TWO-DIMENSIONAL FINITE ELEMENT HEAT TRANSFER MODEL OF SOFTWOOD. PART I. EFFECTIVE THERMAL CONDUCTIVITY

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ABSTRACT

The anisotropy of wood complicates solution of heat and mass transfer problems that require analyses be based on fundamental material properties of the wood structure. Most heat transfer models use average thermal properties across either the radial or tangential direction and do not differentiate the effects of cellular alignment. earlywood/latewood differences, or ring orientation. A model that considers these basic structural characteristics would be more accurate than most models in the literature. which do not consider the anatomical structure of wood. The two-dimensional finite element model descibed here was developed to determine the effective thermal conductivity as a function of cell alignment and cell porosity by modeling the softwood cell structure in either a pure radial or pure tangential orientation. This paper presents the results predicted from the cellular model, from which a new nonlinear regression equation for radial or tangential effective thermal conductivities is determined as a function of density (porosity). The results will be applied to a two-dimensional softwood board model for transient thermal analysis (Part II). Subsequent papers in this series apply and adapt this model to various lumber orientation and sires and to wood at various moisture contents (Part III and IV)

Keywords: Finite element analysis, transient heat transfer, cellular characteristics, porosity, thermal conductivity.

INTRODUCTION

Wood is an anisotropic, porous material with complicated cellular and macro-scale structural features and material properties. The structurally induced anisotropic effects on heat and mass transfer have significant implications for drying lumber, heating logs in veneer mills, and hot-pressing wood composites. Anisotropy is due to the radial, tangential, and longitudinal orientation of the wood fiber (Fig. 1) and the structural differences between the development of early-wood and latewood bands for each annual ring in softwood (Fig. 2). Earlywood cells, which are formed in the fast-growing spring season, are low-density cells with large cavities and thin walls (Fig. 2, left). Latewood cells, which are formed later in the year, are high-density cells characterized by smaller cavities and thick walls (Fig. 2, right). Softwood cells tend to align in straight radial rows because they originate from the same cambial mother cells, but the cells are

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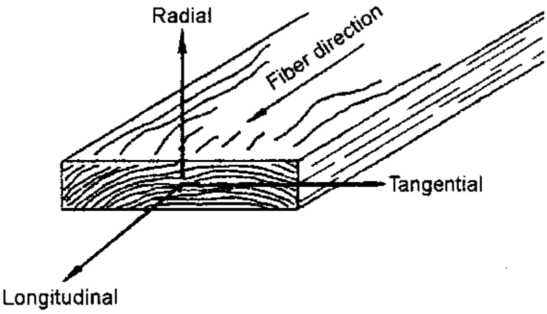

FIG. 1. Three principal axes of wood with respect to fiber direction and growth rings.

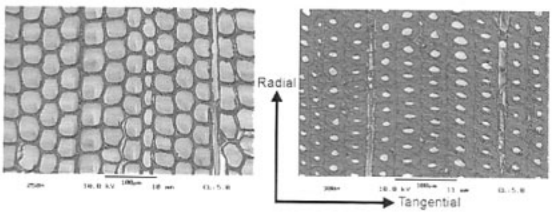

FIG. 2. Microscopic images of softwood structure in earlywood (left) and latewood (right) zones showing principal radial and tangential axes.

not necessarily aligned in tangential rows (Fig. 2). For the tangential direction, alignment can vary from 0% to a 50% offset, which is defined as the maximum misalignment by Hart (1964). Longitudinal differences also occur but are not within the scope of this paper, since the focus is on the development of a two-dimensional softwood cell structure model. Although earlywood and latewood cells vary by wood species and are affected by growing conditions, general assumptions can be made and modeled. For heat transfer modeling, earlywood and latewood cell walls are assumed to be made of essentially the same substance with the same thermal properties. Cell porosity—the percentage of openings in a wood cell—may vary from 90% to 70% in earlywood and from 30% to 10% in latewood (Gu 2001).

Early in the 1940s, MacLean (1941) pointed out that the conductivity of wood with certain moisture content as determined under steady-state conditions does not represent the conducivity of the wood under the original conditions of moisture distribution because the experiment causes redistribution of moisture in the process of reaching the steady state. Therefore, the true thermal conductivity value of wood can be obtained only by theoretical modeling due to the limitation of physical tests. Significant research has been done on measuring thermal conductivities of different species and some regression models have been drawn from the test data (MacLean 1941; Wangaard 1943; Stamm 1960; Hendricks 1962). However, the difference in thermal conductivities between the radial and tangential directions and between earlywood and latewood has not been theoretically studied and fully understood.

Cellular modeling presented in this paper begins to provide such a fundamental insight by presenting a theoretical finite element approach for heat transfer in softwood cells. Two softwood cellular models are used to determine the effective thermal conductivity based on the properties of cell-wall substance and the cellular structure on the micro-scale. This covers the effective thermal conductivity for both earlywood and latewood in an annual ring and orientation in both the radial and tangential directions. In Part II of this series of papers, the effective thermal conductivities from the cellular models will be used to model macro heat transfer effects in a board "cut" from a simulated log. The hoard model can be used to study the effects of growth rate, ring orientation, board size, and board location within a log.

Moisture content has a significant effect on heat transfer rate in wood (Forest Products Laboratory 1999). However, the scope of our initial work was the development of computer models based on fundamental properties of the base cellulose material. The two finite element models presented here assume moisture content to be 0% in order to study transient heat transfer differences from the effect of wood structure alone. The effect of moisture content on the effective thermal conductivity and transient temperatures in a board will be presented in Part III. The goal of the work presented here is to present new finite element models that are being developed as analytical tools to help understand fundamental heat transfer effects in softwood cells based on cellular structure.

PROCEDURE

Both models for this analysis were developed using ANSYS finite element software (ANSYS 2004). Finite element type PLANE35, a two-dimensional 6-node triangular thermal solid element, was used for the heat transfer analysis. The mathematical solution for the conduction heat transfer of this element is based on the first law of thermodynamics, the energy conservation law:

$$\rho C_p \frac{\partial T}{\partial t} = k_{eff,x} \frac{\partial^2 T}{\partial x^2} + k_{eff,y} \frac{\partial^2 T}{\partial y^2} \qquad (1)$$

where r is density of material, C_p is heat capacity, and $k_{eff,x}$, $k_{eff,y}$ are effective thermal conductivities in x, y direction or radial and tangential directions, respectively.

From the cellular structure of softwood observed under the microscope (Fig. 2), two small finite element models were developed to simulate the structural variation of cell porosity and cell alignment/misalignment in softwood. Cell porosity is the fractional void volume of a wood cell. It is assumed in the model to range from 10% to 90%. In softwood, cell porosity ranges from 70% to 90% for earlywood and 10% to 30% for latewood (Gu 2001). To approximate a density value from wood porosity, the following equation was used:

$$\text{Cell density} = \text{cell-wall density} \times (1 - \text{porosity}) \qquad (2)$$

where cell wall density is 1,540 kg/m^3 (Siau 1995).

Softwood cells tend to align in straight radial rows (vertical direction in Fig. 2). In contrast, softwood cells are much less aligned in the tangential direction (horizontal direction in Fig. 2), and this alignment or misalignment between cells varies from 0% to a maximum of 50%. A fully aligned cellular structure model is shown in Fig. 3. Conversely, a fully misaligned cellular structure model (50% offset between two rows of cells) is shown in Fig. 4.

The purpose of the two cellular models was to determine the effective thermal conductivity based on the parameters of porosity and cell alignment. The effective thermal conductivity of wood in the radial and tangential directions was estimated by simulating a simple conduction problem across the models. Cell porosity of 10% to 90% at an increment of 10% for both 0% and 50% misalignment was analyzed. A temperature difference of 80 K across two opposing boundaries for either the radial or tangential direction was analyzed, with the other two boundaries set as adiabatic boundaries. The material properties used as input variables for the cell-wall substance and air in the lumen were obtained from Siau (1995) and Incropera and DeWitt (1981) (Table 1).

Total heat flux was calculated across a random line selected within the models (see Figs. 3

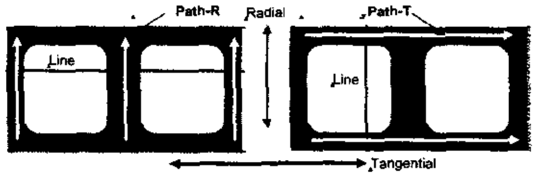

FIG. 3. Model of wood cells with 50% porosity in fully aligned case (0% offset) of cellular structure showing principal radial and tangential axes.

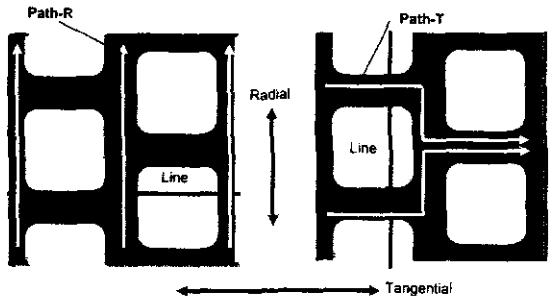

FIG. 4. Model of wood cells with 50% porosity in fully misaligned case (50% offset) of cellular structure showing principal radial and tangential axes.

TABLE 1. *Material property values applied in ANSYS cellular models.*

Input variable	Material properties: Thermal conductivity (W/m · K)	Density (kg.m3)	Specific heat (J/kg · K)
Cell-wall substance	0.41	1,540	1,260
Air in cell lumen	0.026	1.1614	1,007

and 4) to determine the effective thermal conductivity (k_{eff}) using the definition of heat flux (q'_x) (Incropera and DeWitt 1981) in Eq. (3). The effective radial and tangential thermal conductivity at each increment of porosity for 0% and 50% cell misalignment was determined by rearranging the heat transfer equation and solving for k_{eff}:

$$q''_x = k_{\text{eff}} \frac{dT}{dx} \tag{3}$$

$$\Rightarrow k_{\text{eff}} = -\frac{q''_x}{dT/dx}$$

where

q''_x is heat flux (W/m^2),
k_{eff} effective thermal conductivity (W/m · K),
dT temperature change (K), and
dx distance for temperature change (m).

For this paper, 0% moisture content was assumed. The effect of moisture content on thermal conductivity will be discussed in Part III.

ANALYSIS OF RESULTS AND DISCUSSION

Radial and tangential effective thermal conductivity was determined as a function of increasing porosity at increments of 10% for both the aligned and the misaligned cases (Table 2). The ANSYS models only estimated values of fully aligned (0% offset) and fully misaligned (50% offset) cases, which are the two extreme cases for wood structure. In softwood, the cells are aligned between these two cases in the tangential direction. Therefore, the average values of these cases were taken for the effective coefficients (Table 2) and will be used in the macro wood board model for heat transfer analysis (Part II). The results show a range of conductivity values, from –0.046W/m·K for 90% porosity (density of 154 kg/m^3) to ~0.34 W/m · K for 10% porosity (density of 1,386 kg/m^3) (Table 2). The results indicate a significant difference between earlywood and latewood thermal conductivity, assuming 70% to 90% porosity for earlywood and 10% to 30% porosity for latewood (Gu 2001). For earlywood, thermal conductivity averages 0.068 W/m · K and for latewood 0.287 W/m · K, an increase by a factor of 4. This difference will have a considerable effect on the heat transfer in the wood board depending on the orientation of rings and the earlywood/latewood ratio.

For the 0% offset model (fully aligned case), there should not be any difference between radial and tangential thermal conductivity since heat travels the same path in both directions per wood cell (compare Path-R and Path-T in Fig. 3). Small differences in this case are due to slight variations in numerical integration of total flux across cell elements on the line (see Fig. 3). Element size was reduced in the models until less than 1% variation was observed in total heat flux on the line. The misalignment of cells for the tangential heat flow (up to 50% offset) had only a small effect on thermal conductivity (Table 2). The effective thermal conductivity

TABLE 2. *Transverse thermal conductivity of wood with different densities at oven-dry condition.*

		Effective thermal conductivity (W/m · K)						
		0% offset		50% offset		Average of 0 and 50% offset		
Density[a] (kg/m^3)	Void (%)	Tangenital	Radial	Tangential	Radial	Tangential	Radial	Ratio (R/T)
1,540	0	0.410	0.410	0.410	0.410	0.410	0.410	1.00
1,386	10	0.341	0.341	0.341	0.340	0.341	0.341	1.00
1,232	20	0.284	0.284	0.283	0.283	0.283	0.284	1.00
1,078	30	0.236	0.236	0.234	0.235	0.235	0.235	1.00
924	40	0.194	0.194	0.191	0.193	0.193	0.194	1.01
770	50	0.158	0.158	0.154	0.157	0.156	0.157	1.01
616	60	0.125	0.126	0.121	0.124	0.123	0.125	1.01
462	70	0.096	0.097	0.093	0.094	0.095	0.095	1.01
308	80	0.071	0.071	0.068	0.068	0.069	0.070	1.00
154	90	0.047	0.047	0.046	0.047	0.046	0.047	1.01

[a] Density values were calculated based on cell-wall density (1,540 g/m^3) and percentage of void.

values were a little lower in the tangential direction than in the radial because heat flux must travel a longer "crooked" path (compare Path-R and Path-T in Fig. 4) in the tangential direction. The length of the heat flux path increases as porosity increases, up to a maximum of half the cell length as porosity approaches the limit of 100%.

The effect of misalignment is less significant when the cell lumen is filled with air than if the lumen were assumed to contain no conducting medium. Even though air is a low heat conductive medium, some heat is still transferred through the air in the lumen. Misalignment in the tangential direction results in a minimal reduction in thermal conductivity–only 0.1% to 0.5% for moderate cell porosity and 1% to 2% for high cell porosity. Hart (1964) examined this misalignment effect theoretically and found similar results. He demonstrated that misalignment resulted in a reduction in conductivity of about 10% to 12% for wood of moderate density and in even greater reductions for wood of lower density (high porosity). The predicted values (average of the two extreme alignment cases) from the two finite element models (Table 2) show radial thermal conductivity is only 1% higher than tangential conductivity at moderate to high porosities. Griffiths and Kaye (1923) found similar trends in which radial thermal conductivity exceeded that in the tangential direction by 5% to 10%. Rowley (1933) reported that for species with significant differences between springwood and summerwood, tangential conductivity was somewhat greater than radial conductivity, but no appreciable difference was found in wood that possessed a more uniform structure throughout the annual ring. The predictions of the finite element model are consistent with Rowley's observation, where thermal conductivity in cells with uniform structure is not significantly different between the radial and tangential directions. The effects of ring orientation and earlywood/latewood ratio will be demonstrated in the beat transfer board model presented in Part II.

The model clearly shows that cell porosity or density is the driving factor in thermal conductivity. This has been proven experimentally and is the basis for the empirical equation (Eq. (4)) used in the Wood Handbook (Forest Products Laboratory 1999), which shows thermal conductivity as a linear function of average wood density and moisture content:

$$k = G(B + CM) + A \quad (4)$$

where

k = thermal conductivity (W/m · K),
G = specific gravity,
A = 0.024,
B = 0.2002,
C = 0.004032, and
M = moisture content (%).

Calculated thermal conductivity values can vary from 0.094 W/m · K for cottonwood (*Populus deltoides*) to 0.178 W/m · K for hickory (*Carya glabra*), for densities of 350 and 770 kg/m^3, respectively, all at 0% moisture content. These calculated values are from empirical data, which averages the effects of low density (high porosity) and high density (low porosity) earlywood and latewood regions. Using the finite element model at 50% porosity, which is a uniform density of 770 kg/m^3 (half the pure cell-wall substance density of 1,540 kg/m^3), the model calculates an effective thermal conductivity of 0.157 W/m·K (Table 2). The empirical equation (Eq. (4)) calculates k = 0.178 W/m · K, a 12% difference. The estimate of the finite element model and the Wood Handbook estimate are close. However, the lower values from the finite element model may be due to the uniform density analysis of the model, whereas the empirical equation is based on averaged combined earlywood and latewood heat transfer measurements. The higher empirical coefficient value may be a result of the increased heat transfer rate of latewood, which results from its higher density. It is interesting to note that the equation for thermal conductivity (Eq. (4)) is a linear relationship with density or specific gravity, whereas curves for the finite element model (Figs. 5 and 6) show a nonlinear relationship. In the nonlinear regression (Eq. (5)), the k_{eff} (W/m · K) from the model can be estimated as a function of density (kg/m^3):

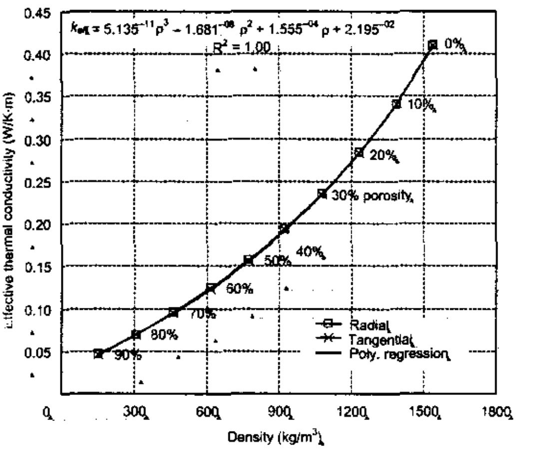

FIG. 5. Effect of density (not averaged) on radial and tangential effective thermal conductivity and polynomial function by regression analysis at zero moisture content.

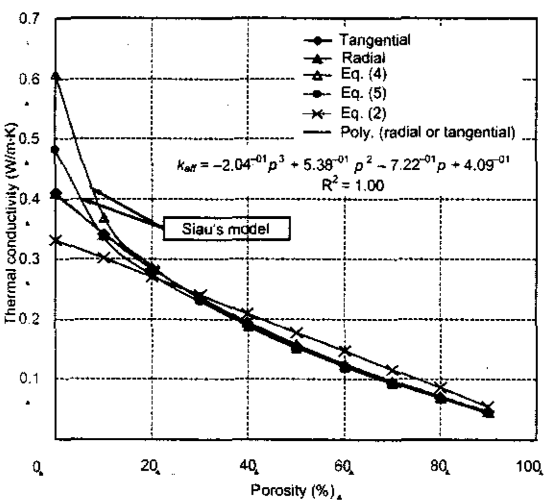

FIG. 6. Wood thermal conductivity for finite element model compared with that for model developed by Siau (1995) and *Wood Handbook* model as a function of porosity.

$$K_{\text{eff}} = 5.135 \times 10^{-11}\,(\text{density})^3 - 1.681 \times 10^{-8}\,(\text{density})^2 + 1.555 \times 10^{-4}\,(\text{density}) + 2.195 \times 10^{-2} \quad (5)$$

Based on the model results, both radial and tangential thermal conductivity values can be calculated from this equation since there was less than 1% difference between the two values (Table 2. Figs. 5 and 6).

The results from this finite element cellular model (Table 2) were also compared with the model developed by Siau (1984) and that from the Wood Handbook (Fig. 6). Siau obtained his model by applying analogous electric circuits to thermal resistance and considering the non-uniformity of the heat flux in the cell sidewalls. Equation (6) was modified from Siau's original equation (eq. 5.17, Siau 1995) by changing the thermal conductivity values for air in the cell lumen (0.042 W/m · K) and the cell-wall substance (0.44 W/m · K) to the values used in this study (Table 1). In his work, Siau simplified equation 5.17 to equation 5.18 as a "useful, empirical linear regression" model by a data fitting technique. The same technique was followed here to simplify Eq. (6) to Eq. (7):

$$\frac{1}{Keff} = \frac{1-a}{(0.6068 - 0.581a)} + \frac{a}{0.41(1-a) + 0.026a} \quad (6)$$

$$Keff = 0.4814 - 0.4626\,a \quad (7)$$

where $a = \sqrt{porosity}$.

Thermal conductivity values obtained from the cellular finite element model developed in this study for both radial and tangential directions were within 1% of Siau's model when the porosity was 10% and greater.

The authors believe that the cellular finite element model can he used to determine effective thermal conductivity values through uniform density earlywood and latewood bands, representing a more realistic characterization of the heat flux path than do thermal conductivity values obtained from either the averaged wood thermal conductivity experiments or thermal resistivity models. The finite element model can also accommodate a more geometrical description of the cell, including the interior radius of the lumen as part of the heat transfer effects, which is not possible at the cellular level for the resistivity models. These individual effective thermal conductivity values can then he applied to a macro wood hoard or composite models.

CONCLUSIONS

Two fundamental finite element based models were developed to present radial and tangential heat transfer based on the fundamental structural characteristics of softwood cells. The models show significant effects of softwood cell density (porosity), no significant differences between radial and tangential directions, and no significant difference related to cell alignment. While this study considered only cell material having 0% moisture content, further analyses of increasing moisture contents are easily adapted and future studies are planned (Part III).

Specific conclusions from this study are as follows:

1. The density (porosity) of softwood cells plays a major role in determining its effective thermal properties. A nonlinear relationship between the effective thermal conductivity and the density of wood was developed by poly regression fitting. Thermal conductivity (at zero moisture content) at any density or porosity value can be calculated and used in macro softwood heat transfer models.
2. Cell misalignment in the wood microstructure has little effect on radial and tangential thermal properties. Only at low densities (high porosities) do slight differences between these properties become evident. For most wood species, however, radial and tangential thermal conductivity can he considered the same because the density (porosity) of wood is not that low (high).
3. Thermal conductivity values for earlywood and latewood are calculated on the basis of their density (porosity).

The new finite element heat transfer models provide the ability to study heat transfer in hoards based on actual structural characteristics of the wood, including ring orientation, growth rate, earlywood and latewood density (porosity), and earlywood/latewood ratio. The specific effects of these characteristics on heat transfer will be presented in Pan II of this series. The effect of moisture content in combination with structural characteristics will be presented in Pan IV. A goal for development of this heat transfer model is to be able to study and gain a better understanding of the heat transfer process for any board, of any size, "cut" from any location in a log, of any species, and from nearly any tree growth condition. Such a fundamental approach to studying heat transfer issues in wood has numerous practical applications, such as optimizing drying schedules for different cut boards, determining heat treatment times to kill insects, and determining heat curing times for solid wood and wood laminates.

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