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WEIGHTING FACTORS FOR COMPUTING THE RELATION BETWEEN TREE VOLUME AND D.B.H. IN THE PACIFIC NORTHWEST

by

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Timber cruising is frequently made easier through use of local volume tables based on d.b.h. alone. These tables are made by establishing the relation between volume and d.b.h. from measurements (including height) made on sample trees in the stand. The sample-tree measurements are converted to volumes through use of standard volume tables, and a volume-diameter curve is constructed.

One form for expressing the relation between tree volume and tree diameter is by a least-squares second-degree curve:

$$V = a+bD+cD^2$$

where V = tree volume

D = d.b.h.

a, b, c = constants

In calculating the constants for this formula, it is assumed that tree volumes for any diameter class are distributed normally and that variance in volume is the same for all diameter classes. Plotting this kind of data, however, will usually show that the second of these assumptions is inadequate. Increasing variation with increasing diameter will probably be obvious. The accepted procedure in

such cases is to weight each observation by the reciprocal or inverse of its variance. The tabulation at the end of this report gives weights for a range of diameters from 5.0 to 100.0 inches, for board feet and for cubic feet. Values used for variance were developed from the following formulas:

1. For board feet--

$$\text{Log } V = 5.61188(\log D) - 3.24659$$

2. For cubic feet--

$$\text{Log } V = 4.68017(\log D) - 2.54636$$

Where V = variance, and D = diameter at breast height.

The weights appear to be fairly constant regardless of species or location in Oregon and Washington. They were developed from trees of several species, including softwoods and hardwoods in both the Douglas-fir and ponderosa pine subregions.

The procedure for using weights in developing the constants a, b, and c for the formula $V = a+bD+cD^2$ or, in the more general form, $Y = a+bX+cX^2$, is as follows: If $x_1\ y_1$, $x_2\ y_2$, ... $x_n\ y_n$ are a series of n pairs of observations with relative weights w_1, w_2, ... w_n, the constants a, b, and c can be expressed as--

$$b = \frac{DE-FG}{HD-F^2}$$

$$c = \frac{HG-FE}{HD-F^2}$$

$$a = \frac{\Sigma w_i y_i}{\Sigma w_i} - b\frac{\Sigma w_i x_i}{\Sigma w_i} - c\frac{\Sigma w_i x_i^2}{\Sigma w_i}$$

where:

$$D = \Sigma w_i x_i^4 - \frac{(\Sigma w_i x_i^2)^2}{\Sigma w_i}$$

$$E = \Sigma w_i x_i y_i - \frac{(\Sigma w_i x_i)(\Sigma w_i y_i)}{\Sigma w_i}$$

$$F = \Sigma w_i x_i^3 - \frac{(\Sigma w_i x_i)(\Sigma w_i x_i^2)}{\Sigma w_i}$$

$$G = \Sigma w_i x_i^2 y_i - \frac{(\Sigma w_i x_i^2)(\Sigma w_i y_i)}{\Sigma w_i}$$

$$H = \Sigma w_i x_i^2 - \frac{(\Sigma w_i x_i)^2}{\Sigma w_i}$$

These constants can be readily developed on the type 650 electronic computer by a program which has been described in a recent publication.[1/] Input data for the program consists of the values for V, D, and the corresponding weighting factor.

Inverse of Variance Weighting Factors

D.b.h.	Cubic feet	Board feet
5.0	.1883923912	--
6.0	.0802561777	--
7.0	.0390085585	--
8.0	.0208807936	--
9.0	.0120322030	--
10.0	.0073484025	--
11.0	.0047040008	.0025259158
12.0	.0031304783	.0015500823
13.0	.0021523753	.0009891783
14.0	.0015215582	.0006526137
15.0	.0011016795	.0004475363
16.0	.0008144715	.0003084696
17.0	.0006132712	.0002195120
18.0	.0004693249	.0001592765
19.0	.0003643996	.0001175921
20.0	.0002866299	.0000881788
21.0	.0002281142	.0000670587
22.0	.0001834832	.0000516505
23.0	.0001490202	.0000402473
24.0	.0001221068	.0000316964
25.0	.0001008710	.0000252068

[1/] Gedney, Donald R., Martin, Dorothy E., and Johnson, Floyd A. Specifications for calculating several equations of relationship between two variables on a type 650 electronic computer. U.S. Forest Serv. Pac. NW. Forest and Range Expt. Sta., 15 pp. 1959.

D.b.h.	Cubic feet	Board feet
26.0	.0000839553	.0000202269
27.0	.0000703621	.0000163662
28.0	.0000593496	.0000133448
29.0	.0000503608	.0000109594
30.0	.0000429719	.0000090608
31.0	.0000368585	.0000075378
32.0	.0000317691	.0000063077
33.0	.0000275080	.0000053073
34.0	.0000239212	.0000044886
35.0	.0000208864	.0000038147
36.0	.0000183066	.0000032569
37.0	.0000161032	.0000027927
38.0	.0000142137	.0000024046
39.0	.0000125867	.0000020784
40.0	.0000111802	.0000018031
41.0	.0000099601	.0000015698
42.0	.0000088977	.0000013712
43.0	.0000079699	.0000012016
44.0	.0000071569	.0000010562
45.0	.0000064424	.0000009310
46.0	.0000058127	.0000008230
47.0	.0000052561	.0000007294
48.0	.0000047629	.0000006481
49.0	.0000043247	.0000005773
50.0	.0000039345	.0000005154
51.0	.0000035863	.0000004612
52.0	.0000032747	.0000004136
53.0	.0000029954	.0000003717
54.0	.0000027445	.0000003350
55.0	.0000025187	.0000003019
56.0	.0000023150	.0000002729
57.0	.0000021309	.0000002471
58.0	.0000019644	.0000002241
59.0	.0000018133	.0000002036
60.0	.0000016762	.0000001853
61.0	.0000015514	.0000001689
62.0	.0000014377	.0000001541
63.0	.0000013340	.0000001409
64.0	.0000012392	.0000001290
65.0	.0000011524	.0000001182
66.0	.0000010730	.0000001085

D.b.h.	Cubic feet	Board feet
67.0	.0000010001	.0000000997
68.0	.0000009331	.0000000918
69.0	.0000008714	.0000000846
70.0	.0000008147	.0000000780
71.0	.0000007624	.0000000720
72.0	.0000007141	.0000000666
73.0	.0000006694	.0000000616
74.0	.0000006289	.0000000571
75.0	.0000005899	.0000000530
76.0	.0000005544	.0000000492
77.0	.0000005215	.0000000457
78.0	.0000004910	.0000000425
79.0	.0000004625	.0000000396
80.0	.0000004361	.0000000369
81.0	.0000004115	.0000000344
82.0	.0000003885	.0000000321
83.0	.0000003671	.0000000300
84.0	.0000003471	.0000000280
85.0	.0000003284	.0000000262
86.0	.0000003109	.0000000246
87.0	.0000002945	.0000000230
88.0	.0000002792	.0000000216
89.0	.0000002648	.0000000203
90.0	.0000002513	.0000000190
91.0	.0000002386	.0000000179
92.0	.0000002267	.0000000168
93.0	.0000002155	.0000000158
94.0	.0000002050	.0000000149
95.0	.0000001951	.0000000141
96.0	.0000001858	.0000000133
97.0	.0000001770	.0000000125
98.0	.0000001687	.0000000118
99.0	.0000001609	.0000000112
100.0	.0000001535	.0000000105