Analysis of K-Factor for Five-Ply Plywood

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Abstract

K-factor has long been used as a modifier of section modulus in the calculation of bending moment for plywood beams. No comparable modification of moment of inertia is included in current recommendations for calculating plywood deflection at span-depth ratios of 30:1 or greater. Results of limited testing lend support to the authors' contention that, where "shear" strains are significant, the same *K* value would be appropriate for predicting both the deflection and bending moment of plywood. If these results are confirmed by further investigation, a method for determining an appropriate value for *K* based on separate calculations of deflections due to shear and bending is suggested.

CURRENT DESIGN PROCEDURES for plywood loaded in bending are based on an "effective" moment of inertia (I) and an "effective" section modulus (KS). These section properties are defined in current design specifications published by the American Plywood Association (1).

Due to the lower modulus of elasticity of plies stressed perpendicular to the grain, the effective moment of inertia of a plywood section is less than that of a section composed of all parallel-laminated plies of the same material. The effectiveness of a plywood section is accounted for by calculating its I as the summation of the I of the plies stressed-parallel-to-grain and 1/35 I of the perpendicular plies, all taken about the neutral axis of the section. The Plywood Design Specification includes tabulated values for effective I for standard plywood constructions (1).

Deflection of a plywood beam is calculated by incorporating the effective I into one of the usual engineering formulas. For example, deflection (in.) of a simple beam supporting a center load P is calculated as

$$\Delta = \frac{Pl^3}{48EI} \qquad [1]$$

where

P = load (lb.)
l = span (in.)
E = modulus of elasticity in bending of parallel-stressed plies (psi)
I = effective moment of inertia (in.^{4}) = $\Sigma(I_{\parallel} + 1/35\ I_{\perp})$

Although calculation of deflection by this method appears to ignore shear, it should be noted that the usual shear effect encountered in standard testing of solid wood beams with a span/depth ratio of 14:1 (on the order of 10 percent of total deflection) is included in the value for modulus of elasticity when this value is taken from standard sources. For the normal span-depth ratios of 30:1 to 50:1 at which plywood sheathing is most commonly used, equations ignoring any additional effect of shear are usually considered adequate (1). This conclusion appears to be based on extensive tests conducted more than 30 years ago at the Forest Products Laboratory (6), although those tests were limited to a span-depth ratio of 48:1 when outer plies were stressed parallel to grain.

Section modulus (S), normally calculated as I/c, where c is the distance from the neutral plane to the outermost fiber, is modified still further in the case of plywood beams. Limiting consideration here to plywood in which the outer plies are stressed parallel to the grain, effective section modulus (KS) involves a reduction factor K as well as effective I in the expression I/c; $KS = KI/c$. For the loading condition just described, tabulated values for KS appearing in the Plywood Design Specification (1) incorporate a value of 0.85 for K based on experimental results obtained, as noted in the preceding paragraph, at the Forest Products Laboratory (6). The test data were evaluated at the proportional limit level as well as at the level of failure. This value for K, applicable to calculation of bending moment for plywood through the relationship

$$M = fKI/c \text{ or } fKS \qquad [2]$$

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has appeared in a number of other publications (3, 4, 5, 7) and has acquired the credibility of age. In this equation

M = bending moment (lb.-in.)
f = stress in outermost fiber of face plies (psi)
KS = effective section modulus (in.^{3})

Although other factors, such as moments associated with excessive deflections of plywood beams at 48:1 span-depth ratio, may have been involved in the original introduction of a value of 0.85 for K in Equation [2], its more general application at lower span-depth ratios where shear effects cannot be ignored calls for new interpretation.

Presumably, under these conditions, the incorporation of K into the expression for effective section modulus accounts for some departure from the theoretical assumption of linear distribution in strain ranging from zero at the neutral axis to maximum in the outermost fiber even within the proportional limit. Such an assumption is implicit in the unmodified flexure formula $M = fI/c$ which, as Freas (6) noted, is applicable only to the extent that shear deformations may be ignored. The acceptance of the reduction factor K, then, would appear to acknowledge the existence of a "shear" effect, beyond that experienced in parallel-laminated veneer, insofar as the development of resisting moment is concerned. As discussed earlier, no comparable modification of standard engineering formulas is included in current recommendations for calculation of plywood deflection at span-depth ratios commonly employed with plywood sheathing.[1]

It has long seemed inconsistent to the senior author that this situation could exist. To whatever extent "shear" strains deform beam sections from theoretical planes, they modify equally the effective moment of inertia (I) involved in the deflection calculation and the effective section modulus (I/c or S) used in the calculation of bending moment. It would seem from this logic, that the same K factor should be involved in both calculations; i.e., we propose that the deflection equation cited previously should read:

$$\Delta = \frac{Pl^3}{48EKI} \qquad [3]$$

analogous to

$$M = fKS \qquad [4]$$

with an equal value for K—not necessarily 0.85—in both equations. For other loading conditions the appropriate deflection equation should be modified similarly.

Recently some plywood test data collected as part of two other studies being conducted at Colorado State University became available for analysis. Both studies involved a direct comparison of plywood properties with those of parallel-laminated veneer specimens fabricated from the same grades of veneer. All veneers were 1/10-inch thick and all test specimens were five-ply construction. Ponderosa pine specimens — both plywood and laminated — were made with C grade face and D grade inner and back plies corresponding to C-D grade plywood. Engelmann spruce specimens were made with a C grade face and inner plies and D grade back, representative of a mill run consisting of approximately 80 percent C grade and 20 percent D grade veneer. Layups of both species were hot-pressed in two series using an interior type glue line in one and an exterior type in the other. Bending specimens were 12 inches wide and 20 inches long and were tested by center loading over a 16-inch span. Rate of crosshead travel of the testing machine was standard for a one-half-inch deep beam at this span. The resulting span-depth ratio of 32:1 is in the range of recommended spans for this thickness of plywood.

Average results obtained from testing approximately 25 specimens of each type are shown in Tables 1 and 2. Values in these tables have been calculated as follows for plywood:

$$E = \frac{Pl^3}{48\Delta I} \qquad [5]$$

P and Δ are load (lb.) and corresponding deflection (in.), respectively, within the proportional limit
l = span (in.)
I = effective moment of inertia (in.^{4})
E = modulus of elasticity of parallel-stressed plies (psi)

and

$$f = \frac{M}{S} \qquad [6]$$

where

f = stress in outermost fiber of face plies, psi at either the level of proportional limit or failure
M = bending moment (lb.-in.) at the appropriate level of stress
S = section modulus (effective I/c, in.^{3})

For parallel-laminated veneer, moment of inertia and section modulus were calculated in the usual manner for a solid homogeneous material.

Values for K, then, were simply computed as ratios of the elastic and strength properties for the plywood and laminated veneer. These are the values which would allow the accurate prediction of plywood deflections and bending moments by means of Equations [3] and [4]. As shown in Tables 1 and 2, average K-factor values for ponderosa pine plywood were 0.92 based on modulus of elasticity, 0.925 based on stress at proportional limit, and 0.92 based on modulus of rupture. For Engelmann spruce plywood corresponding average values for K-factor were 0.835, 0.89, and 0.835, respectively.

Conclusions

These results lend support to the basic premise of this study, i.e., where "shear" strains are significant, the same K value is appropriate for predicting the deflection and bending moment of plywood from the properties of parallel-laminated veneer. This conclusion is, of course, subject to experimental limitations, i.e., ponderosa pine and Engelmann spruce five-ply plywood with

[1]When very short spans are involved, as in the case of concrete forms, deflections due to shear and bending are calculated separately and added together.

Table 1. — K-FACTORS FOR PONDEROSA PINE PLYWOOD BASED ON BENDING TESTS OF FIVE-PLY PLYWOOD AND PARALLEL-LAMINATED SPECIMENS OF C-D VENEER.[1]

	MOE (psi)	K	FSPL (psi)	K	MOR (psi)	K
			Interior Adhesive			
Plywood	730,000	0.91	3,606	0.93	5,586	0.88
Parallel-laminated veneer	799,000		3,878		6,380	
			Exterior Adhesive			
Plywood	786,000	0.93	3,835	0.92	6,132	0.96
Parallel-laminated veneer	841,000		4,165		6,411	
			Average (Interior and Exterior)			
Plywood		0.92		0.925		0.92

[1]All data adjusted to 12 percent moisture content and to an average original wood specific gravity of 0.40 after taking into account weight of adhesive and process densification. One ply C grade, 4 plies D grade veneer.

Table 2. — K-FACTORS FOR ENGELMANN SPRUCE PLYWOOD BASED ON BENDING TESTS OF FIVE-PLY PLYWOOD AND PARALLEL-LAMINATED SPECIMENS OF C-D VENEER.[1,2]

	MOE (psi)	K	FSPL (psi)	K	MOR (psi)	K
			Interior Adhesive			
Plywood	1,010,000	0.84	3,780	0.84	6,380	0.84
Parallel-laminated veneer	1,205,000		4,490		7,580	
			Exterior Adhesive			
Plywood	1,110,000	0.83	3,850	0.94	6,790	0.83
Parallel-laminated veneer	1,344,000		4,104		8,176	
			Average (Interior and Exterior)			
Plywood		0.835		0.89		0.835

[1]All data adjusted to 12 percent moisture content and to an average original wood specific gravity of 0.34 after taking into account weight of adhesive and process densification. Four plies C grade, 1 ply D grade veneer.

[2]Adapted from "Evaluation of Full-Sized Engelmann Spruce Plywood Panels" by John P. Schuldt and H. E. Troxell. 1970. Wood Science Laboratory, Colorado State University. Unpublished report.

outer plies stressed parallel to grain and at a span-depth ratio of 32:1. Experimental confirmation is needed to extend this conclusion to other conditions. A seeming discrepancy in the data for Engelmann spruce at proportional limit is probably due to experimental difficulties in measuring this property accurately. The differences in *K* shown by the data for pine and spruce may represent the effect of species, grade, and manufacturing variables such as veneer tightness. The last of these is particularly worthy of further study as it appears likely that shear deformation in the TR plane of the crossplies, already low in modulus of rigidity, is associated with the prominence of lathe checks. This could be an important contributor to the magnitude of *K*-factor although this remains to be proved.

It is interesting to note that Biblis and Chiu (2) have reported an increasing proportion of shear deflection with decreasing span for five-ply plywood beams of southern pine. Expressed as a percentage of pure bending deflection (free of shear), shear deflection amounted to 5.4 percent at a span-depth of 48:1, 21:8 percent at 24:1, and 63.9 percent at 14:1. By interpolation shear deflection at a span-depth ratio of 32:1 would have been 15 percent — equivalent to a *K*-factor of 0.87 in the expression of effective moment of inertia (*KI*). Such evidence, pointing to shear as the underlying explanation for *K*-factor, suggests further an expectation for variation in *K* with span-depth ratio. Shear deformations in plywood at spans commonly encountered in applications such as roof sheathing and subflooring are obviously sufficient to warrant further study and perhaps more sophisticated analysis.

If further investigation should substantiate our contention that "shear" strains are to a considerable degree responsible for *K*-factor, it would seem reasonable to calculate shear and bending deflections separately as is now recommended by the American Plywood Association in the case of concrete forms.[3] The appropriate value for *K*, then, in Equations [3] and [4] would be

$$\frac{1}{\cdot\;\cdot\;\cdot} \text{ or, more simply } \frac{\Delta_b}{\cdot\;\cdot\;\cdot}$$

where

Δ_s = deflection due to shear
Δ_b = deflection due to pure bending

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[3]American Plywood Association. 1971. Plywood for concrete forming. 31 pp.