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STANDARD ERROR OF ESTIMATED AVERAGE TIMBER VOLUME PER ACRE UNDER POINT SAMPLING WHEN TREES ARE MEASURED FOR VOLUME ON A SUBSAMPLE OF ALL POINTS

by

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This report assumes a knowledge of the principles of point sampling as described by Grosenbaugh,[1] Bell and Alexander,[2] and others.

Whenever trees are counted at every point in a sample of points (large sample) and measured for volume at a portion (small sample) of these points, the sampling design could be called ratio double sampling. If the large sample is either a completely random or a systematic sample of the area being cruised, and if the small sample is either a completely random or a systematic sample from the large sample, formula 1 (below) is appropriate for calculating an estimate of average volume per acre. Formula 1 is already available in the literature on point sampling, and is shown here merely to introduce the standard error of estimated average volume (formula 2) which is not well known. Formula 2 is strictly appropriate only for completely random samples. If used with systematic samples, it must be considered an approximation.

1/Grosenbaugh, L. R. Plotless timber estimates--new, fast, easy. Jour. Forestry 50: 32-37, illus.

2/Bell, John F., and Alexander, Lucien B. Application of the variable plot method of sampling forest stands. Oreg. State Bd. Forestry Res. Note 30, 22 pp., illus. 1957.

ESTIMATED AVERAGE VOLUME PER ACRE

For estimates of average volume per acre under ratio double sampling, we have:

$$\bar{u}_d = F \frac{\overset{n}{\Sigma} w_i}{n} \frac{\overset{k}{\Sigma} x_i}{\overset{k}{\Sigma} w_i} \quad \text{- - - - - - - - - - - - - - - - - - - -} \quad (1)$$

where $\bar{u}_d$ = estimated average volume per acre (the subscript d denotes double sampling)

F = basal area factor

w_i = tree count on any point

n = total number of points (i.e., count points plus measured points)

k = number of points at which trees were measured for volume (i.e., measured points only)

x_i = Σz_{ij} for any point where z_{ij} = volume of the jth tree on the ith point divided by its basal area.

Note that in formula 1 the ratio $(\Sigma x_i/\Sigma w_i)$ from the small sample is multiplied by average count $(\Sigma w_i/n)$ from the large sample, and the factor F adjusts the product to an acre basis.

The advantage of the ratio double sampling method results from the time saved in merely counting trees at some of the points instead of measuring trees at all points. This advantage may disappear if the time saved is not appreciable or if x_i and w_i are weakly correlated.

STANDARD ERROR OF ESTIMATED AVERAGE VOLUME

Standard error of estimated average volume per acre, $\bar{u}_d$, can be found by:

$$s_{\bar{u}_d} = F \sqrt{\frac{\overset{k}{\Sigma}(x_i - \bar{x})^2}{n(k-1)} + \frac{\overset{k}{\Sigma}(x_i - \hat{R}w_i)^2}{k(k-1)} \left[\frac{n-k}{n}\right]} \quad \text{- - - - - -} \quad (2)$$

where $$\bar{x} = \frac{\sum^{k} x_i}{k}$$

$$\hat{R} = \frac{\sum^{k} x_i}{\sum^{k} w_i}$$

and where, for calculation purposes,

$$\sum^{k}(x_i - \bar{x})^2 = \sum^{k} x_i{}^2 - \frac{(\sum^{k} x_i)^2}{k}$$

$$\sum^{k}(x_i - \hat{R}w_i)^2 = \sum^{k} x_i^2 + \hat{R}^2 \sum^{k} w_i^2 - 2\hat{R}\sum^{k} x_i w_i$$

Note that when trees are measured for volume at all points (i.e., n = k), the second term under the radical sign is zero and formula 2 becomes:

$$s_{\bar{u}_a} = F\sqrt{\frac{\sum^{n}(x_i - \bar{x})^2}{n(n-1)}} \quad \text{- - - - - - - - - - - - - - - - - -} \quad (3)$$

In this case,

$$\bar{u}_a = F \frac{\sum^{n} x_i}{n} \quad \text{- -} \quad (4)$$

With double sampling, $\bar{u}_d$ is, in a sense, an estimate of $\bar{u}_a$, where $\bar{u}_a$ is in turn an estimate of $\bar{U}$, the true average volume per acre. Thus when n = k, $\bar{u}_a$ is known without sampling error and formula 3 reflects only the failure of $\bar{u}_a$ to be $\bar{U}$. When k is less than n, the second term under the radical sign in formula 2 reflects the failure of $\bar{u}_d$ to be $\bar{u}_a$, and it is, therefore, a necessary additional component of total sampling error.

Palley and Horwitz develop the equivalent of formula 2 in an excellent report on the statistical theory underlying point sampling.[3/] They use different notation, and their equation has a different form. Cochran[4/] uses still another form, but formula 2 may be more convenient computationally than either the Palley-Horwitz version or the Cochran version.

Bell and Alexander[5/] offer a different standard error formula for this sampling problem. Individual trees are assumed to be independent for one part of their formula, but trees are obviously not independent. They are tallied in clusters at points.

However, if the usual systematic sample of points has been taken, formula 2 can also be challenged. In that event the required independence for points is lacking. Although formula 2 and the Bell-Alexander formula both leave something to be desired, formula 2 seems to require fewer and more acceptable assumptions.

EXAMPLE

For an actual timber cruise, trees were counted at 62 points (i.e., $n = 62$). Trees were measured for volume on 25 of these 62 points (i.e., $k = 25$). Basal area factor (F) was 38.55. Values of z_{ij}, x_i, and w_i for each measured point and values of w_i for each count point are listed below:

3/ Palley, M. N., and Horwitz, Leah G. Properties of some random and systematic point sampling estimators. (Manuscript accepted for publication in Forest Sci., vol. 7. 1960.)

4/ Cochran, W. G. Sampling techniques. 330 pp. New York. 1953.

5/ See footnote 2.

Measured Points

Plot no.	z_{ij} Tree volume in bd. ft. divided by basal area						w_i Trees per point	$x_i = \Sigma z_{ij}$
1	274	274	261				3	809
2	269	319	323				3	911
3	327	317	319	319	266	266	6	1814
4	378	319	272	269			4	1238
5							0	0
6	317	373	317				3	1007
7	317	273					2	590
8	319	310					2	629
9							0	0
10	277	269					2	546
11							0	0
12	281	323	319				3	923
13	274						1	274
14	280						1	280
15	222	319					2	541
16	273	269	328	273	323		5	1466
17	319	319	323	272			4	1233
18	261						1	261
19							0	0
20	327	378					2	705
21	328	281	329				3	938
22	272	272	266	273			4	1083
23							0	0
24	319	323	319	319	323		5	1603
25	273	266	273	319	323		5	1454

Additional Count Points

Plot No.	w_i
1	1
2	2
3	3
4	1
5	1
6	2
7	0
8	2
9	1
10	3
11	4
12	2

Plot No.	w_i
13	5
14	3
15	3
16	0
17	3
18	5
19	4
20	1
21	4
22	3
23	6
24	0

Plot No.	w_i
25	8
26	3
27	4
28	4
29	8
30	5
31	1
32	2
33	1
34	0
35	2
36	0
37	0

The above information was then used to calculate:

$$\sum^{n} w_i = 158$$

$$\sum^{k} w_i = 61$$

$$\sum^{k} x_i = \Sigma z_{ij} = 18,305$$

$$\sum^{k} w_i^2 = 227$$

$$\sum^{k} x_i^2 = 20,632,683$$

$$\sum^{k} x_i w_i = 68,316$$

$$\sum^{k} (x_i - \bar{x})^2 = 7,229,762$$

$$\hat{R} = 300.082$$

$$\frac{n-k}{n} = 0.597$$

$\bar{u}_d$ = 29,480 bd. ft. per acre from formula 1

$s_{\bar{u}_d}$ = 2,707 bd. ft. per acre from formula 2