Development of a Transition Pathway Model Using Three Traditional Variables to Describe the Main Structural Characteristics of a Forest Stand Type, Size, and Density

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Introduction

The Central Hardwood Region is a unique forest region in the United States, encompassing 340 million acres of land, of which 100 million acres are forested (Parker 1993). The region contains one-fourth of the U.S. population, with approximately 90 percent of the land in private ownership (Parker 1993). In general, private forested lands in the Central Hardwood Region tend to be small, non-industrial forests with little forest management (Nyland 1992). The tendency of non-industrial private forest (NIPF) landowners to eschew professional guidance when managing their forests is cause for concern. The sustainability of Central Hardwood Region forests with regards to economic, environmental, and social benefits may depend on educating and encouraging NIPF landowners to seek professional forest management advice.

The most difficult part of the decision making process often lies in identifying and choosing goals. Since the purpose of modern forest management is to achieve diverse landowner goals, reasoned management cannot be practiced without those goals (Wang and others 2002). Private landowners have many interests pertaining to their lands; they may be interested in increasing the value of timber production, producing better wildlife habitat or creating aesthetically pleasing forests. In all cases, management options are available to the landowner to increase or decrease the inherent value of the forest relative to a particular goal. We created a decision support tool designed to assist private landowners in identifying management goals based upon probable outcomes and impacts associated with forest management decisions.

As a general rule, existing forest management tools are technical and complex. In designing a new tool for NIPF landowners, we wanted to keep the tool relatively simple while ensuring compatibility with existing tools. To this end, a transition pathway model was developed using three traditional variables to describe the main structural characteristics of a forest stand: type, size, and density. The transition pathway model predicts the probability of the current stand transitioning to a new Type-Size-Density class given various forest management treatments. Most NIPF landowners are not aware how Type-Size-Density variables relate to their management interests. Thus, several forest attributes were assigned qualitative values for each Type-Size-Density class describing the suitability of that class for specific commodity and non-commodity resource uses. The assigned qualitative attributes were designed to give the NIPF landowner a greater appreciation for different attribute potentials while describing likely structural changes from different management decisions. An option-based decision support tool was then developed based on the transition pathway model and forest resource attributes. A complete description of the project can be found in Larson (2004).

Option-based decision tools provide a series of options given the initial conditions, describe the options, and provide a number of metrics to rate the options against each other. However, there is a dearth of simple option-based forest decision tools (but see Marquis and Ernst 1992, Twery and others 2000). Our underlying premise is that many landowners may have trouble stating specific goals for their land; but, if presented with a number of options, landowners would have little trouble choosing between them. The attributes assigned to each Type-Size-Density class provide the landowner with the means to

Table 1. The Type, Size, and Density variables defined by the categories for each variable and a brief description of each category.

Variable	Category	Description
Type[a]	White Oak	White oak species group is dominant
	Red Oak	Red oak species group is dominant
	Pine	Pine species group is dominant
	Other Hardwood	All other species group is dominant
	Mixed	No species group is dominant
Size	Sapling	Less than 6 inches DBH
	Pole	6 to 10 inches DBH
	Sawlog	Greater than 10 inches DBH
Density	High	Greater than 750 trees per acre
	Medium	250 to 750 trees per acre
	Low	Less than 250 trees per acre

[a] Species group dominance is defined as >40% relative dominance by basal area.

evaluate management options using the criteria deemed most important by the landowner. Landowners may then proceed to existing forest growth models and decision tools with a clear understanding of their own desires and goals, as well as knowledge of the effect professional management may have on their forest.

Methods

Transition Pathway Model

Study area

The study area consisted of the St. Francis River and Black River Watersheds, located in southeastern Missouri. Both watersheds were chosen because over 90 percent of the land area is forested. Combined, the watersheds encompass ~1.75 million acres and include portions of St. Francois, Madison, Iron, Reynolds, Wayne, Carter, Ripley and Butler counties (fig. 1).

Data sources

The primary presupposition made in designing the decision support tool was that a simple model with easily defined variables was the best method for fulfilling the purpose of the study. However, in order to prevent placing unnecessary limits on the possible applications of the tool, the underlying model was based on simulations run on the Forest Vegetation Simulator (FVS), a standard forest growth and yield model of the U.S. Forest Service (Stage 1973, Wykoff and others 1982).

Forest Inventory and Analysis (FIA) data (Miles and others 2001) from 1987 and 1988 (582 plots) were input into FVS. FIA data and FVS simulation output were summarized into three variables: Type, Size, and Density. Within a stand, Type refers to the dominant tree species group, Size refers to the average tree diameter at breast height (DBH), and Density refers to the number of trees per acre (TPA). The species groups for Type were based on oak dominance in the study area and the economic importance of oak and pine species in Missouri. Categories for the Type variable species groups were White Oak, Red Oak, Pine, Other Hardwood, and Mixed species. A species group was considered dominant if the relative dominance by basal area was >40 percent. Quadratic mean diameter was used to create three Size categories: Sapling (<6" DBH), Pole (6"-10" DBH), and Sawlog (>10" DBH). Size categories were based on industry standards and professional expertise. Density was also classified into three categories: Low (<250 TPA), Medium (250-750 TPA), and High (>750 TPA). The Density categories were chosen after initial analysis of the FIA data and were based on professional expertise.

The FIA plots were grown in FVS using the Suppose graphical user interface (Crookston 1997, Dixon 2003). Plots were grown using the following management options: no treatment, thinning from below, and diameter limit cut. Predictions were limited to 20 years, which is long enough for potential forest change and short enough

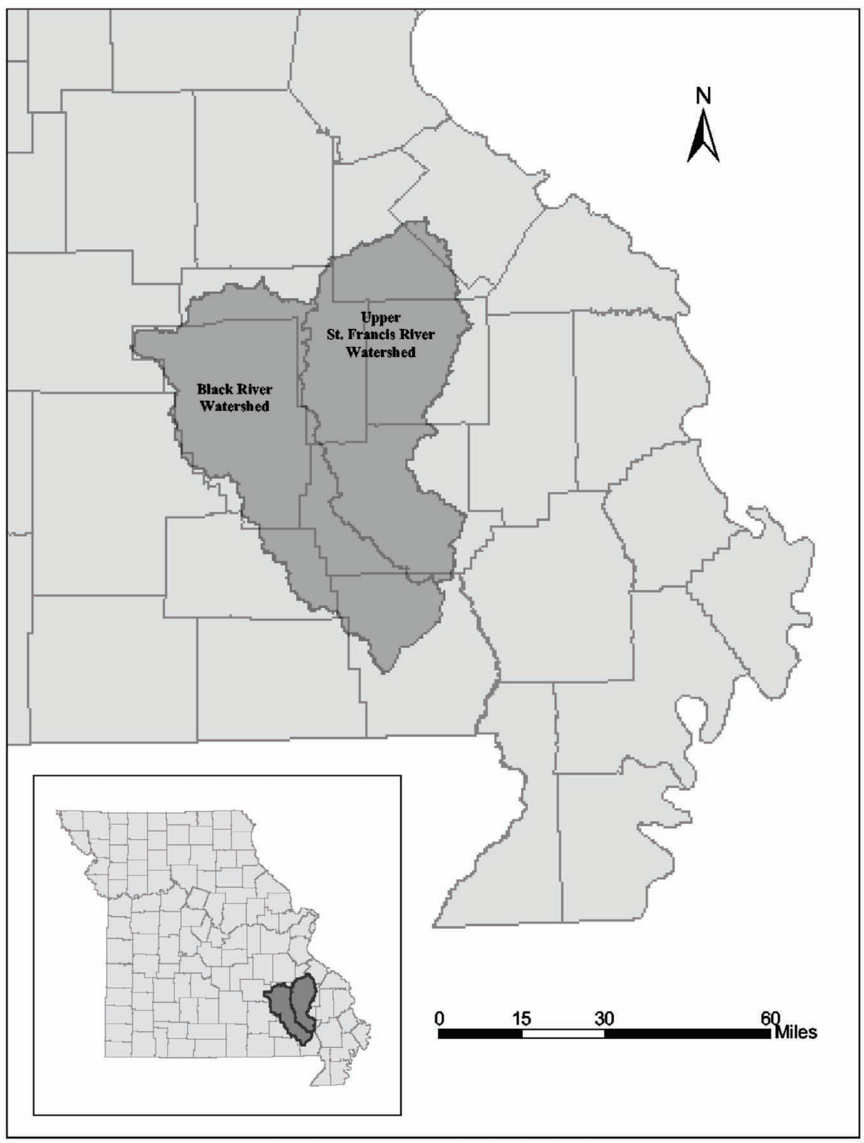

Figure 1. The study area in southeastern Missouri as defined using USGS 8-digit hydrologic units (Seaber 1984).

for landowners to retain a vested interest in the land. For the No Treatment option, FIA plots were grown in FVS with no management treatment. The Thinning From Below and Diameter Limit Cut management options used FVS predefined management treatments: "thinning from below" to a basal area target of 55 ft^2/acre (Thinning From Below) and "thinning from above" to a desired diameter of 10 inches DBH with 100 percent removal (Diameter Limit Cut). All management options conducted the management prescription at year zero and then grew the sample plot for 20 years with no further management activity.

Transition pathways

The transition pathway model was built to predict the probability of an initial Type-Size-Density class transitioning to a new Type-Size-Density class. Based on categories defining the Type-Size-Density variables, all possible combinations of variables were developed (table 2); however, the large number of Type-Size-Density classes created an overly complex model with insufficient data to support the development of a full transition matrix. Therefore, transition pathways were designed based on two assumptions for the No Treatment option. First, it was assumed that the dominant species group (Type) at year zero would have a low probability of changing in 20 years, thus all transitions to a different Type were combined into the same probability class of "other." Second, it was assumed that Size and Density were negatively correlated; thus, over 20 years, the plots would tend to grow larger in diameter (Size increases) while becoming smaller in density (Density decreases).

A matrix was created for each management option with the initial classes (year 0) in rows and counts of the transition classes (year 20) in columns. Transition probabilities were calculated by dividing the count for each transition class by the total number of plots in the initial class. The transition pathways were then created by collating the initial classes by Type category, keeping intact all transition class probabilities of the initial Type, and combining all transition class probabilities for new Type categories into a new probability class called "other."

Model testing

Model testing consisted of verifying that the transition pathway model predictions were consistent with the FVS model being summarized. Test plots were selected from FIA plots outside the study area but within the same FIA survey unit. Ten test plots were chosen for each Type-Size-Density class using a random number generator (even distribution from 0 to 1), except in those classes with ≤10 samples. Data were grown in FVS, Type-Size-Density classes of initial test data and resulting FVS output were calculated, and results were compared to predictions of the transition pathway model. The chi-square goodness of fit test (Zar 1999) was used to determine if a significant difference existed between the actual FVS output classes and the predicted transition classes (all $\alpha = 0.05$). Two methods were used to determine if a significant difference existed between the FVS test results and the transition pathway model predictions. First, the chi-square value was compared to the appropriate critical value. Second, the test plots used to calculate the chi-square value were analyzed to determine if any plots transitioned to a class that was not predicted by the model.

Table 2. Combined Type-Size-Density classes. Blue font indicates classes not used in the building of the transition pathway model.

Class	Type	Size	Density	n
1	White Oak	Sapling	High	37
2	White Oak	Sapling	Med	94
3	White Oak	Sapling	Low	3
4	White Oak	Pole	High	0
5	White Oak	Pole	Med	20
6	White Oak	Pole	Low	15
7	White Oak	Sawlog	High	0
8	White Oak	Sawlog	Med	0
9	White Oak	Sawlog	Low	5
10	Red Oak	Sapling	High	31
11	Red Oak	Sapling	Med	99
12	Red Oak	Sapling	Low	1
13	Red Oak	Pole	High	0
14	Red Oak	Pole	Med	31
15	Red Oak	Pole	Low	8
16	Red Oak	Sawlog	High	0
17	Red Oak	Sawlog	Med	0
18	Red Oak	Sawlog	Low	5
19	Pine	Sapling	High	11
20	Pine	Sapling	Med	9
21	Pine	Sapling	Low	1
22	Pine	Pole	High	0
23	Pine	Pole	Med	3
24	Pine	Pole	Low	3
25	Pine	Sawlog	High	0
26	Pine	Sawlog	Med	0
27	Pine	Sawlog	Low	1
28	Other Hardwood	Sapling	High	35
29	Other Hardwood	Sapling	Med	69
30	Other Hardwood	Sapling	Low	5
31	Other Hardwood	Pole	High	0
32	Other Hardwood	Pole	Med	8
33	Other Hardwood	Pole	Low	4
34	Other Hardwood	Sawlog	High	0
35	Other Hardwood	Sawlog	Med	0
36	Other Hardwood	Sawlog	Low	2
37	Mixed	Sapling	High	21
38	Mixed	Sapling	Med	36
39	Mixed	Sapling	Low	1
40	Mixed	Pole	High	0
41	Mixed	Pole	Med	7
42	Mixed	Pole	Low	5
43	Mixed	Sawlog	High	0
44	Mixed	Sawlog	Med	0
45	Mixed	Sawlog	Low	0

Forest Resource Attributes

A forest resource attribute describes an inherent characteristic of a forest. The complexity of analysis was limited by the simple model design and simple variables; however, we wanted to provide landowners with general impressions of forest potential, and for that purpose qualitative attribute values were defined as a function of Type-Size-Density.

Attributes were developed for income potential (e.g., sawtimber), wildlife habitat suitability (e.g., gray squirrel), and risk susceptibility (e.g., insect/disease). The sawtimber attribute was assigned qualitative values based on professional expertise and Missouri logging sale charts, with tree species and tree diameter influencing the attribute values. Gray squirrel (Sciurus carolinensis) habitat attributes were assigned qualitative values based on a habitat suitability index (HSI) model (Larson and others 2003), with forage and nesting cover influencing the attribute values. The insect/disease risk attribute was assigned qualitative values based on professional opinion and the work of Wargo, Houston and LaMadeleine (1983), with stress from competition for resources influencing the attribute values.

The first step in assigning a qualitative attribute value to a Type-Size-Density class was to determine which variable(s) was the most influential for that attribute and then ranking all classes on a relative scale of 1 to 9. The second step was to indicate the degree of confidence in the assigned ranking for each class. The degree of confidence (high, medium, low) was a measure of the importance of outside factors not captured by Type-Size-Density in determining the suitability and potential of a particular forest for the attribute in question.

Decision Support Tool

The predictions and attributes developed in the transition pathway model were organized in a Microsoft Access database. A transition table was created for each management option containing the two largest transition probabilities for each initial class along with the probability of that class staying the same. The attribute tables in the database consisted of the relative ranking and degree of confidence for all classes. A table with representative photographs of each class was added to the database for additional landowner edification. A user interface was built in the same Microsoft Access database using a series of forms, command buttons, and macros, and was designed to provide landowners with a series of screens giving them the ability to choose the Type-Size-Density class of their forest from drop down menus, select management options, view attributes, and maneuver back and forth through the screens (fig. 2).

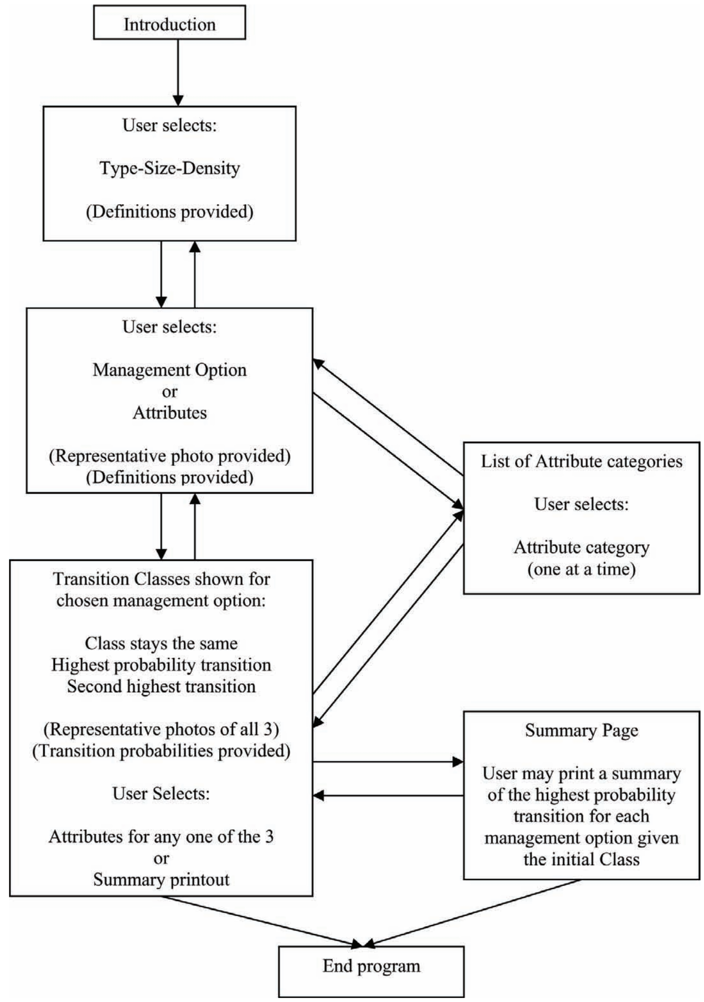

Figure 2. Design logic of the user interface for the decision support tool.

Results

Transition Pathway Model

Initial classes were calculated for the 582 FIA data plots. Analysis of initial sample counts (n) prompted the removal of all Size-Density combinations of Sapling-Low and Pole-High classes from the database due to low n (table 2). In addition, Sawlog classes and plots containing no live trees were removed. The 546 remaining FIA data plots were used to build the transition pathway model predictions.

The White Oak category included 166 sample plots. The transition pathway probabilities (table 3) affirmed the assumptions made for the No Treatment management option; the White Oak, No Treatment management option demonstrated that Size and Density were negatively correlated and Type at year zero had a low probability of changing in 20 years. The largest sample size for White Oak, White Oak-Sapling-Medium (class 2), was 94 plots and only 7.4 percent of the plots changed Type, while

Table 3. Transition pathway probabilities (in percent) for the White Oak category.

Management Treatment	Initial Class[a]	Transition Class[a] (percent) 1	2	3	5	6	9	Other
No Treatment	1	10.8	29.7	0.0	51.4	0.0	0.0	8.1
	2	0.0	8.5	1.1	78.7	4.3	0.0	7.4
	5	0.0	0.0	0.0	50.0	35.0	15.0	0.0
	6	0.0	0.0	0.0	0.0	33.3	46.7	20.0
Thinning From Below	1	0.0	13.5	0.0	10.8	5.4	40.5	29.7
	2	0.0	7.4	1.1	17.0	14.9	48.9	10.6
	5	0.0	0.0	0.0	0.0	10.0	65.0	25.0
	6	0.0	0.0	0.0	0.0	13.3	66.7	20.0
Diameter Limit Cut	1	8.1	45.9	0.0	21.6	0.0	0.0	24.3
	2	0.0	26.6	2.1	44.7	3.2	0.0	23.4
	5	0.0	0.0	5.0	20.0	30.0	0.0	45.0
	6	0.0	0.0	0.0	0.0	40.0	6.7	53.3

[a] The Type-Size-Density classes are: 1-White Oak-Sapling-High, 2-White Oak-Sapling-Medium, 3-White Oak-Sapling-Low, 5-White Oak-Pole-Medium, 6-White Oak-Pole-Low, 9-White Oak-Sawlog-Low.

78.7 percent changed Size to White Oak-Pole-Medium (class 5). The No Treatment transition pathway for White Oak is illustrated in figure 3.

The No Treatment management options for Red Oak, Pine, and Other Hardwood followed similar patterns as White Oak and further affirmed the assumptions made in the design of the transition pathways; the Sapling classes in particular showed strong probabilities to increase in Size, Density, or both, while showing low probabilities of changing Type. However, the Mixed category averaged >30 percent probability of changing Type. This tendency was expected due to the lack of a dominant species group in the Mixed category and is consistent with forest stand dynamics theory, as competition for resources allows tree species with a competitive edge to achieve dominance over time.

The Thinning From Below management option showed a stronger tendency for classes to change Type than the No Treatment option (White Oak example, table 3), with the exception of the Red Oak category. Thinning proscriptions were carried out in FVS with no regard to tree species; thus, the dominance of tree species groups (Type) was potentially shifted at year zero. Any shifts were then exaggerated over the 20 year simulation, resulting in the stronger tendency to change Type.

The Diameter Limit Cut management option generally showed an even stronger tendency for classes to change Type than the Thinning From Below option (White Oak example, table 3), with the exception of the Other Hardwood category. Harvesting the largest trees in a stand at year zero had an even greater potential to shift tree species group dominance than thinning.

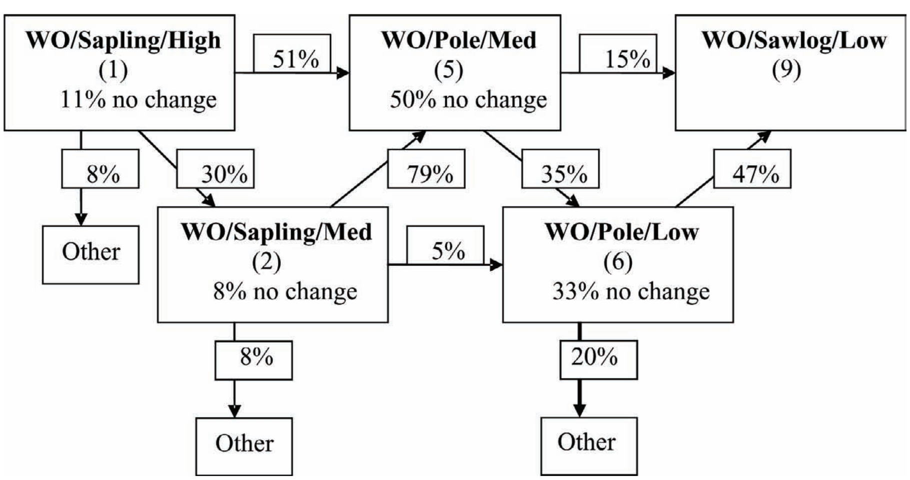

Figure 3. Transition pathway for the White Oak Type category.

Model Validation

The chi-square goodness of fit test was conducted 84 times and used 255 test plots. The White Oak and Red Oak classes were tested twice each with replacement. In total, 31 of the 84 tests (37 percent) were significantly different from the transition pathway model predictions (White Oak results, table 4). Each management option had 28 tests, with all three options showing <40 percent significant difference. Of the 31 tests that showed a significant difference, 23 were significantly different due to one or more test plot transitions having a predicted transition probability of zero, while only 8 (10 percent of total) were significantly different due to the chi-square value being greater than the critical value. Analyzing the chi-square tests by Type shows that

Table 4. Results of the predicted White Oak transition pathway model chi-square tests.

Management Treatment	Class[a]	n	χ^2 Test 1	χ^2 Test 2	Critical Value
No Treatment	1	10	2.721	0.061	7.815
	2	10	1.503	0.601	7.815
	5	10	*	*	5.991
	6	10	0.046	3.153	5.991
Thinning From Below	1	10	2.297	4.232	9.488
	2	10	1.276	3.440	11.07
	5	10	1.138	1.138	5.991
	6	10	0.450	2.796	5.991
Diameter Limit Cut	1	10	1.003	8.207	7.815
	2	10	0.950	1.994	9.488
	5	10	1.500	4.222	7.815
	6	10	*	2.019	5.991

[a] The Type-Size-Density classes are: 1-White Oak-Sapling-High, 2-White Oak-Sapling-Medium, 5-White Oak-Pole-Medium, 6-White Oak-Pole-Low.

* - Indicates one or more observed test plots transitioned to a Class that was not predicted in the study area.

Pine and Red Oak transition pathways were the weakest with 58 percent and 54 percent significantly different, respectively. The Other Hardwood transition pathway had 42 percent significantly different. The White Oak and Mixed transition pathways were the best with both having only 17 percent significantly different.

Forest Resource Attributes

The most influential variable for sawtimber was Size, with the Sawlog category receiving the highest ranking (table 5). The highest value sawtimber species in Missouri are oaks and pines, and thus the Sawlog classes for White Oak, Red Oak, and Pine received the highest ranking of 9. However, the confidence for the ranking of 9 was assigned as Low due to the Type-Size-Density variables being unable to take into account the quality and grade of lumber. The classes for Other Hardwood and Mixed were ranked slightly lower than the White Oak, Red Oak, and Pine across all classes.

The most influential variable for gray squirrel species was Size, as related to forage and nesting (table 4). Hard mast production and nesting habitat were considered highest in the Sawlog classes, particularly for White Oak and Red Oak. The confidence values for the best gray squirrel habitats were high due to forage and nesting requirements being met by the same classes of forest.

Insect/disease risk was most influenced by Density combined with Size (table 4). Competition for moisture and nutrients was the main factor increasing tree susceptibility to insect/disease. The confidence levels assigned were Low to Medium due to the importance of landscape position, frost injury, and drought.

Conclusion

The decision support tool was based on the idea of producing a simple forest growth model capable of predicting forest change while providing descriptive forest resource attributes to assist forest landowners in developing management goals. The additional goal, to maintain compatibility between the developed simple forest growth model and existing models, was a major design consideration. A transition pathway model utilizing the three traditional variables, Type, Size, and Density, was designed and tested for compatibility against the FVS model on which it was based.

The management treatments used (No Treatment, Thinning From Below, and Diameter Limit Cut) all produced differing transition pathways showing that the model design is capable of capturing select effects of forest management. The No Treatment scenario showed that as trees grow larger in size, stand density goes down, and the dominant tree species group (Type), once established, is unlikely to change in 20 years. The Thinning From Below scenario demonstrated that trees grow larger more rapidly when competition for resources is eased. The Diameter Limit Cut scenario demonstrated that the forest stand will continue to grow and transition after management actions have been taken; however, the Diameter Limit Cut scenario also demonstrated that removing the largest trees in a forest stand does not guarantee replacement by similar species.

The transition pathway model was reasonably compatible with the FVS model. Over 60 percent of all

Table 5. Qualitative values assigned to forest resource attributes.

	Sawtimber		Gray Squirrel		Insect/Disease[a]	
Class	**Rank**	**Confidence**	**Rank**	**Confidence**	**Rank**	**Confidence**
1	1	High	1	High	1	High
2	1	High	1	High	1	High
5	4	Med	6	Med	4	Low
6	4	Med	6	Med	3	Med
9	9	Low	9	High	4	Med
10	1	High	1	High	2	High
11	1	High	1	High	2	High
14	4	Med	6	Med	5	Low
15	4	Med	6	Med	4	Med
18	9	Low	9	High	6	Low
19	1	High	1	High	1	High
20	1	High	1	High	1	High
23	4	Med	4	Med	4	High
24	4	Med	4	Med	3	High
27	9	Low	6	Med	5	Med
28	1	High	1	High	1	Med
29	1	High	1	High	1	Med
32	3	Med	4	Low	5	Low
33	3	Med	4	Low	4	Low
36	8	Low	6	Low	5	Low
37	1	High	1	High	1	High
38	1	High	1	High	1	High
41	3	Med	3	Low	3	Med
42	3	Med	4	Low	3	Med
45	7	Low	6	Low	4	Med

[a]Higher rankings for Insect/Disease indicate the class is at greater risk.

chi-square tests showed no significant difference from the model predictions. Of the 31 tests showing a significant difference, 23 were due to one or more test plots transitioning to a Type-Size-Density class that was not predicted by the transition pathway model (prediction = 0.0 percent). However, no model can predict all transitions for every possible forest combination, and the number of significantly different results is acceptable for this type of model.

The forest resource attributes captured the general aspects of forest potential. The simple model design limited the amount of information that could be shown using attributes; however, when taken with the descriptive explanations in the decision tool, landowners have access to knowledge of forest potential that is capable of aiding in the development of management goals.

The decision support tool software, as currently developed, will provide landowners with the means to predict forest change, analyze forest resource potential, and aid in the development of management goals. Overall, the decision support tool is designed largely for NIPF landowner education and information. Future developments will seek to expand management options and forest attributes, produce an internet based version, apply the methodology to data sets from different geographical regions, and distribute the software to federal and state agencies for use with landowner consultations.

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