular functional form for P and $P_2(x)$ for a different form.

Occasionally, letter subscripts are used in the same way.

The modules and their interactions are described in the logical sequence shown in figure 2. Then, fire occurrence weighting will be used to give the final model outputs.

Fire Behavior

For each IPC, the fire behavior module will calculate probabilities for quantities such as fire spread rate, intensity, and tree crown scorch height needed to calculate effectiveness of detection and suppression activities and fire effects on resources (Salazar 1981).

Let B represent the desired set of fire behavior descriptors and β a value vector for B . Then, current modeling capabilities [sic] (Albini 1976, Rothmel 1972) can be represented by the function:

$$\beta = F_B(u, f, w, m_f) \quad (1)$$

where

- u = slope of terrain
- f = fuel model (a set of fuel characteristics)
- w = windspeed

m_f = fuel moisture vector for dead fuel particle size classes and live fuels

This function is useful in the model only if fire occurrence probabilities can be defined for the independent variable cell $[u, f, w, m_f]$. Data availability does not permit this directly. The weather-dependent variables w and m_f are not recorded, in general, in historic fire records and the fuel model may have to be inferred from other data sources. It appears feasible to use proxy variables which are included in occurrence data and weather records to get probabilities:

$$P_w(w, m_f | t_d, t_y, \hat{u}, v; s)$$

where

- t_d = time of day class
- t_y = time of year class
- s = value vector for FMS parameter set S
- $\hat{u}$ = [slope, aspect, elevation]
- v = vegetative cover type

Direct fuel model information also is missing from historic fire records, so that f must be inferred from other information. A vegetative cover type (v) is generally included in individual fire reports. We assume that a probability function $P_f(f | v; s)$ can be defined from available data to give probabilities for fires in fuel model f , given that they have occurred in vegetative cover type v in FMS(s).

Then probabilities for sets $\tilde{b}$ of values for the fire behavior parameters B are calculated by summing the product of P_w and P_f over all values of (w, m_f, f) for which β from equation 1 lies within $\tilde{b}$. That is

$$P_1(\tilde{b} | t_d, t_y, \hat{u}, v; s) = \sum_{(w, m_f, f) \in C_1} P_w(w, m_f | t_d, t_y, \hat{u}, v; s) P_f(f | v; s) \quad (2)$$

where $\tilde{b}$ is a set of values for B and C_1 is the set of (w, m_f, f) values such that the β calculated by equation 1 falls within $\tilde{b}$. Using the notation previously explained, P_1 can be written

$$P_1(\tilde{b} | e_B; s)$$

where

$$e_B = (t_d, t_y, \hat{u}, v)$$

Fire Detection Module

Fire detection activities A_D result in probabilities at the time of report for fire size classes d . Intuitively, these probabilities depend on fire type h , fire behavior [sic] $\tilde{b}$, and a parameter value set e_D . Assume a probability function:

$$P_2(d | \tilde{b}, e_D; h, A_D, s)$$

where

- e_D is assumed to be the same as e_B
- h = fire type, i.e., cause and access class

It is useful to combine P_1 and P_2 to get

$$P_3(d, \tilde{b} | e_D; h, A_D, s) = P_2(d | \tilde{b}, e_D; h, A_D, s) P_1(\tilde{b} | e_B; s) \quad (3)$$

Size at detection (d) and fire behavior ($\tilde{b}$) are used in the next section as independent variables for fire suppression calculations.

It should be noted that no comprehensive model of the kind assumed here for fire detection is currently available. Simplified assumptions will be made in the initial FEES development (Salazar 1981). The FEES model will provide a means for sensitivity analysis of the detection assumptions.

Fire Suppression and Costs

Analyses of fire suppression require that fires be described by both intensive and extensive variables. The fire suppression modules (*fig. 2*, initial attack and large fire modules) use primarily fire intensity and rate of fire spread, which are lumped into "fire behavior" here. Fire size at the time of beginning suppression action also is important; this can be related to d , the size at the time of report. Another kind of variable used in suppression analyses deals with effectiveness of suppression forces and activities. Examples are dispatching rules and fireline construction rates.

Fire suppression activity analysis is of two kinds: *initial attack analysis* (IAA) and *escaped fire analysis* (EFA). IAA uses computer modeling to produce joint probabilities for final fire sizes, suppression times, fire behavior, and suppression costs. For a given IPC, limits are set on fire size and suppression time beyond which IAA methods are invalid and the fires are said to have "escaped initial attack." For IPC's with a significant escape probability, EFA uses a combination of subjective and computer methods to estimate probabilities for final fire sizes, behaviors, and costs. EFA methods also are required for IPC's with significant probabilities for fire behavior parameters or fire size when reported which exceed certain limits. Behaviors and/or sizes beyond these limits predetermine a very high probability of escape from initial attack, making IAA unnecessary.

For IAA, assume a probability function

$$P_4(a_i, t_i, c_i | d, \tilde{b}_i, v, u, t_d; h, A_s, s)$$

where

- a_i = fire size class resulting from IAA
- t_i = suppression time class for IAA
- $\tilde{b}_i$ = fire behavior, assumed constant during IAA
- c_i = suppression cost calculated in IAA (Gonzalez-Caban 1981)
- u = slope class
- A_s = suppression program including kinds and quantities of suppression resources
- Other quantities are as previously defined.

Deterministic versions of initial attack models of this kind are currently being used in the FOCUS fire planning model (Bratten and others 1981) and in the Forest Service's national fire analysis system for forest planning (U.S. Dep. Agric., Forest Serv. 1982).

A major source of variability not shown explicitly in P_4 is fire force productivity (e.g., fireline construction rates) (Haven and others 1981, Hunter 1980). Another source is arrival time of attack units. The FEES initial attack module will include these kinds of variability along with the explicit input distributions for detection area and fire behavior (Hunter 1980).

The notation for the IAA probability function is shortened to

$$P_4(a_i, t_i, c_i | d, \tilde{b}_i, e_s; h, A_s, s) \quad (4)$$

where

$$e_s = [v, u, t_d] \quad (5)$$

In the fire effects and cost calculations to follow, a_i , $\tilde{b}_i$, and c_i are needed, but d is no longer needed. Similarly, fire rate of spread r , a component of b_i , is no longer needed. P_3 and P_4 are combined and summed on d and r to give for IAA:

$$P_5(a_i, t_i, b_i, c_i | e; h, A_D, A_s, s) \quad (6)$$

$$= \sum_d \sum_r P_3(d, \tilde{b}_i | e_D; A_D, s) P_4(a_i, t_i, c_i | d, \tilde{b}_i, e_s; h, A_s, s)$$

where b_i is $\tilde{b}_i$ without the r component and e is the union of sets e_D and e_s . That is

$$e = [\hat{u}, v, t_d, t_y] \quad (7)$$

Define a special fire size-suppression time class Ψ_{ie} as the "escape" class of IAA results for IPC (e), which will indicate further analysis using EFA methods (Hunter 1981). EFA also may be required for a set of initial size and fire behavior conditions Ψ_o which will exist with a known probability prior to IAA. For IPC (e) let

$$P_{Ei}(\Psi_{ie} | e; h, A_D, A_s, s) = \text{the probability for escape in IAA}$$

$$P_{Eo}(\Psi_o | e; h, A_D, s) = \text{the probability of prior conditions requiring EFA.}$$

Then the total probability for requiring EFA in IPC (e) is

$$P_6(EFA | e; h, A_D, A_s, s) = P_{Eo}(\Psi_o | e; h, A_D, s) \quad (8)$$

$$+ (1 - P_{Eo}(\Psi_o | e; h, A_D, s)) P_{Ei}(\Psi_{ie} | e; h, A_D, A_s, s)$$

We evaluate P_{Eo} and P_{Ei} by summing probabilities over sets Ψ_o and Ψ_{ie} . P_{Eo} is calculated by summing P_3 over set Ψ_o :

$$P_{Eo}(\Psi_o | e; h, A_D, s) = \sum_{d, \tilde{b}_i \in \Psi_o} P_3(d, \tilde{b}_i | e; h, A_D, s) \quad (9)$$

Similarly, P_{Ei} is further defined using P_5 :

$$P_{Ei}(\Psi_{ie} | e; h, A_D, A_s, s) \quad (10)$$

$$= \sum_{a_i, t_i \in \Psi_{ie}} \sum_{b_i, c_i} P_5(a_i, t_i, b_i, c_i | e; h, A_D, A_s, s)$$

In IAA, homogeneity of the simulated fire situation is assumed. That is, a single fire behavior and IPC will apply throughout the analysis, and a single fire size affecting all resources will result. The invalidity of this assumption after the fire attains a certain size or elapsed time is precisely what makes EFA necessary. In EFA, a fire may move through several. IPC's or equivalent before containment. Therefore, it is necessary to describe the results in inhomogeneous terms. The means used here is to describe fire behaviors and burned areas by vectors with a component for each affected resource. Let

- a_E = escaped fire burned area vector, with a component for each affected resource
- b_E = escaped fire behavior array, with a behavior vector for each affected resource
- C_E = escaped fire cost class, including cost of prior IAA if pertinent.

Then we assume that EFA will give probability functions of the form:

$$P_7(a_E, b_E, C_E | e; h, A_D, A_S, s) \quad (11)$$

Suppression time t_i was used in the IAA escape probability calculations (Equations 6, 10), but will not be needed in further calculations. We define (a_i, b_i, c_i) as the class of nonescape IAA results, i.e., without Ψ_{ie} . We sum P_5 over t_i to give:

$$P_{5a}(a_i, b_i, c_i | e; h, A_D, A_S, s) \quad (11a)$$

$$= \sum_{t_i} P_5(a_i, t_i, b_i, c_i | e; h, A_D, A_S, s) \div [1 - P_{Ei}(\Psi_{ie} | e; h, A_D, A_S, s)]$$

Let

$$\alpha_i = (a_i, b_i, c_i)$$

$$\alpha_E = (a_E, b_E, C_E)$$

where the a, b, c 's are as previously defined. Also, assume α_i and α_E to be mutually exclusive. Then, using P_5 , P_6 , and P_7 , the final suppression area, behavior, and cost probability function is

$$P_8(\alpha | e; h, A_D, A_S, s) \quad (12)$$

$$= \begin{cases} P_{5a}(\alpha | e; h, A_D, A_S, s)(1 - P_6(EFA | e; h, A_D, A_S, s))\alpha \in \alpha_i \\ P_7(\alpha | EFA, e, h, A_D, A_S, s)P_6(EFA | e; h, A_D, A_S, s), \alpha \in \alpha_E \end{cases}$$

Fire size and behavior are used in the next section to calculate probabilities for fire effects on land resources.

Fire Effects

The effects of fire on land resources and the measures of productivity vary greatly between resources. Calculations of the immediate impacts of fire on resource quantities require transformations from the physical effects of fire

(Peterson 1981). For example, a physical effect is area of grazing range burned; the corresponding resource effect is animal grazing capacity lost. The immediate resource effects must be further evaluated, over time, to obtain the final fire impacts: total animal-unit-months lost or gained, for the range example.

The FMS parameter set S includes R , the resource management emphasis vector with a component for each resource type. Fire effects are calculated for types having nonzero emphasis. In fire effects valuation, e.g., deriving dollar values for fire effects, the component values in R imply specific prefire and postfire management procedures.

Immediate physical resource effects probabilities are given by the FEES fire effects module in the form:

$$P_9(g | a, b, e; \Gamma(s), s) \quad (13)$$

where

g = vector of immediate physical effects per unit burned area for emphasized resources

$\Gamma(s)$ = array of prefire resource characteristics needed to calculate fire effects in s

Other quantities are as previously defined. In particular, it should be recalled that a and b are fire area and behavior vectors with components corresponding to those of g .

Probabilities P_8 and P_9 are combined and summed on b , which is no longer needed, to give P_{10} which will be used later after defining probabilities for resource changes. Then

$$P_{10}(a, g, c | e; h, A_D, A_S, \Gamma(s), s) \quad (14)$$

$$= \sum_b P_8(a, b, c | e; h, A_D, A_S, s) P_9(g | a, b, e, \Gamma(s), s)$$

Resource Changes

Persistence or delay of fire effects on resources from year to year implies that resource changes in any year t , from a particular previous fire, must be correlated with changes from that fire in preceding years. However, we assume that resource changes from any fire are independent of those from all other fires. Then, the probability of resource change per unit burned area in year t after a fire in e with effects g is of the form:

$$P_{11}^t(q_t | q_{t-1}, \dots, q_1, g, e, a; \Gamma(s), \Omega^t(s), s)) \quad (15)$$

where

q_t = resource change vector per unit burned area in year t after a fire causing effects g in year 1

$\Omega^t(s)$ = postfire resource management program for years 1 through t for the fire-affected resources

In the mathematical development of the model which follows, the year t of resource changes is treated as an integer variable up to some year T . In application of the model, fire effects on resources are confined generally to a

small number of years. In considering fire effects on timber output, for example, the effects of a single fire can be lumped into, say, 2 years--the year of positive effect due to salvage, if any, and the future year of negative effects on planned or actual harvest. One or two thinning harvests may be included between. The point is that even though a total time span T of 50 or 100 years may be involved, the effects are null in all but a few years.

Similarly, significant effects on range, recreation, wildlife, or watershed are confined generally to a small number of years following a fire.

The conclusions are that computational problems which may appear insuperable for $t = 1, 2, \dots, 100$, say actually are tractable because only a few years are of significance. The probabilities of effects g are obtained by summing P_{10} on costs (which are not needed for resource change calculations). The result is combined with P_{11} and summed over the range of effects g and fire area vector a (as it affects resource changes per unit fire area):

$$P_{12}^t(q_t|q_{t-1}, \dots, q_1, e; h, \Gamma(s), A_D, A_S, \Omega^t(s), s) \quad (16)$$

$$\begin{aligned} &= \sum_{a, g} [P_{11}^t(q_t|q_{t-1}, \dots, q_1, g, e, a; \Gamma(s), \Omega^t(s), s) \\ &\quad \times \sum_c P_{10}(a, b, c|e; h, A_D, A_S, \Gamma(s))] \end{aligned}$$

To shorten notation, the "given" parameters and programs indicated following the semicolon and h in the expression for P_{12} will be abbreviated using the notation $\Phi^t(s)$. That is

$$\Phi^t(s) = [\Gamma(s), A_D, A_S, \Omega^t(s), s] \quad (17)$$

Resource output changes are assumed to be additive over the years following a fire. Given a set of resource change vectors $[q_1, q_2, \dots, q_T]$ over T years, the cumulative total change per unit burned area is

$$Q_T = \sum_{t=1}^T q_t \quad (18)$$

The probability for a particular Q_T is just the probability of the corresponding set of q_t values. Using P_{12}^t from equation 16, the joint probability for the q_t 's is

$$\begin{aligned} P_{13}^T(q_T, \dots, q_1 | e; h, \Phi^T(s)) \\ = \prod_{t=1}^T P_{12}^t(q_t|q_{t-1}, \dots, q_1, e; h, \Phi^t(s)) \end{aligned} \quad (19)$$

To gain computational efficiency, we rewrite equation 19 in recursive form, for $k = 2, 3, \dots, T$:

$$\begin{aligned} P_{13}^k(q_k, \dots, q_1 | e; h, \Phi^k(s)) \\ = P_{13}^{k-1}(q_{k-1}, \dots, q_1 | e; h, \Phi^{k-1}(s)) \times P_{12}^k(q_k|q_{k-1}, \dots, q_1, e; h, \Phi^k(s)) \end{aligned} \quad (20)$$

with

$$P_{13}^1(q_1|e; h, \Phi^1(s)) = P_{12}^1(q_1|e; h, \Phi^1(s)) \quad (21)$$

From the definition of Q_T (equation 18), we note that

$$q_t = Q_t - Q_{t-1}$$

for $t > 1$, and $q_1 = Q_1$ so that we can write:

$$\begin{aligned} P_{14}^k(Q_k, \dots, Q_1 | e; h, \Phi^k(s)) \\ = P_{13}^k(Q_k - Q_{k-1}, \dots, Q_1 | e; h, \Phi^k(s)) \end{aligned} \quad (22)$$

with

$$\begin{aligned} P_{14}^1(Q_1|e; h, \Phi^1(s)) &= P_{13}^1(Q_1|e; h, \Phi^1(s)) \\ &= P_{12}^1(Q_1|e; h, \Phi^1(s)) \end{aligned} \quad (23)$$

Then, equations 20 and 21 can be rewritten entirely in terms of the Q 's:

$$\begin{aligned} P_{14}^k(Q_k, \dots, Q_1 | e; h, \Phi^k(s)) &= P_{14}^{k-1}(Q_{k-1}, \dots, Q_1 | e; h, \Phi^{k-1}(s)) \\ &\quad \times P_{12}^k(Q_k - Q_{k-1}, \dots, Q_1 | Q_{k-1} - Q_{k-2}, \dots, Q_1, e; h, \Phi^k(s)) \end{aligned} \quad (24)$$

with

$$P_{14}^1(Q_1|e; h, \Phi^1(s)) = P_{12}^1(Q_1|e; h, \Phi^1(s)) \text{ and } k = 2, 3, \dots, T$$

When P_{14}^T has been calculated, we can sum on the Q 's for $t < T$ to get the probability for total resource change per unit burned area, Q_T , over T years following a fire in e :

$$P_{15}^T(Q_T | e; h, \Phi^T(s)) = \sum_{Q_1} \dots \sum_{Q_{T-1}} P_{14}^T(Q_T, \dots, Q_1 | e; h, \Phi^T(s)) \quad (25)$$

Total resource change vector Q_T^* is calculated by multiplying the resource change per unit area by the fire area, i.e., $Q_T^* = Q_T a$. Then the probabilities for Q_T^* are given by

$$P_{16}(Q_T^* | e; h, \Phi^T(s)) \quad (25a)$$

$$= \sum_t [P_{15}^T(Q_T^* | e; h, \Phi^T(s)) \sum_{bc} P_8(a, \tilde{b}, c | e; h, A_D, A_S, s)]$$

where P_{16} is defined for values of Q_T^* such that (Q_T^*/a) is in the domain of P_{15}^T and a is in the domain of P_8 .

We now have probabilities for total resource changes due to fires in each IPC for years 1 through T , where year 1 is the year of fire occurrence. We must combine these changes and probabilities over all IPC's in the FMS, and then use probabilities for annual numbers of fires to get probabilities for total annual resource changes in the FMS.

Fire Occurrence

Assume a function $P_{17}(e; h, x, A_P, A_f^x, s)$, giving the probability that a fire of type h , in year x , occurs in parameter cell e , given prevention program A_P and fuels treatment program A_f^x through years 1 to x .

The probability of total resource change vector $Q_{T_{xh}}$ over T years, following a fire of type h in year x is

$$P_{18}(Q_{T_{xh}}; h, x, T, \Phi^T(s, x)) \quad (26)$$

$$= \sum P_{16}^T(Q_T^* | e; h, \Phi^T(s)) P_{17}(e; h, x, A_P, A_f^x, s)$$

where A_P , A_f^x , and $\Phi^T(s)$ have been included in $\Phi^T(s, x)$.

Given a total of N_{xh} fires of type h in year x and assuming independence of effects between fires, the distribution of these fires over the $Q_{T_{xh}}$ values is given by

$$P_{19}(N_{Q_{T_{xh}}} | N_{xh}; h, x, T, \Phi^T(s, x)) \quad (27)$$

$$= M(N_{Q_{T_{xh}}} | N_{xh}; P_{18}(Q_{T_{xh}}; h, x, T, \Phi^T(s, x)))$$

where $N_{Q_{T_{xh}}}$ is the number of fires in year x of type h giving resource changes $Q_{T_{xh}}$ over T years. M is a multinomial distribution of the N_{xh} fires over the $Q_{T_{xh}}$ values which have probabilities P_{18} .

Assume a probability distribution $P_{20}(N_{xh}; h, x, A_P, s)$ for N_{xh} , the number of fires of type h in year x with fire prevention program A_P . Then, the total probability per year for $N_{Q_{T_{xh}}}$ is

$$P_{21}(N_{Q_{T_{xh}}}; h, x, T, \Phi^T(s, x)) \quad (28)$$

$$= \sum_{N_{xh}} P_{19}(N_{Q_{T_{xh}}} | N_{xh}; h, x, T, \Phi^T(s, x)) P_{20}(N_{xh}; h, x, A_P, s)$$

where prevention program A_P is included in $\Phi^T(s, x)$ in P_{21} and hereafter.

Let $N_{Q_{T_{xh}}}^j$ represent a vector (j) of numbers of fires over the $Q_{T_{xh}}$ values. Then, the total resource change vector for this number vector is $\tilde{Q}_{T_{xh}}^j$ given by multiplying each component of the resource change vector by the corresponding number of fires. We assume that the effects of individual fires are independent and additive.

$$\tilde{Q}_{T_{xh}}^j = Q_{T_{xh}} N_{Q_{T_{xh}}}^j \quad (29)$$

(Formally $N_{Q_{T_{xh}}}^j$ should be a diagonal matrix here rather than a vector. We assume that the meaning is clear.)

The probability for $\tilde{Q}_{T_{xh}}^j$ is just the probability for fire number vector $N_{Q_{T_{xh}}}^j$:

$$P_{22}(\tilde{Q}_{T_{xh}}^j; h, x, T, \Phi^T(s, x)) \quad (30)$$

$$= P_{21}(N_{Q_{T_{xh}}}^j; h, x, T, \Phi^T(s, x))$$

These probabilities are aggregated and summed over arrangements j to give

$$P_{23}(\tilde{Q}_{T_{xh}}; h, x, T, \Phi^T(s, x)) \quad (31)$$

$$= \sum P_{22}(\tilde{Q}_{T_{xh}}^j; h, x, T, \Phi^T(s, x))$$

$$\tilde{Q}_{T_{xh}}^j \in \tilde{Q}_{T_{xh}}$$

The year of fire occurrence x and the number of years T over which fire effects are accumulated have been treated

as parameters. We now wish to form an expression for resource changes due to fires of type h occurring in years 1 through Z , say. Let $\hat{Q}_{Zh}$ be this vector of changes. Then

$$\hat{Q}_{Zh} = \sum_{x=1}^Z \hat{Q}_{(Z-x+1)xh} \quad (Z-x+1)xh \quad (32)$$

Let $\tilde{Q}_{(Z-x+1)xh}^k$ be a particular set of values for $\tilde{Q}$ over years $x = 1$ to Z in an arrangement k . Then the probability for $\tilde{Q}_{Zh}^k$ resulting from this set of values is

$$P_{24}(\hat{Q}_{Zh}^k; h, Z, \Phi^Z(s, Z)) \quad (33)$$

$$= \prod_{x=1}^Z P_{23}(\hat{Q}_{(Z-x+1)xh}^k; h, x, Z-x+1, \Phi^{Z-x+1}(s, x))$$

Aggregating and summing on arrangement k gives

$$P_{25}(\hat{Q}_{Zh}; h, Z, \Phi^Z(s, Z)) \quad (34)$$

$$= \sum P_{24}(\hat{Q}_{Zh}^k; h, Z, \Phi^Z(s, Z))$$

$$\hat{Q}_{Zh}^k \in \hat{Q}_{Zh}$$

Finally, we sum over fire types h to get

$$\hat{Q}_Z^m = \sum_h \hat{Q}_{Zh}^m \quad (35)$$

for arrangement m . Then the probabilities for sets of values $\hat{Q}_Z^m$ are given for this arrangement by

$$P_{26}(\hat{Q}_Z^m; Z, \Phi^Z(Z)) = \prod_h P_{25}(\hat{Q}_{Zh}^m; h, Z, \Phi^Z(s, Z)) \quad (35a)$$

Summing probabilities on m and aggregating gives

$$P_{27}(\hat{Q}_Z; Z, \Phi^Z(s, Z)) = \sum P_{26}(\hat{Q}_Z^m; Z, \Phi^Z(s, Z)) \quad (36)$$

$$\hat{Q}_Z^m \in \hat{Q}_Z$$

Function P_{27} gives probabilities for the total resource change vector $\hat{Q}_Z$ over Z years. Marginal distributions for individual resource components of $\hat{Q}_Z$ are easily obtained by summing P_{27} over the other components. Expected values are calculated by the usual procedures.

Valuation of Resource Changes

The financial impacts of resource changes due to fire are calculated using a process quite similar to that outlined above for amounts of resource changes. The differences are that suppression costs must be carried through the calculations and that dollar values are discounted to present values.

We pick up the previous calculations at equation (16) where the summation on c is omitted, giving a probability function:

$$P_{28}^t(q_t, c | q_{t-1}, \dots, q_1, e; h, \Phi^t(s)) \quad (37)$$

where $\Phi^t(s)$ is defined in equation 17. Expression (37) gives the joint probability for resource changes per unit burned area in year t following a fire and suppression costs for the

fire, given resource changes for previous years and parameter set e for the fire location.

Let $\hat{v}_t$, be the dollar value vector per unit burned area for resource change vector q_t (Althaus and Mills 1982). Assume a function f_t such that $\hat{v}_t = f_t(q_t)$, and the inverse function F_t giving $q_t = F_t(\hat{v}_t)$.

Discounted changes, due to fire, in the dollar values of resource outputs are assumed to be additive over the years following a fire. Given a set of resource change vectors ($q_1, q_2, \dots, q_T$) over T years, the total present value vector (in year 1) for the sum of these changes using discount rate I is:

$$V_T = \sum_{t=1}^T f_t(q_t)/(1+I)^{t-1} \quad (38)$$

The probability of a particular V_T is just the probability of the corresponding set of q_i . Using P_{28}^1 from equation 37:

$$P_{29}^T(q_T, \dots, q_1, c|e; h, \Phi^T(s)) = \prod_{i=1}^T P_{28}^i(q_i, c|q_{i-1}, q_{i-2}, \dots, q_1, e; h, \Phi^i(s)) \quad (39)$$

In recursive form:

$$P_{29}^k(q_k, \dots, q_1, c|e; h, \Phi^k(s)) = P_{29}^{k-1}(q_{k-1}, \dots, q_1, c|e; h, \Phi^{k-1}(s)) P_{28}^k(q_k, c|q_{k-1}, \dots, q_1, e; h, \Phi^k(s)) \quad (40)$$

for $k = 2, 3, \dots, T$, and with

$$P_{29}^1(q_1, c|e; h, \Phi^1(s)) = P_{28}^1(q_1, c|e; h, \Phi^1(s)) \quad (41)$$

From equation 38,

$$f_i(q_i) = (V_i - V_{i-1})(1+I)^{i-1}$$

so that

$$q_1 = F_1(V_1)$$

and

$$q_i = F_i[(V_i - V_{i-1})(1+I)^{i-1}]$$

for $i \geq 2$. Then we can write

$$P_{30}^k(V_k, \dots, V_1, c|e; h, \Phi^k(s)) = P_{29}^k(F_k[V_k - V_{k-1}](1+I)^{k-1}, \dots, F_1(V_1), c|e; h, \Phi^k(s)) \quad (42)$$

with

$$P_{30}^1(V_1, c|e; h, \Phi^1(s)) = P_{29}^1(F_1(V_1), c|e; h, \Phi^1(s)) \quad (43)$$

Equations 40 and 41 can be written entirely in terms of the V 's, using equations 42 and 43:

$$P_{30}^k(V_k, \dots, V_1, c|e; h, \Phi^k(s)) = P_{30}^{k-1}(V_{k-1}, \dots, V_1, c|e; h, \Phi^{k-1}(s)) P_{28}^k(F_k[(V_k - V_{k-1})(1+I)^{k-1}], c|e; h, \Phi^k(s)) \quad (44)$$

for $k = 2, 3, \dots, T$ and with P_{30}^1 given by equation 43.

After P_{30}^T is calculated, the probabilities are summed over the V_i 's for $i < T$ to give the probability of total dollar impact of resource changes per unit burned area over T years, discounted to the year of the fire:

$$P_{31}^T(V_T, c|e; h, \Phi^T(s)) = \sum_{V_{T-1}} \dots \sum_{V_1} P_{30}^T(V_T, \dots, V_1, c|e; h, \Phi^T(s)) \quad (45)$$

From this point in the calculations, the development parallels that under Fire Occurrence in the previous section from after equation 25 through equation 31. The equivalent of function P_{23} here is

$$P_{32}(\tilde{V}_{Txh}, \tilde{c}_{xh}; h, x, T, \Phi^T(s, x))$$

This function gives the joint probability of resource value change vector $\tilde{V}_{Txh}$ and suppression cost $\tilde{c}_{xh}$ for a fire of type h , where the value changes include the time stream from year x to year $x+T$, discounted to the year of the fire, x .

The resource change values and suppression costs in year x are further discounted to present values:

$$V_{Txh} = \tilde{V}_{Txh}/(1+I)^{x-1} \text{ and } c_{xh} = \tilde{c}_{xh}/(1+I)^{x-1} \quad (46)$$

The corresponding probabilities, using P_{32} are

$$P_{33}(V_{Txh}, c_{xh}; h, x, T, \Phi^T(s, x)) = P_{32}(V_{Txh}(1+I)^{x-1}, c_{xh}(1+I)^{x-1}; h, x, T, \Phi^T(s, x)) \quad (47)$$

Resource change values and suppression costs are all discounted to present values and can now be summed over all years of fire occurrence x . If the sum is through year Z , the number of years for accumulation of values, T , is $Z-x+1$ for each x . Then

$$\tilde{V}_{Zh} = \sum_{X=1}^Z V_{(Z-X+1)xh} \text{ and } \hat{c}_{Zh} = \sum_{X=1}^Z c_{xh} \quad (48)$$

Let $V_{(Z-x+1)xh}^j$ and c_{xh}^j be a particular set of values, in an arrangement j . Then the probability for the corresponding sums is

$$P_{34}(\hat{V}_{Zh}^j, \hat{c}_{Zh}^j; h, Z, \Phi^Z(s, Z)) = \prod_{x=1}^Z P_{33}(V_{(Z-x+1)xh}^j, c_{xh}^j; h, x, Z-x+1, \Phi_x(s, x)) \quad (49)$$

These probabilities are summed over all arrangements j and aggregated along with $\hat{V}^j$ and $\hat{c}^j$ to give

$$P_{35}(\hat{V}_{Zh}, \hat{c}_{Zh}; h, Z, \Phi^Z(s, Z)) = \sum_{\hat{V}_{Zh}^j \in \hat{V}_{Zh}, \hat{c}_{Zh}^j \in \hat{c}_{Zh}} P_{34}(\hat{V}_{Zh}^j, \hat{c}_{Zh}^j; h, Z, \Phi^Z(s, Z)) \quad (50)$$

Resource value change vectors and costs are next summed over fire type h to give

$$\hat{V}_Z = \sum_h \hat{V}_{Zh} \text{ and } \hat{c}_Z = \sum_h \hat{c}_{Zh} \quad (51)$$

If $(\hat{V}_{Zh}^k, \hat{c}_{Zh}^k)$ is a particular arrangement k of values, the probability for the corresponding sum is

$$P_{36}(\hat{V}_Z^k, \hat{c}_Z^k; Z, \Phi^Z(s, Z)) = \prod_{35}(\hat{V}_{Zh}^k, \hat{c}_{Zh}^k; h, Z, \Phi^Z(s, Z)) \quad (52)$$

Summing over arrangements k and aggregating:

$$\begin{aligned} P_{37}(\hat{V}_Z, \hat{c}_Z; Z, \Phi^Z(s, Z)) \\ = \sum_{\substack{\hat{V}_Z^k \in \hat{V}_Z \\ \hat{c}_Z^k \in \hat{c}_Z}} P_{36}(\hat{V}_Z^k, \hat{c}_Z^k; Z, \Phi^Z(s, Z)) \end{aligned} \quad (53)$$

This function gives the probability for present value of suppression costs and value change for each resource type over a planning period of Z years. The "bottom line" FEES model output is "total cost plus net value change" (Mills and Bratten 1982). To get this quantity, the components of $\hat{V}_Z$ must be added to suppression costs $\hat{c}_Z$ and the present value of fixed costs associated with $\Phi^Z(s, Z)$ added to the total.

Let $\hat{V}_{Z1}, \hat{V}_{Z2}, \dots, \hat{V}_{ZN}$ be the components of $\hat{V}_Z$ for resources 1, 2, ..., N. Then, net value change plus suppression cost is

$$\hat{V}_Z = \sum_{i=1}^N \hat{V}_{Zi} + \hat{c}_Z \quad (54)$$

Let $\hat{V}_{Zi}^j$ and $\hat{c}_Z^j$ be a particular arrangement j of values. Then, the probability for the corresponding sum is

$$P_{38}(\hat{V}_Z^j; Z, \Phi^Z(s, Z)) = \prod_{37}(\hat{V}_{Zi}^j, \hat{c}_Z^j; Z, \Phi^Z(s, Z)) \quad (55)$$

Summing over arrangements and aggregating gives

$$P_{39}(\hat{V}_Z; Z, \Phi^Z(s, Z)) = \sum_{\hat{V}_Z^j \in \hat{V}_Z} P_{38}(\hat{V}_Z^j; Z, \Phi^Z(s, Z)) \quad (56)$$

Let the present value, fixed cost of the programs in $\Phi^Z(s, Z)$ be $C(s, Z)$. Then, adding these costs to $\hat{V}_Z$ gives total cost plus net value change

$$D_Z = \hat{V}_Z + C(s, Z)$$

and

$$P_{40}(D_Z; Z, \Phi^Z(s, Z)) = P_{39}(D_Z - C(s, Z); Z, \Phi^Z(s, Z)) \quad (57)$$

The expected value for D_Z is

$$\bar{D}_Z = \sum_{D_Z} D_Z P_{40}(D_Z; \Phi^Z(s, Z)) \quad (58)$$

OUTLOOK FOR MODEL IMPLEMENTATION

The overall probability model has been defined in functional mathematical form. The independent and dependent variables and parameters for the various modules have been specified to the degree possible.

Implementation of the model could proceed using various methods and levels of completeness for the modules. For example, the initial attack module could be constructed as an analytical model, assuming the availability of data or physical models for the required probability functions. Alternatively a Monte-Carlo procedure could be used to calculate the probabilities using a deterministic model with stochastic inputs.

Some of the functions and relationships may require interpretation to fit unforeseen situations. In particular, the escaped fire analysis procedures and some of the fire effects analyses will become more clearly defined as additional research efforts proceed. The overall model has been defined in sufficiently general terms that it should readily adapt to the final forms taken by these and the other modules.

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Bratten, Frederick W. **Probability model for analyzing fire management alternatives: theory and structure.** Gen. Tech. Rep. PSW-66. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 1982. 11 p.

A theoretical probability model has been developed for analyzing program alternatives in fire management. It includes submodels or modules for predicting probabilities of fire behavior, fire occurrence, fire suppression, effects of fire on land resources, and financial effects of fire. Generalized "fire management situations" are used to represent actual fire management areas. The model serves as the framework for the Fire Economics Evaluation System now being designed at the Pacific Southwest Forest and Range Experiment Station. Outputs from the model will consist of probability distributions for changes in resources caused by fire and for a measure of economic efficiency over a planning period.

Retrieval Terms: Fire Economics Evaluation System, fire economics, fire management, probability model