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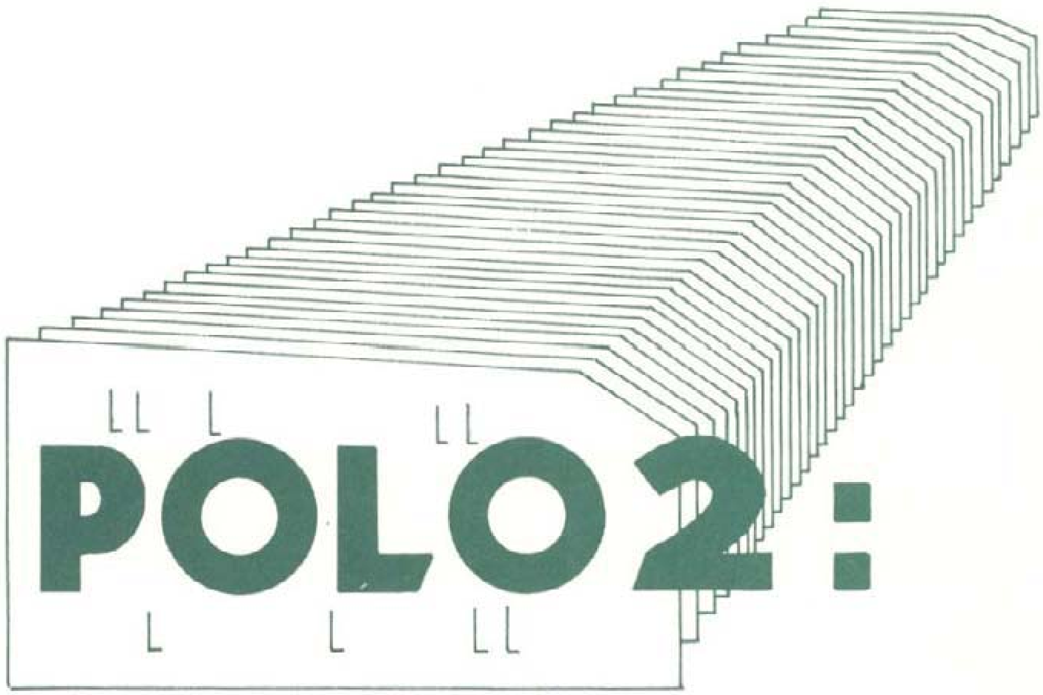

a user's guide to multiple Probit Or LOgit analysis

Robert M. Russell, N. E. Savin, Jacqueline L. Robertson

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POLO2:

a user's guide to multiple Probit Or LOgit analysis

Robert M. Russell, N. E. Savin, Jacqueline L. Robertson

CONTENTS

Many studies involving quantal response include more than one explanatory variable. The variables in an insecticide bioassay, for example, might be the dose of the chemical as well as the body weight of the test subjects. POLO2 is a computer program developed to analyze binary quantal response models with one to nine explanatory variables. Such models are of interest in insecticide research as well as in other subject areas. For examples of other applications, texts such as those by Domencich and McFadden (1975) and Maddala (1977) should be consulted.

For models in which only one explanatory variable (in addition to the constant) is present, another program, POLO (Russell and others 1977, Savin and others 1977, Robertson and others 1980) is available. However, the statistical inferences drawn from this simple model may be misleading if relevant explanatory variables have been omitted. A more satisfactory approach is to begin the analysis with a general model which includes all the explanatory variables suspected as important in explaining the response of the individual. One may then test whether certain variables can be omitted from the model. The necessary calculations for carrying out these tests are performed by POLO2. If the extra variables are not significant in the multiple regression, a simple regression model may be appropriate.

The statistical documentation of POLO2, descriptions of its statistical features, and examples of its application are described in articles by Robertson and others (1981 a, b), and Savin and others (1981).

The POLO2 program is available upon request to:

Director
Pacific Southwest Forest and Range Experiment Station
P.O. Box 245
Berkeley, California 94701
Attention: Computer Services Librarian

A magnetic tape with format specifications should be sent with the request. The program is currently operational on the Univac 1100 Series, but can be modified for use with other large scientific computers. The program is not suitable for adaptation to programmable desk calculators.

This guide was prepared to assist users of the POLO2 program. Selected statistical features of the program are described by means of a series of examples chosen from our work and that of others. A comprehensive description of all possible situations or experiments amenable to multivariate analyses is beyond the scope of this guide. For experiments more complex than those described here, a statistician or programmer, or both, should be consulted regarding the appropriate use of POLO2.

1. GENERAL STATISTICAL FEATURES

Consider a sample of I individuals indexed by $i = 1,...,I$. For individual i there is an observed J x 1 vector $s_i' = (s_{1i},\ldots, s_{Ji})$ of individual characteristics. In a binary quantal response model the individual has two responses or choices. These can be denoted by defining the binomial variable

$$f_i = 1 \text{ if the first response occurs (if alternative 1 is chosen),}$$
$$f_i = 0 \text{ if the second response occurs (if alternative 2 is chosen).}$$

For example, in a bioassay of toxicants the individuals are insects and the possible responses are dead or alive. The measured characteristics may include the dose of the toxicant, the insect's weight and its age.

The probability (P) that $f_i = 1$ is

$$P_i = F(\beta' z_i)$$

where F is a cumulative distribution function (CDF) mapping points on the real line into the unit interval, $\beta' = (\beta_1, \ldots, \beta_K)$ is a K x 1 vector of unknown parameters, $z_{ki} = z_k(s_i)$ is a numerical function of s_i, and $z_i' = (z_{1i}, \ldots, z_{Ki})$ is K x 1 vector of these numerical functions. If, for instance, weight is one of the measured characteristics, then the function z_{ki} may be the weight itself, the logarithm of the weight or the square of the weight.

For the probit model

$$P_i = F(\beta' z_i) = \Phi(\beta' z_i)$$

where Φ is the standard normal CDF. For the logit model

$$P_i = F(\beta' z_i) = 1/[1 + e^{-\beta' z_i}].$$

POLO2 estimates both models by the maximum likelihood (ML) method with grouped as well as ungrouped data.

The ML procedure can be applied to the probability function $P_i = F(\beta' z_i)$ where F is any CDF. Since f_i is a binomial variable, the log of the probability of observing a given sample is

$$L = \sum_{i=1}^{I} [f_i \log P_i + (1 - f_i) \log(1 - P_i)]$$

where L is referred to as the log likelihood function. The ML method selects as an estimate of β that vector which maximizes L. In other words, the ML estimator for β maximizes the calculated probability of observing the given sample.

When the data are grouped there are repeated observations for each vector of values of the explanatory variables. With grouped data, we change the notation as follows. Now let I denote the number of groups and i=1,...,I denote the levels (z_i, s_i) of the explanatory variables. Let n_i denote the number of observations at level i and r_i denote the number of times that the first response occurs. The log likelihood function for grouped data is then

$$L = \sum_{i=1}^{I} [r_i \log P_i - (n_i - r_i) \log(1 - P_i)]$$

Again, the ML method selects the vector that maximizes the log likelihood function L as an estimate of β. For further discussion of the estimation of probit and logit models with several explanatory variables, see Finney (1971) and Domencich and McFadden (1975).

The maximum of the log likelihood is reported to facilitate hypothesis testing. Two indicators of goodness-of-fit are routinely calculated. One is the prediction success table (Domencich and McFadden 1975), which compares the results predicted by the multiple regression model with the results actually observed. The other goodness-of-fit indicator is the calculation of the likelihood ratio statistic for testing the hypothesis that all coefficients in the regression are equal to zero. Finally, a general method for transformation of variables is included.

2. DATA INPUT FORMAT

2.1 Starter Cards

Every POLO2 run starts with five cards that call the program from a tape (*fig. 1*).These cards reflect the current Univac 1100 implementation and would be completely different if POLO2 were modified to run on a different computer. All of the remaining input described in *sections 2.2-2.9* would be the same on any computer.

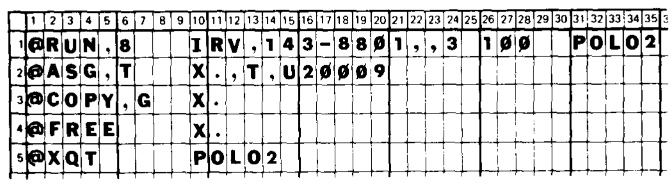


Figure 1.

Cards 2-5 must be punched as shown. In card 1, the user's identification and account number should be placed in columns 10-21. Column 24 is the time limit in minutes; the page limit is listed in columns 26-28. Both time and page limits may be changed to meet particular needs.

2.2 Title Card

Each data set begins with a title card that has an equal sign (=) punched in column 1. Anything desired may be placed in columns 2-80 (*fig. 2*). This card is useful in documenting the data, the model, the procedures used for the analysis, or equivalent information. The information is reprinted at the top of every page of the output. Only one title card per data set may be used.

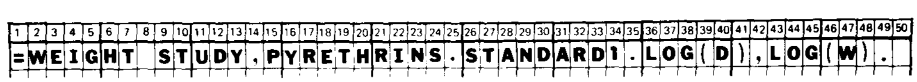


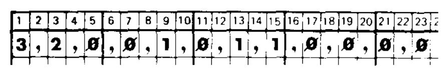
Figure 2.

2.3 Control Card

The information on this card controls the operation of the program. All items are integers, separated by commas (*fig. 3*). These numbers need not occur in fixed fields (specific columns on a card). Extra spaces may be inserted before or after the commas as desired (*fig. 4*). Twelve integers must be present. These are, from left to right:

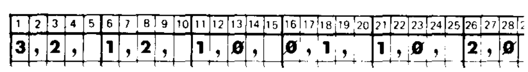
Position	*Internal program designation*	*Explanation*
1	NVAR	Number of regression coefficients in the model, including the constant term, but not including natural response.
2	NV	Number of variables to be read from a data card. This number corresponds to the number of F's and I's on the Format Card (*sec. 2.7*). Normally, NV=NVAR-1 because the data do not include a constant 1 for the constant term. NV may differ from NVAR-1 when transformations are used in the analysis.
3	LOGIT	LOGIT=0 if the probit model is to be used; LOGIT=1 if the logit model is desired.
4	KONTROL	One of the explanatory variables (for example, dose) will be zero, for the group when a control group is used. KONTROL is the index of that parameter within the sequence of parameters used, with 2 ≤ KONTROL ≤ NVAR. This limit indicates that any parameter except the constant term may have a control group.
5	ITRAN	ITRAN=1 if variables are transformed and a transformation card will be read. ITRAN=0 if the variables are to be analyzed as is and there will be no transformation card.
6	ISTARV	ISTAR V=1 if starting values of the parameters are to be input; ISTARV=0 if they will be calculated automatically. The ISTARV=1 option should be used only if the automatic method fails.
7	IECHO	IECHO=1 if all data are to be printed back for error checking. If the data have been scrupulously checked, the IECHO=0 option may be used and the data will not be printed back in their entirety. A sample of the data set will be printed instead.
8	NPARN	NPARN=0 if natural response, such as that which occurs without the presence of an insecticide, is present and is to be calculated as a parameter. If natural response is not a parameter, NPARN=1.
9	IGROUP	IGROUP=0 if there is only one test subject per data card; IGROUP=1 if there is more than one (that is, if the data are grouped).
10	NPLL	Number of parallel groups in the data (see *sec. 4.1* for an example). NPLL=0 is read as if it were NPLL=1; in other words, a single data set would compose a group parallel to itself.
11	KERNEL	When a restricted model is to be computed, some variables will be omitted from the model and the regression will be rerun. The KERNEL control instructs the program how may variables to retain. Variables 1 through KERNEL are retained, where 2 ≤ KERNEL ≤ NVAR; variables KERNEL+1 through NVAR are dropped. Note that variables can be rearranged as desired through the use of transformation or the "T" editing control on the format card (*sec. 2.7*). When a restricted model is not desired, KERNEL=0.
12	NITER	This integer specifies the number of iterations to be done in search of the maximum log likelihood. When NITER=0, the program chooses a suitable value. If starting values are input (ISTARV=1), NITER=0 will be interpreted as no iterations and starting values become the final values. Unless the final values are known and can be used as the starting values, NITER=50 should achieve maximization.

Figure 3



Control card specifying three regression coefficients, two variables to be read from a data card, probit model to be used, no controls, a transformation card to be read, and starting values of the parameters to be calculated automatically. Data will be printed back, natural response is not a parameter, there is one subject per data card, no parallel groups, no restricted model will be calculated, and the program will select the number of iterations to be done to find the maximum values of the likelihood function.

Figure 4



Control card specifying three regression coefficients, two variables to be read from each data card, and the logit model to be used. The second variable defines a control group, a transformation card to be read, starting values of the parameters will be calculated automatically, data will not be printed back, natural response is not a parameter, data are grouped with more than one individual per card, and there are no parallel groups. A restricted model with the first and second variables will be calculated, and the program will select the number of iterations to be done to find the maximum value of the likelihood function. All integers will be read as intended because each is separated from the next by a comma, despite the presence or absence of a blank space.

2.4 Transformation Card

This card contains a series of symbols written in Reverse "Polish" Notation (RPN) (see *sec. 2.4.1*) which defines one or more transformations of the variables. This option is indicated by ITRAN=1 on the control card (*sec. 2.3*). If ITRAN=0 on the control card, the program will not read a transformation card.

2.4.1 Reverse Polish Notation

Reverse Polish Notation is widely used in computer science and in Hewlett-Packard calculators. It is an efficient and concise method for presenting a series of arithmetic calculations without using parentheses. The calculations are listed in a form that can be acted on directed by a computer and can be readily understood by the user.

The central concept of RPN is a "stack" of operands (numbers). The "stack" is likened to a stack of cafeteria trays. We specify that a tray can only be removed from the top of the stack; likewise, a tray can only be put back in the stack at the top. Reverse Polish Notation prescribes all calculations in a stack of operands. An addition operation, for example, calls the top two operands from the stack (reducing the height by two), adds them together, and places the sum back on the stack. Subtraction, multiplication, and division also take two operands from the stack and return one. Simple functions like logarithms and square roots take one operand and return one.

How do numbers get into the stack? Reverse Polish Notation consists of a string of symbols (the operators) and the operands. The string is read from left to right. When an operand is encountered, it is placed on the stack. When an operator is encountered, the necessary operands are removed, and the result is returned to the stack. At the end of the scan, only one number, the final result, remains.

To write an RPN string, any algebraic formula should first be rewritten in linear form.

For example, $\frac{a+b}{c}$ is rewritten (a + b)/c.

The operands are written in the order in which they appear in the linear equation. The operators are interspersed in the string in the order in which the stack operates, that is, ab+c/. No parentheses are used. When this string is scanned, the following operations occur: (1) a is put on the stack, (2) b is put on the stack, (3) + takes the top two stack items (a,b) and places their sum back on the stack, (4) c is put on the stack, (5) / takes the top stack item (c), divides it into the next item (a+b), and places this result back on the stack. In cases where the subtraction operator in an algebraic formula only uses one operand (for example, -a+b), a single-operand negative operator such as N can be used. The string is then written aNb+.

Once a string has been scanned, a means must exist to begin another stack for another transformation. This is achieved by an operator =, which disposes of the final result. For example (a+b) / c=d becomes the string ab+c / d=. The operator = takes two items from the stack and returns none; the result is stored and the stack is empty.

2.4.2 Operators

The operators used in POLO2 are:

Operator	*Number of operands*	*Number of results*	*Operation*
+	2	1	addition
-	2	1	subtraction
*	2	1	multiplication
/	2	1	division
N	1	1	negation
E	1	1	exponentiation (10^x)
L	1	1	logarithm (base 10)
S	1	1	square root
=	2	0	store result

2.4.3 Operands

The operands are taken from an array of values of the variables, that is, x_1, x_2, x_3, ... ,x_n. These variables are simply expressed with the subscripts (1,2,3,...,n); the subscripts are the operands in the RPN string. The symbols in the string are punched one per column, with no intervening blanks. If several new variables are formed by transformations, their RPN strings follow one after another on the card. The first blank terminates the transformations.

The transformations use x_1, x_2, x_3,...,x_n, to form new variables that must replace them in the same array. To avoid confusion, the x array is copied into another array, y. A transformation then uses operands from x and stores the result in y. Finally, the y is copied back into x and the transformations are done.

The first NPLL numbers in the x array are dummies, or the constant term (1.0) if NPLL=1. Transformations, therefore, are done on x_2, x_3,...,x_n, and not on the constant term or the dummy variables.

2.4.4 Examples

Several examples of transformations from algebraic notation to RPN are the following:

Algebraic Notation	*RPN*
$\log(x_2/ x_3) = y_2$	23/ L2 =
$(x_2+x_3)(x_4+x_5) = y_5$	23+45+*5=
$(x_2)^2+2x_2x_3+(x_3)^2 = y_2$	22*23*+23*+33*+2 =
$(x_2)^2 = y_2\ (x_2)^4 = y_3$	22*2 = 22*22**3 =
$-x_3+(x_3)^2-x_2x_4 = y_2$	3N33*24*-S+2 =
$x_2(10x^3+ 10x^4) = y_2$	23E4E+*2 =

For another example, let the variables in a data set be x_1, x_2, x_3, and x_4; x_1 is the constant term. We require the transformations $y_2 = \log(x_2/x_3)$, $y_3 = \log(x_3)$, $y_5 = (x_2)^2$, and $y_6 = x_2x_3$; x_4 is left unchanged. The RPN strings are 23/L2=, 3L3=, 22*5=, and 23*6=. The appropriate transformation card for this series is shown in *figure 5*.

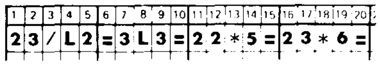


Figure 5.

2.5 Parameter Label Card

This card contains the descriptive labels for all NVAR parameters. These labels are used on the printout. The parameters include the constant term or dummy variables, other explanatory variables, and natural response, if it is present. Labels apply to the explanatory variables after any transformations. Each label is 8 characters long. Use columns 1-8 for the first label, columns 9-16 for the second, and so on (*fig. 6*). If a label does not fill the 8 spaces, begin the label in the leftmost space available (columns 1, 9, 17, 15, 33, and so on).

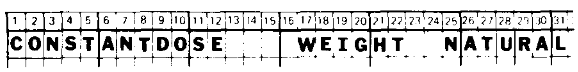


Figure 6.

2.6 Starting Values of the Parameters Card

This card is used only under special circumstances, such as quickly confirming calculations in previously published experiments. If this card is to be used, ISTARV=1 on the control card (*sec. 2.3*). The parameters are punched in 10F8.6 format (up to 10 fields of 8 columns each); in each field there is a number with six digits to the right of the decimal point and two to the left. The decimal point need not be punched. The parameters on this card are the same in number and order as on the label card (*fig. 6, 7*).

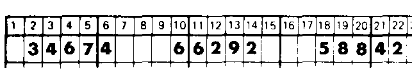


Figure 7.

In this example, the constant is 3.4674, β_1 is 6.6292, and β_2 is 5.8842. All POLO2 calculations will be done on the basis of these parameter values if ISTARV=1 and NITER=0 on the control card (*sec. 2.3*).

2.7 Format Card

This card contains a standard FORTRAN format statement with parentheses but without "FORMAT" punched on the card (*fig. 8*). This statement instructs the program how to read the data from each card. A variable occupies specific columns—a field—on each data card; this area is specified by "F" followed by a number giving the field width. After that number is a decimal point and another number telling where the decimal point is located, if it is not punched on the data cards. For example, "F7.2" means that the data item requires a 7-column field; a decimal point occurs between columns 5 and 6. "F7.0" means whole numbers. When a decimal point is actually punched on a data card, the computer ignores what the format card might say about its location. (For more information, see any FORTRAN textbook.)

Besides the variables, the other items on a data card, such as the group number (K), number of subjects (N), and number of subjects responding (M) must be specified on the format card in "I" (integer) format. Formal editing controls "X" and "T" may be used to skip extraneous columns on a data card or to go to a particular column. For example, "3X" skips 3 columns and "16T resets the format scan to column 16 regardless of where the scan was previously. All steps in the format statement are separated by commas, and the statement is enclosed in parentheses.

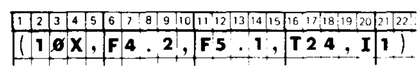


Figure 8.

Format card instructing program to skip the first 10 columns of each data card, read the first variable within the next 4 columns assuming 2 decimal places, read the second variable within the next 5 columns assuming 1 decimal place, then go to column 24 and read a single integer, M (M=1 for response; M=0 for no response).

2.8 Data Cards

Punch one card per subject or per group of subjects grouped at identical values of one of the independent variables. All individuals treated with the same dose of an insecticide, for example, might be grouped on a single card, or each might have its own data card. Values of the NV variables are punched, followed by N (the number of subjects) and M (the number responding). If there is only one subject per card (IGROUP=0) (see *sec. 2.3*), N should be omitted.

If parallel groups are being compared (NPLL > 1), the data card must also contain the group number K (K = 1,2,3,. . .,NPLL) punched before the variables. In summary, a data card contains $K,x_1,x_2,x_3,\ldots,x_{NV},N,M$ with K omitted when NPLL = 0 or 1, and N omitted if IGROUP=0.

Figures illustrating these alternatives will be provided in the examples to follow. If data have already been punched, but are to be used in a different order, the order may be altered in the format card by use of the "T" format editing control. This control will permit the scan to jump backwards, as necessary.

2.9 END Card

To indicate the end of a problem if more problems are to follow in the same job, "END" should be punched in columns 1-3 (*fig. 9*). If only one problem is analyzed, this card is not necessary.

3. LIMITATIONS

No more than 3000 test subjects may be included in a single analysis. This counts all subjects in grouped data. Including the constant term(s), no more than nine explanatory variables may be used.

4. DATA OUTPUT EXAMPLES

Examples illustrating POLO2 data output and uses of the program's special features for hypotheses testing follow. Each problem is presented in its entirety, from data input through hypotheses testing, with statistics from the output.

4.1 Toxicity of Pyrethrum Spray and Film

Data from experiments of Tattersfield and Potter (1943) are used by Finney (1971, p. 162-169) to illustrate the calculations for fitting parallel probit planes. Insects (*Tribolium castaneum*) were exposed to pyrethrum, a botanical insecticide, either as a direct spray or as a film deposited on a glass disc. We use these data to illustrate multivariate analysis of grouped data, use of dummy variables, use of transformations, the likelihood ratio test for parallelism of probit planes, and the likelihood ratio test for equality of the planes.

4.1.1 Models

The probit model expressing the lethal effect of pyrethrum spray is

$$y_s = \alpha_s + \beta_{s1}x_1 + \beta_{2s}x_2 \qquad [1]$$

where y_s is the probit of percent mortality, x_1 is the concentration of pyrethrum in mg/ ml, and x_2 is the weight (deposit) in mg/ cm^2. The regression coefficients are α_s for the constant, β_{1s} for spray concentration, and β_{2s} for weight. Similarly, the model for the lethal effect of pyrethrum film is

$$y_f = \alpha_f + \beta_{if}x_1 + \beta_{2f}x_2 \qquad [2]$$

where y_f is the probit of percent mortality, x_1 is concentration, and x_2 is weight. The regression coefficients are α_f for the constant, β_{1f} for concentration, and β_{2f} for weight (deposit) of pyrethrum in the film.

4.1.2 Hypotheses

The likelihood ratio (LR) procedure will be used to test three hypotheses. These hypotheses are that the spray and film planes are parallel, that the planes are equal given the assumption that the planes are parallel, and that the planes are equal with no assumption of parallelism.

The LR test compares two values of the logarithm of the likelihood function. The first is the maximum value of the log likelihood when it is maximized unrestrictedly. The second is the maximum value when it is maximized subject to the restrictions imposed by the hypothesis being tested. The unrestricted maximum of the log likelihood is denoted by $L(\Omega)$ and the restricted maximum by $L(\omega)$.

The hypothesis of parallelism is H:(P): $\beta_{1s} = \beta_{1f}$, $\beta_{2s} = \beta_{2f}$. Let L_s and L_f denote the maximum value of the log likelihood for models [1] and [2], respectively. The value $L(\Omega)$ is the sum of L_s and L_f.

(i) L_s: = ML estimation of [1].
L_f: = ML estimation of [2].
$L(\Omega) = L_s + L_f$.

The model with the restrictions imposed is

$$y = \alpha_s x_s + \alpha_f x_f + \beta x_1 + \beta_2 x_2 \qquad [3]$$

In this restricted model, dummy variables are used. The dummy variables x_s and x_f are defined as follows:

$x_s = 1$ for spray; $x_f = 1$ for film;
$x_s = 0$ for film; $x_f = 0$ for spray;

The value $L(\omega)$ is obtained by estimating [3] by ML.

(ii) $L(\omega)$: ML estimation of [3].

When H is true, asymptotically,

$$LR = 2[L(\Omega) - L(\omega)] \sim \chi^2\ (2).$$

In other words, for large samples the LR test statistic has approximately a chi-square distribution with 2 degrees of freedom (df). The df is the number of restrictions imposed by the hypothesis, which in this situation equals the number of parameters constrained to be the same. The LR test accepts H(P) at significance level α if

$$LR \le \chi_\alpha^2\ (n)$$

where $\chi^2(n)$ denotes the upper significance point of a chi-square distribution with n df.

The hypothesis of equality given parallelism is H(E|P): $\alpha_s = \alpha_f$. Now the unrestricted model is [3] and the restricted model is

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 \qquad [4]$$

In this model the coefficients for spray and film are restricted to be the same. The required maximum log likelihoods are $L(\Omega)$ and $L(\omega)$.

(i) $L(\Omega)$: ML estimation of [3].
(ii) $L(\omega)$: ML estimation of [4].

When H(E|P) is true, asymptotically,

$$LR = 2[L(\Omega) \sim L(\omega)] \sim \chi^2 (1).$$

The hypothesis H(E| P) is accepted at significance level α if

$$LR \leq \chi^2 (1).$$

Once H(P) is accepted we may wish to test H(E|P). Note the H(E|P) assumes that H(P) is true. Of course, H(P) can be accepted even if it is false. This is the well known Type II error of hypothesis testing.

The hypothesis of equality is H(E): $\alpha_s = \alpha_f$, $\beta_{1s} = \beta_{1f}$, $\beta_{2s} = \beta_{2f}$. Here the unrestricted model consists of [1] and [2] and the restricted model is [4]. The required maximum log likelihoods are $L(\Omega)$ and $L(\omega)$.

(i) $L(\Omega) = L_S + L_f$: ML estimation of [1] and [2].
(ii) $L(\omega)$: ML estimation of [4].

When H(E) is true, asymptotically,

$$LR = 2[L(\Omega) - L(\omega)] \sim \chi^2 (3).$$

The hypothesis H(E) is accepted if

$$LR \leq \chi^2 (3).$$

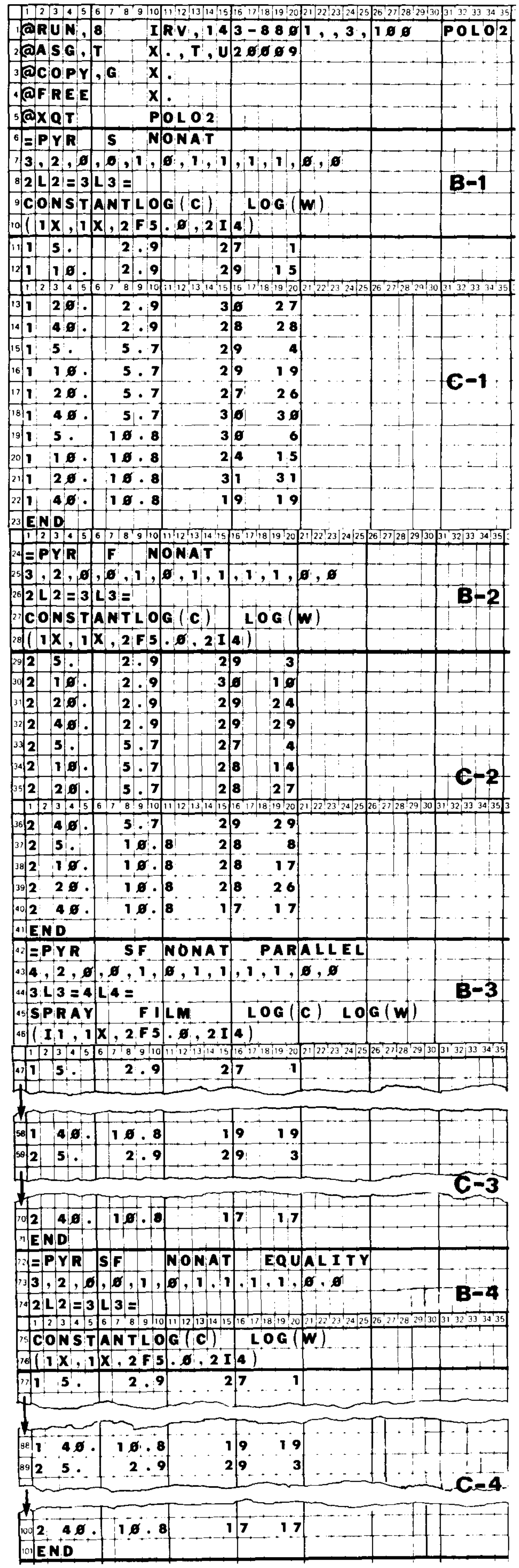
4.1.3 Analyses Required

The data must be analyzed for models [1]-[4] to perform the statistical tests described in *section 4.1.2.* In addition, model [3] including natural response as a parameter, which is referred to as model [5], will be analyzed. The estimation of [5] permits a direct comparison with Finney's (1971) calculations. A total of five analyses, therefore, are provided in this example.

4.1.4 Input

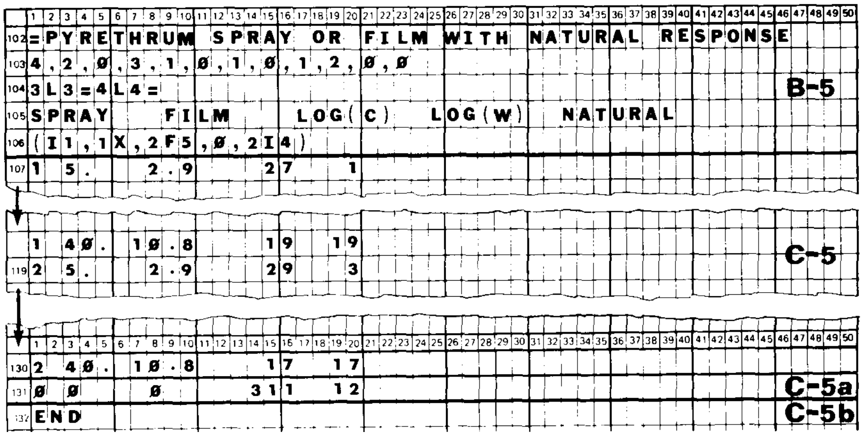
The input for these analyses consists of 132 cards (*fig. 9*). The starter cards (*fig. 9-A*) are followed by the first set of program cards (*fig. 9-B-1*) for the pyrethrum spray application (model [1]). The data cards are next (*fig. 9-C-1*); an "END" card indicates that another problem follows. The next problem, pyrethrum film (model [2]), begins with its program cards (*fig. 9-B-2*), followed by the data and an END card (*fig. 9-C-2*).

Except for the title cards (cards 6 and 24), the program cards for the first two data sets are identical. Each control card (cards 7 and 25) specifies three regression coefficients, and two variables to be read from each data card. The



```
1   @RUN,8       IRV,143-88Ø1,,3,1ØØ   POLO2
2   @ASG,T       X.,T,U2ØØØ9
3   @COPY,G      X.
4   @FREE        X.
5   @XQT         POLO2
6   =PYR  S      NONAT
7   3,2,Ø,Ø,1,Ø,1,1,1,1,Ø,Ø
8   2L2=3L3=
9   CONSTANTLOG(C)   LOG(W)
10  (1X,1X,2F5.Ø,2I4)
11  1   5.   2.9   27   1
12  1  1Ø.   2.9   29  15
13  1  2Ø.   2.9   3Ø  27
14  1  4Ø.   2.9   28  28
15  1   5.   5.7   29   4
16  1  1Ø.   5.7   29  19
17  1  2Ø.   5.7   27  26
18  1  4Ø.   5.7   3Ø  3Ø
19  1   5.  1Ø.8   3Ø   6
20  1  1Ø.  1Ø.8   24  15
21  1  2Ø.  1Ø.8   31  31
22  1  4Ø.  1Ø.8   19  19
23  END
24  =PYR  F      NONAT
25  3,2,Ø,Ø,1,Ø,1,1,1,1,Ø,Ø
26  2L2=3L3=
27  CONSTANTLOG(C)   LOG(W)
28  (1X,1X,2F5.Ø,2I4)
29  2   5.   2.9   29   3
30  2  1Ø.   2.9   3Ø  1Ø
31  2  2Ø.   2.9   29  24
32  2  4Ø.   2.9   29  29
33  2   5.   5.7   27   4
34  2  1Ø.   5.7   28  14
35  2  2Ø.   5.7   28  27
36  2  4Ø.   5.7   29  29
37  2   5.  1Ø.8   28   8
38  2  1Ø.  1Ø.8   28  17
39  2  2Ø.  1Ø.8   28  26
40  2  4Ø.  1Ø.8   17  17
41  END
42  =PYR  SF     NONAT   PARALLEL
43  4,2,Ø,Ø,1,Ø,1,1,1,1,Ø,Ø
44  3L3=4L4=
45  SPRAY   FILM    LOG(C)  LOG(W)
46  (I1,1X,2F5.Ø,2I4)
47  1   5.   2.9   27   1
58  1  4Ø.  1Ø.8   19  19
59  2   5.   2.9   29   3
70  2  4Ø.  1Ø.8   17  17
71  END
72  =PYR  SF     NONAT   EQUALITY
73  3,2,Ø,Ø,1,Ø,1,1,1,1,Ø,Ø
74  2L2=3L3=
75  CONSTANTLOG(C)   LOG(W)
76  (1X,1X,2F5.Ø,2I4)
77  1   5.   2.9   27   1
88  1  4Ø.  1Ø.8   19  19
89  2   5.   2.9   29   3
100 2  4Ø.  1Ø.8   17  17
101 END
```


Figure 9



102 =PYRETHRUM SPRAY OR FILM WITH NATURAL RESPONSE
103 4,2,Ø,3,1,Ø,1,Ø,1,2,Ø,Ø
104 3L3=4L4=
105 SPRAY FILM LOG(C) LOG(W) NATURAL
106 (I1,1X,2F5,Ø,2I4)
107 1 5. 2.9 27 1
B-5

1 4Ø. 1Ø.8 19 19
119 2 5. 2.9 29 3
C-5

130 2 4Ø. 1Ø.8 17 17
131 Ø Ø Ø 311 12
C-5a
132 END
C-5b

Figure 9—Continued

probit model will be used, none of the variables defines a control group, transformations will be used, starting values will be calculated automatically, data will be printed back, natural response is not a parameter, there is more than one subject per data card, no parallel groups, a restricted model will not be computed, and the program will select a suitable number of iterations in search of ML estimates. Each transformation card (card 8, 26) defines y_2 as the logarithm of x_2, and y_3 as the logarithm x_3. Parameter labels (cards 9 and 27) are x_1=constant, x_2=logarithm of concentration, and x_3=logarithm of deposit weight. Both format cards (cards 10 and 28) instruct the program to skip the first two columns of a data card, read two fields of five digits with a decimal point in each field, then read two 4-column fields of integers. The data cards for the two experiments are in identical format (*fig. 9-C-1,2*). Column 1 contains a "1" in the spray experiments and a "2" in the film experiments. Columns 3-5 list the concentration of pyrethrum in mg/ 0.1 ml; columns 7-10 contain the deposit weight in mg/ 0.1 cm^2.

The next analysis computes coefficients and test statistics for model [3]. The program cards (*fig. 9-B-3*) reflect the complexity of this model compared to the first two simpler models. After the title card (card 42), the control card (card 43) specifies four regression coefficients, two explanatory variables to be read from each data card (note that the two dummy variables are not included in NV), the probit model, no control group included for any parameter, transformations will be used, starting values will be calculated automatically, data will be printed back, natural response is not a parameter, data are grouped, there are two parallel groups, a restricted model will not be computed, and the program will choose the number of iterations necessary for ML estimates. The transformation card (card 44) specifies that log $(x_3)=y_3$ and log $(x_4)=y_4$. Parameter labels (card 45) are: x_1=spray (the first dummy variable), x_2=film (the second dummy variable), x_3=logarithm of concentration, and x_4=logarithm of (deposit) weight. The format card (card 46) tells the program to read the integer in column 1, skip the next column, read two fields of five digits with a decimal point in each, then read two 4-column fields of integers. The data, shown in abbreviated form in *fig. 9-C-3,* follow; the data from models [1] (*fig. 9-C-1*) and [2] (*fig. 9-C-2*) have been combined into a single set followed by an END card (card 71).

In the next analysis, coefficients and statistics for model [4] are computed. The program cards (*fig. 9-B-4*) specify the computations. After the title card (card 72), the control card (card 73) states that there will be three regression coefficients, two variables to be read from each data card, the probit model will be used, no control group is present for either explanatory variable, transformations will be used, starting values will be calculated automatically, data will be printed back, natural response is not a parameter, data are grouped, there is no comparison of parallel groups, a restricted model will not be computed, and the program will choose the number of iterations to be done for ML estimation. The transformation card (card 74) specifies that log $x_2=y_2$ and log $x_3=y_3$. The three parameter labels (card 75) are x_1=constant, x_2=logarithm of concentration, and x_3=logarithm of (deposit) weight. The format card (card 76) instructs the program to skip the first 2 columns, read each of two fields of five digits with decimal points punched, then read two 4-column fields of integers. The data are combined data for models [1] and [2] (*fig. 9-C-4*). These cards are followed by an END card (card 101).

The final analysis, for model [5], begins with program cards (*fig. 9-B-5*). The title (card 102) describes the analysis. The control card (card 103) is the same as that for analysis of model [3], with the following exceptions: the fourth integer (KONTROL) specifies that the log (concentration) parameter includes a control group; the eighth integer (NPARN), is equal to zero because natural response will be calculated as a parameter. The transformation (card 104) is the same as that for model [3]: log $x_3=y_3$ and log $x_4=y_4$. The parameters (card 105) are labeled as: x_1=spray (first dummy variable), x_2=film (second dummy variable), x_3=logarithm of pyrethrum concentration, x_4=logarithm of (deposit) weight and x_5=natural response. The format statement (card 106) instructs the program to read the first column of integers, skip the next column, read two 5-digit fields each of which includes a decimal point, then read two 4-column fields of integers. The data cards (*fig. 9-C-5*) are followed by the natural response data card (*fig. 9-C-5a*), then the END card (*fig. 9-C-5b*).

4.1.5 Output

The output for the five analyses is shown in *figure 10.* Except where noted, each analysis is shown in its entirety. The title of the analysis is printed as the first line on each page (*fig. 10-1*, lines 1 and 56; *fig. 10-2,* lines 94 and 149; *fig. 10-3*, lines 184, 239, 297; fig. *10-4*, lines 324, 378, and 435; *fig. 10-5*, lines 459, 514, and 573). Next, each control card is listed (*fig. 10-1*, line 2; *fig. 10-2*, line 95; *fig. 10-3*, line 185; *fig. 10-4*, line 325; *fig. 10-5*, line 460). The subsequent section of the printout describes the specifications of the analysis and reflects the information on the control card (*fig. 10-1*, lines 3-11; *fig. 10-2,* lines 96-104; *fig. 10-3*, lines 186-195; *fig. 10-4*, lines 326-334; *fig. 10-5*, lines 461-471).

Transformations are reproduced in RPN, just as they were punched on the transformation card (*fig. 10-1*, line 12; *fig. 10-2*, line 105; *fig. 10-3*, line 196; *fig. 10-4*, line 335; *fig. 10-5*, line 472). Parameter labels are reproduced next (*fig. 10-1*, lines 13-16; *fig. 10-2*, lines 106-109; *fig. 10-3*, lines 197-201; *fig. 10-4*, lines 336-339; *fig. 10-5*, lines 473-478), followed by the format statement (*fig. 10-1*, line 17; *fig. 10-*2, line 110; *fig. 10-3*, line 202; *fig. 10-4*, line 340; *fig. 10-5*, line 479).

In the next section, input data are listed as punched on the data cards (*fig. 10-1*, lines 18-30; *fig. 10-2*, lines 111-123; *fig. 10-3*, lines 203-227; *fig. 10-4*, lines 341-363; *fig. 10-5*, lines 480-505). The transformed data are listed after the data input. For the purposes of computation of the prediction success table and other statistics, grouped data are now listed as individual cases. This section of the output has been abbreviated in the figure (*fig. 10-1*, lines 31-65; *fig. 10-2*, lines 124-155; *fig. 10-3*, lines 228-292; *fig. 10-4*, lines 364-429; *fig. 10-5*, lines 506-570). At the end of the transformed data, the total number of cases (observations plus controls) is summarized (*fig. 10-1*, line 66; *fig. 10-2*, line 156; *fig. 10-3*, line 293; *fig. 10-4*, line 430; *fig. 10-5*, line 571). Proportional control mortality is also printed (*fig. 10-5*, line 572).

Initial estimates of the parameters that were computed by the program are printed (*fig. 10-1,* lines 67-68; *fig. 10-2,* lines 157-158; *fig. 10-3,* lines 294-295; *fig. 10-4,* lines 431-432; *fig. 10-5*, lines 574-577), followed by the initial value of the log likelihood (*fig. 10-1*, line 69; *fig. 10-2*, line 159; *fig. 10-3*, line 296; *fig. 10-4*, line 433; *fig. 10-5*, line 578). The program begins with logit iterations that are computationally simpler, then switches to probit calculations. The number of iterations of each type are listed (*fig. 10-1*, lines 70-71; *fig. 10-2*, lines 160-161; *fig. 10-3,* lines 298-299; *fig. 10-4,* lines 434, 436; *fig. 10-5*, lines 579-580) preceding the final value of the log likelihood (*fig. 10-1*, line 72; *fig. 10-2*, line 162; *fig. 10-3*, line 300; *fig. 10-4*, line 437; *fig. 10-5*, line 581).

Parameter values, their standard errors, and their t-ratios (parameter values divided by its standard error) are then presented (*fig. 10-1,* lines 73-76; *fig. 10-2,* lines 163-166; *fig. 10-3*, lines 301-305; *fig. 10-4,* lines 438-441; *fig. 10-5*, lines 582-587). The t-ratios are used to test the significance of each parameter in the regression. The hypothesis that a regression coefficient is zero is rejected at the $\alpha = 0.05$ significance level when the absolute values of the t-ratio is greater than $t = 1.96$, that is, the upper $\alpha = 0.05$ significance point of a t distribution with ∞ df. All parameters in each of the five analyses were significant in this example. These statistics are followed by the covariance matrix for the analysis (*fig. 10-1,* lines 77-81; *fig. 10-2,* lines 167-171; *fig. 10-3,* lines 306-311; *fig. 10-4,* lines 442-446; *fig. 10-5,* lines 588-594).

A prediction success table (*fig. 10-1,* lines 82-91; *fig. 10-*2, lines 172-181; *fig. 10-3,* lines 312-321; *fig. 10-4,* lines 447-456; *fig. 10-5,* lines 595-604) lists the number of individual test subjects that were predicted to be alive and were actually alive, predicted to be alive but were actually dead, predicted to be dead and were actually dead, and predicted to be dead but were actually alive. The numbers are calculated by using maximum probability as a criterion (Domencich and McFadden 1975). The percent correct prediction, rounded to the nearest percent, is calculated as the number predicted correctly (that is, alive when predicted alive, or dead when predicted dead) divided by the total number predicted in that category, times 100. In the pyrethrum spray experiment (*fig. 10-1*), for example, 89 individuals were correctly forecast as alive, but 26 others were dead when they had been predicted to be alive. The correct percent alive is

$$\frac{89}{89+26} \text{x}100,$$

which is 77 percent (*fig. 10-1,* line 90). The overall percent correct (OPC) equals the total number correctly predicted divided by the total number of observations, times 100, rounded to the nearest percent. In the pyrethrum spray example, OPC equals

$$\frac{89+195}{333} \text{x}100,$$

or 85 percent (*fig. 10-1,* line 91). On the basis of random choice, the correct choice should be selected for about 50 percent of the observations. In the five analyses, OPC values indicate reliable prediction by the models used, since all were greater than 80 percent.

The last portion of the output tests the significance of the regression coefficients in each regression (*fig. 10-1*, lines 92-93; *fig. 10-2*, lines 182-183; *fig. 10-3*, lines 322-323; *fig. 10-4*, lines 457-458; *fig. 10-5*, lines 605-606). The hypothesis tested is that all the regression coefficients equal zero. This hypothesis implies that the probability of death is 0.5 at all spray concentrations and film deposits. The log likelihood

$L(\omega)$ is calculated for this restricted model and compared to the maximized log likelihood $L(\Omega)$ for the unrestricted model. When the hypothesis is true, asymtotically,

$$LR = 2[L(\Omega) - L(\omega)] \sim \chi^2\,(df)$$

where df equals the number of parameters in the unrestricted model. The hypothesis is accepted at the α level of significance if

$$LR \leq \chi_\alpha^2\,(df).$$

In each of the five, analyses in this example, the hypothesis was rejected at the α = 0.05 significance level. All regressions were highly significant.

Figure 10-1

```
 1   PYR  S  NONAT                                                       POLO2          26 AUG 80  14:06:40    4     1

 2   CONTROL CARD: 3,2,0,0,1,0,1,1,1,1,0,0

 3   NUMBER OF VARIABLES: 3
 4   NUMBER OF VARIABLES ON INPUT CARDS: 2
 5   NUMBER OF PARAMETERS: 3
 6   TYPE OF MODEL: PROBIT
 7   THERE ARE TRANSFORMATIONS
 8   INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
 9   ALL RAW DATA WILL BE PRINTED
10   NATURAL RESPONSE IS NOT A PARAMETER
11   MULTIPLE SUBJECTS PER DATA CARD

12   TRANSFORMATIONS: 2L2=3L3=

13   PARAMETER LABELS
14     1  CONSTANT
15     2  LOG(C)
16     3  LOG(W)

17   FORMAT: (1X,1X,2F5.0,2I4)

18   INPUT DATA
19   1  5.    2.9    27   1
20   1 10.    2.9    29  15
21   1 20.    2.9    30  27
22   1 40.    2.9    28  28
23   1  5.    5.7    29   4
24   1 10.    5.7    29  19
25   1 20.    5.7    27  26
26   1 40.    5.7    30  30
27   1  5.   10.8    30   6
28   1 10.   10.8    24  15
29   1 20.   10.8    31  31
30   1 40.   10.8    19  19

31   TRANSFORMED DATA
32      1.  1      1.00000          .698970          .462398
33      2.  0      1.00000          .698970          .462398
34      3.  0      1.00000          .698970          .462398
35      4.  0      1.00000          .698970          .462398
36      5.  0      1.00000          .698970          .462398
37      6.  0      1.00000          .698970          .462398
38      7.  0      1.00000          .698970          .462398
39      8.  0      1.00000          .698970          .462398
40      9.  0      1.00000          .698970          .462398
41     10.  0      1.00000          .698970          .462398
42     11.  0      1.00000          .698970          .462398
43     12.  0      1.00000          .698970          .462398
44     13.  0      1.00000          .698970          .462398
45     14.  0      1.00000          .698970          .462398
46     15.  0      1.00000          .698970          .462398
47     16.  0      1.00000          .698970          .462398
48     17.  0      1.00000          .698970          .462398
49     18.  0      1.00000          .698970          .462398
50     19.  0      1.00000          .698970          .462398
51     20.  0      1.00000          .698970          .462398
52     21.  0      1.00000          .698970          .462398
53     22.  0      1.00000          .698970          .462398
54     23.  0      1.00000          .698970          .462398
55     24.  0      1.00000          .698970          .462398
-------------------------------------------------------------------------------------------------------------------------
56   PYR  S  NONAT                                                       POLO2          26 AUG 80  14:06:43    5     7

57    325.  1      1.00000          1.60206          1.03342
58    326.  1      1.00000          1.60206          1.03342
59    327.  1      1.00000          1.60206          1.03342
60    328.  1      1.00000          1.60206          1.03342
61    329.  1      1.00000          1.60206          1.03342
62    330.  1      1.00000          1.60206          1.03342
63    331.  1      1.00000          1.60206          1.03342
```

Figure 10-1—Continued

```
64   332.  1      1.00000        1.60206        1.03342
65   333.  1      1.00000        1.60206        1.03342

66   333 OBSERVATIONS + 0 CONTROLS = 333 TOTAL

67   INITIAL ESTIMATES OF THE PARAMETERS
68              -4.327488 CONSTANT              3.934193 LOG(C)              .7871704 LOG(W)

69   STARTING LIKELIHOOD = -103.9408

70   7 LOGIT EVALUATIONS
71   5 PROBIT EVALUATIONS

72   MAXIMUM LOG LIKELIHOOD = -101.3851

73                       PARAMETER            STD ERROR            T RATIO
74   CONSTANT            -5.46446             .644240              -8.48203
75   LOG(C)               4.92555             .484850              10.1589
76   LOG(W)               1.09323             .445611              2.45334

77   COVARIANCE MATRIX
78                      CONSTANT       LOG(C)        LOG(W)
79     CONSTANT         .4150454     -.2669031     -.2008408
80     LOG(C)          -.2669031      .2350799     .5576819-01
81     LOG(W)          -.2008408     .5576819-01    .1985690

82   PREDICTION SUCCESS TABLE
83                                 PREDICTED OUTCOME
84                                 -----------------
85   ACTUAL OUTCOME                  ALIVE     DEAD
86   -----------------------------------------------
87   ALIVE                            89        23
88   DEAD                             26       195
89   -----------------------------------------------
90   PERCENT CORRECT                  77        89
91   OVERALL PERCENT CORRECT: 85

92   TESTING HYPOTHESIS THAT THE 3-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
93      LIKELIHOOD RATIO TEST:  CHI-SQUARED = 258.8652   DF=3   TAIL PROBABILITY = .000
```

Figure 10-2

```
94   PYR  F  NONAT                                                  POLO2        26 AUG 80  14:06:47    8     8

95   CONTROL CARD: 3,2,0,0,1,6,1,1,1,1,0,0

96   NUMBER OF VARIABLES: 3
97   NUMBER OF VARIABLES ON INPUT CARDS: 2
98   NUMBER OF PARAMETERS: 3
99   TYPE OF MODEL: PROBIT
100  THERE ARE TRANSFORMATIONS
101  INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
102  ALL RAW DATA WILL BE PRINTED
103  NATURAL RESPONSE IS NOT A PARAMETER
104  MULTIPLE SUBJECTS PER DATA CARD

105  TRANSFORMATIONS: 2L2=3L3=

106  PARAMETER LABELS
107    1  CONSTANT
108    2  LOG(C)
109    3  LOG(W)

110  FORMAT: (1X,1X,2F5.0,2I4)

111  INPUT DATA
112  2 5.    2.9    29   3
113  2 10.   2.9    30  10
114  2 20.   2.9    29  24
115  2 40.   2.9    29  29
116  2 5.    5.7    27   4
117  2 10.   5.7    28  14
118  2 20.   5.7    28  27
119  2 40.   5.7    29  29
120  2 5.    10.8   28   8
121  2 10.   10.8   28  17
122  2 20.   10.8   28  26
123  2 40.   10.8   17  17
```

Figure 10-2—Continued

```
TRANSFORMED DATA
    1.  1     1.00000          .698970          .462398
    2.  1     1.00000          .698970          .462398
    3.  1     1.00000          .698970          .462398
    4.  0     1.00000          .698970          .462398
    5.  0     1.00000          .698970          .462398
    6.  0     1.00000          .698970          .462398
    7.  0     1.00000          .698970          .462398
    8.  0     1.00000          .698970          .462398
    9.  0     1.00000          .698970          .462398
   10.  0     1.00000          .698970          .462398
   11.  0     1.00000          .698970          .462398
   12.  0     1.00000          .698970          .462398
   13.  0     1.00000          .698970          .462398
   14.  0     1.00000          .698970          .462398
   15.  0     1.00000          .698970          .462398
   16.  0     1.00000          .698970          .462398
   17.  0     1.00000          .698970          .462398
   18.  0     1.00000          .698970          .462398
   19.  0     1.00000          .698970          .462398
   20.  0     1.00000          .698970          .462398
   21.  0     1.00000          .698970          .462398
   22.  0     1.00000          .698970          .462398
   23.  0     1.00000          .698970          .462398
   24.  0     1.00000          .698970          .462398
-----------------------------------------------------------------------------------------------------------------------------------

PYR  F  NONAT                                                                   POL02         26 AUG 80  14:06:49    9     14


  325.  1     1.00000         1.60206          1.03342
  326.  1     1.00000         1.60206          1.03342
  327.  1     1.00000         1.60206          1.03342
  328.  1     1.00000         1.60206          1.03342
  329.  1     1.00000         1.60206          1.03342
  330.  1     1.00000         1.60206          1.03342

330 OBSERVATIONS + 0 CONTROLS = 330 TOTAL

INITIAL ESTIMATES OF THE PARAMETERS
           -4.009471 CONSTANT                 3.357726 LOG(C)                        .9834336 LOG(W)

STARTING LIKELIHOOD = -124.0496

7 LOGIT EVALUATIONS
6 PROBIT EVALUATIONS

MAXIMUM LOG LIKELIHOOD = -122.4908

                        PARAMETER             STD ERROR              T RATIO
CONSTANT                -4.77369              .535735               -8.91055
LOG(C)                   3.99017              .381101               10.4701
LOG(W)                   1.18636              .395729               2.99792

COVARIANCE MATRIX
                      CONSTANT       LOG(C)         LOG(W)
   CONSTANT           .2870119     -.1689206      -.1474890
   LOG(C)            -.1689206      .1452383     .3136549-01
   LOG(W)            -.1474890    .3136549-01       .1566018


PREDICTION SUCCESS TABLE
                                   PREDICTED OUTCOME
                                   -----------------
ACTUAL OUTCOME                      ALIVE     DEAD
-----------------------------------------------------
ALIVE                                89        33
DEAD                                 25       183
-----------------------------------------------------
PERCENT CORRECT                      78        85
OVERALL PERCENT CORRECT: 82


TESTING HYPOTHESIS THAT THE 3-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
   LIKELIHOOD RATIO TEST:  CHI-SQUARED = 212.4950    DF=3    TAIL PROBABILITY = .000
```

Figure 10-3

```
184  PYR  SF  NONAT  PARALLEL                                  POLO2        26 AUG 80  14:06:52   11    15

185  CONTROL CARD: 4,2,0,0,1,0,1,1,1,2,0,0

186  NUMBER OF VARIABLES: 4
187  NUMBER OF VARIABLES ON INPUT CARDS: 2
188  NUMBER OF PARAMETERS: 4
189  TYPE OF MODEL: PROBIT
190  THERE ARE TRANSFORMATIONS
191  INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
192  ALL RAW DATA WILL BE PRINTED
193  NATURAL RESPONSE IS NOT A PARAMETER
194  MULTIPLE SUBJECTS PER DATA CARD
195  NUMBER OF PARALLEL GROUPS: 2

196  TRANSFORMATIONS: 3L3=4L4=

197  PARAMETER LABELS
198    1  SPRAY
199    2  FILM
200    3  LOG(C)
201    4  LOG(W)

202  FORMAT: (I1,1X,2F5.0,2I4)

203  INPUT DATA
204  1 5.    2.9    27   1
205  1 10.   2.9    29  15
206  1 20.   2.9    30  27
207  1 40.   2.9    28  28
208  1 5.    5.7    29   4
209  1 10.   5.7    29  19
210  1 20.   5.7    27  26
211  1 40.   5.7    30  30
212  1 5.    10.8   30   6
213  1 10.   10.8   24  15
214  1 20.   10.8   31  11
215  1 40.   10.8   19  19
216  2 5.    2.9    29   5
217  2 10.   2.9    30  10
218  2 20.   2.9    29  24
219  2 40.   2.9    29  29
220  2 5.    5.7    27   4
221  2 10.   5.7    28  14
222  2 20.   5.7    26  27
223  2 40.   5.7    29  24
224  2 5.    10.8   26   6
225  2 10.   10.8   28  17
226  2 20.   10.8   28  26
227  2 40.   10.8   17  17

228  TRANSFORMED DATA
229     1.  1     1.00000      .000000      .698970      .462398
230     2.  0     1.00000      .000000      .698970      .462398
231     3.  0     1.00000      .000000      .698970      .462398
232     4.  0     1.00000      .000000      .698970      .462398
233     5.  0     1.00000      .000000      .698970      .462398
234     6.  0     1.00000      .000000      .698970      .462398
235     7.  0     1.00000      .000000      .698970      .462398
236     8.  0     1.00000      .000000      .698970      .462398
237     9.  0     1.00000      .000000      .698970      .462398
238    10.  0     1.00000      .000000      .698970      .462398
--------------------------------------------------------------------------------------------------------
239  PYR  SF  NONAT  PARALLEL                                  POLO2        26 AUG 80  14:06:55   13    26

240    611.  0    .000000      1.00000      1.00000      1.03342
241    612.  0    .000000      1.00000      1.00000      1.03342
242    613.  0    .000000      1.00000      1.00000      1.03342
243    614.  0    .000000      1.00000      1.00000      1.03342
244    615.  0    .000000      1.00000      1.00000      1.03342
245    616.  0    .000000      1.00000      1.00000      1.03342
246    617.  0    .000000      1.00000      1.00000      1.03342
247    618.  0    .000000      1.00000      1.00000      1.03342
248    619.  1    .000000      1.00000      1.30103      1.03342
249    620.  1    .000000      1.00000      1.30103      1.03342
250    621.  1    .000000      1.00000      1.30103      1.03342
251    622.  1    .000000      1.00000      1.30103      1.03342
252    623.  1    .000000      1.00000      1.30103      1.03342
253    624.  1    .000000      1.00000      1.30103      1.03342
254    625.  1    .000000      1.00000      1.30103      1.03342
255    626.  1    .000000      1.00000      1.30103      1.03342
256    627.  1    .000000      1.00000      1.30103      1.03342
257    628.  1    .000000      1.00000      1.30103      1.03342
258    629.  1    .000000      1.00000      1.30103      1.03342
259    630.  1    .000000      1.00000      1.30103      1.03342
260    631.  1    .000000      1.00000      1.30103      1.03342
261    632.  1    .000000      1.00000      1.30103      1.03342
262    633.  1    .000000      1.00000      1.30103      1.03342
```

Figure 10-3—Continued

```
634.  1    .000000     1.00000     1.30103     1.03342
635.  1    .000000     1.00000     1.30103     1.03342
636.  1    .000000     1.00000     1.30103     1.03342
637.  1    .000000     1.00000     1.30103     1.03342
638.  1    .000000     1.00000     1.30103     1.03342
639.  1    .000000     1.00000     1.30103     1.03342
640.  1    .000000     1.00000     1.30103     1.03342
641.  1    .000000     1.00000     1.30103     1.03342
642.  1    .000000     1.00000     1.30103     1.03342
643.  1    .000000     1.00000     1.30103     1.03342
644.  1    .000000     1.00000     1.30103     1.03342
645.  0    .000000     1.00000     1.30103     1.03342
646.  0    .000000     1.00000     1.30103     1.03342
647.  1    .000000     1.00000     1.60206     1.03342
648.  1    .000000     1.00000     1.60206     1.03342
649.  1    .000000     1.00000     1.60206     1.03342
650.  1    .000000     1.00000     1.60206     1.03342
651.  1    .000000     1.00000     1.60206     1.03342
652.  1    .000000     1.00000     1.60206     1.03342
653.  1    .000000     1.00000     1.60206     1.03342
654.  1    .000000     1.00000     1.60206     1.03342
655.  1    .000000     1.00000     1.60206     1.03342
656.  1    .000000     1.00000     1.60206     1.03342
657.  1    .000000     1.00000     1.60206     1.03342
658.  1    .000000     1.00000     1.60206     1.03342
659.  1    .000000     1.00000     1.60206     1.03342
660.  1    .000000     1.00000     1.60206     1.03342
661.  1    .000000     1.00000     1.60206     1.03342
662.  1    .000000     1.00000     1.60206     1.03342
663.  1    .000000     1.00000     1.60206     1.03342

663 OBSERVATIONS + 0 CONTROLS = 663 TOTAL

INITIAL ESTIMATES OF THE PARAMETERS
          -4.101413 SPRAY           -4.209135 FILM           3.631875 LOG(C)       .8847998 LOG(W)

STARTING LIKELIHOOD = -228.9565
---------------------------------------------------------------------------------------------------

PYR  SF  NONAT  PARALLEL                                     POL02    26 AUG 80  14:07:00   15   27

7 LOGIT EVALUATIONS
5 PROBIT EVALUATIONS

MAXIMUM LOG LIKELIHOOD = -225.1918

                     PARAMETER          STD ERROR           T RATIO
SPRAY                -4.97434           .411879            -12.0772
FILM                 -5.12994           .417825            -12.2777
LOG(C)                4.38963           .299519             14.6556
LOG(W)                1.12907           .294399             3.83517

COVARIANCE MATRIX
                    SPRAY          FILM          LOG(C)         LOG(W)
  SPRAY           .1696447       .1630720     -.1016954     -.8344456-01
  FILM            .1630720       .1745781     -.1039320     -.8417556-01
  LOG(C)         -.1016954      -.1039320     .8971165-01    .1988976-01
  LOG(W)       -.8344456-01   -.8417556-01    .1988976-01    .8667098-01

PREDICTION SUCCESS TABLE
                               PREDICTED OUTCOME
                               -----------------
ACTUAL OUTCOME                  ALIVE    DEAD
-------------------------------------------
ALIVE                            178      56
DEAD                              51     378
-------------------------------------------
PERCENT CORRECT                   78      87
OVERALL PERCENT CORRECT: 84

TESTING HYPOTHESIS THAT THE 4-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
   LIKELIHOOD RATIO TEST:  CHI-SQUARED = 468.7280    DF=4    TAIL PROBABILITY = .000
```

Figure 10-4

```
324   PYR  SF  NONAT  EQUALITY                                              POLO2        26 AUG 80  14:07:02  17   28

325   CONTROL CARD: 3,2,0,0,1,0,1,1,1,1,0,0

326   NUMBER OF VARIABLES: 3
327   NUMBER OF VARIABLES ON INPUT CARDS: 2
328   NUMBER OF PARAMETERS: 3
329   TYPE OF MODEL: PROBIT
330   THERE ARE TRANSFORMATIONS
331   INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
332   ALL RAW DATA WILL BE PRINTED
333   NATURAL RESPONSE IS NOT A PARAMETER
334   MULTIPLE SUBJECTS PER DATA CARD

335   TRANSFORMATIONS: 2L2=3L3=

336   PARAMETER LABELS
337     1  CONSTANT
338     2  LOG(C)
339     3  LOG(W)

340   FORMAT: (1X,1X,2F5.0,2I4)

341   INPUT DATA
342   1 5.    2.9     27   1
341a  1 10.   2.9     29  15
342a  1 20.   2.9     30  27
343   1 40.   2.9     28  28
344   1 5.    5.7     29   4
345   1 10.   5.7     29  19
346   1 20.   5.7     27  26
347   1 40.   5.7     30  30
348   1 5.    10.8    30   6
349   1 10.   10.8    24  15
350   1 20.   10.8    31  31
351   1 40.   10.8    19  19
352   2 5.    2.9     29   3
353   2 10.   2.9     30  10
354   2 20.   2.9     29  24
355   2 40.   2.9     29  29
356   2 5.    5.7     27   4
357   2 10.   5.7     26  14
358   2 20.   5.7     28  27
359   2 40.   5.7     29  29
360   2 5.    10.8    28   8
361   2 10.   10.8    28  17
362   2 20.   10.8    28  26
363   2 40.   10.8    17  17

364   TRANSFORMED DATA
365      1.  1     1.00000      .698970      .462398
366      2.  0     1.00000      .698970      .462398
367      3.  0     1.00000      .698970      .462398
368      4.  0     1.00000      .698970      .462398
369      5.  0     1.00000      .698970      .462398
370      6.  0     1.00000      .698970      .462398
371      7.  0     1.00000      .698970      .462398
372      8.  0     1.00000      .698970      .462398
373      9.  0     1.00000      .698970      .462398
374     10.  0     1.00000      .698970      .462398
375     11.  0     1.00000      .698970      .462398
377     12.  0     1.00000      .698970      .462398
--------------------------------------------------------------------------------------------------------------------
378   PYR  SF  NONAT  EQUALITY                                              POLO2        26 AUG 80  14:07:05  19   39

379    613.  0     1.00000      1.00000      1.03342
380    614.  0     1.00000      1.00000      1.03342
381    615.  0     1.00000      1.00000      1.03342
382    616.  0     1.00000      1.00000      1.03342
383    617.  0     1.00000      1.00000      1.03342
384    618.  0     1.00000      1.00000      1.03342
385    619.  1     1.00000      1.30103      1.03342
386    620.  1     1.00000      1.30103      1.03342
387    621.  1     1.00000      1.30103      1.03342
388    622.  1     1.00000      1.30103      1.03342
389    623.  1     1.00000      1.30103      1.03342
390    624.  1     1.00000      1.30103      1.03342
391    625.  1     1.00000      1.30103      1.03342
392    626.  1     1.00000      1.30103      1.03342
393    627.  1     1.00000      1.30103      1.03342
394    628.  1     1.00000      1.30103      1.03342
395    629.  1     1.00000      1.30103      1.03342
396    630.  1     1.00000      1.30103      1.03342
397    631.  1     1.00000      1.30103      1.03342
398    632.  1     1.00000      1.30103      1.03342
399    633.  1     1.00000      1.30103      1.03342
400    634.  1     1.00000      1.30103      1.03342
401    635.  1     1.00000      1.30103      1.03342
```

Figure 10-4—Continued

```
402   636.  1     1.00000      1.30103      1.03342
403   637.  1     1.00000      1.30103      1.03342
404   638.  1     1.00000      1.30103      1.03342
405   639.  1     1.00000      1.30103      1.03342
406   640.  1     1.00000      1.30103      1.03342
407   641.  1     1.00000      1.30103      1.03342
408   642.  1     1.00000      1.30103      1.03342
409   643.  1     1.00000      1.30103      1.03342
410   644.  1     1.00000      1.30103      1.03342
411   645.  0     1.00000      1.30103      1.03342
412   646.  0     1.00000      1.30103      1.03342
413   647.  1     1.00000      1.60206      1.03342
414   648.  1     1.00000      1.60206      1.03342
415   649.  1     1.00000      1.60206      1.03342
416   650.  1     1.00000      1.60206      1.03342
417   651.  1     1.00000      1.60206      1.03342
418   652.  1     1.00000      1.60206      1.03342
419   653.  1     1.00000      1.60206      1.03342
420   654.  1     1.00000      1.60206      1.03342
421   655.  1     1.00000      1.60206      1.03342
422   656.  1     1.00000      1.60206      1.03342
423   657.  1     1.00000      1.60206      1.03342
424   658.  1     1.00000      1.60206      1.03342
425   659.  1     1.00000      1.60206      1.03342
426   660.  1     1.00000      1.60206      1.03342
427   661.  1     1.00000      1.60206      1.03342
428   662.  1     1.00000      1.60206      1.03342
429   663.  1     1.00000      1.60206      1.03342

430   663 OBSERVATIONS + 0 CONTROLS = 663 TOTAL

431   INITIAL ESTIMATES OF THE PARAMETERS
432             -4.544105 CONSTANT              3.947060 LOG(C)                1.018223 LOG(W)

433   STARTING LIKELIHOOD = -226.9796

434   6 LOGIT EVALUATIONS
------------------------------------------------------------------------------------------------------

435   PYR  SF  NONAT  EQUALITY                                           POLO2   26 AUG 80   14:07:13   22    40

436   6 PROBIT EVALUATIONS

437   MAXIMUM LOG LIKELIHOOD = -225.8631

438                          PARAMETER            STD ERROR           T RATIO
439   CONSTANT              -5.03989               .408145           -12.3483
440   LOG(C)                 4.37818               .298517            14.6664
441   LOG(W)                 1.12486               .293838            3.82817

442   COVARIANCE MATRIX
443                         CONSTANT        LOG(C)         LOG(W)
444     CONSTANT           .1665827      -.1021096    -.8328212-01
445     LOG(C)            -.1021096      .8911216-01   .1960563-01
446     LOG(W)           -.8328212-01    .1960563-01   .8634052-01

447   PREDICTION SUCCESS TABLE
448                                     PREDICTED OUTCOME
449                                     -----------------
450   ACTUAL OUTCOME                     ALIVE    DEAD
451   -------------------------------------------------
452   ALIVE                               178      56
453   DEAD                                 51     378
454   -------------------------------------------------
455   PERCENT CORRECT                      78      87
456   OVERALL PERCENT CORRECT: 84

457   TESTING HYPOTHESIS THAT THE 3-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
458      LIKELIHOOD RATIO TEST:  CHI-SQUARED = 467.3855   DF=3   TAIL PROBABILITY = .000
```

Figure 10-5

```
459   PYRETHRUM SPRAY OR FILM WITH NATURAL RESPONSE            POLO2          28 AUG 80   09:33:01   10     4

460   CONTROL CARD: 4,2,0,3,1,0,1,0,1,2,0,0

461   NUMBER OF VARIABLES: 4
462   NUMBER OF VARIABLES ON INPUT CARDS: 2
463   NUMBER OF PARAMETERS: 5
464   TYPE OF MODEL: PROBIT
465   THERE ARE TRANSFORMATIONS
```

Figure 10-5—Continued

```
466   A CONTROL GROUP IS INDICATED BY VARIABLE 5 BEING ZERO
467   INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
468   ALL RAW DATA WILL BE PRINTED
469   NATURAL RESPONSE IS A PARAMETER
470   MULTIPLE SUBJECTS PER DATA CARD
471   NUMBER OF PARALLEL GROUPS: 2

472   TRANSFORMATIONS: 3L3=4L4=

473   PARAMETER LABELS
474     1  SPRAY
475     2  FILM
476     3  LOG(C)
477     4  LOG(W)
478     5  NATURAL

479   FORMAT: (I1,1X,2F5.0,2I4)

480   INPUT DATA
481   1 5.    2.9     27   1
482   1 10.   2.9     29  15
483   1 20.   2.9     30  27
484   1 40.   2.9     28  28
485   1 5.    5.7     29   4
486   1 10.   5.7     29  19
487   1 20.   5.7     27  26
488   1 40.   5.7     30  30
489   1 5.    10.8    30   6
490   1 10.   10.8    24  15
491   1 20.   10.8    31  31
492   1 40.   10.8    19  19
493   2 5.    2.9     29   3
494   2 10.   2.9     30  10
495   2 20.   2.9     29  24
496   2 40.   2.9     29  29
497   2 5.    5.7     27   4
498   2 10.   5.7     28  14
499   2 20.   5.7     28  27
500   2 40.   5.7     29  29
501   2 5.    10.8    28   8
502   2 10.   10.8    28  17
503   2 20.   10.8    28  26
504   2 40.   10.8    17  17
505   0 0     0      311  12

506   TRANSFORMED DATA
507      1.  1    1.00000       .000000       .698970       .462398
508      2.  0    1.00000       .000000       .698970       .462398
509      3.  0    1.00000       .000000       .698970       .462398
510      4.  0    1.00000       .000000       .698970       .462398
511      5.  0    1.00000       .000000       .698970       .462398
512      6.  0    1.00000       .000000       .698970       .462398
513      7.  0    1.00000       .000000       .698970       .462398
-----------------------------------------------------------------------------------------------
514   PYRETHRUM SPRAY OR FILM WITH NATURAL RESPONSE                   POL02   28 AUG 80   09:33:02   12   15

515    608.  0    .000000       1.00000       1.00000       1.03342
516    609.  0    .000000       1.00000       1.00000       1.03342
517    610.  0    .000000       1.00000       1.00000       1.03342
518    611.  0    .000000       1.00000       1.00000       1.03342
519    612.  0    .000000       1.00000       1.00000       1.03342
520    613.  0    .000000       1.00000       1.00000       1.03342
521    614.  0    .000000       1.00000       1.00000       1.03342
522    615.  0    .000000       1.00000       1.00000       1.03342
523    616.  0    .000000       1.00000       1.00000       1.03342
524    617.  0    .000000       1.00000       1.00000       1.03342
525    618.  0    .000000       1.00000       1.00000       1.03342
526    619.  1    .000000       1.00000       1.30103       1.03342
527    620.  1    .000000       1.00000       1.30103       1.03342
528    621.  1    .000000       1.00000       1.30103       1.03342
529    622.  1    .000000       1.00000       1.30103       1.03342
530    623.  1    .000000       1.00000       1.30103       1.03342
531    624.  1    .000000       1.00000       1.30103       1.03342
532    625.  1    .000000       1.00000       1.30103       1.03342
533    626.  1    .000000       1.00000       1.30103       1.03342
534    627.  1    .000000       1.00000       1.30103       1.03342
535    628.  1    .000000       1.00000       1.30103       1.03342
536    629.  1    .000000       1.00000       1.30103       1.03342
537    630.  1    .000000       1.00000       1.30103       1.03342
538    631.  1    .000000       1.00000       1.30103       1.03342
539    632.  1    .000000       1.00000       1.30103       1.03342
540    633.  1    .000000       1.00000       1.30103       1.03342
541    634.  1    .000000       1.00000       1.30103       1.03342
542    635.  1    .000000       1.00000       1.30103       1.03342
543    636.  1    .000000       1.00000       1.30103       1.03342
544    637.  1    .000000       1.00000       1.30103       1.03342
545    638.  1    .000000       1.00000       1.30103       1.03342
546    639.  1    .000000       1.00000       1.30103       1.03342
547    640.  1    .000000       1.00000       1.30103       1.03342
```

Figure 10-5—Continued

```
641.  1    .000000    1.00000    1.30103    1.03342
642.  1    .000000    1.00000    1.30103    1.03342
643.  1    .000000    1.00000    1.30103    1.03342
644.  1    .000000    1.00000    1.30103    1.03342
645.  0    .000000    1.00000    1.30103    1.03342
646.  0    .000000    1.00000    1.30103    1.03342
647.  1    .000000    1.00000    1.60206    1.03342
648.  1    .000000    1.00000    1.60206    1.03342
649.  1    .000000    1.00000    1.60206    1.03342
650.  1    .000000    1.00000    1.60206    1.03342
651.  1    .000000    1.00000    1.60206    1.03342
652.  1    .000000    1.00000    1.60206    1.03342
653.  1    .000000    1.00000    1.60206    1.03342
654.  1    .000000    1.00000    1.60206    1.03342
655.  1    .000000    1.00000    1.60206    1.03342
656.  1    .000000    1.00000    1.60206    1.03342
657.  1    .000000    1.00000    1.60206    1.03342
658.  1    .000000    1.00000    1.60206    1.03342
659.  1    .000000    1.00000    1.60206    1.03342
660.  1    .000000    1.00000    1.60206    1.03342
661.  1    .000000    1.00000    1.60206    1.03342
662.  1    .000000    1.00000    1.60206    1.03342
663.  1    .000000    1.00000    1.60206    1.03342

663 OBSERVATIONS + 311 CONTROLS = 974 TOTAL
MORTALITY IN CONTROLS: .039
-----------------------------------------------------------------------------------------------

PYRETHRUM SPRAY OR FILM WITH NATURAL RESPONSE                         POLO2   28 AUG 80   09:33:04   14   16

INITIAL ESTIMATES OF THE PARAMETERS
          -4.101413 SPRAY               .8847998 LOG(W)
          -4.209135 FILM               .3858521-01 NATURAL
           3.631875 LOG(C)

STARTING LIKELIHOOD = -281.2558

10 LOGIT EVALUATIONS
5 PROBIT EVALUATIONS

MAXIMUM LOG LIKELIHOOD = -275.5481

                 PARAMETER          STD ERROR        T RATIO
SPRAY            -5.41446           .504502          -10.7323
FILM             -5.61829           .525942          -10.6823
LOG(C)            4.70390           .365933           12.8546
LOG(W)            1.22311           .327875            3.73041
NATURAL           .395908-01        .110382-01         3.58671

COVARIANCE MATRIX
                  SPRAY          FILM          LOG(C)         LOG(W)        NATURAL
  SPRAY         .2545221      .2543949     -.1591772      -.1135423    -.1383305-02
  FILM          .2543949      .2766148     -.1676289      -.1163738    -.1537166-02
  LOG(C)       -.1591772     -.1676289      .1339068      .3393508-01   .1019743-02
  LOG(W)       -.1135423     -.1163738     .3393508-01     .1075022     .2416572-03
  NATURAL     -.1383305-02  -.1537166-02   .1019743-02   .2416572-03   .1218416-03

PREDICTION SUCCESS TABLE
                                   PREDICTED OUTCOME
                                   -----------------
ACTUAL OUTCOME                      ALIVE     DEAD
--------------------------------------------------
ALIVE                                178       56
DEAD                                  51      378
--------------------------------------------------
PERCENT CORRECT                       78       87
OVERALL PERCENT CORRECT: 84

TESTING HYPOTHESIS THAT THE 4-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
   LIKELIHOOD RATIO TEST:  CHI-SQUARED = 455.2636    DF=4   TAIL PROBABILITY = .000
```

4.1.6 Hypotheses Testing

The maximized log likelihood values needed to test the hypotheses outlined in *section 4.1.2* are:

Model	*L*	*Source*
[1]	-101.3851	*fig. 10-1,* line 72
[2]	-122.4908	*fig. 10-2,* line 162
[3]	-225.1918	*fig. 10-3,* line 300
[4]	-225.8631	*fig. 10-4,* line 437

The LR tests of the three hypotheses are the following: (1) Hypothesis H(P) of parallelism.

$$L(\Omega) = -101.3851 + -122.4908$$
$$= -223.8759,$$
$$L(\Omega) = -225.1918,$$
$$LR = 2[L(\Omega) - L(\omega)] = 2[-223.8959 + 225.1918]$$
$$= 2[l.3159] = 2.6318.$$

The hypothesis H(P) is accepted, at significance level $\alpha = 0.05$ if

$$LR \le \chi^2_{.05}(2) = 5.99.$$

Since $2.6318 < 5.99$, we accept H(P).
(2) Hypothesis H(E|P) of equality given parallelism.

$$L(\Omega) = -225.1918,$$
$$L(\omega) = -225.8631,$$
$$LR = 2[L(\Omega) - L(\omega)] = 2[-225.1918 + 225.8631]$$
$$= 2[0.6713] = 1.3426.$$

The hypothesis H(E|P) is accepted at level $\alpha = 0.05$ if

$$LR \le \chi^2_{.05}(1)=3.84.$$

Since $1.3426 < 3.84$, we accept H(E| P).
(3) Hypothesis H(E) of equality.

$$L(\Omega) = -223.8759,$$
$$L(\omega) = -225.8631,$$
$$LR = 2[L(\Omega) - L(\omega)] = 2[-223.8759 + 225.8631]$$
$$= 2[1.9872] = 3.9744.$$

The hypothesis H(E) is accepted at level $\alpha = 0.05$ if

$$LR \le \chi^2_{.05}(3) = 7.81.$$

We also accept this hypothesis since $3.9744 < 7.81$.

4.1.7 Comparison with Published Calculations

The analyses of models [1]-[4] cannot be compared directly with those described by Finney. The parameter values for model [5] confirm those of Finney's equations (8.27) and (8.28) (Finney 1971, p. 169).

4.2 Vaso-Constriction

Finney (1971, p. 183-190) describes a series of measurements of the volume of air inspired by human subjects, their rate of inspiration, and whether or not a vaso-constriction reflex occurred in the skin of their fingers. These experiments were reported originally by Gilliatt (1947). A feature of this example is that the data are ungrouped. We use the example to illustrate the analysis of ungrouped data, the likelihood ratio test for equal regression coefficients, and the use of transformations.

4.2.1 Models, Hypothesis, and Analyses Required

The model expressing the probability of the vaso-constriction reflex is

$$Y=\alpha+\beta_1 x_1 + \beta_2 x_2 \qquad [1]$$

where y is the probit or logit of the probability, α is the constant term, x_1 is the logarithm volume of air inspired in liters, x_2 is the logarithm of rate of inspiration in liters per second, β_1 is the regression coefficient for volume, and β_2 is the regression coefficient for rate.

The hypothesis is H: $\beta_1=\beta_2$ which states that the regression coefficients for rate and volume of air inspired are the same. The unrestricted model is [1] and the restricted model is

$$Y = \alpha + \beta(x_l+x_2) = \alpha + \beta_x \qquad [2]$$

The required maximum log likelihoods are $L(\Omega)$ and $L(\omega)$.

(i) $L(\Omega)$: ML estimation of [1].
(ii) $L(\omega)$: ML estimation of [2].

When the hypothesis H is true, asymptotically,

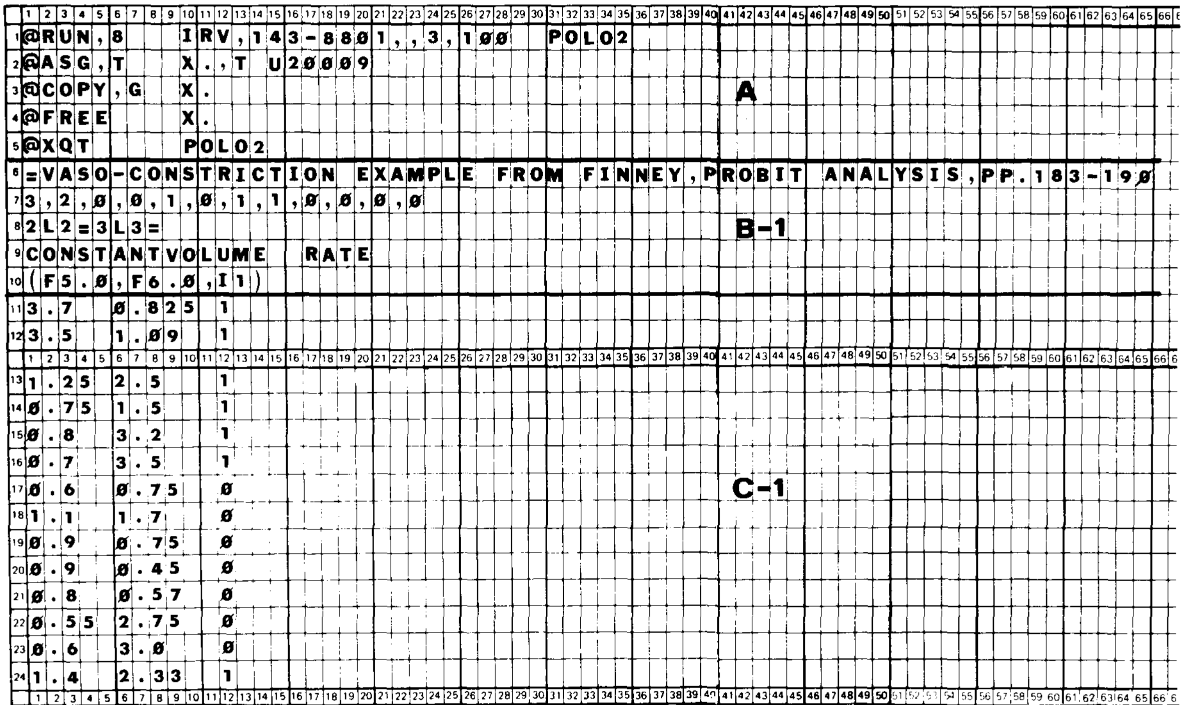
$$LR = 2[L(\Omega) - L(\omega)] \sim \chi 2\ (1)$$

so that the hypothesis H is accepted at the α level of significance if

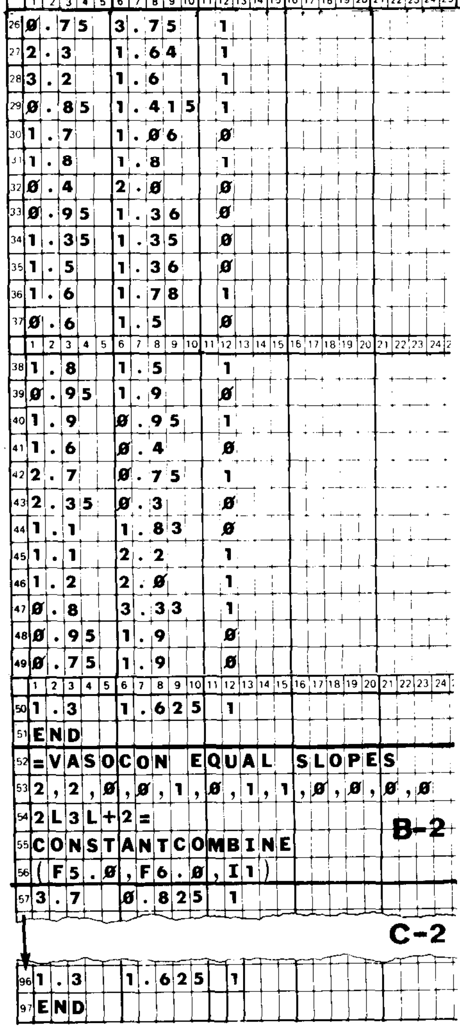
$$LR \le \chi_\alpha^2\ (1).$$

4.2.2 Input

The input for analyses of the two required models consists of 97 cards (*fig. 11*). After the starter cards (*fig. 11-A*), program cards specify the analysis of model [1] (*fig. 11-B-1*). The title card (card 6) cites the source of the data; the control card (card 7) specifies three regression coefficients, two variables to be read from each data card, the probit model to be used, none of the explanatory variables contains a control group, transformations will be used, starting values of the parameters will be calculated automatically, data will be printed back, natural response is not a parameter, there is one subject per data card, there are no parallel groups, a restricted model will not be computed, and the program will select a suitable number of iterations in serach of an ML estimate. The transformation card (card 8) defines y_2 as the logarithm of x_2, and y_3 as the logarithm of x_3. The three parameters (card 9) are labeled constant (x_1), volume (x_2), and rate (x_3). The format statement (card 10) instructs the program to read a 5-column field including a decimal point (volume), and a 6-column field including a decimal point (rate), and finally, a single column of integers (l=constricted, 0=not constricted). The data, consisting of 39 individual records,



```
 1 @RUN,8     IRV,143-88Ø1,,3,1ØØ   POLO2
 2 @ASG,T     X.,T U2ØØØ9
 3 @COPY,G    X.                              A
 4 @FREE      X.
 5 @XQT       POLO2
 6 =VASO-CONSTRICTION EXAMPLE FROM FINNEY,PROBIT ANALYSIS,PP.183-19Ø
 7 3,2,Ø,Ø,1,Ø,1,1,Ø,Ø,Ø,Ø
 8 2L2=3L3=                                   B-1
 9 CONSTANTVOLUME    RATE
10 (F5.Ø,F6.Ø,I1)
11 3.7   Ø.825  1
12 3.5   1.Ø9   1
13 1.25  2.5    1
14 Ø.75  1.5    1
15 Ø.8   3.2    1
16 Ø.7   3.5    1
17 Ø.6   Ø.75   Ø                             C-1
18 1.1   1.7    Ø
19 Ø.9   Ø.75   Ø
20 Ø.9   Ø.45   Ø
21 Ø.8   Ø.57   Ø
22 Ø.55  2.75   Ø
23 Ø.6   3.Ø    Ø
24 1.4   2.33   1
```



```
26 Ø.75  3.75   1
27 2.3   1.64   1
28 3.2   1.6    1
29 Ø.85  1.415  1
30 1.7   1.Ø6   Ø
31 1.8   1.8    1
32 Ø.4   2.Ø    Ø
33 Ø.95  1.36   Ø
34 1.35  1.35   Ø
35 1.5   1.36   Ø
36 1.6   1.78   1
37 Ø.6   1.5    Ø
38 1.8   1.5    1
39 Ø.95  1.9    Ø
40 1.9   Ø.95   1
41 1.6   Ø.4    Ø
42 2.7   Ø.75   1
43 2.35  Ø.3    Ø
44 1.1   1.83   Ø
45 1.1   2.2    1
46 1.2   2.Ø    1
47 Ø.8   3.33   1
48 Ø.95  1.9    Ø
49 Ø.75  1.9    Ø
50 1.3   1.625  1
51 END
52 =VASOCON EQUAL SLOPES
53 2,2,Ø,Ø,1,Ø,1,1,Ø,Ø,Ø,Ø
54 2L3L+2=                 B-2
55 CONSTANTCOMBINE
56 (F5.Ø,F6.Ø,I1)
57 3.7   Ø.825  1
                           C-2
96 1.3   1.625  1
97 END
```

Figure 11

follows (*fig. 11-C-1*). After the END card (card 51), the input for analysis of model [2] follows.

The program cards for model [2] (*fig. 11-B-2*) begin with a descriptive title (card 52); the control card (card 53) differs from that for model [1] only in the NVAR position (integer 1). In this analysis, there are only two regression coefficients in addition to the constant term. The transformations are also different (card 54). The variable y_2=x is defined as the sum of the logarithm of x_2 and the logarithm of x_3. The two parameter labels are "constant" and "combine" (card 55). The format statement (card 56) and the data (*fig. 11-C-2*) are identical to that in the analysis of model [1].

4.2.3 Output

The analyses, in their entirety, are shown in *figure 12*. Titles for the analyses are reprinted at the top of each page (*fig. 12*, lines 1, 57, 110, 128, 184, and 235). Integers from the control cards (*fig. 12*, lines 2 and 129) begin each printout, followed by specification statements for each analysis (*fig. 12*, lines 3-11 and 130-138). Each transformation card is reproduced (*fig. 12*, lines 12 and 139), after which the parameter labels are stated (*fig. 12*, lines 13-16; lines 140-142). Format statements (*fig. 12*, lines 17 and 143) precede listings of data in both raw and transformed versions (*fig. 12*, lines 18-56 and 58-98, lines 144-183 and 185-224). From left to right, the columns in the transformed data listing are chronological number of the individual, its response, sample size (=1 is all cases), logarithm (base 10) of volume, and logarithm (base 10) of rate. The summary of total observations concludes the descriptive portion of each printout (*fig. 12*, lines 99 and 225.

Initial parameter estimates, starting ML values, and iteration statements (*fig. 12*, lines 100-104; lines 226-230) begin the statistical portion of each printout. The final ML estimate follows (*fig. 12*, lines 105 and 231); parameter values, their standard errors, and t-ratios (*fig. 12*, lines 106-109; lines 232-234) are printed next. The covariance matrix and prediction success table follow (*fig. 12*, lines 111-125; lines 236-249). Note that the program's automatic dead-alive category labels are not appropriate for this experiment; labels such as constricted and not constricted would be more appropriate. The LR test for significance of the model coefficients ends each analysis (*fig. 12*, lines 126-127; lines 250-251).

The t-ratios of the parameters for both models indicate that each parameter is significant in the regression. The OPC values indicate that each model is a good predictor of observed results, and the LR tests indicate that both regressions are highly significant ($\alpha = 0.05$).

4.2.4 Hypothesis Testing

The maximum log likelihoods needed to test the hypothesis H: $\beta_1 =: \beta_2$ are:

Model	*L*	*Source*
[1]	-14.6608	*fig. 12*, line 105
[2]	-14.7746	*fig. 12*, line 231

In this example

$$L(\Omega) = -14.6608,$$
$$L(\omega) = -14.7746,$$
$$LR = 2[L(\Omega) - L(\omega)] = 2[0.1138] = 0.2276.$$

For a test at the 0.05 significance levels, the χ^2 critical value is 3.84. Hence we accept the hypothesis H.

Figure 12

```
 1   VASO-CONSTRICTION,EXAMPLE FROM FINNEY,PROBIT ANALYSIS,PP.183-190      POLO2      28 AUG 80   09:33:07   17   17

 2   CONTROL CARD: 3,2,0,0,1,0,1,1,0,0,0,0

 3   NUMBER OF VARIABLES: 3
 4   NUMBER OF VARIABLES ON INPUT CARDS: 2
 5   NUMBER OF PARAMETERS: 3
 6   TYPE OF MODEL: PROBIT
 7   THERE ARE TRANSFORMATIONS
 8   INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
 9   ALL RAW DATA WILL BE PRINTED
10   NATURAL RESPONSE IS NOT A PARAMETER
11   ONE SUBJECT PER DATA CARD

12   TRANSFORMATIONS: 2L2=3L3=

13   PARAMETER LABELS
14      1  CONSTANT
15      2  VOLUME
16      3  RATE

17   FORMAT: (F5.0,F6.0,I1)

18   INPUT DATA
19   3.7  0.825 1
20   3.5  1.09  1
21   1.25 2.5   1
22   0.75 1.5   1
23   0.8  3.2   1
24   0.7  3.5   1
25   0.6  0.75  0
26   1.1  1.7   0
27   0.9  0.75  0
28   0.9  0.45  0
29   0.8  0.57  0
30   0.55 2.75  0
31   0.6  3.0   0
32   1.4  2.33  1
33   0.75 3.75  1
34   2.3  1.64  1
35   3.2  1.6   1
36   0.85 1.415 1
37   1.7  1.06  0
38   1.8  1.8   1
39   0.4  2.0   0
40   0.95 1.36  0
41   1.35 1.35  0
42   1.5  1.36  0
43   1.6  1.78  1
```

Figure 12—Continued

```
0.6  1.5   0
1.8  1.5   1
0.95 1.9   0
1.9  0.95  1
1.6  0.4   0
2.7  0.75  1
2.35 0.3   0
1.1  1.83  0
1.1  2.2   1
1.2  2.0   1
0.8  3.33  1
0.95 1.9   0
0.75 1.9   0
--------------------------------------------------------------------------------------------------------------
VASO-CONSTRICTION,EXAMPLE FROM FINNEY,PROBIT ANALYSIS,PP.183-190     POLO2      28 AUG 80   09:33:08   17    18

1.3  1.625 1

TRANSFORMED DATA
   1.  1      1.00000        .568202       -.835460-01
   2.  1      1.00000        .544068        .374265-01
   3.  1      1.00000        .969100-01     .397940
   4.  1      1.00000       -.124939        .176091
   5.  1      1.00000       -.969100-01     .505150
   6.  1      1.00000       -.154902        .544068
   7.  0      1.00000       -.221849       -.124939
   8.  0      1.00000        .413927-01     .230449
   9.  0      1.00000       -.457575-01    -.124939
  10.  0      1.00000       -.457575-01    -.346787
  11.  0      1.00000       -.969100-01    -.244125
  12.  0      1.00000       -.259637        .439333
  13.  0      1.00000       -.221849        .477121
  14.  1      1.00000        .146128        .367356
  15.  1      1.00000       -.124939        .574031
  16.  1      1.00000        .361728        .214844
  17.  1      1.00000        .505150        .204120
  18.  1      1.00000       -.705811-01     .150756
  19.  0      1.00000        .230449        .253059-01
  20.  1      1.00000        .255273        .255273
  21.  0      1.00000       -.397940        .301030
  22.  0      1.00000       -.222764-01     .133539
  23.  0      1.00000        .130334        .130334
  24.  0      1.00000        .176091        .133539
  25.  1      1.00000        .204120        .250420
  26.  0      1.00000       -.221849        .176091
  27.  1      1.00000        .255273        .176091
  28.  0      1.00000       -.222764-01     .278754
  29.  1      1.00000        .278754       -.222764-01
  30.  0      1.00000        .204120       -.397940
  31.  1      1.00000        .431364       -.124939
  32.  0      1.00000        .371068       -.522879
  33.  0      1.00000        .413927-01     .262451
  34.  1      1.00000        .413927-01     .342423
  35.  1      1.00000        .791812-01     .301030
  36.  1      1.00000       -.969100-01     .522444
  37.  0      1.00000       -.222764-01     .278754
  38.  0      1.00000       -.124939        .278754
  39.  1      1.00000        .113943        .210853

39 OBSERVATIONS + 0 CONTROLS = 39 TOTAL

INITIAL ESTIMATES OF THE PARAMETERS
            -.2624056 CONSTANT                  1.637666 VOLUME                   1.240651 RATE

STARTING LIKELIHOOD = -20.5856

14 LOGIT EVALUATIONS
8 PROBIT EVALUATIONS

MAXIMUM LOG LIKELIHOOD = -14.6608

                      PARAMETER           STD ERROR            T RATIO
CONSTANT              -1.53306             .611100            -2.50869
VOLUME                 6.63059            2.08127              3.18584
RATE                   5.88569            2.06139              2.85521
--------------------------------------------------------------------------------------------------------------

VASO-CONSTRICTION,EXAMPLE FROM FINNEY,PROBIT ANALYSIS,PP.183-190    POLO2      28 AUG 80   09:33:08   18    19

COVARIANCE MATRIX
                    CONSTANT        VOLUME          RATE
  CONSTANT          .3734435       -.9039205      -1.128364
  VOLUME           -.9039205       4.331670        3.066827
  RATE            -1.128364        3.066827        4.249311
```

Figure 12—Continued

```
116      PREDICTION SUCCESS TABLE
117                                     PREDICTED OUTCOME
118                                     -----------------
119      ACTUAL OUTCOME                   ALIVE    DEAD
120      ------------------------------------------------
121      ALIVE                             14        5
122      DEAD                               2       18
123      ------------------------------------------------
124      PERCENT CORRECT                   88       78
125      OVERALL PERCENT CORRECT: 82

126     TESTING HYPOTHESIS THAT THE 3-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
127         LIKELIHOOD RATIO TEST:  CHI-SQUARED = 24.7440   DF=3   TAIL PROBABILITY = .000
--------------------------------------------------------------------------------------------------------
128     VASOCON EQUAL SLOPES                                              POL02   28 AUG 80  09:33:08   18   20

129     CONTROL CARD: 2,2,0,0,1,0,1,1,0,0,0,0

130     NUMBER OF VARIABLES: 2
131     NUMBER OF VARIABLES ON INPUT CARDS: 2
132     NUMBER OF PARAMETERS: 2
133     TYPE OF MODEL: PROBIT
134     THERE ARE TRANSFORMATIONS
135     INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
136     ALL RAW DATA WILL BE PRINTED
137     NATURAL RESPONSE IS NOT A PARAMETER
138     ONE SUBJECT PER DATA CARD

139     TRANSFORMATIONS: 2L3L+2=

140     PARAMETER LABELS
141       1  CONSTANT
142       2  COMBINE

143     FORMAT: (F5.0,F6.0,I1)

144     INPUT DATA
145     3.7  0.825 1
146     3.5  1.09  1
147     1.25 2.5   1
148     0.75 1.5   1
149     0.8  3.2   1
150     0.7  3.5   1
151     0.6  0.75  0
152     1.1  1.7   0
153     0.9  0.75  0
154     0.9  0.45  0
155     0.8  0.57  0
156     0.55 2.75  0
157     0.6  3.0   0
158     1.4  2.33  1
159     0.75 3.75  1
160     2.3  1.64  1
161     3.2  1.6   1
162     0.85 1.415 1
163     1.7  1.06  0
164     1.8  1.8   1
165     0.4  2.0   0
166     0.95 1.36  0
167     1.35 1.35  0
168     1.5  1.36  0
169     1.6  1.78  1
170     0.6  1.5   0
171     1.8  1.5   1
172     0.95 1.9   0
173     1.9  0.95  1
174     1.6  0.4   0
175     2.7  0.75  1
176     2.35 0.3   0
177     1.1  1.83  0
178     1.1  2.2   1
179     1.2  2.0   1
180     0.8  3.33  1
181     0.95 1.9   0
182     0.75 1.9   0
183     1.3  1.625 1
--------------------------------------------------------------------------------------------------------
184     VASOCON EQUAL SLOPES                                      POL02           28 AUG 80  09:33:08   18   21

185     TRANSFORMED DATA
186        1.  1      1.00000       .484656
187        2.  1      1.00000       .581495
188        3.  1      1.00000       .494850
```

Figure 12—Continued

```
189        4.  1       1.00000         .511525-01
190        5.  1       1.00000         .408240
191        6.  1       1.00000         .389166
192        7.  0       1.00000        -.346787
193        8.  0       1.00000         .271842
194        9.  0       1.00000        -.170696
195       10.  0       1.00000        -.392545
196       11.  0       1.00000        -.341035
197       12.  0       1.00000         .179695
198       13.  0       1.00000         .255273
199       14.  1       1.00000         .513484
200       15.  1       1.00000         .449093
201       16.  1       1.00000         .576572
202       17.  1       1.00000         .709270
203       18.  1       1.00000         .801754-01
204       19.  0       1.00000         .255755
205       20.  1       1.00000         .510545
206       21.  0       1.00000        -.969100-01
207       22.  0       1.00000         .111263
208       23.  0       1.00000         .260668
209       24.  0       1.00000         .309630
210       25.  1       1.00000         .454540
211       26.  0       1.00000        -.457575-01
212       27.  1       1.00000         .431364
213       28.  0       1.00000         .256477
214       29.  1       1.00000         .256477
215       30.  0       1.00000        -.193820
216       31.  1       1.00000         .306425
217       32.  0       1.00000        -.151811
218       33.  0       1.00000         .303844
219       34.  1       1.00000         .383815
220       35.  1       1.00000         .380211
221       36.  1       1.00000         .425534
222       37.  0       1.00000         .256477
223       38.  0       1.00000         .153815
224       39.  1       1.00000         .324797

225   39 OBSERVATIONS + 0 CONTROLS = 39 TOTAL

226   INITIAL ESTIMATES OF THE PARAMETERS
227              -.7302226 CONSTANT                3.077000 COMBINE

228   STARTING LIKELIHOOD = -16.7708

229   9 LOGIT EVALUATIONS
230   5 PROBIT EVALUATIONS

231   MAXIMUM LOG LIKELIHOOD = -14.7746

232                        PARAMETER          STD ERROR          T RATIO
233   CONSTANT             -1.61648           .602371            -2.68353
234   COMBINE               6.30556           1.93851             3.25279
------------------------------------------------------------------------------------------------------------
235   VASOCON EQUAL SLOPES                                            POL02        28 AUG 80   09:33:09  19     22

236   COVARIANCE MATRIX
237                          CONSTANT       COMBINE
238     CONSTANT             .3628504      -1.049731
239     COMBINE             -1.049731       3.757826

240   PREDICTION SUCCESS TABLE
241                                    PREDICTED OUTCOME
242                                    -----------------
243   ACTUAL OUTCOME                    ALIVE    DEAD
244   -----------------------------------------------
245   ALIVE                              13        6
246   DEAD                                2       18
247   -----------------------------------------------
248   PERCENT CORRECT                    87       75
249   OVERALL PERCENT CORRECT: 79

250   TESTING HYPOTHESIS THAT THE 2-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
251      LIKELIHOOD RATIO TEST:  CHI-SQUARED = 24.5162    DF=2    TAIL PROBABILITY = .000
```

4.3 Body Weight as a Variable: Higher Order Terms

Robertson and others (1981) described a series of experiments designed to test the hypothesis that the response of an insect (*Choristoneura occidentalis* Freeman) is proportional to its body weight. Three chemicals, including mexacarbate, were used. We use data for tests with mexacarbate to illustrate the use of individual data to test the significance of a higher order term in a polynomial model. This example demonstrates the use of the restricted model option of POLO2 (see *section 2.3*).

Briefly, each insect in this experiment was selected at random from a laboratory colony, weighed, and treated with 1 μl of mexacarbate dissolved in acetone. Mortality was tallied after 7 days. Individual records for 253 insects were kept. Because a printback of all the data is too voluminous for this report, we use the program option of IECHO=0 (see *section 2.3*).

4.3.1 Models and Hypothesis

The polynomial model is

$$y = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \delta_3 z$$

where y is the probit or logit of response, x, is the logarithm of dose (in μg), x_2 is the logarithm of body weight (in mg), and z is the square of the logarithm of body weight ($z = x_2^2$). The regression coefficients are δ_0 for the constant, δ_1 for log dose, δ_2 for log weight, and δ_3 for the square of log weight.

The hypothesis is H: $\delta_3 = 0$, that is, the coefficient of the higher order term in weight equals zero. The unrestricted model is [1] and the restricted model is

$$Y = \delta_0 + \delta_1 x_1 + \delta_2 x_2 \qquad [2]$$

The required maximum log likelihoods are L(Ω) and L(ω).

(i) L(Ω): ML estimation of [1].
(ii) L(ω): ML estimation of [2].

When H is true, asymptotically,

$$LR = 2[L(\Omega) - L(\omega)] \sim \chi^2 (1),$$

so that H is accepted at level α if

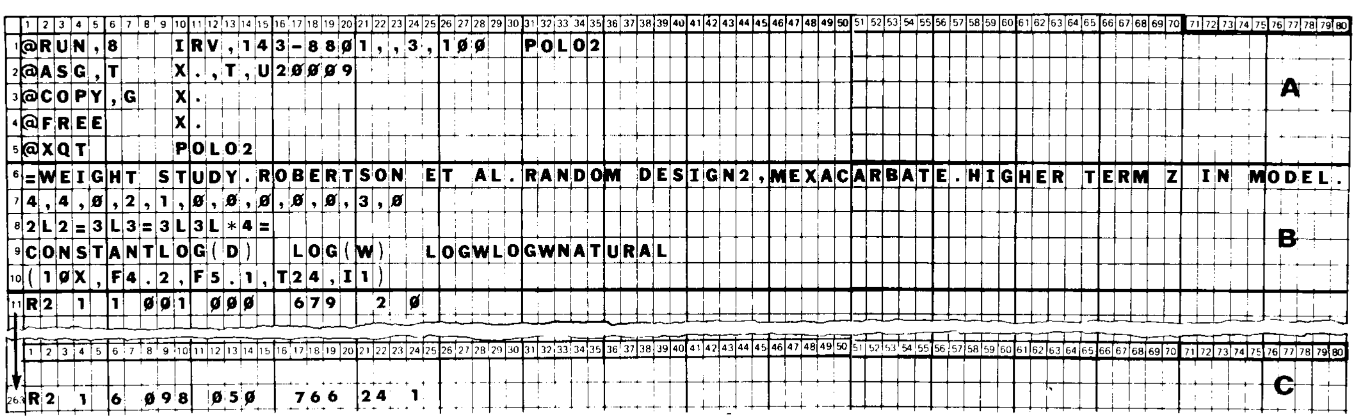
$$LR \leq \chi^2 (1).$$

The program will automatically conduct an LR test of H: $\delta_3 = 0$.

4.3.2 Input

The input for this analysis consists of 263 cards (*fig. 13*). The program cards (*fig. 13-B*) follow the usual starter cards (*fig. 13-A*). Following the title (card 6), the control card specifies four regression coefficients, two variables to be read from each data card, the probit model is to be used, the second parameter (log (D)) contains a control group, transformations will be used, starting values will be calculated automatically, data printback will be suppressed, natural response is a parameter, there is one subject per data card, there are no parallel groups, a restricted model retaining three variables will be computed, and the program will choose the number of iterations for M L estimates. The transformations defined (card 8) are: y_2 equals the logarithm of x_2, y_3 equals the logarithm of x_3, and y_4 equals the square of the logarithm of x_3. Parameters are x_1="constant," x_2="logarithm of dose," x_3="logarithm of weight," x_4="(logarithm of weight)^2," and x_5="natural response." Suggestive labels appear on card 9.

The germane information on each data card (card 11-263, *fig. 13-C*) is listed in columns 12-14 (dose), 17-19 (weight), and 24 (dead or alive). The format statement (card 10), therefore, instructs the program to skip the first 10 columns, read a 4-column field with two digits to the right of the decimal point, read a 5-column field with one digit to the right of the decimal point, then go to column 24 to read an integer. No END card is needed after the data assuming that no other analysis follows.



```
1   @RUN,8   IRV,143-88Ø1,,3,1ØØ  POLO2
2   @ASG,T   X.,T,U2ØØØ9
3   @COPY,G  X.                                          A
4   @FREE    X.
5   @XQT     POLO2
6   =WEIGHT STUDY.ROBERTSON ET AL.RANDOM DESIGN2,MEXACARBATE.HIGHER TERM Z IN MODEL.
7   4,4,Ø,2,1,Ø,Ø,Ø,Ø,Ø,3,Ø
8   2L2=3L3=3L3L*4=
9   CONSTANTLOG(D)  LOG(W)  LOGWLOGWNATURAL              B
10  (1ØX,F4.2,F5.1,T24,I1)
11  R2 1  1 ØØ1 ØØØ  679  2 Ø

263 R2 1  6 Ø98 Ø5Ø  766 24 1                            C
```

Figure 13

4.3.3 Output

The title is repeated at the top of each page of the printout (*fig. 14*, lines 1, 56, and 101). The initial portions of the analysis demonstrate the same features noted in previous examples: the control card listing (*fig. 14*, line 2) is followed by specification statements (*fig. 14*, lines 3-12), the transformation statement (*fig. 14*, line 13), parameter labels (*fig. 14*, lines 14-19), and the format statement (*fig. 14*, line 20). Because the data printback option was not used (IECHO=0), the program prints only the first 20 data cards in raw and transformed versions (*fig. 14*, lines 21-55 and 57-65). This permits the user to check a sample to assure that the data are being transformed correctly. The observations summary (*fig. 14*, line 66) is followed by the proportional mortality observed in the controls (*fig. 14*, line 67).

The statistical portion of the printout begins with initial parameter estimates (*fig. 14*, line 68-71), the initial log likelihood values (*fig. 14*, line 72), iterations totals (*fig. 14*, lines 73-74) and the final log likelihood value (*fig. 14*, line 75). These precede the table of parameter values, their standard errors, and t-ratios (*fig. 14*, lines 76-81). In this example, the only parameter with a significant t-ratio is log(D); the values of the ratios for all other parameters fall below the critical t = 1.96 tabular value. The covariance matrix (*fig. 14*, lines 82-88) and prediction success table (*fig. 14*, lines 89-98) follow. The OPC indicates good prediction success. Finally, the significance of the full model is tested (*fig. 14*, lines 99-100).

The restricted model [2] is computed next, with the (log W)^2 parameter omitted (*fig. 14*, line 102). The initial estimates of the parameters that are retained are listed (*fig. 14*, lines 103-104), followed by the initial log likelihood value (*fig. 14*, line 105) and the iteration summary (*fig. 14*, lines 106-107). The final log likelihood value (*fig. 14*, line 108) and parameter estimates with their standard errors and t-ratios (*fig. 14*, lines 109-113) follow. In the restricted model, the t-ratios of all the parameters except natural response are now significant; this contasts with the lack of significance of all parameters expect dose in model [1]. The next portion of the printout is the usual presentation of the covariance matrix (*fig. 14*, lines 114-119), followed by the prediction success table (*fig. 14*, lines 120-129) and LR test of the hypothesis that the regression coefficients of the restricted model equal zero (*fig. 14*, lines 130-131). The hypothesis is rejected. Finally, the LR test of the hypothesis H: δ_3=0, which was outlined in *section 4.3.1*, is presented (*fig. 14*, lines 132-133).The maximum log likelihood for the unrestricted model, the model including (log weight)^2, is $L(\Omega)$ = -93.6792 and for the restricted model, the one excluding (log weight)^2, is $L(\omega)$ = -94.9086. Since LR = $2[L(\Omega) - L(\omega)] = 2(1.2294) = 2.4589 < 3.84$, the hypothesis H is accepted at the 0.05 significance level. Consequently, we conclude that (log weight)^2 is not a relevant explanatory variable in the regression.

Figure 14

```
 1   WEIGHT STUDY.ROBERTSON ET AL.RANDOM DESIGN2,MEXACARBATE. HIGHER TERM Z IN MODEL POLO2  28 AUG 80  09:32:55  4  1

 2   CONTROL CARD: 4,4,0,2,1,0,0,0,0,0,3,0

 3   NUMBER OF VARIABLES: 4
 4   NUMBER OF VARIABLES ON INPUT CARDS: 4
 5   NUMBER OF PARAMETERS: 5
 6   TYPE OF MODEL: PROBIT
 7   THERE ARE TRANSFORMATIONS
 8   A CONTROL GROUP IS INDICATED BY VARIABLE 2 BEING ZERO
 9   A RESTRICTED MODEL OF THE FIRST 3 VARIABLES IS TO BE COMPUTED ALSO
10   INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
11   NATURAL RESPONSE IS A PARAMETER
12   ONE SUBJECT PER DATA CARD

13   TRANSFORMATIONS: 2L2=3L3=3L3L*4=

14   PARAMETER LABELS
15      1  CONSTANT
16      2  LOG(D)
17      3  LOG(W)
18      4  LOGWLOGW
19      5  NATURAL
```

Figure 14—Continued

```
FORMAT: (10X,F4.2,F5.1,T24,I1)

INPUT DATA
R2 1 1 001 000  679  2 0
R2 1 1 002 000  861  2 0
R2 1 1 003 000  762  2 0
R2 1 1 004 000  635  2 0
R2 1 1 005 000  973  2 0
R2 1 1 006 000  604  2 0
R2 1 1 007 000  963  2 0
R2 1 1 008 000  745  2 0
R2 1 1 009 005  760  2 0
R2 1 1 010 005  791  2 0
R2 1 1 011 005  766  2 0
R2 1 1 012 005  930  2 0
R2 1 1 013 005  727  2 0
R2 1 1 014 005  614  2 0
R2 1 1 015 005  838  2 0
R2 1 1 016 010  815  2 0
R2 1 1 017 010  728  2 0
R2 1 1 018 010  769  2 0
R2 1 1 019 010  599  2 1
R2 1 1 020 010  652  2 1
 ETC.

TRANSFORMED DATA
    1.  0     1.00000      -1.30103        1.88081       3.53746
    2.  0     1.00000      -1.30103        1.89818       3.60307
    3.  0     1.00000      -1.30103        1.88423       3.55032
    4.  0     1.00000      -1.30103        1.96848       3.87493
    5.  0     1.00000      -1.30103        1.86153       3.46531
    6.  0     1.00000      -1.30103        1.78817       3.19755
    7.  0     1.00000      -1.30103        1.92324       3.69887
    8.  0     1.00000      -1.00000        1.91116       3.65252
    9.  0     1.00000      -1.00000        1.86213       3.46753
   10.  0     1.00000      -1.00000        1.88593       3.55672
   11.  1     1.00000      -1.00000        1.77743       3.15925
   12.  1     1.00000      -1.00000        1.81425       3.29149
-----------------------------------------------------------------------------------------------------------------

WEIGHT STUDY.ROBERTSON ET AL.RANDOM DESIGN2,MEXACARBATE. HIGHER TERM Z IN MODEL  POLO2  28 AUG 80  09:32:57  6  2

   13.  1     1.00000      -1.00000        1.84572       3.40668
   14.  0     1.00000      -1.00000        1.80414       3.25492
   15.  0     1.00000      -1.00000        1.83696       3.37441
   16.  1     1.00000      -1.00000        1.77305       3.14372
   17.  0     1.00000      -1.00000        1.81954       3.31074
   18.  0     1.00000      -1.00000        1.82478       3.32981
   19.  1     1.00000      -.698970        1.89763       3.60099
   20.  0     1.00000      -.698970        1.91009       3.64845
   ETC.

207 OBSERVATIONS + 46 CONTROLS = 253 TOTAL
MORTALITY IN CONTROLS: .022

INITIAL ESTIMATES OF THE PARAMETERS
              61.92830 CONSTANT              13.86360 LOGWLOGW
              2.314442 LOG(D)            .2173913-01 NATURAL
             -58.12285 LOG(W)

STARTING LIKELIHOOD = -97.5068

13 LOGIT EVALUATIONS
7 PROBIT EVALUATIONS

MAXIMUM LOG LIKELIHOOD = -93.6792

                       PARAMETER          STD ERROR          T RATIO
CONSTANT               99.4907            55.5623            1.79062
LOG(D)                 3.41625            .456533            7.48303
LOG(W)                -94.6854            58.0475           -1.63117
LOGWLOGW               22.8966            15.1593            1.51040
NATURAL                .144445-01         .142699-01         1.01223

COVARIANCE MATRIX
                      CONSTANT        LOG(D)          LOG(W)          LOGWLOGW        NATURAL
   CONSTANT           3087.166        6.196166       -3224.037        841.1348       .1926991-01
   LOG(D)             6.196166        .2084226       -6.060385        1.509324       .5030425-03
   LOG(W)            -3224.037       -6.060385        3369.508       -879.6802      -.1834847-01
   LOGWLOGW           841.1348        1.509324       -879.6802        229.8032       .4319225-02
   NATURAL          .1926991-01     .5030425-03    -.1834847-01     .4319225-02    .2036306-03

PREDICTION SUCCESS TABLE
                            PREDICTED OUTCOME
                            -----------------
ACTUAL OUTCOME               ALIVE    DEAD
----------------------------------------------
```

Figure 14—Continued

```
ALIVE                              72        24
DEAD                               23        88
-------------------------------------------------
PERCENT CORRECT                    76        79
OVERALL PERCENT CORRECT: 77

TESTING HYPOTHESIS THAT THE 4-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
   LIKELIHOOD RATIO TEST:  CHI-SQUARED = 108.9986    DF=4    TAIL PROBABILITY = .000
```

```
WEIGHT STUDY.ROBERTSON ET AL.RANDOM DESIGN2,MEXACARBATE. HIGHER TERM Z IN MODEL  POLO2 28 AUG 80  09:32:59 9   3

RESTRICTED MODEL WITH 3 VARIABLES AND 4 PARAMETERS

INITIAL ESTIMATES OF THE PARAMETERS
              11.99335 CONSTANT        2.303810 LOG(D)       -5.455093 LOG(W)          .2173913-01 NATURAL

STARTING LIKELIHOOD = -98.2843

9 LOGIT EVALUATIONS
7 PROBIT EVALUATIONS

MAXIMUM LOG LIKELIHOOD = -94.9086

                        PARAMETER             STD ERROR            T RATIO
CONSTANT                15.9899               2.84374              5.62283
LOG(D)                  3.30682               .430669              7.67834
LOG(W)                 -7.18592               1.43618             -5.00348
NATURAL                 .148626-01            .147205-01           1.00965

COVARIANCE MATRIX
                     CONSTANT       LOG(D)         LOG(W)         NATURAL
  CONSTANT          8.086880       .5906353      -4.061704      .4634708-02
  LOG(D)            .5906353       .1854755      -.2444996      .4560381-03
  LOG(W)           -4.061704      -.2444996       2.062625     -.2457787-02
  NATURAL          .4634708-02    .4560381-03   -.2457787-02    .2166941-03

PREDICTION SUCCESS TABLE
                                 PREDICTED OUTCOME
                                 -----------------
ACTUAL OUTCOME                   ALIVE     DEAD
-------------------------------------------------
ALIVE                              72        24
DEAD                               23        88
-------------------------------------------------
PERCENT CORRECT                    76        79
OVERALL PERCENT CORRECT: 77

TESTING HYPOTHESIS THAT THE 3-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
   LIKELIHOOD RATIO TEST:  CHI-SQUARED = 106.5107    DF=3    TAIL PROBABILITY = .000

TESTING HYPOTHESIS THAT THE 4-BETA-PARAMETER MODEL AND THE 3-BETA-PARAMETER MODEL ARE THE SAME
   LIKELIHOOD RATIO TEST:  CHI-SQUARED = 2.4589    DF=1    TAIL PROBABILITY = .117
```

4.4 Body Weight as a Variable: PROPORTIONAL Option

Dosage estimates are the primary objective of many toxicological investigations. The topical application technique described by Savin and others (1977), for example, is used to obtain precise estimates of the amounts of chemicals necessary to affect 50 or 90 percent of test subjects. The quality of chemical applied is known, as is the weight of test subjects. In the previous example, we tested the significance of a higher order term in a polynominal model. On the basis of an LR test, we conclude that the higher order term was not relevant. The PROPORTIONAL option permits the user to test the hypothesis that the response of test subjects is proportional to their body weight. If the hypothesis of proportionality is correct, LD_{50} and LD_{90} estimates at weights chosen by the investigator can be calculated.

4.4.1 Models and Hypotheses

We now consider the model

$$y = \delta_0 + \delta_1 x_1 + \delta_2 x_2$$

Where y = the probit or logit of the response, x_1 = logarithm of the dose (log D), and x_2 = logarithm of the weight (log W). Let $\delta_0 = \beta_0$, $\delta_1 = \beta_2$, and $\beta_2 = \delta_1 + \delta_2$. Then the model [1] can be rewritten as

$$y = \beta_0 + \beta_1 \log (D/W) + \beta_2 \log W.$$

The hypothesis of proportionality is H: $\beta_2 = 0$ (Robertson and others 1981). The unrestricted model is [1] and the restricted model is

$$y = \beta_0 + \beta_1 \log (D/W)$$

The required maximum log likelihoods are $L(\Omega)$ and $L(\omega)$.

(i) $L(\Omega)$: ML estimation of [2].
(ii) $L(\omega)$: ML estimation of [3].

When the hypothesis H is true, asymptotically,

$$LR = 2[L(\Omega) - L(\omega)] \sim \chi^2 (1)$$

so that the hypothesis H is accepted at the α significance level if

$$LR \leq \chi^2(1).$$

The necessary calculations are automatically performed when the PROPORTIONAL option is chosen. Note that the hypothesis can also be tested using the t-ratio for β_2. If the hypothesis is accepted, the user may obtain LD_{50} and LD_{90} estimates for any body weight desired with the method described by Savin and others (1981). If the hypothesis is rejected, the BASIC option (*sec. 4.5*) should be used.

4.4.2 Input

The input (*fig. 15*) may begin with the usual starter cards (*fig. 15-A*) unless the PROPORTIONAL option is used after another analysis (except another PROPORTIONAL option set or the BASIC option—no other analysis may follow the use of either). In this example, input begins with the program cards (*fig. 15-B*).

The program card must have PROPORTIONAL in some position from columns 2-80 (card 6). The control card (card 7) must have NVAR=3 and KERNEL=2 so that the proportionality hypothesis will be tested. If the LR test of proportionality is not needed, NVAR=2 and KERNEL=0 may be specified; however, we suggest that the LR test be performed unless there is ample evidence that proportional response can be assumed. NPLL must be zero in either case, but other integers on the control card may vary as needed.

In this example, the transformations specified (card 8) are $y_2 = \log (x_2/x_3)$ and $y_3 = \log x_3$. If the LR test of proportionality is not requested and NVAR=2, KERNEL=0 are present on the control card, the transformation $y_2 = \log(x_2/x_3)$ alone- should be used. Parameters are: constant, log dose divided by log weight,

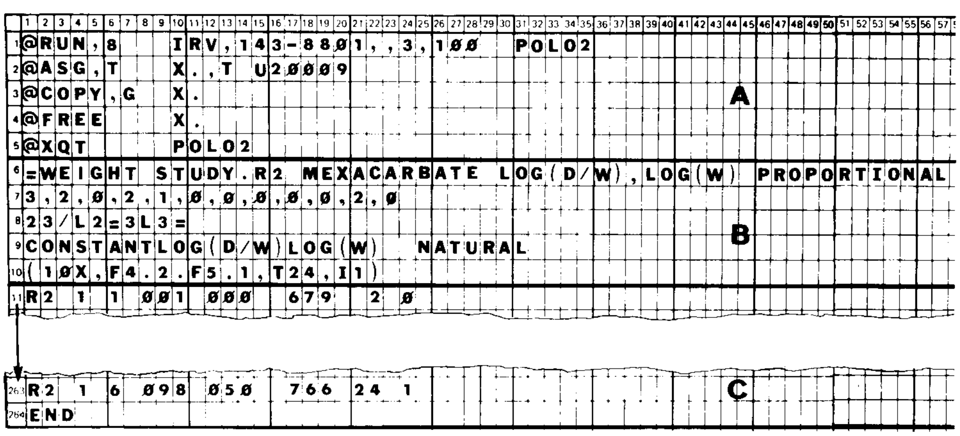


Figure 15

log weight, and natural response. Card 9 has labels suggestive of these names. The format card (card 10) instructs the program to read dose (*fig. 15-C*, columns 11-14), weight (*fig. 15-C*, columns 15-19), and response (*fig. 15-C*, column 24) from the data cards. The data cards are followed by an END card (card 264).

If the proportionality hypothesis is accepted, weights specified by the user may be placed behind the END card to obtain LD_{50} and LD_{90} estimates. One weight can be punched on each card in free field format.

4.4.3 Output

The output follows the usual pattern until the last portion. Titles appear at the top of each page (*fig. 16*, lines 1, 56 and 96). The descriptive section lists specification statements (*fig. 16*, line 3-12), transformation statement (*fig. 16*, line 13), parameter labels (*fig. 16*, lines 14-18), format statement (*fig. 16*, line 19), abbreviated raw and transformed data listing (*fig. 16*, lines 20-55 and 57-64), the observation summary (*fig. 16*, line 65), and the natural mortality statement (*fig. 16*, line 66).

The statistical portion of the printout begins, as usual, with the initial parameter estimates (*fig. 16*, lines 67-68), starting log likelihood value (*fig. 16*, line 69), and the iteration summary (*fig. 16*, lines 70-71) preceding the final log likelihood value (*fig. 16*, line 72). Parameter values and statistics (*fig. 16*, lines 73-77), covariance matrix (*fig. 16*, lines 78-83), prediction success table with OPC (*fig. 16*, lines 84-93) and test of significance of coefficients in the full model [2] follow (*fig. 16*, lines 94-95). The statistics and LR test for the restricted model [3] are printed next (*fig. 16*, lines 97-124).

4.4.4 Hypothesis Testing

The LR statistic is

$$LR = 2[L(\Omega) - L(\omega)] = 2[-94.9086 + 99.4884]$$
$$= 2[4.5788] = 9.1596.$$

The hypothesis of proportionality is rejected at the 0.05 significance level because the χ^2 critical value is 3.84. Note that the hypothesis is also rejected by the t-test because the t-ratio for β_2 is -2.92. The calculations in the last section of the printout are statistics based on average weight; these should be disregarded unless the proportionality hypothesis was accepted (see *sec. 4.5.2* for an explanation of the printout).

4.5 Body Weight as a Variable: BASIC Option

The BASIC option estimates lethal doses D in the equation

$$y = \beta_0+\beta_1(\log D)+\beta_2(\log W)$$

when $\beta_1 \neq \beta_2$. This model is appropriate when the proportionality hypothesis has been rejected, as in the previous example (*sec. 4.4*). Calculations are described by Savin and others (1981).

Figure 16

```
1    WEIGHT STUDY.R2 MEXACARBATE LOG(D/W),LOG(W) PROPORTIONAL          POLO2    28 AUG 80   12:06:38    4     1

2    CONTROLCARD: 3,2,0,2,1,0,0,0,0,0,2,0

3    NUMBER OF VARIABLES: 3
4    NUMBER OF VARIABLES ON INPUT CARDS: 2
5    NUMBER OF PARAMETERS: 4
6    TYPE OF MODEL: PROBIT
7    THERE ARE TRANSFORMATIONS
8    A CONTROL GROUP IS INDICATED BY VARIABLE 2 BEING ZERO
9    A RESTRICTED MODEL OF THE FIRST 2 VARIABLES IS TO BE COMPUTED ALSO
10   INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
11   NATURAL RESPONSE IS A PARAMETER
12   ONE SUBJECT PER DATA CARD

13   TRANSFORMATIONS: 23/L2=3L3=

14   PARAMETER LABELS
15      1  CONSTANT
16      2  LOG(D/W)
17      3  LOG(W)
18      4  NATURAL

19   FORMAT: (10X,F4.2,F5.1,T24,I1)

20   INPUT DATA
21   R2 1 1 001 000   679  2 0
22   R2 1 1 002 000   861  2 0
23   R2 1 1 003 000   762  2 0
24   R2 1 1 004 000   635  2 0
25   R2 1 1 005 000   973  2 0
```

Figure 16—Continued

```
26   R2 1 1 006 000  604  2 0
27   R2 1 1 007 000  963  2 0
28   R2 1 1 008 000  745  2 0
29   R2 1 1 009 005  760  2 0
30   R2 1 1 010 005  791  2 0
31   R2 1 1 011 005  766  2 0
32   R2 1 1 012 005  930  2 0
33   R2 1 1 013 005  727  2 0
34   R2 1 1 014 005  614  2 0
35   R2 1 1 015 005  836  2 0
36   R2 1 1 016 010  815  2 0
37   R2 1 1 017 010  728  2 0
38   R2 1 1 018 010  769  2 0
39   R2 1 1 019 010  599  2 1
40   R2 1 1 020 010  652  2 1
41    ETC.

42   TRANSFORMED DATA
43      1.  0     1.00000      -3.18184      1.88081
44      2.  0     1.00000      -3.19921      1.89818
45      3.  0     1.00000      -3.18526      1.88423
46      4.  0     1.00000      -3.26951      1.96848
47      5.  0     1.00000      -3.16256      1.86153
48      6.  0     1.00000      -3.08920      1.78817
49      7.  0     1.00000      -3.22427      1.92324
50      8.  0     1.00000      -2.91116      1.91116
51      9.  0     1.00000      -2.86213      1.86213
52     10.  0     1.00000      -2.88593      1.88593
53     11.  1     1.00000      -2.77743      1.77743
54     12.  1     1.00000      -2.81425      1.81425
55     13.  1     1.00000      -2.84572      1.84572
--------------------------------------------------------------------------------------------------------
56   WEIGHT STUDY.R2 MEXACARBATE LOG(D/W),LOG(W) PROPORTIONAL          POL02   28 AUG 80  12:06:42   6    2

57     14.  0     1.00000      -2.80414      1.80414
58     15.  0     1.00000      -2.83696      1.83696
59     16.  1     1.00000      -2.77305      1.77305
60     17.  0     1.00000     -2.81954       1.81954
61     18.  0     1.00000      -2.82478      1.82478
62     19.  1     1.00000      -2.59660      1.89763
63     20.  0     1.00000      -2.60906      1.91009
64    ETC.

65   207 OBSERVATIONS + 46 CONTROLS = 253 TOTAL
66   MORTALTIY IN CONTROLS: .022

67   INITIAL ESTIMATES OF THE PARAMETERS
68               12.22245 CONSTANT          2.269062 LOG(D/W)   -3.317775 LOG(W)     .2173913-01 NATURAL

69   STARTING LIKELIHOOD = -98.4603

70   12 LOGIT EVALUATIONS
71   7 PROBIT EVALUATIONS

72   MAXIMUM LOG LIKELIHOOD = -94.9086

73                       PARAMETER           STD ERROR          T RATIO
74   CONSTANT            15.9899             2.84379            5.62273
75   LOG(D/W)            3.30682             .430670            7.67832
76   LOG(W)             -3.87910             1.32634           -2.92467
77   NATURAL             .148627-01          .147207-01         1.00965

78   COVARIANCE MATRIX
79                       CONSTANT      LOG(D/W)       LOG(W)        NATURAL
80     CONSTANT          8.087159      .5906401     -3.471210     .4634957-02
81     LOG(D/W)          .5906401      .1854768   -.5902480-01    .4560508-03
82     LOG(W)           -3.471210   -.5902480-01    1.759175    -.2001868-02
83     NATURAL         .4634957-02   .4560508-03  -.2001868-02    .2166987-03

84   PREDICTION SUCCESS TABLE
85                                 PREDICTED OUTCOME
86                                 -----------------
87   ACTUAL OUTCOME                  ALIVE    DEAD
88   -----------------------------------------------
89   ALIVE                            72       24
90   DEAD                             23       88
91   -----------------------------------------------
92   PERCENT CORRECT                  76       79
93   OVERALL PERCENT CORRECT: 77

94   TESTING HYPOTHESIS THAT THE 3-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
95      LIKELIHOOD RATIO TEST:  CHI-SQUARED = 106.5108   DF=3   TAIL PROBABILITY = .000
```

Figure 16—Continued

```
 96  WEIGHT STUDY.R2 MEXACARBATE LOG(D/W),LOG(W) PROPORTIONAL            POLO2            28 AUG 80  12:06:44    8     3

 97  RESTRICTED MODEL WITH 2 VARIABLES AND 3 PARAMETERS

 98  INITIAL ESTIMATES OF THE PARAMETERS
 99                 7.334388 CONSTANT               2.791762 LOG(D/W)              .2173913-01 NATURAL

100  STARTING LIKELIHOOD = -100.4000

101  10 LOGIT EVALUATIONS
102  11 PROBIT EVALUATIONS

103  MAXIMUM LOG LIKELIHOOD = -99.4884

104                        PARAMETER            STD ERROR            T RATIO
105  CONSTANT              8.59942              1.06703              8.05919
106  LOG(D/W)              3.28190              .412611              7.95398
107  NATURAL               .139061-01           .137678-01           1.01004

108  COVARIANCE MATRIX
109                         CONSTANT      LOG(D/W)       NATURAL
110    CONSTANT            1.138559      .4380162    .5864396-03
111    LOG(D/W)            .4380162      .1702479    .3247294-03
112    NATURAL          .5864396-03   .3247294-03    .1895527-03

113  PREDICTION SUCCESS TABLE
114                                PREDICTED OUTCOME
115                                -----------------
116  ACTUAL OUTCOME                  ALIVE     DEAD
117  -----------------------------------------------
118  ALIVE                             66       28
119  DEAD                              19       92
120  -----------------------------------------------
121  PERCENT CORRECT                   78       77
122  OVERALL PERCENT CORRECT: 77

123  TESTING HYPOTHESIS THAT THE 2-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
124     LIKELIHOOD RATIO TEST:  CHI-SQUARED = 97.4199    DF=2   TAIL PROBABILITY = .000


125  PROPORTIONAL MODEL

126  WBAR=77.367111, LOG10(WBAR)=1.888556

127  A=2.4013643, B=3.2819001

128  V11=.0913368, V12=.1164934, V22=.1702479

129  SE(A)=.3022199, SE(B)=.4126111

130  G=.060719, T=1.95996

131  LD50=.1854815, LIMITS=.1576298,.2151935

132  LD90=.4558178, LIMITS=.3707691,.6206283
```

4.5.1 Input

The input (*fig. 17*) begins with the usual starter cards (*fig. 17-A*) unless the BASIC option follows another analysis (except another BASIC or PROPORTIONAL set). In that case, input would begin with the program cards (*fig. 17-B*). The program cards must have:

1. BASIC is some position on the title card (card 6).
2. A control card (card 7) with NVAR=3, NPLL=0, and KERNEL=0. The basic model is limited to three regression coefficients (NVAR), no parallel groups, and no test of a restricted model. The other integers may vary, as required.

In this example, the transformation card (card 8) states that y_2=log x_2 and y_3=log x_3. The parameters (card 9) are labeled: "constant," "log(D)," and "log(W)." Natural response is a parameter is this example, but need not be present in each use of the BASIC option. The format statement (card 10) instructs the program to read only dose (*fig. 17-C*, columns 11-14), body weight (*fig. 17-C*, columns 15-19), and response (*fig. 17-C*, column 24). The data are followed by an END card (*fig. 17-D*, card 264) and weight cards (*fig. 17-D*, cards 265-269).

4.5.2 Output

The BASIC output follows the usual pattern until the last section. Title repetition on each page (*fig. 18*, lines 1, 56, 99, and 125), control card listing (*fig. 18*, line 2), specification statements (*fig. 18*, lines 3-11), transformation listing (*fig. 18*, line 12), parameter labels (*fig. 18*, lines 13-17), format statement listing (*fig. 16*, line 18), an abbreviated raw and transformed data listing (*fig. 18*, lines 19-55 and 57-63), observations summary (*fig. 18*, line 64), and natural response statement (*fig. 18*, line 65) form the descriptive portion of the output.

The statistical portion contains the usual initial parameter estimates (*fig. 18*, lines 66-67), initial log likelihood value (*fig. 18*, line 68), iteration summary (*fig. 18*, lines 69-70) final log likelihood value (*fig. 18*, line 71), parameter values and statistics (*fig. 18*, lines 72-76), covariance matrix (*fig. 18*, lines 77-82), prediction success table (*fig. 18*, lines 83-92), and LR test of the full model (*fig. 18*, lines 93-94).

Statistics for the model using average weight of the test subjects as the value of W follows (*fig. 18*, lines 95-98 and 100-102). The terminology of these statistics is as follows. WBAR is average weight, and LOGIO (WBAR) is the logarithm of WBAR to the base 10 (*fig. 18*, line 96). The parameters are called "A" and "B"; A is the intercept of the line calculated at WBAR, and B is the slope of the line (*fig. 18*, line 97). The variances and covariances of the parameters are listed next (*fig. 18*, line 98), followed by the standard errors of intercept and slope (*fig. 18*, line 100). Values of *g* and *t*, used to calculate confidence limits (Finney 1971) for point estimates on a probit or logit line, appear next (*fig. 16*, line 101). Finally, values of the lethal dose necessary for 50 and 90 percent mortality at WBAR, together with their 95 percent confidence limits, are printed (*fig. 18*, lines 101-102). Next, statistics for each weight specified on the weight cards (*fig. 17-D*, cards 265-269) are printed (*fig. 18*, lines 104-124 and 126-139).

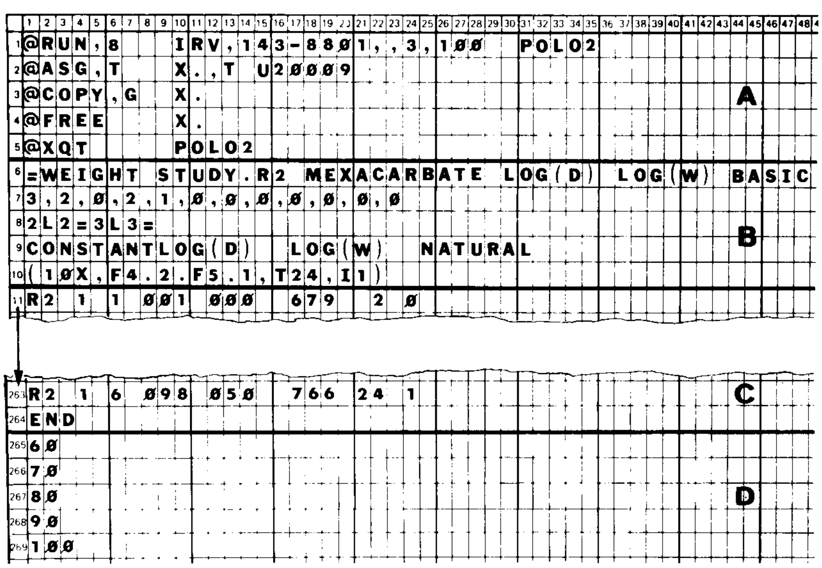

```
1   @RUN,8    IRV,143-88Ø1,,3,1ØØ  POLO2
2   @ASG,T    X.,T U2ØØØ9
3   @COPY,G   X.                                A
4   @FREE     X.
5   @XQT      POLO2
6   =WEIGHT STUDY.R2 MEXACARBATE LOG(D) LOG(W) BASIC
7   3,2,Ø,2,1,Ø,Ø,Ø,Ø,Ø,Ø,Ø
8   2L2=3L3=
9   CONSTANTLOG(D)  LOG(W)  NATURAL             B
10  (1ØX,F4.2.F5.1,T24,I1)
11  R2  1  1  ØØ1  ØØØ  679  2  Ø
263 R2  1  6  Ø98  Ø5Ø  766  24  1             C
264 END
265 6Ø
266 7Ø
267 8Ø                                          D
268 9Ø
269 1ØØ
```

Figure 17.

Figure 18

```
1   WEIGHT STUDY.R2 MEXACARBATE LOG(D) LOG(W) BASIC                    POLO2   17 SEP 80  12:55:39   4   1

2   CONTROL CARD: 3,2,0,2,1,0,0,0,0,0,0,0

3   NUMBER OF VARIABLES: 3
4   NUMBER OF VARIABLES ON INPUT CARDS: 2
5   NUMBER OF PARAMETERS: 4
6   TYPE OF MODEL: PROBIT
7   THERE ARE TRANSFORMATIONS
8   A CONTROL GROUP IS INDICATED BY VARIABLE 2 BEING ZERO
9   INITIAL ESTIMATES OF THE PARAMETERS ARE TO BE CALCULATED AUTOMATICALLY
10  NATURAL RESPONSE IS A PARAMETER
11  ONE SUBJECT PER DATA CARD

12  TRANSFORMATIONS: 2L2=3L3=

13  PARAMETER LABELS
14    1  CONSTANT
15    2  LOG(D)
16    3  LOG(W)
17    4  NATURAL

18  FORMAT: (10X,F4.2,F5.1,T24,I1)

19  INPUT DATA
20  R2 1 1 001 000  679  2 0
21  R2 1 1 002 000  861  2 0
22  R2 1 1 003 000  762  2 0
23  R2 1 1 004 000  635  2 0
24  R2 1 1 005 000  973  2 0
25  R2 1 1 006 000  604  2 0
26  R2 1 1 007 000  963  2 0
27  R2 1 1 008 000  745  2 0
28  R2 1 1 009 005  760  2 0
29  R2 1 1 010 005  791  2 0
30  R2 1 1 011 005  766  2 0
31  R2 1 1 012 005  930  2 0
32  R2 1 1 013 005  727  2 0
33  R2 1 1 014 005  614  2 0
34  R2 1 1 015 005  838  2 0
35  R2 1 1 016 010  815  2 0
36  R2 1 1 017 010  728  2 0
37  R2 1 1 018 010  769  2 0
38  R2 1 1 019 010  599  2 1
39  R2 1 1 020 010  652  2 1
40   ETC.

41  TRANSFORMED DATA
42     1.  0      1.00000      -1.30103       1.88081
43     2.  0      1.00000      -1.30103       1.89818
44     3.  0      1.00000      -1.30103       1.88423
45     4.  0      1.00000      -1.30103       1.96848
46     5.  0      1.00000      -1.30103       1.86153
47     6.  0      1.00000      -1.30103       1.78817
48     7.  0      1.00000      -1.30103       1.92324
49     8.  0      1.00000      -1.00000       1.91116
50     9.  0      1.00000      -1.00000       1.86213
51    10.  0      1.00000      -1.00000       1.88593
52    11.  1      1.00000      -1.00000       1.77743
53    12.  1      1.00000      -1.00000       1.81425
54    13.  1      1.00000      -1.00000       1.84572
55    14.  0      1.00000      -1.00000       1.80414
----------------------------------------------------------------------------------------------------------
56  WEIGHT STUDY.R2 MEXACARBATE LOG(D) LOG(W) BASIC                    POLO2   17 SEP 80  12:55:41   6   2

57    15.  0      1.00000      -1.00000       1.83696
58    16.  1      1.00000      -1.00000       1.77305
59    17.  0      1.00000      -1.00000       1.81954
60    18.  0      1.00000      -1.00000       1.82478
61    19.  1      1.00000      -.698970       1.89763
62    20.  0      1.00000      -.698970       1.91009
63     ETC.

64  207 OBSERVATIONS + 46 CONTROLS = 253 TOTAL
65  MORTALITY IN CONTROLS: .022

66  INITIAL ESTIMATES OF THE PARAMETERS
67            11.99335 CONSTANT         2.303810 LOG(D)  -5.455093 LOG(W)    .2173913-01 NATURAL

68  STARTING LIKELIHOOD = -98.2843

69  9 LOGIT EVALUATIONS
70  7 PROBIT EVALUATIONS
```

Figure 18—Continued

```
71   MAXIMUM LOG LIKELIHOOD = -94.9086

72                          PARAMETER            STD ERROR            T RATIO
73   CONSTANT               15.9899              2.84374              5.62283
74   LOG(D)                 3.30682              .430669              7.67834
75   LOG(W)                 -7.18592             1.43618              -5.00348
76   NATURAL                .148626-01           .147205-01           1.00965

77   COVARIANCE MATRIX
78                          CONSTANT        LOG(D)          LOG(W)          NATURAL
79     CONSTANT             8.086880        .5906353        -4.061704       .4634708-02
80     LOG(D)               .5906353        .1854755        -.2444996       .4560381-03
81     LOG(W)               -4.061704       -.2444996       2.062625        -.2457787-02
82     NATURAL              .4634708-02     .4560381-03     -.2457787-02    .2166941-03

83   PREDICTION SUCCESS TABLE
84                                  PREDICTED OUTCOME
85                                  -----------------
86   ACTUAL OUTCOME                   ALIVE     DEAD
87   ------------------------------------------------
88   ALIVE                              72       24
89   DEAD                               23       88
90   ------------------------------------------------
91   PERCENT CORRECT                    76       79
92   OVERALL PERCENT CORRECT: 77

93   TESTING HYPOTHESIS THAT THE 3-BETA-PARAMETER MODEL AND THE 0-BETA-PARAMETER MODEL ARE THE SAME
94      LIKELIHOOD RATIO TEST:  CHI-SQUARED = 106.5107    DF=3    TAIL PROBABILITY = .000

95   BASIC MODEL

96   WBAR=77.367111, LOG10(WBAR)=1.888556

97   A=2.4188659, B=3.3068204

98   V11=.1020167, V12=.1288841, V22=.1854755
```

```
99   WEIGHT STUDY.R2 MEXACARBATE LOG(D) LOG(W) BASIC            POLO2          17 SEP 80   12:55:44    8     3

100  SE(A)=.3194006, SE(B)=.4306687

101  G=.065157, T=1.95996

102  LD50=.1855761, LIMITS=.1573355,.2163299

103  LD90=.4529707, LIMITS=.3654136,.6284598


104  W=60.000000, LOG10(W)=1.778151

105  A=3.2122284, B=3.3068204

106  V11=.1638840, V12=.1558781, V22=.1854755

107  SE(A)=.4048259, SE(B)=.4306687

108  G=.065157, T=1.95996

109  LD50=.1068083, LIMITS=.0799298,.1368499

110  LD90=.2607072, LIMITS=.2007445,.3676450


111  W=70.000000, LOG10(W)=1.845098

112  A=2.7311541, B=3.3068204

113  V11=.1203684, V12=.1395096, V22=.1854755

114  SE(A)=.3469415, SE(B)=.4306687

115  G=.065157, T=1.95996

116  LD50=.1493085, LIMITS=.1230761,.1768952

117  LD90=.3644454, LIMITS=.2943531,.4990451
```

Figure 18—Continued

```
W=80.000000, LOG10(W)=1.903090
A=2.3144286, B=3.3068204
V11=.0976179, V12=.1253306, V22=.1854755
SE(A)=.3124386, SE(B)=.4306687
G=.065157, T=1.95996
LD50=.1995743, LIMITS=.1692023,.2335790
LD90=.4871386, LIMITS=.3905918,.6827091
------------------------------------------------------------------------------------------------
WEIGHT STUDY.R2 MEXACARBATE LOG(D) LOG(W) BASIC          POLO2        17 SEP 80  12:55:44    9    4

W=90.000000, LOG10(W)=1.954243
A=1.9468505, B=3.3068204
V11=.0890662, V12=.1128239, V22=.1854755
SE(A)=.2984396, SE(B)=.4306687
G=.065157, T=1.95996
LD50=.2577876, LIMITS=.2112662,.3165341
LD90=.6292309, LIMITS=.4848818,.9305364


W=100.000000, LOG10(W)=2.000000
A=1.6180408, B=3.3068204
V11=.0905631, V12=.1016362, V22=.1854755
SE(A)=.3009370, SE(B)=.4306687
G=.065157, T=1.95996
LD50=.3241126, LIMITS=.2507269,.4269428
LD90=.7911227, LIMITS=.5783192,1.2488830

@FIN
```

5. ERROR MESSAGES

Error messages clearly indicate mistakes in the input. For example:

Message	*Reason*
CONTROL CARD: 4,4,0,2,1,0,0,0,0,3,0 THERE SHOULD BE 12 NUMBERS, NOT 11	One of the integers is missing from the control card.
FORMAT: (10x,F4.2,F5.I(T24,I1) ILLEGAL FORMAT CHARACTERS WERE ACCEPTED AS BLANKS	The parenthesis preceding T24 should have been a comma.
TRANSFORMATIONS: L2=3L3=3L33L3*4= STACK MUST BE EMPTY AT END OF POLISH STRING	An extra "3" was punched ("33" should be "3"), so an extra x_3 was put into the stack. The stack, therefore, was not empty at the end of the scan. See *section 2.4.1.*
CONTROL CARD: 4,4,0,2,1,0,0,0,0,3,0,0 THERE ARE NOW 16750372454 PARALLEL GROUPS, EXCEEDING 3	The 3 belonged in the KERNEL column, not in the NPLL column.

Along with the error message, the user will receive a message guaranteed to catch the eye.

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Russell, Robert M.; Savin, N.E.; Robertson, Jacqueline L. **POLO2: a user's guide to multiple Probit Or LOgit analysis.** Gen. Tech. Rep. PSW-55. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 1981. 37 p.

This guide provides instructions for the use of POLO2, a computer program for multivariate probit or logic analysis of quantal response data. As many as 3000 test subjects may be included in a single analysis. Including the constant term, up to nine explanatory variables may be used. Examples illustrating input, output, and uses of the program's special features for hypothesis testing are included.

Retrieval terms: multiple probit analysis, multiple logit analysis, multivariate analysis