Effect of Forming Stresses On the Strength of Curved Laminated Beams Of Loblolly Pine

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Abstract

Curvature-stress factors reflecting the effect of forming stresses in producing curved beams of thin vertical-grain laminations of clear wood have been determined for loblolly pine. Strength retention of curved beams decreases with increasing severity of curvature but not to the degree suggested by the Wilson equation commonly used in design. Curved beams loaded on the convex face (stresses reversed) are stronger than beams loaded on the concave face (stresses additive) and curvature-stress factors applicable to these two conditions are recommended. Wood quality characteristics favoring higher retention of curved-beam strength are suggested.

Laminated wood has an outstanding advantage over sawn timber in that it can easily be adapted to the production of curved structural forms. Current design practice (1), however, imposes rather severe restrictions on the ratio of lamination thickness to radius of curvature (t/R.) One of these limitations is that t/R shall not exceed 1/100 for hardwoods and southern pine or 1/125 for other softwoods. Another is that for the curved portion of structural members, the allowable unit stress in bending must be modified by multiplication by a curvature-stress factor:[1]

$$1 - 2000\ (t/R)^2$$

where t = thickness of individual lamination
R = radius of curvature

These limitations are based on recognition of the fact that stresses of considerable magnitude are developed in bending individual laminae in the laminating process and that these stresses increase as the ratio of lamination thickness to radius of curvature increases. For certain species these limitations have been shown to be unnecessarily restrictive with respect to the maximum t/R ratio for thin laminations of clear wood, and the reduction formula developed by Wilson (9) at the U.S. Forest Products Laboratory, Madison, Wis., has been shown to be conservative for the curvature-stress factor at the higher t/R ratios (4, 5).

Work by Kostukevich and Wangaard (5) and others (6, 7, 8) indicates that the response to pre-stress varies considerably among different species and also that the direction of loading, *i.e.*, whether the bending moment increases or decreases the radius of curvature, has an effect. The latter effect might be anticipated, as convex loading (increasing radius) acts to reverse the prestress in the outermost fibers, whereas concave loading (decreasing radius) has an additive effect. The response is

[1]Ratio of curved-beam strength to that of a matched straight beam that has not been subjected to pre-stress.

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also different when considering different levels of stress such as that measured at proportional limit or maximum load.

Objectives

The primary purpose of this study was to derive curvature-stress factors for thin vertical-grain laminations of clear wood of loblolly pine (*Pinus taeda*) at various degrees of curvature, and also to determine the influence of recognizable qualities of southern pine on curvature-stress factor. Density, rate of growth, stiffness, and other properties were used as indices of wood quality.

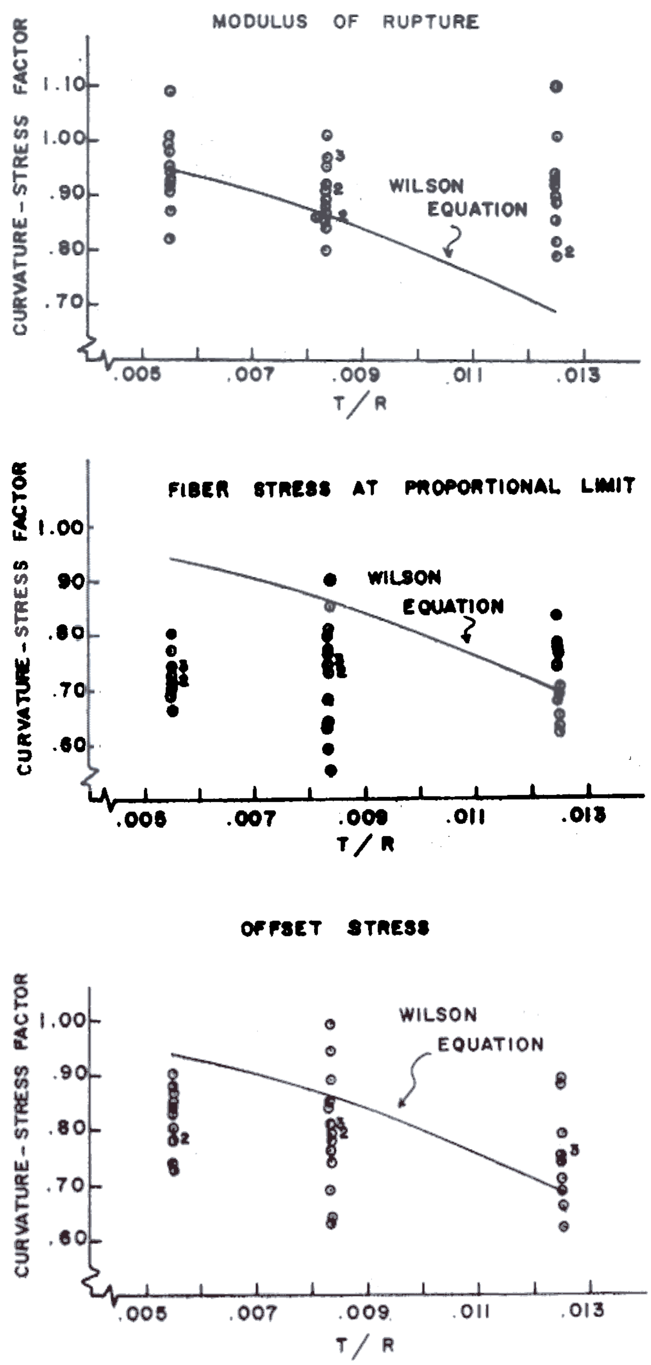

Figure 1. — Distribution of curvature-stress factors for individual concave beams at different t/R ratios.

Table 1. — THICKNESS AND NUMBER OF LAMINAE AND TOTAL BEAM DEPTH CORRESPONDING TO VARIOUS t/R RATIOS. (ALL BEAMS FORMED TO A UNIFORM INNER RADIUS OF 40 IN.)

Individual lamination thickness (in.)	No. of laminae	Total beam depth (in.)	t/R ratio for curved beams
0.222	9		1/180
.333	6	1.998	1/120
.500	4	2.000	1/80

Preparation of Material

This study was concerned with laminating matched sets of straight and curved beams at t/R ratios of 1/180, 1/120, and 1/80[1] and testing them in static bending, loading one curved beam in each set on the convex face and another on the concave face. Four sets of beams (a set consisting of one straight, one concave-, and one convex-loaded beam) were tested at each t/R ratio for three density classes employing a broad range of stiffnesses in each density class.

One hundred and eight defect-free beams were made up having material of uniform density and stiffness throughout their depth. Another 12 beams, also free from defect, were fabricated with a density and stiffness gradient (high density and stiffness in the outer laminations, low density and stiffness in the interior laminae) making a total of 120 beams. The beams having a density and stiffness gradient were included in the final analysis since the results showed no marked difference in curvature-stress factor.

Table 1 shows the thickness and number of individual laminations corresponding to the t/R ratios used in the study.

Material for the study consisted of partially seasoned flat-sawn loblolly pine from Louisiana in the form of 3- by 6-inch planks 12 feet in length. Care was taken in selection of material to use only clear straight-grained cuttings. The material contained considerable amounts of blue stain that apparently developed in transit and it was impossible to eliminate this discoloration from all cuttings, but close examination of the beams after failure gave no indication that blue stain had in any way affected their strength properties.

Following a preliminary separation into three density classes, the flatsawn planks were ripped oversize into vertical-grain laminae according to the assigned t/R ratio. Laminae were matched in either the lengthwise or edgewise direction and only adjacent pieces were used as matched material. Rough stock for the straight-beam assemblies was cut to 33-inch lengths and curved-beam stock was trimmed to 45-inch lengths. All laminae were 2-1/2-inches wide. Following rough cutting all the

[1]Ratios obtained by varying the thickness of lamination at a constant bending radius of 40 inches.

laminating stock was brought to moisture equilibrium in a humidity-controlled room maintained at 12 percent equilibrium conditions.

Upon attainment of constant weight each lamination was edged to 2-inch width and planed to 0.050 inch greater than its final thickness.

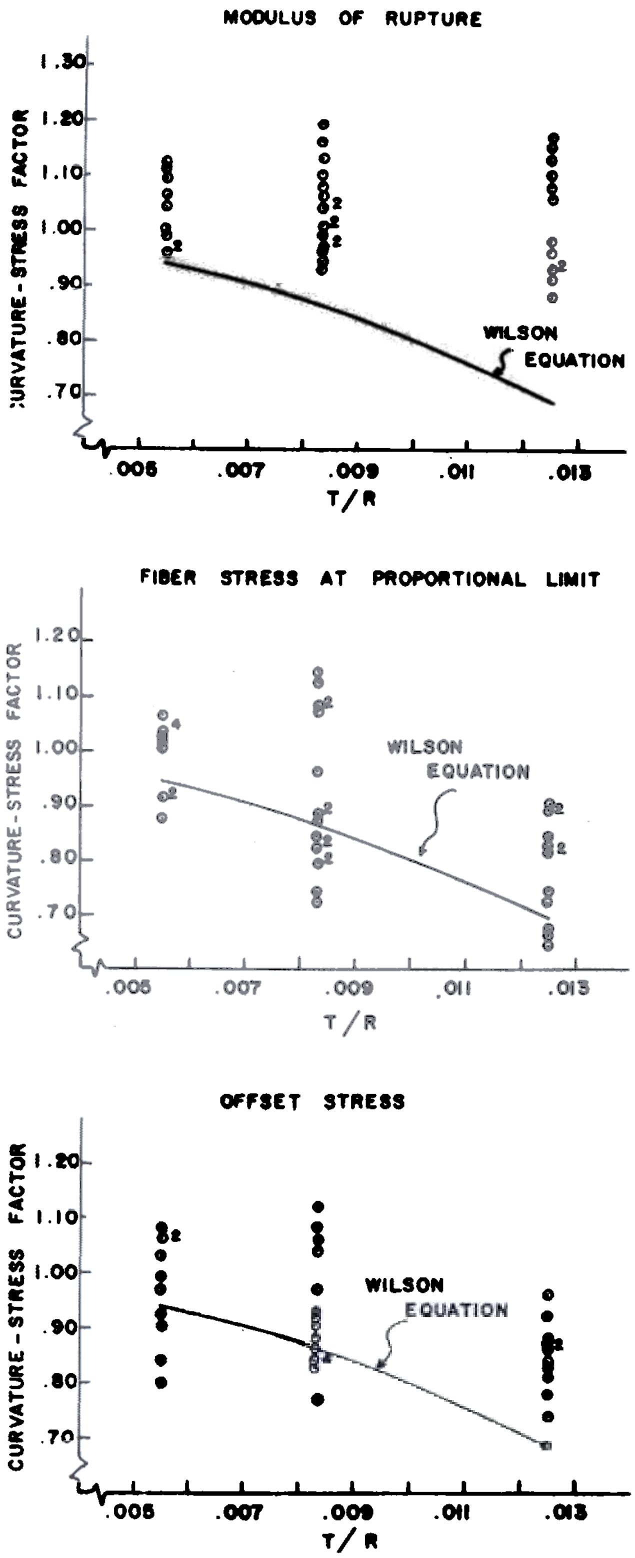

Figure 2. — Distribution of curvature-stress factors for individual convex beams at different t/R ratios.

Measurement of Stiffness

Stiffness was used as a means of classifying the laminating stock. One lamination from each matched set of three (intended for incorporation in straight, concave- and convex-loaded beams, respectively) was tested for stiffness. A preliminary check showed fair uniformity throughout a set matched in this way. The measuring apparatus consisted of a jig with a 40-inch span across which the individual lamination was placed. A zero deflection reading was taken and then a bag of lead shot of known weight was center-loaded on the piece. Measurement of the instantaneous deflection under a load well within proportional limit gave a reasonable estimate of modulus of elasticity.

Table 2. — CURVATURE-STRESS FACTOR FOR INDIVIDUAL BEAMS OF LOBLOLLY PINE.

	Concave-loaded Beams			Convex-loaded Beams		
Straight Beam No.	MR	FSPL	Offset Stress	MR	FSPL	Offset Stress
1	0.79	0.70		0.98	0.82	0.87
4	......	......		1.17	.81	.81
7	.90	.68		1.10	.72	.86
10	.94	.62		.93	.90	.83
13	.82	.74		1.08	.81	.88
16	1.10	.69		1.15	.89	.87
19	.79	.63		.91	.67	.74
22	.93	.77		1.06	.66	.92
25	1.01	.83		1.13	.89	.96
28	.92	.76		.93	.84	.84
31	.86	.78		.88	.64	.69
34	.89	.65		.96	.74	.78
37	0.86	.064		0.93	0.79	0.86
40	.91	.74		.97	.82	.84
43	.87	.74		.99	1.08	1.04
46	.95	.90		1.13	1.12	1.06
49	.88	.77		1.10	.96	.97
52	.92	.85		.97	.88	.84
55	.97	.80		1.04	1.07	.93
58	.80	.59		1.16	1.14	1.12
61	.84	.63		1.19	1.08	1.08
64	.85	.55		1.01	.87	.84
67	.85	.76		1.06	.84	.88
70	.97	.68		1.08	.72	.83
x-7	1.01	.81		1.01	.79	.90
x-10	.89	.74		.94	.82	.84
x-13	.91	.73		1.04	.88	.92
x-16	.97	.73		.96	.74	.77
73	0.87	0.74		0.99	0.91	0.84
76	.95	.72		1.12	1.02	.99
79	.99	.80		1.06	1.03	.97
82	.93	.74		1.04	1.00	.92
85	.82	.69		1.00	1.01	1.03
88	.94	.70		.96	.91	.90
94	1.09	.77		1.34	1.06	1.06
97	.91	.66		.96	.87	.80
100	.92	.74		1.11	1.03	1.17
103	1.01	.71		1.21	1.03	1.06
106	.98	.72		1.09	1.03	1.08

Fabrication of Beams

Immediately before gluing, each lamination was planed to final thickness. A room-temperature setting phenol-resorcinol-formaldehyde adhesive (Penacolite G-4422)[3] designed specifically for softwood lumber laminating was used. Manufacturer's recommendations were followed in using the adhesive. Pressure of approximately 150 psi was applied to the low-density material whereas 200 psi was used with the higher density assemblies. Pressure was applied to the straight beam assemblies by means of a jack-screw press with the aid of a calibrated torque wrench. Curved-beam pressure was applied with bar clamps until an equivalent "squeeze-out" of adhesive was observed. This practice proved to be successful since no glue-line failures were observed throughout the testing of all beams. The beam assemblies remained under

Table 3. — INDEPENDENT VARIABLES USED IN REGRESSION ANALYSIS. (ALL VARIABLES REPRESENT STRAIGHT-BEAM PROPERTIES).

t/R	—	Ratio of thickness of individual laminae to radius of curvature.
SP x 10^3	—	Strain at proportional limit (fiber stress at proportional limit/modulus of elasticity).
RW	—	Ring width (in.).
(W/R) x 10^4	—	Work to maximum load (in-lb/cu. in.)/modulus of rupture.
E x 10^{-3}	—	Modulus of elasticity.
D	—	Effective specific gravity.
FS/R	—	Fiber stress at proportional limit/modulus of rupture.
SD x 10^3	—	Departure strain (strain at maximum load minus ratio of modulus of rupture over modulus of elasticity).
(E/D) x 10^{-3}	—	Modulus of elasticity/effective specific gravity.
t/R x D	—	Cross product.
t/R x FS/R	—	Cross product.
t/R x E x 10^{-3}	—	Cross product.
t/R x SP x 10^3	—	Cross product.
t/R x SD x 10^3	—	Cross product.
t/R x (W/R) x 10^4	—	Cross product.

pressure overnight at room temperature and were then replaced in the humidity-controlled room maintained at 12 percent equilibrium moisture content conditions. All beams were conditioned for 3-4 weeks before testing.

Testing Procedure

Following the conditioning period, the excess glue was scraped off and the beams trimmed to an appropriate length for testing over a 28-inch span. All beams were tested in general conformance to ASTM Standards (2) for center-loaded beams. Loads were applied by means of a Baldwin hydraulic testing machine. Rate of platen travel was 0.10 inch per minute with deflection measured at midspan to the nearest 0.001 inch and load to the

[3]Acknowledgement is made to Koppers Co., Inc., Pittsburgh, Pa. for supplying the adhesive used in this study.

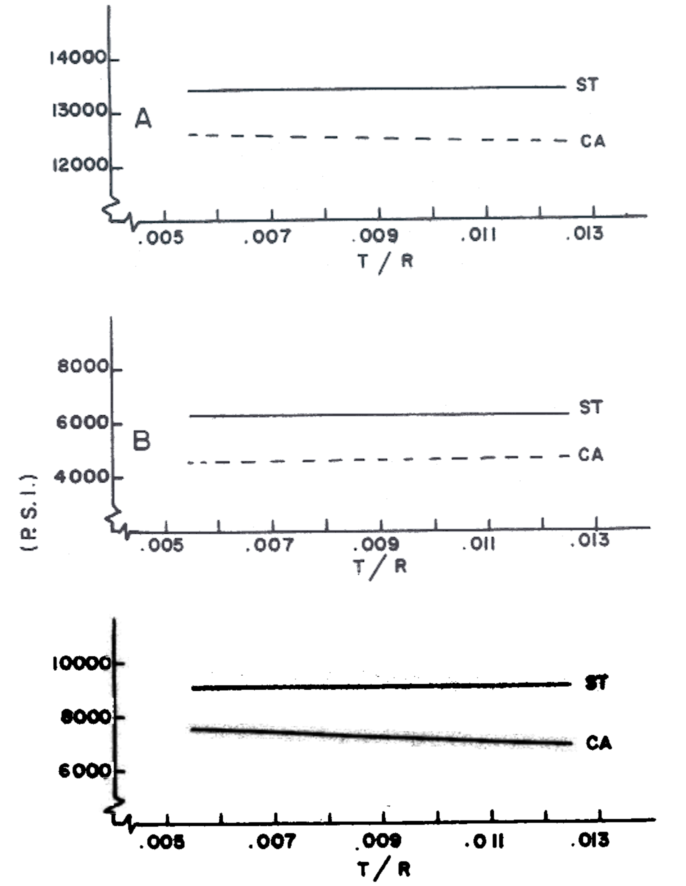

Figure 3. — Effect of t/R on strength of concave-loaded curved beams (ST, straight beams; CA, concave beams).
A. Modulus of rupture.
B. Fiber stress at proportional limit.
C. Offset stress.

nearest 10 pounds. In the case of the straight beams, deflection measurements were taken to failure.

Immediately following static bending, two cross-sectional samples were removed from each beam near the point of failure. Moisture content was determined on one sample and expressed as the average for the entire cross section. Individual laminations were separated by sawing the other sample along the glue lines, and specific gravity determinations based on ovendry weight and ovendry volume were made on each individual lamination. Effective specific gravity was expressed according to the method outlined by Finnorn and Rapavi (3):

$$G = \frac{G_i\, I_i}{\;}$$

where G = effective specific gravity of the beam.

G_i = average specific gravity as determined for each pair of laminae equidistant from the neutral axis.

I_i = moment of inertia of each of the above pairs of laminae.

$I = \frac{bd^3}{12}$ = moment of inertia of beam.

Table 4. — MULTIPLE-REGRESSION EQUATIONS FOR STRAIGHT LAMINATED BEAMS (PSI).[1]

Equation no.	Variables	Coefficients		Significance: Overall by F-test	Significance: Individual variable by "t" test	Cumulative regression coefficient	R^2 (X 100)	Standard error of residual psi
				Modulus of Rupture				
		a	−593					
1	$E \times 10^{-3}$	b_1	3.141		6.51**	0.900		
	D	b_2	13741	167.41**	5.89**	.950	90%	734
		a	3437					
2	$E \times 10^{-3}$	b_1	5.329	157.20**	12.54**	.900	81%	1014
		a	−1622					
3	D	b_1	25448	138.10**	11.75**	.888	79%	1068
				Fiber Stress at Proportional Limit				
		a	−806					
4	$E \times 10^{-3}$	b_1	1.810		4.96**	0.866		
	D	b_2	6343	79.98**	3.59**	.903	82%	555
		a	1054					
5	$E \times 10^{-3}$	b_1	2.820	111.24**	10.55**	.866	75%	638
		a	−1399					
6	D	b_1	13090	82.62**	9.09**	.831	69%	710
				Offset Stress				
		a	−335					
	$E \times 10^{-3}$	b_1	2.867		8.25**	0.926		
	D	b_2	6890	168.66**	4.10**	.950	90%	528
		a	1686					
	$E \times 10^{-3}$	b_1	3.965	224.58**	14.98**	.926	86%	631
		a	−1274					
9	D	b_1	17577	95.67**	9.78**	.849	72%	886

[1] Equations of the form $Y = a + b_1 x_1 + b_2 x_2$

** highly significant at 1% level.

Strength and stiffness (stiffness only in the case of straight beams) were calculated by standard equations (5).

A problem was encountered in selecting the proportional limit from the load-deflection curves particularly in the case of curved beams. Departure from the modulus line was typically very gradual, making it difficult to choose a meaningful proportional limit. Another method was consequently used to avoid this difficulty, still measuring a meaningful stress in the neighborhood of proportional limit. This stress value, designated as "offset stress," was obtained by drawing a line parallel to the straight-line portion of the load-deflection diagram and offset from it by 0.020 inch deflection (equivalent to .03 percent unit strain). The load at which this line intersected the load-deflection curve was called the offset load and "offset stress" was calculated using the standard flexure formula.

Curvature-stress factor was expressed as the ratio of curved-beam strength to the matched straight-beam strength after adjusting for differences in effective specific gravity. In most cases the adjustments were small indicating that matching in terms of specific gravity was fairly good.

All but 12 beams were fabricated with all laminae in a set of straight, concave, and convex beams of approximately the same stiffness and density. The straight beams in this group averaged 0.58(0.46-.76) in effective specific gravity, 6,230 psi (4,030-9,040) in fiber stress at proportional limit, 13,350 psi (8,280-18,770) in modulus of rupture, 9,020 psi (5,890-13,260) in offset stress, and 1,840,000 psi (840,000-2,660,000) in modulus of elasticity. Variation within the species was well represented. The 12 beams assembled with stiffness and density gradients from outer to inner laminae with higher stiffness and density in the outer laminae ranged in modulus of elasticity from approximately 3,000,000 psi in the face laminae to approximately 1,000,000 in the innermost laminae. The effective modulus of elasticity for the straight beams in this group ranged from 2,190,000 psi to 2,320,000 psi.

Table 2 gives the curvature-stress factors calculated for each curved beam. A check on the beams with a stiffness gradient (nos. X-7 to X-16) showed them to be within the range of values for curvature-stress factor in similarly loaded beams without such a gradient. This similarity was the basis for retaining these 12 beams in the analysis assuming that the observed stiffness gradients had no effect on curvature-stress factor.

Figures 1 and 2 illustrate the distribution of curvature-stress factors for individual beams at each t/R ratio. Figure 1 shows the distribution for modulus of rupture, fiber stress at proportional limit, and offset stress for concave-loaded beams. The same properties are shown for convex-loaded beams in Figure 2. The curve of the Wilson equation is included in each figure as a means of comparing these data with the currently used reduction equation.

The data presented in Figures 1 and 2 show that the effect of increasing t/R is not so pronounced as suggested by the Wilson equation, and that convex-loaded curved beams retain greater strength than concave-loaded beams of comparable quality.

Regression Analysis of Results

The data were treated in two separate analyses. Multiple-regression analysis was first conducted using psi values for both straight and curved beams to determine the effects of t/R and quality parameters on actual beam strength. The second analysis was based on curvature-stress factors from Table 2. The first analysis will be referred to as the "Psi Method" and the second as the "Ratio Method." All computations were carried out by means of an IBM 7090-7094 computer employing a standard program for multiple regression.

Psi Method

Multiple-regression analysis was first used to predict the strength of curved and straight beams. Fifteen independent variables (eight quality variables obtained from the straight beams, t/R, and six cross products involving t/R and certain quality variables as shown in Table 3) were used in the original predicition of individual beam strength. Most of these variables could be left out of the prediction equation without losing significance. Dropping out those variables that contributed the least in accounting for strength variation, final prediction equations were obtained containing only two to four variables. Tables 4, 5, and 6 include the final equations for the prediction of the strength of straight,

Table 5. — MULTIPLE-REGRESSION EQUATIONS FOR CONCAVE-LOADED CURVED BEAMS (PSI).

Equation no.	Variables	Coefficients		Significance: Overall by F-test	Significance: Individual variable by "t" test	Cumulative regression coefficient	R^2 (X 100)	Standard error of residual psi
		a	—945					
10	t/R	b_1	—25931		0.483	0.041		
	$Ex10^{-3}$	b_2	1.617		2.51*	.803		
	D	b_3	17663	56.23**	6.16**	.912	83%	874
		a	4193					
11	t/R	b_1	—79235		1.04	.041		
	$Ex10^{-3}$	b_2	4.683	31.84**	7.97**	.803	64%	1253
		a	—1371					
12	t/R	b_1	—2411		.042	.041		
	D	b_2	23218	70.55**	11.86**	.895	80%	938
				Fiber Stress at Proportional Limit				
		a	—3954					
	t/R	b_1	7503		0.234	0.170		
	$SPx10^3$	b_2	1139		3.67**	.262		
	$Ex10^{-3}$	b_3	2.083		4.51**	.874		
	D	b_4	1182	27.07**	.594	.875	76%	478
		a	540					
14	t/R	b_1	37958		1.00	.170		
	$Ex10^{-3}$	b_2	1.975	23.99**	6.75**	.760	58%	624
		a	—1371					
15	t/R	b_1	69486		1.93	.170		
	D	b_2	9077	27.96**	7.20**	.784	61%	596
				Offset Stress				
		a	—2742					
16	t/R	b_1	—100542		2.31*	0.106		
	$SPx10^3$	b_2	1078		2.56*	.162		
	$Ex10^{-3}$	b_3	2501		3.99**	.873		
	D	b_4	4109	28.88**	1.52	.882	78%	649
		a	2380					
	t/R	b_1	—80745		1.64	.106		
	$Ex10^{-3}$	b_2	2.918	29.79**	7.65**	.794	63%	813
		a	—438					
	t/R	b_1	—34179		0.737	.106		
	D	b_2	13400	35.60**	8.37**	.819	67%	767

* significant at 5% level.

** highly significant at 1% level.

concave-, and convex-loaded beams, respectively. The variables used are indentified in Table 3.

The results obtained from the "Psi method" are shown graphically in Figures 3 and 4.

Modulus of Rupture (Concave-loaded)

Figure 3,A, derived from Equations 1 and 10 (Tables 4 and 5), involves modulus of elasticity (E) and effective specific gravity (D) for straight beams, and t/R in addition to E and D for concave-loaded beams. The lines were plotted by varying t/R and holding the other independent variables at their mean values. These mean values were 1,911,000 psi for E and 0.60 for D. The broken line for concave (CA) beams indicates that t/R did not show a significant effect on modulus of rupture even though the equation was highly significant and accounted for 83 percent of the total variation in curved-beam strength. Throughout this paper a broken line will be used to represent a non-significant relationship with t/R.

Equations 2 and 11 relate both straight- and concave-beam strength to modulus of elasticity. Equations 3 and 12 similarly relate beam strength to density. The dependence of modulus of rupture of both curved and straight beams on E and D is clearly shown in these equations. Nonsignificant trends were shown with t/R even though the equations were highly significant and accounted for 64 and 80 percent of the total variation in curved-beam strength, respectively.

Fiber Stress at Proportional Limit (Concave-loaded)

Figure 3,B represents Equations 4 and 13 (Tables 4 and 5) and shows a nonsignificant trend with t/R in the curve for concave beams. In plotting these curves, E, D, and strain at proportional limit (SP) were held at their mean values. The mean value for SP was 0.00338. The broken line is essentially horizontal and indicates no effect of t/R on the strength of concave beams at this level of stress.

Equations 5 and 14 show the effect of E and Equations 6 and 15 show the effect of density on proportional limit

Table 6. — MULTIPLE-REGRESSION EQUATIONS FOR CONVEX-LOADED CURVED BEAMS (PSI).

Equation no.	Variables	Coefficients		Significance: Overall by F-test	Significance: Individual variable by "t" test	Cumulative regression coefficient	R^2 (X 100)	Standard error of residual psi
		a	−1216					
19	t/R	b_1	−66893		0.989	0.145		
	$E \times 10^{-3}$	b_2	1.338		1.78	.774		
	D	b_3	22507	49.85**	6.17**	.899	81%	1141
		a	5805					
20	t/R	b_1	−112691		1.18	.145		
	$E \times 10^{-3}$	b_2	4.914	26.97**	7.21**	.774	60%	1624
		a	−1741					
21	t/R	b_1	−58812		.846	.145		
	D	b_2	27521	68.04**	11.51**	.889	79%	1174
			Fiber Stress at Proportional Limit					
		a	1027					
22	t/R	b_1	−113213		2.56*	0.316		
	$E \times 10^{-3}$	b_2	1.048		2.13*	.743		
	D	b_3	6133	19.27**	2.57*	.789	62%	745
		a	2941					
23	t/R	b_1	−125693		2.66*	.316		
	$E \times 10^{-3}$	b_2	2.022	22.14**	6.02**	.743	55%	801
		a	616					
24	t/R	b_1	−106885		2.31*	.316		
	D	b_2	10059	24.23**	6.33**	.757	57%	781
			Offset Stress					
		a	901					
25	t/R	b_1	−152784		3.13**	0.328		
	$E \times 10^{-3}$	b_2	1.085		2.00*	.783		
	D	b_3	11297	34.30**	4.29**	.864	75%	822
		a	4425					
26	t/R	b_1	−175771		2.97**	.328		
	$E \times 10^{-3}$	b_2	2.880	28.49**	6.85**	.783	61%	1002
		a	475					
27	t/R	b_1	−146232		2.88**	.328		
	D	b_2	15362	45.64**	8.81**	.847	72%	857

* significant at 5% level.

** highly significant at 1% level.

Table 7. — CORRELATIONS BETWEEN INDEPENDENT VARIABLES USED FOR ESTIMATING CURVATURE-STRESS FACTORS FOR CURVED BEAMS.[1]

	t/R	SPx10³	R.W.	W/Rx10⁴	Ex10⁻³	D	FS/R	SDx10³	E/Dx10⁻³
t/R		0.255	−0.158	0.022	0.026	−0.066	0.363	0.011	
SPx10³		1.000	.170	.247	−.352	.062	.492	.040	
R.W.			1.000	.161	−.454	−.247	−.220	.146	
W/Rx10⁴				1.000	−.131	.074	−.190	.915	
Ex10⁻³				....	1.000	.768	.252	−.079	
D				....	....	1.000	.183	.022	
FS/R				....	....	....	1.000	−.202	
SDx10³				....	....	....	....	1.000	
E/Dx10⁻³					......	....	....	....	

[1]To be significant at the 1 and 5% probability levels, correlation coefficients must be greater than 0.418 and 0.325, respectively.

strength. The strength of both straight and curved beams was positively influenced by E and D.

Offset Stress (Concave-loaded)

Figure 3,C represents Equations 7 and 16 (Tables 4 and 5) with all variables except t/R held at their means. Equation 16 indicates that t/R had a significant effect on the strength of curved beams after the variation due to the other three variables was taken out. However, t/R was not significant when used only with E (Equation 17) or with D (Equation 18).

Modulus of Rupture (Convex-loaded)

Figure 4,A illustrates Equations 1 and 19 when E and D are held at their mean values and t/R is varied. No significant effect of t/R was found at the level of modulus of rupture for convex beams, but the interesting thing about this graph is that the convex beams at modulus of rupture were stronger than the straight beams over the entire range of t/R.

Again the positive effects of E and D on the strength of both straight and curved beams are clear from Equations 2 and 20 and 3 and 21, respectively.

Fiber Stress at Proportional Limit (Convex-loaded)

Figure 4,B, derived from Equations 4 and 22 (with E and D held at their mean values), illustrates a significant trend with t/R for convex beams at this level of stress. The curve for convex beams lies below the line for straight beams showing that *FSPL* is more sensitive than modulus of rupture to increasing pre-stress as indicated by increasing t/R.

Equations 5 and 23 illustrate the positive effect of modulus of elasticity on strength of straight and convex beams at this level of stress. Equations 6 and 24 show a similar positive effect of density.

Offset Stress (Convex-loaded)

Equations 7 and 25 are shown graphically in Figure 4,C based on mean values for E and D. The solid line (CX) indicates a significant trend of decreasing curved-beam strength with increasing t/R.

Positive effects on straight and curved beam strength are shown by E and D in Equations 8 and 26 and in Equations 9 and 27, respectively. High E and D favor high strength beams, both straight and curved.

Prediction of Curved and Straight Beam Strength

The final multiple-regression equations appearing in Tables 4, 5, and 6 were highly significant and, with the exception of Equation 22, accounted for more than 75 percent of the total variation in the strength properties of straight and curved beams. The high degree of accountability of E and D for straight-beam properties (82-90 percent) and the low standard error of residuals are of interest from the standpoint of non-destructive testing.

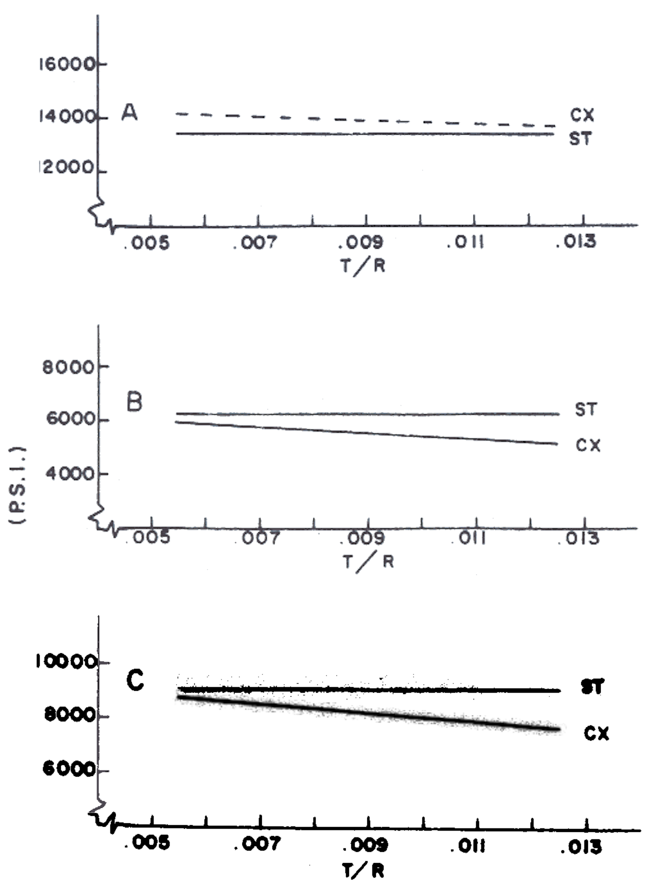

Figure 4. — Effect of t/R on strength of convex-loaded curved beams (ST, straight beams; CX, convex beams).
A. Modulus of rupture.
B. Fiber stress at proportional limit.
C. Offset stress.

For each level of stress, equations for straight beams are shown in Table 4 based on modulus of elasticity alone and on density alone. Equations for curved beams (Tables 5 and 6), involving t/R together with modulus of elasticity and t/R together with density, are also shown for each level of stress. Combined with t/R, density is the better predictor for curved-beam strength at every level of stress. The higher R^2 values indicate that more variation was accounted for by density than by modulus of elasticity. The opposite was true in the case of straight beams. Modulus of elasticity in simple regressions accounted for more variation in straight-beam strength than did density.

Ratio Method

In the ratio method the data were analyzed directly as ratios of curved-to straight-beam strength (curvature-stress factors) instead of as strength values. The 15 variables shown in Table 3 were also used in this analysis and, as in the psi method, variables that contributed the least in accounting for variation were dropped from the final multiple-regression equations. Some of the "independent" variables used to predict curvature-stress factor were correlated with one another as shown in Table 7. When such correlations exist, it is not possible to isolate the independent influence of an individual variable by holding the other variables in the equation constant. What can be shown by this procedure is the "added" effect of any one variable after the effect of the other variables in the equation has been considered.

The more meaningful results of the analysis by the ratio method are shown in Tables 8 and 9 and in Figures 5-6. The tables include the constants and coefficients for the final prediction equations for concave- and convex-loaded beams. The level of significance and the standard error of the residual are shown for each equation. The significance of the contribution of individual variables in combination with other specified variables is shown by the conventional "t" test, which evaluates the variable considered as the final term in the regression equation; *i.e.*, after the variation due to the other variables has been taken out.

From Tables 8 and 9 several variables were significant by the "t"-test. In the following discussion the added effect of variables shown to be significant is considered when the other variables are held at their mean values.

Modulus of Rupture (Concave-loaded)

Equation 28 in Table 8 is shown graphically in Figure 5,A. Each variable except t/R was held at its mean and t/R was allowed to vary. The mean for each independent variable is shown in Table 8. The broken line indicates that the effect of t/R was not significant. (See also Fig. 3,A.) Although the overall regression was significant at the 5 percent level, it accounted for only 31.4 percent of the total variation in curvature-stress factor. The quality of the prediction is, however, indicated by the low value of 0.066 for the standard error of residual. The predicted curvature-stress factor for modulus of rupture of convex-loaded beams is within ±0.066 of the actual value in two beams out of three. The Wilson equation is also shown in Figure 5,A to permit comparing the

Table 8. — MULTIPLE REGRESSION EQUATIONS FOR CURVATURE-STRESS FACTOR FOR CONCAVE BEAMS.[1]

Equation no.	Variables	Items	Coefficients		Overall regression by F-test	Individual variables by "t" test	Cumulative regression coefficient R	R^2 (x100)	Standard error of residual
			a						
28	1	t/R	b_1			1.27	0.196		
	2	$SD\times10^3$	b_2			2.02*	.331		
	3	FS/R	b_3			1.98	.404		
	4	R.W.	b_4			0.94	.479		
	5	$(E/D)\times10^{-3}$	b_5			2.00*	.561	31.4%	
			a	0.801					
29	1	t/R	b_1	-4.771		0.425	0.075		
	2	$SD\times10^3$	b_2	.00710		.705	.182		
	3	$E/D\times10^{-3}$	b_3	-3.06×10^{-5}		.898	.235		
	4	$t/R\times D$	b_4	5.874	.510	.331	.241	5.8%	
					Offset Stress				
			a	1.140					
30	1	t/R	b_1	-10.314		2.17*	0.324		
	2	$SP\times10^3$	b_2	$-.1528$		2.17*	.385		
	3	$SD\times10^3$	b_3	0.0204		2.06*	.453		
	4	FS/R	b_4	1.491		2.99**	.487		
	5	$E/D\times10^{-3}$	b_5	-1.679×10^{-4}	4.34**	2.99**	.636	40.4%	0.071

[1] Mean values for independent variables were as follows: SD, 0.00441; FS/R, 0.471; R. W., 0.19; E/D, 3,202,000; SP, 0.00338; D, 0.60.

* significant at 5% level.

** highly significant at 1% level.

Table 9. — MULTIPLE-REGRESSION EQUATIONS FOR CURVATURE-STRESS FACTOR FOR CONVEX BEAMS[1]

Equation no.	Variables	Items	Coefficients		Significance: Overall regression by F-test	Significance: Individual variables by "t" test	Cumulative regression coefficient R	R^2 (x100)	Standard error of residual
				Modulus of Rupture					
			a	1.902					
31	1	t/R	b_1	−28.717		1.51	0.212		
	2	$E/D \times 10^{-3}$	b_2	-1.35×10^{-4}		4.22**	.450		
	3	$SD \times 10^{3}$	b_3	0.02425		0.982	.612		
	4	$W/R \times 10^{4}$	b_4	−.05517		2.38*	.678		
	5	t/RxFS/R	b_5	44.108	6.16**	1.22	.695	48.3%	0.075
				Fiber Stress at Proportional Limit					
			a	1.193					
32	1	t/R	b_1	−27.900		4.97**	0.609		
	2	$E/D \times 10^{-3}$	b_2	4.418×10^{-6}		0.100	.627		
	3	$W/R \times 10^{4}$	b_3	−.02388		2.08*	.670		
	4	R.W.	b_4	0.688	10.81**	2.93**	.748	56.0%	0.095
				Offset Stress					
			a	−1.955					
33	1	t/R	b_1	−16.469		3.30**	0.512		
	2	$E \times 10^{-3}$	b_2	-1.617×10^{-3}		3.46**	.564		
	3	D	b_3	5.266		3.48**	.618		
	4	$E/D \times 10^{-3}$	b_4	8.93×10^{-4}		3.18**	.675		
		R.W.	b_5	0.558	7.45**	2.28*	.728	53.0%	0.082

[1] Mean values for independent variables were as follows: E/D, 3,164,000; SD, 0.00436; W/R, 0.000846; FS/R, 0.472; R. W., 0.20; E, 1,884,000; D, 0.59.

* Significant at 5% level.

** highly significant at 1% level.

regression line fitted to the data from this study to the curve currently used in design. The Wilson formula lies well below the regression at the higher t/R ratios.

The negative influences $(-)$[4] of departure strain (*SD*) and of E/D on curvature-stress factor were significant as shown in Equation 28.

Fiber Stress at Proportional Limit (Concave-loaded)

Figure 5,B, derived from Equation 29 with t/R varied and all other variables at their means, shows a nonsignificant effect of t/R at proportional limit. (See also Fig. 3,B.) The overall regression equation was also nonsignificant. Although only a small part of the variation was accounted for, the standard error of residual was only 0.074. Two-thirds of the time, the prediction given by this equation is within ±0.074 of the actual curvature-stress factor. Curvature-stress factor at the level of proportional limit is overestimated by the Wilson equation except at the most severe curvature.

Offset Stress (Concave-loaded)

Offset stress is a level of stress somewhat higher than fiber stress at proportional limit. This level of stress is proposed as a substitute for proportional limit stress in curved beams since, as previously mentioned, the actual proportional limit is very difficult to define due to the gradual departure of deflection from linearity with increasing load. This level of stress should be given more weight than fiber stress at proportional limit in evaluating the effect of curvature on strength retention in curved beams.

Equation 30 was used to plot Figure 5,C holding each variable at its mean and varying t/R. The significant regression line (See also Fig. 3,C) lies below the Wilson equation at milder curvatures but crosses over and lies slightly above it at the most severe curvature, indicating the slope of the Wilson equation to be too steep. The overall regression is significant at the 1 percent level and each independent variable is significant by the "t"-test. Variation accounted for by the regression was 40.4 percent of the total and the standard error of residual was 0.071.

Equation 30 also shows the influence of other significant variables. Curvature-stress factor decreases with increasing strain at proportional limit (*SP*) whereas increasing departure strain (*SD*) and FS/R result in an increase in curvature-stress factor at this level of stress. A low E/D ratio is also favorable in terms of curvature-stress factor.

It should be noted from Table 7 that *SP* is itself correlated with FS/R (+) and E/D $(-)$ and consequently that the "added" effects indicated by the signs of the coefficients in Equation 30 are applicable only to this combination of variables.

[4] Positive and negative slopes of the regression lines are indicated by (+) and (−), respectively.

Modulus of Rupture (Convex-loaded)

Equation 31 is illustrated in Figure 6,A with t/R varied and other variables held at their means. Mean values of the independent variables are shown in Table 9. The overall regression was highly significant but t/R failed to show a significant trend. (See also Fig. 4,A.) It is obvious from the graph that the Wilson equation is much too conservative for convex beams at modulus of rupture over the entire range of t/R. The standard error of residual (Table 9) is 0.075 and the regression accounted for 48.3 percent of the total variation.

This equation also shows the influence of E/D (−) and work to maximum load/modulus of rupture (−) on curvature-stress factor at the level of modulus of rupture. Both of these variables were significant.

Fiber Stress at Proportional Limit (Convex-loaded)

Figure 6,B is based on Equation 32 and shows a significant effect (−) of t/R. (See also Fig. 4,B.) This equation predicts curvature-stress factors for convex-beams at fiber stress at proportional limit with a standard error of ±0.095. The regression accounted for 56 percent of the total variation. The Wilson equation lies well below the regression line for this level of stress throughout the entire t/R range.

Equation 32 shows that more elastic material with a low ratio of W/R is favorable for strength retention and also indicates increasing strength retention with increasing ring width (RW.)

Offset Stress (Convex-loaded)

Figure 6,C, derived from Equation 33, shows the Wilson equation curve to lie well below and to have a greater slope than the regression line for convex beams at this level of stress throughout the range of t/R. The regression was highly significant and accounted for 53 percent of the total variation. (See also Fig. 4,C.) The quality of the prediction equation is shown by the standard error of residual of 0.082. All variables in Equation 33 were significant by the "t"-test and the coefficients in this equation show the effects on strength retention of modulus of elasticity (−), density (+), and ring width (+) in this particular combination of variables. Equations, such as Equation 33, containing correlated variables have been treated as though none of the variables were correlated. When the "t"-test indicated variables to be significant, even after the variation had been taken out for some correlated variable, it seemed reasonable to show this effect.

Summary and Conclusions

The results of this study furnish prediction equations for the strength of straight and curved beams, made from clear, thin, vertical-grain laminations of loblolly pine in terms of actual strength values. Equations are also given for predicting curvature-stress factors; *i.e.*. strength retention of curved beams as compared to matched straight beams, for concave- and convex-loaded beams.

Curvature-stress factors for individual beams were obtained by two methods. From the equations for predicted strengths (Psi method), curvature-stress factor was calculated from the ratio of predicted curved-beam strength to predicted straight-beam strength. In the "Ratio method," predictions of curvature-stress factor were obtained directly. A comparison of the average residual between actual and predicted curvature-stress factors for all curved beams by the two methods showed the "Ratio method" to be superior in predicting the strength retention of individual beams. The "Psi method" was nevertheless successful in that it gave very good estimations of the strength of curved and straight beams that were influenced predominantly by E and D as shown in the analysis.

Significant effects of t/R on curvature-stress factor were shown for concave-loaded beams at the level of offset stress and for convex-loaded beams at fiber stress at proportional limit and offset stress. At the level of modulus of rupture strength retention of curved beams appears to be less sensitive to t/R than at the level of proportional limit.

Expressed as modulus of rupture, the results of this study indicate that, over a range in t/R from 1/180 to 1/80, curved laminated beams of lobolly pine loaded on the concave surface are on the average 92 percent as

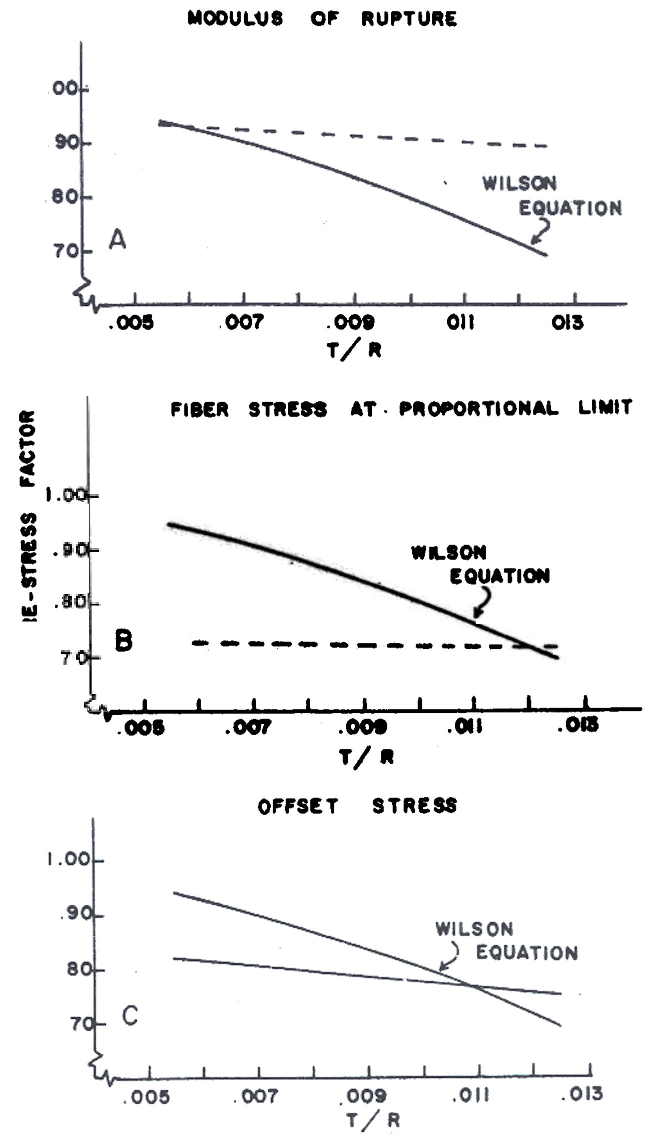

Figure 5. — Effect of t/R on strength retention of concave-loaded curved beams.

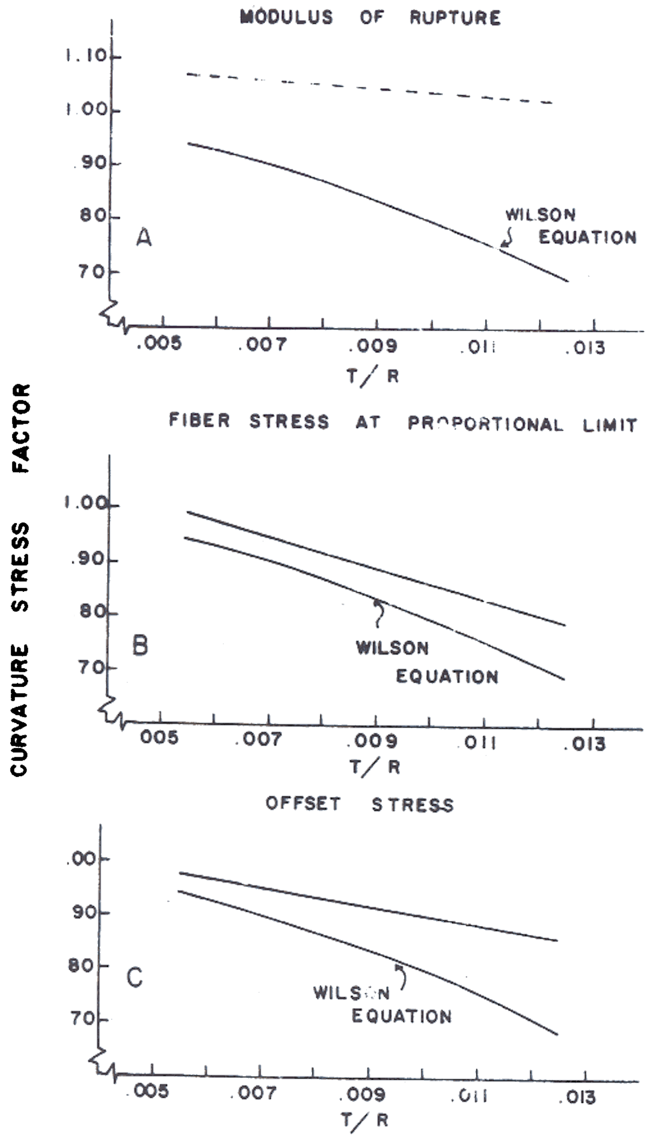

Figure 6. — Effect of t/R on strength retention of convex-loaded curved beams.

strong as matched straight beams. Convex loaded beams were on the average about 4 percent stronger than matched straight beams.

Concave-loaded beams, at the level of fiber stress at proportional limit, showed an average curvature-stress factor of 0.72. As noted previously, the proportional limit was very difficult to locate on the load-deflection curve for concave beams and it is felt that offset stress is more dependable in showing the effect of t/R on these beams. Offset stress showed average curvature-stress factors of 0.82 and 0.75 at t/R ratios of 1/180 and 1/80, respectively. At the same t/R ratios the Wilson equation predicts curvature-stress factors of 0.94 and 0.69 and thus appears as a much steeper curve than these data show.

Convex-loaded beams at *FSPL* and offset stress showed significant trends with t/R. At the level of *FSPL*, convex-loaded beams gave average curvature-stress factors of 0.99 and 0.79 at t/R ratios of 1/180 and 1/80, respectively, and offset stress gave average values of 0.98 and 0.86 at these two ratios. The Wilson equation predicts values of 0.94 at $t/R = 1/180$ and 0.69 at $t/R = 1/80$.

Based on the trends with t/R shown by the regression lines in Figures 5 and 6 and averaging curvature-stress factors at the levels of modulus of rupture and offset stress, the following curvature factors are suggested as suitable for thin vertical-grain laminations of clear wood of loblolly pine in lieu of the Wilson equation values currently used in design:

t/R	1/180 (.00555)	1/160 (.00625)	1/140 (.00714)	1/120 (.00833)	1/100 (.0100)	1/80 (.0125)
Concave-loaded beams	0.88	0.87	0.86	0.85	0.84	0.8[illegible]
Convex-loaded beams	1.02	1.01	1.00	.99	.97	.94
Wilson equation	.94	.92	.90	.86	.80	.69

The following wood quality variables, measured from the straight beams, have been shown in this study to have significant effects upon curvature-stress factor for concave-loaded beams: at modulus of rupture—*SD* (−) and *E/D* (−); at offset stress—*SP* (−), *SD* (+), *FS/R* (+), and *E/D* (−).

Significant effects upon curvature-stress factor for convex-loaded beams were found in the following: at modulus of rupture—*E/D* (−), and *W/R* (−); at fiber stress at proportional limit—*W/R* (−) and *RW* (+); at offset stress—*E* (−), *D* (+), and *RW* (+).

The significance of the foregoing variables lies in their effects in combination with other variables as shown in Tables 8 and 9. By selecting a favorable combination of these variables, wood qualities characterized by better than average retention of curved-beam strength can be identified.

Literature Cited

1. AMERICAN INSTITUTE OF TIMBER CONSTRUCTION. 1966. Timber Construction Manual. John Wiley and Sons, Inc., New York.
2. AMERICAN SOCIETY FOR TESTING MATERIALS. 1959 ASTM Standards on Wood, Wood Preservatives, and Related Materials, ASTM Committee D-7 on Wood.
3. FINNORN, W. J., and A. RAPAVI. 1959. Effect of lamination thickness on strength of curved beams. For. Prod. Jour. 9:248-251.
4. HUDSON, W. M. 1960. The effect of initial bending on the strength of curved laminated timber beams. Wood (London) 25:234-236.
5. KOSTUKEVICH, C., and F. F. WANGAARD. 1964. Curvature-stress factor in laminated wood beams. For. Prod. Jour. 14:44-50.
6. MURRAY, M. R., and F. F. WANGAARD. 1964. Unpublished data on "Curvature-stress factor in laminated wood beams." Yale School of Forestry, New Haven, Conn.
7. WANGAARD, F. F., and G. E. WOODSON. 1965. Unpublished data on "Curvature-stress factor in laminated wood beams." Yale School of Forestry, New Haven, Conn.
8. ———, ———, and M. R. MURRAY. 1968. Species response to pre-stress in curved laminated wood beams. For. Prod. Jour. 18(1):49-56.
9. WILSON, T. R. C. 1939. The glued laminated wooden arch. U.S. Dept. of Agr. Tech. Bull. No. 691., Washington, D.C.

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