

Finite Element Analyses of Two Dimensional, Anisotropic Heat Transfer in Wood

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Abstract

The anisotropy of wood creates a complex problem for solving heat and mass transfer problems that require analyses be based on fundamental material properties of the wood structure. Inputting basic orthogonal properties of the wood material alone are not sufficient for accurate modeling because wood is a combination of porous fiber cells that are aligned and mis-aligned in low- and high-density regions called annual rings. Modeling heat transfer requires the development of effective thermal conductivities as a function of those parameters associated with the rings. Effective 2D thermal conductivities were determined by modeling the wood structure at the cellular scale as a function of cell alignment and cell openness or density. Effective heat transfer coefficients were then applied to a macro scale wood board model. This macro-scale model was developed to study the transient heat transfer effects for any specific location, board dimension, and annual ring dimensions located on the cross section of a log. Using ANSYS Parametric Design Language program we were able to easily generate the geometrical specifications and enter the appropriate heat transfer coefficients determined from the micro wood cell model. Traditional initial and convective boundary conditions were applied as if a dry wood board were being heated in a convection oven. These were used to examine the transient heat transfer and temperature rise in the core of the wood board “cut” from various positions in a log. Significant differences in transient core temperature and heat conductive paths were observed for the various board configurations and annual ring geometry.

Introduction

Wood is an anisotropic, axisymmetric, porous material with complicated cellular and macro scale structure features and material properties. The structure induced anisotropic effects for heat and mass transfer have significant implications for drying lumber, heating logs in veneer mills, or hot pressing wood composites. Anisotropy of wood is due to wood fiber’s radial, tangential, and longitudinal orientation (Figure 1) and the structural differences between the development of earlywood and latewood bands for each annual ring in softwood. Earlywood cells are formed in the fast growing spring season and are low-density wood cells with large cavities and thin walls (Figure 2, left). Latewood cells are formed later in the year and are high-density cells, characterized by smaller cavities and thick walls (Figure 2, right). Softwood cells tend to align in straight radial rows because they originate from the same cambial mother cells, but cells are not aligned in tangential rows (Figure 2). For the tangential direction the alignment can vary from 0% up to a 50% offset. Longitudinal differences also occur but are not within the scope of this paper. While growing conditions and tree species impart numerous variations within earlywood and latewood cells, general assumptions can be made and modeled. For heat transfer modeling, earlywood and latewood cells are made of essentially the same material having the same thermal properties. Cell porosity or openness may vary from 90% to 70% in earlywood and from 30% to 10% in latewood (Gu, 2001).

Heat and mass transfer in wood on a macro wood board scale has been studied extensively which has resulted in numerous empirical and theoretical models. Development of these models has been reviewed by Kamke & Vanek 1992 and 1994. Most heat transfer models for wood are 2D and ignore the longitudinal heat transfer effects because of its relatively long heat transfer path along the length of a board compared to radial and tangential paths from the sides of the board. For some boards, the rings can curve

such that the radial directions could act both horizontally and vertically in the same board. Most models do not differentiate thermal properties with respect to the ideal axial symmetry, radial, or tangential directions, but assume averaged thermal heat transfer characteristics (Forest Products Laboratory, 1999 and MacLean, 1941) based on a species-density relationship. However, thermal properties can change depending on where the board was cut and its size. Earlywood and latewood differentiation of the thermal characteristics have only been studied in limited efforts in the modeling field. Significant thermal differences can occur based on ring orientation, ring density, radial, and tangential cell alignment. There is a need to develop heat transfer models which considers these parameters in order to more accurately determine the heat transfer effects in wood, especially for transient heating and cooling conditions.

This paper presents two theoretical heat transfer finite element models. The first is used to determine the effective heat transfer coefficients based on the properties of cell wall substance, cell wall thickness, and cell alignment. This would cover the effective coefficients for both the earlywood and latewood portions of each annual ring and the orientations in both the radial and tangential directions. The second model uses the results from this cellular analysis to predict the macro-heat transfer effects in wood boards where growth rate, ring orientation, board size, and location within a log are input parameters. It is well documented in the literature that moisture content has a significant effect on heat transfer rates in wood (Forest Products Laboratory, 1999). However, the scope of this investigation was intended only to develop simplified finite element models that can be used to study transient heat transfer effects from wood at constant moisture content conditions. These first two models assume moisture content to be 0% to study transient heat transfer differences based on the structure of wood. Further studies will include the effect of moisture content. The goal of this paper is to present finite element analyses that are being developed as analytical tools to help study and understand the fundamental heat transfer effects in a board of wood based on its cellular- and macro-structure.

Procedure

Micro: cellular model

Based on the microstructure of wood observed under the microscope (Figure 2), two small ANSYS parametric design language (APDL) programmed models were developed to simulate the structural variation of cell porosity and alignment. The porosity, percentage of openings in a wood cells, can range from 10% to 90%. In wood, cell porosities range from 70% to 90% for earlywood and 10% to 30% for latewood (Gu, 2001). Softwood cells tend to align in straight radial rows (vertical in Figure 2). Cells are not perfectly aligned in the tangential direction (horizontal in Figure 2) and this alignment between cells varies from 0% to 50% offset. Fully aligned cells in tangential direction are modeled as in Figure 3. Fully mis-aligned or 50% offset between the two rows of cells is modeled as in Figure 4. The purpose of these two models was to determine the effective heat transfer coefficients based on porosity and cell alignment. Effective thermal conductivities of wood in radial and tangential direction were estimated by simulating a simple conduction problem across the wood cell models. The models were programmed to simulate cellular porosities of 10% to 90% porosity at increments of 10% for both cellular alignment of either 0% or 50% offset. A temperature difference of 80K was applied to two opposing boundaries either in the radial or tangential direction with the other two boundaries set as adiabatic boundaries. Thermal conductivity of cell wall substance is 0.41(W/m·K), density is 1.54 (Kg/m³), and specific heat is 2.805×10³ (KJ/Kg·K) (Kollmann and Malmquist, 1956). A random line was selected within the model and was defined to sum the total heat flux across the line in order to calculate the effective thermal conductivity, k_{eff} , by the heat transfer equation (Incropera & DeWitt, 1981) in *Equa. 1*. By rearranging the heat transfer equation and solving for k_{eff} we are able to determine effective radial and tangential heat transfer coefficients at each increment of porosity for 0% and 50% cell offset.

$$q_x = -k_{eff} \frac{dT}{dx} \Rightarrow k_{eff} = -q_x / \frac{dT}{dx} \dots\dots\dots Equa. 1$$

Where, q_x = heat flux (W/m²), k_{eff} = effective thermal conductivity (W/mK), dT = temperature change (K), dx = distance for the temperature change (m).

Macro: wood board model

When wood is cut from a log it can have significantly different number and orientation of rings depending on the location from where it was cut (Figure 5). Within each growth ring, the heat transfer properties are different from earlywood to latewood due to different density (or porosity), and also between radial and tangential directions due to the wood cell alignment. An ANSYS APDL programmed model was developed to simulate a log of any size from which any size board could be “cut” from any location and be analyzed. The log model was created using a series of concentric circles that were partitioned into rings. Earlywood and latewood rings were alternatively assigned using representative dimensions. The wide rings in Figure 5 represent earlywood. The narrow rings represent latewood. Earlywood occupies 70 to 80% of each ring and latewood only takes about 20 to 30% according to the general structure of softwood (Gu, 2001). Effective radial and tangential heat transfer coefficients, k_{eff} , from the micro-wood cellular models were assigned axisymmetrically to the earlywood and latewood sections. A rectangular board 44.45 by 88.9 mm (1.75 by 3.5 inches) was generated at a specified location. The program retained the inside area of the rectangle for the wood board model and discarded the areas outside the board. The element coordinate system was concurrent with the log cylindrical system. Boundary conditions were set up as a convective heat flow simulating the cooling surface temperature condition in a practical wood drying operation. The convective heat transfer coefficients are $9.15 \text{ W/m}^2\cdot\text{K}$ for the side boundaries and $12.93 \text{ W/m}^2\cdot\text{K}$ for the top and bottom boundaries calculated based on the equations for external flow over a flat plate (Incropera & DeWitt 1981). Details of the calculation can be found in Gu's dissertation (Gu 2001). A temperature of 100°C was applied to all outside boundaries with the board initial temperature of the board set at 20°C . A series of seven transient heat transfer analyses were conducted for boards “cut” from 7 different locations in the log (Figure 5). Temperature rise at the core of each board was determined and plotted over time for each board.

Analysis Results & Discussion

All the models used for this analysis were built using APDL programming language in ANSYS. Input variables were used in the program for easy model modification and analysis. Element type PLANE35 was used for the heat transfer analysis. Element PLANE35 is a 2-D 6-node triangular thermal solid element.

Effective thermal conductivity of wood: micro cellular model

From Figure 2 it is easy to see that the cell alignment is generally aligned in the radial direction and for the tangential direction it varies from fully aligned (0% offset) to fully misaligned (50% offset). The radial and tangential effective coefficients were determined as a function of increasing porosity at increments of 10% for both the aligned and the misaligned cases (Figures 6 and 7).

Effective heat transfer coefficients are plotted as a function of porosity in Figures 6 and 7. The thermal conductivity values ranged from $\sim 0.02 \text{ (W/m}\cdot\text{K)}$ for 90% porosity to $\sim 0.33 \text{ (W/m}\cdot\text{K)}$ for 10% porosity of wood. Both figures show there would be significant thermal conductivity differences between earlywood (70% to 90% porosity) and latewood (10 to 30% porosity). From the 0% offset model, earlywood thermal conductivity is approximately $0.046 \text{ W/m}\cdot\text{K}$ (for 70 to 90% porosity), while latewood thermal conductivity is approximately $0.27 \text{ W/m}\cdot\text{K}$ (for 10 to 30% porosity), an increase by a factor of 5. This difference will have a considerable effect on the heat transfer into the wood board depending on the orientation rings and relative thicknesses of the earlywood and latewood.

The radial and tangential thermal conductivity did not show a difference in the case of 0% offset in tangential cell alignment, as was expected, since the heat travels the same path length in both directions (Compare Path-R and Path-T in Figure 3). Small differences in this case are due to slight variations in numerical integration of the total flux across the cell elements. Element size was reduced until only small variations were observed in the total heat flux. The 50% offset in the tangential cell alignment has a small effect on the thermal conductivities, Figure 7. The effective tangential thermal conductivity is lower than the aligned radial thermal conductivity because heat flux must travel a longer crooked path (compare Path-R and Path-T, Figure 4). As porosity increases the heat flux path length increases up to a maximum of $\frac{1}{2}$ cell length as porosity approaches the limit of 100% porosity. The 0% and 50% offset cases are two

extreme examples for the cell structure in wood. Usually wood cells are aligned between these two cases. Therefore, for the macro-wood board model we used the effective thermal conductivity coefficients for the radial direction from the aligned cell cases and the effective thermal conductivity coefficients from the average of the 0% and the 50% offset conditions for the tangential direction. In all cases porosity is the driving factor in the thermal conductivity functions.

From the literature (Forest Products Laboratory, 1999), the equation (Equa. 2) for radial and tangential wood heat transfer coefficient was empirically derived and varies as a function of the average wood density (or by tree species) and moisture content.

$$k = G(B+CM) + A \dots\dots\dots \text{Equa. 2}$$

Where k = thermal conductivity (W/m^2); G = specific gravity; $A = 0.01864$; $B = 0.1941$; $C = .004064$; and M = Moisture content (%). Calculated heat transfer coefficients can vary from 0.087 W/m-K for cottonwood (*Populus deltoides*) to 0.17 W/m-K for hickory (*Carya glabra*), for densities of 350 and 780 kg/m^3 respectively all at 0% moisture content. These calculated values are from empirical data, which averages the effects of low (high porosity) and high-density (low porosity) earlywood and latewood regions and ring orientation. Using our model at 50% porosity, which is a uniform density of 770 kg/m^3 , the model calculates an effective heat transfer coefficient of 0.135 W/m-K . The empirical equation calculates a $k = 0.168 \text{ W/m-K}$, a 20% difference. Our model's estimate is very close to the empirical values. Our lower value may be due to the model's uniform density analysis whereas the empirical equation is based on an averaged combined earlywood and latewood heat transfer measurement. It's possible the latewood (denser) cells from the empirical tests may have influenced the data due to the latewood's increased heat transfer rate resulting in a higher empirical coefficient value. We believe the use of these cellular finite element models will help in understanding the heat transfer fundamentals at the cellular level and can be applied to larger macro wood models. It is interesting to note that the equation for thermal conductivity in the literature is a linear relationship, whereas our curves (Figures 6 and 7) show it is a non-linear relationship. A further evaluation of this non-linear relationship and moisture effects on the heat transfer coefficients will be developed in future publications.

Two dimensional transient heat transfer: macro board model

The transient heat transfer in wood boards, 44.45 by 88.9 mm (1.75 by 3.5 inches), "cut" from seven different locations in the log (Figure 5) was modeled, solved, and plotted to examine the effects of wood structure on the transient heat transfer process.

Figure 8 shows the path heat flows in a wood board when it is subjected to a conventional heating process. Relative heat flux lines and directions show significant heat flux following the latewood rings. This is because latewood has a significantly higher effective thermal conductivity than the earlywood due to its high density (low porosity). Therefore heat transfers through the latewood part first, and then across earlywood from one latewood ring to another latewood ring (see the magnified plot in Figure 8).

Figure 9 shows temperature profiles at several time steps for the boards from location 1 and location 5 in the log. Board 1 was from the pith (center) of a log and board 5 was from the log peripheral area with rings characterized as perpendicular to the top and bottom faces. The heat flows faster in board 5 because the rings are oriented parallel to the shorter pathway -- thickness of the board. Higher transfer rates occur because the latewood cells are able to transfer more energy from the boundaries to the core, whereas the heat transfer rate to the core in board 1 is lower because the energy must pass through the lower thermally conductive earlywood bands.

The ring orientation on a wood board has a considerable effect on the rate of heat transfer. Comparisons of the seven simulated boards simulated as cut from a log (Figure 5) having significantly different ring patterns show this effect. Core temperature vs. time for each board is plotted in Figure 10. Board 1 has the slowest heat transfer rate and board 4 has the fastest heat transfer rate. Board 2 is often called a true quarter-sawn board and board 3 is a true flat-sawn board. A flat-sawn board has a slower heat transfer rate than the quarter-sawn board because the heat transfer path follows the rings and the rings are aligned parallel to the longer (width) component of the board dimensions in the flat-sawn board. Board 4 has a faster heat transfer than board 5 due to the higher density of latewood bands in the pith area of the log.

Comparing board 3 with board 5 showed that board 3 might take more than an hour to reach the same temperature at the core as board 5. This is a result of the higher heat transfer paths (latewood) aligning in the different directions. Boards 6 and 7 have heat transfer rates between boards 5 and 2.

This wood board model provides the ability to study the effects of growth rate and earlywood/latewood ratio on the effects on heat transfer in wood. The core temperature rise of slow-growth-rate wood board is higher than that of fast-growth-rate wood board as shown in Figure 10: the temperature curve on the top is for the board from location 5 but with higher frequency of rings (slow annual growth) as it is in the fast-growth-rate log. Figure 11 shows a visual time lapse comparison of temperature vs. time plots for two boards “cut” from a log with different growth rates and earlywood/latewood ratio. For this simulated condition, wood with slow growth rate (high ring density) transferred heat faster than wood with a fast growth rate (low ring density).

Summary & Conclusion

Two flexible finite element models have been developed that are based on the fundamental structural characteristics of wood. The models show the significant effects of cell alignment, cell density, ring width, and ring orientation. While this study only looked at a wood boards having 0% moisture content, further analyses at increasing moisture contents are easily adapted and future studies are planned. Such a fundamental approach to studying heat transfer issues in wood has numerous practical applications. These could include: optimizing drying schedules for different cut boards; determining heat treatment times to kill insects; and determining heat curing times for solid wood and composites.

Specific conclusions from this study are:

1. The porosity of wood plays a major role in determining the effective thermal properties. The higher the porosity, the lower the effective thermal conductivity. Effective thermal conductivities can be developed as equations or a look-up table for further use in macro wood models.
2. Radial and tangential thermal properties are different due to the structure of wood, but the effect of the radial vs. tangential direction is small. Only at high porosities do the differences start to become considerable. For most wood species the porosity would not be that high.
3. Heat transfer in a piece of wood is significantly affected by the ring density and orientation. Heat transfers faster in latewood than earlywood due to the high density of latewood cells. The flow path for heat transfer in wood is first through the latewood rings and then from the hot latewood to the cool earlywood.
4. Quarter-sawn boards have higher heat transfer rate than flat-sawn boards due to the shorter heat pathway through the latewood cells. The length of the heat flow path determines the heat transfer rate in a piece of wood.
5. Growth rate of a tree significantly affects heat transfer rates. Boards with slow growth rate (high density rings) transfer heat faster than boards with fast growth rate (low density rings).
6. These Finite Element models provide valuable analysis tools to not only examine the fundamental thermal conductivity differences for radial and tangential heat transfer at the micro level, but also to estimate the transient heat transfer effects at the macro level or in a board of wood.

Acknowledgements

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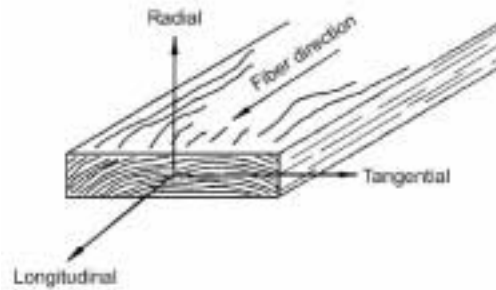


Figure 1. Three principal axes of wood with respect to fiber direction and growth rings.

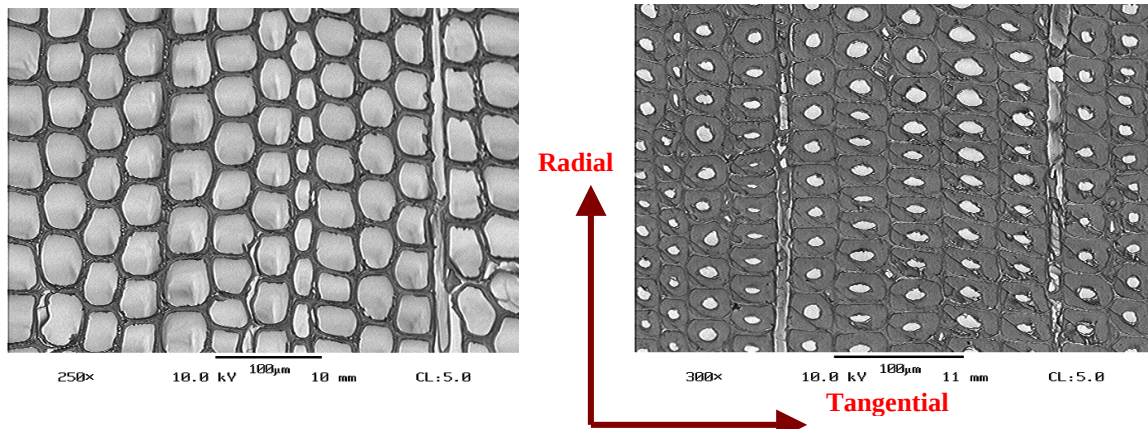


Figure 2. Microscope images of wood structure in earlywood (left) and latewood (right) zones. (the species of wood is southern yellow pine).

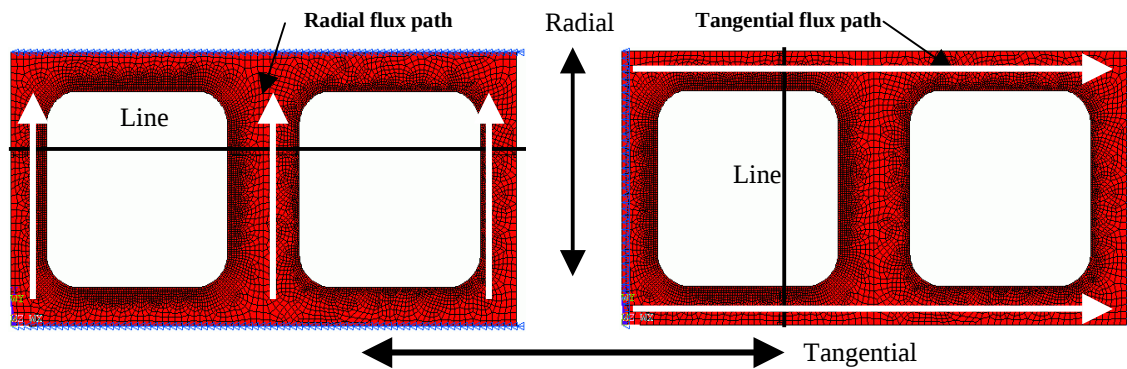


Figure 3. Model of wood cells with 50% porosity and 0% offset between cell alignments in tangential direction.

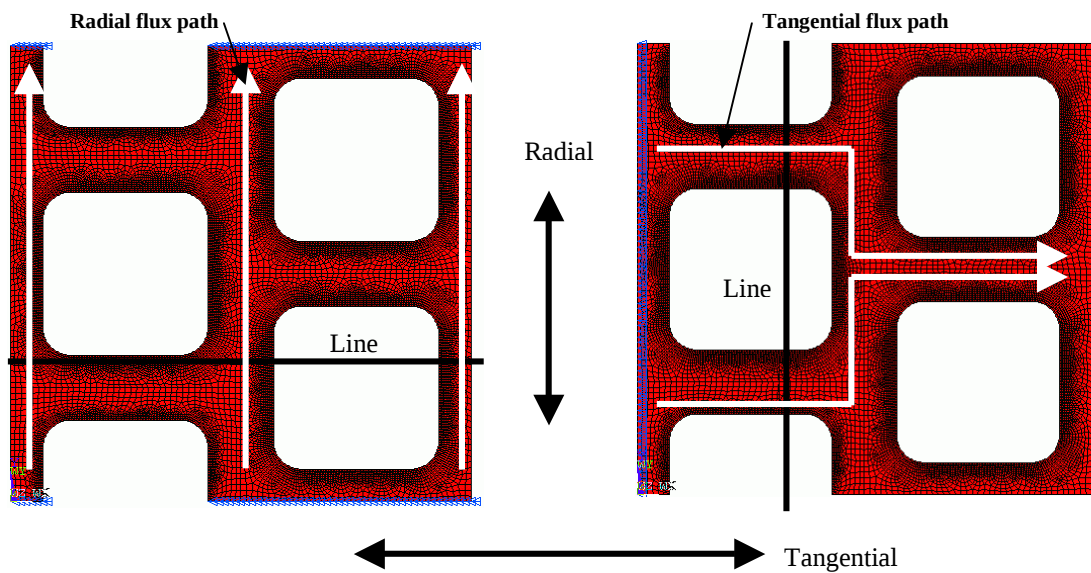


Figure 4. Model of wood cells with 50% porosity and 50% offset between cell alignments in tangential direction.

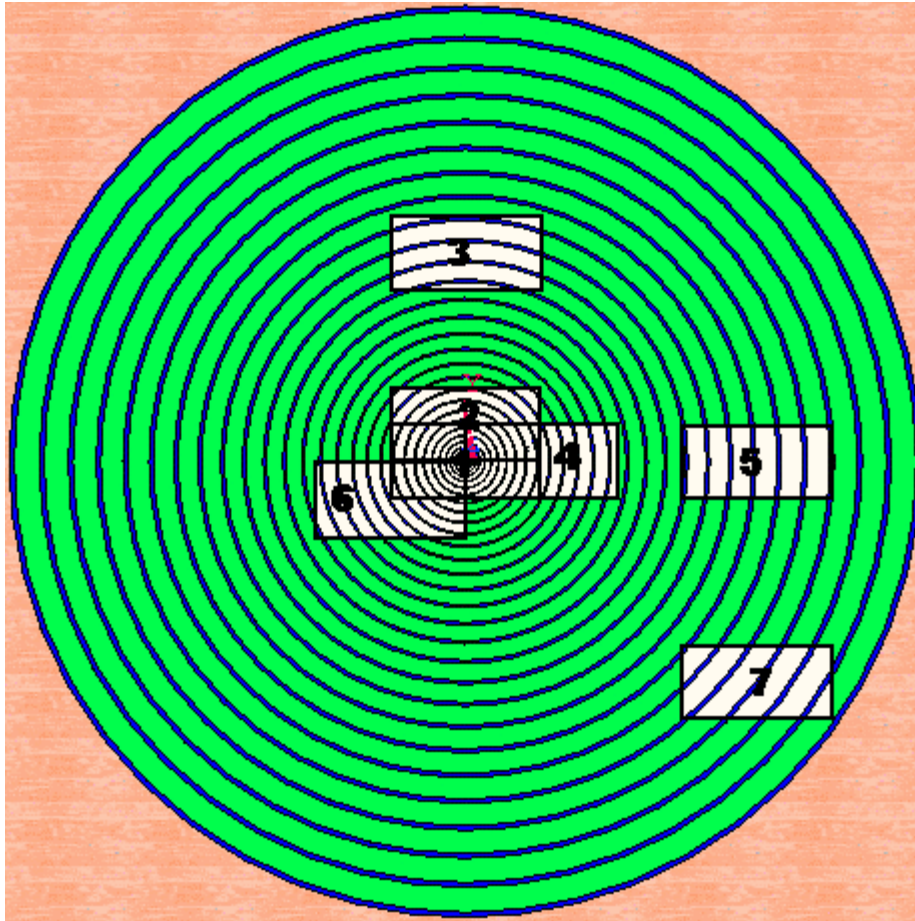


Figure 5. Model of wood boards with different number and orientation of rings cut from a log.

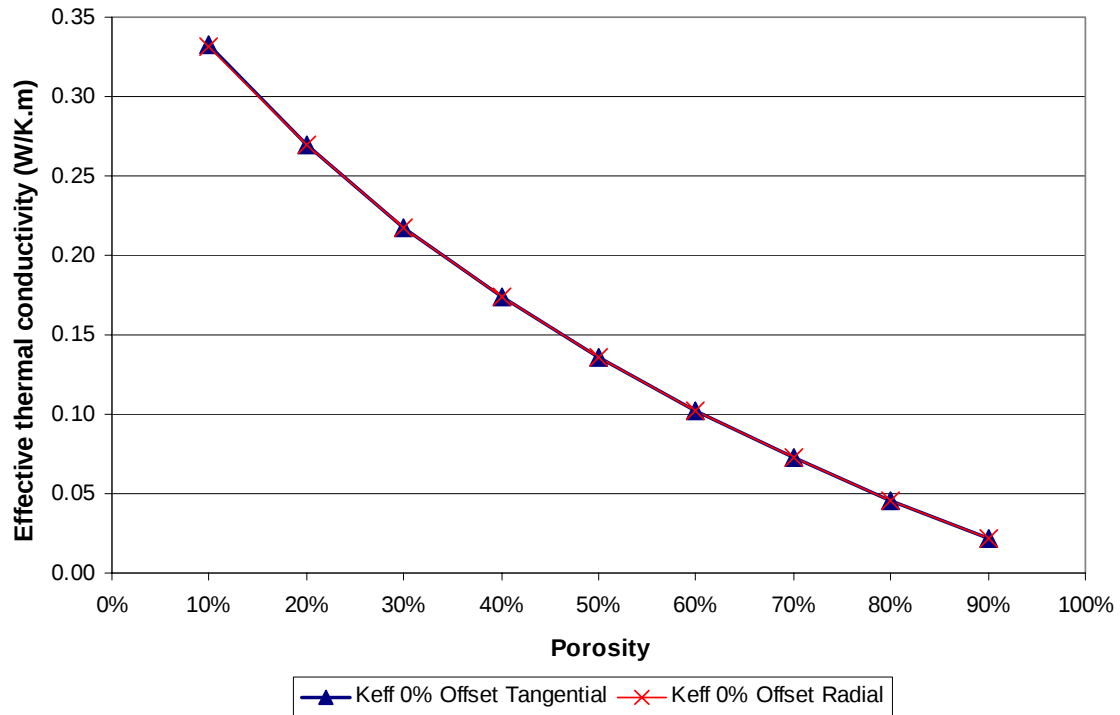


Figure 6. Effective thermal conductivities (radial and tangential) change with the percent porosity in cell structure for 0% offset case.

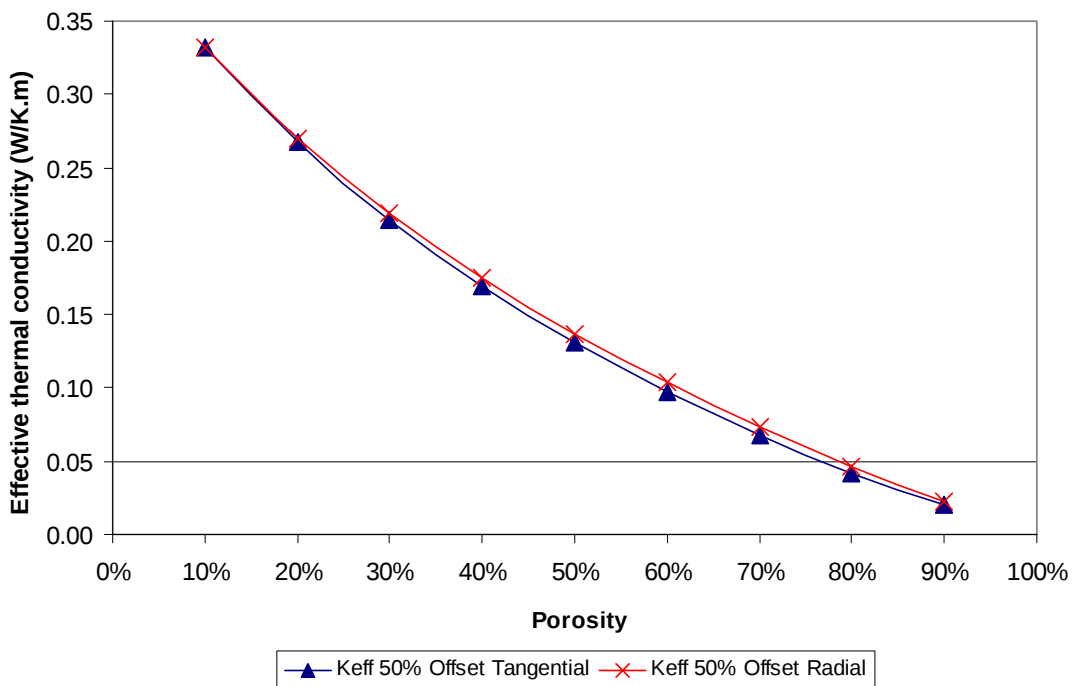


Figure 7. Effective thermal conductivities (radial and tangential) change with the percent porosity in cell structure for 50% offset case.

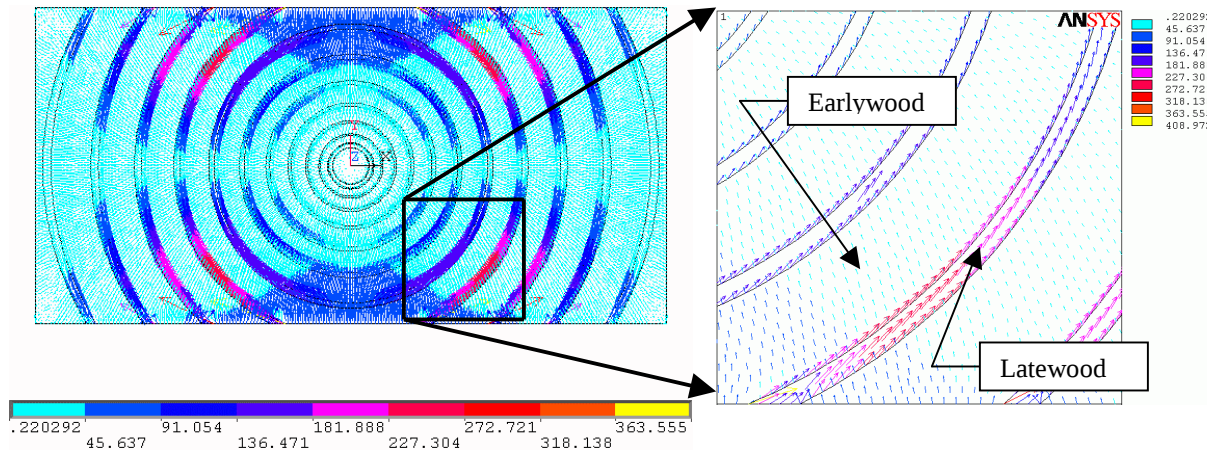
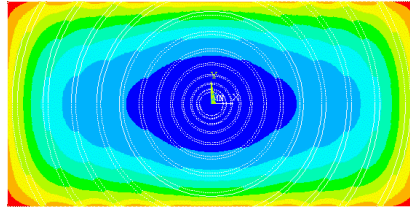


Figure 8. Vector plot of heat flux for Board 1. (left: entire plot for the board; right: magnified plot showing high and low heat flux regions in earlywood and latewood).

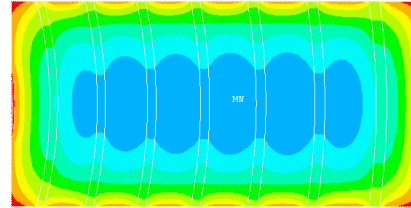
Wood board from Location 1

Wood board from Location 5

at time 1892 seconds (~30 minutes)

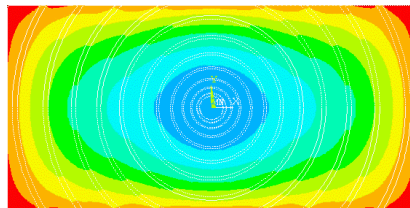


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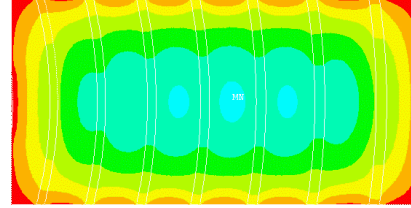


(min=29.774 max=96.697)

at time 3748 seconds (~60 minutes)

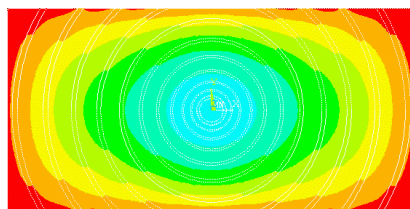


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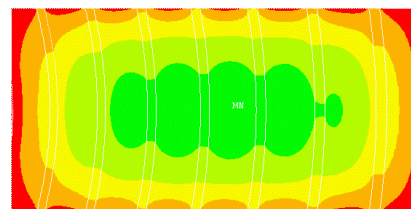


(min=45.222 max=98.314)

at time 5604 seconds (~90 minutes)

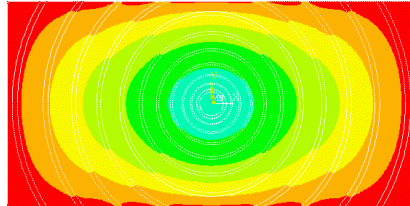


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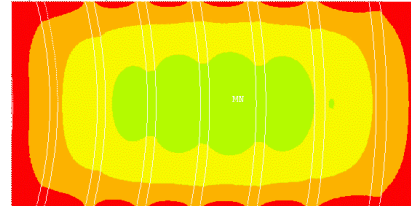


(min=57.842 max=98.913)

at time 7459 seconds (~2 hours)

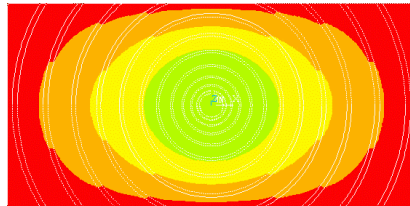


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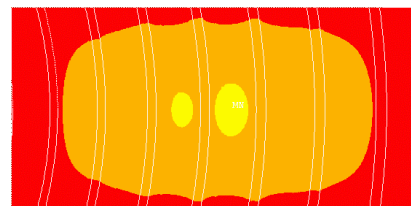


(min=67.573 max=99.256)

at time 10800 seconds (~3 hours)



(min=65.533 max=99.698)



(min=81.112 max=99.640)

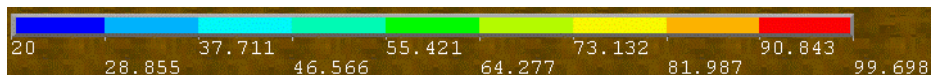


Figure 9. Contour plots of transient heat temperature ($^{\circ}\text{C}$) profiles for wood boards at location no. 1 and 5 with different ring structures.

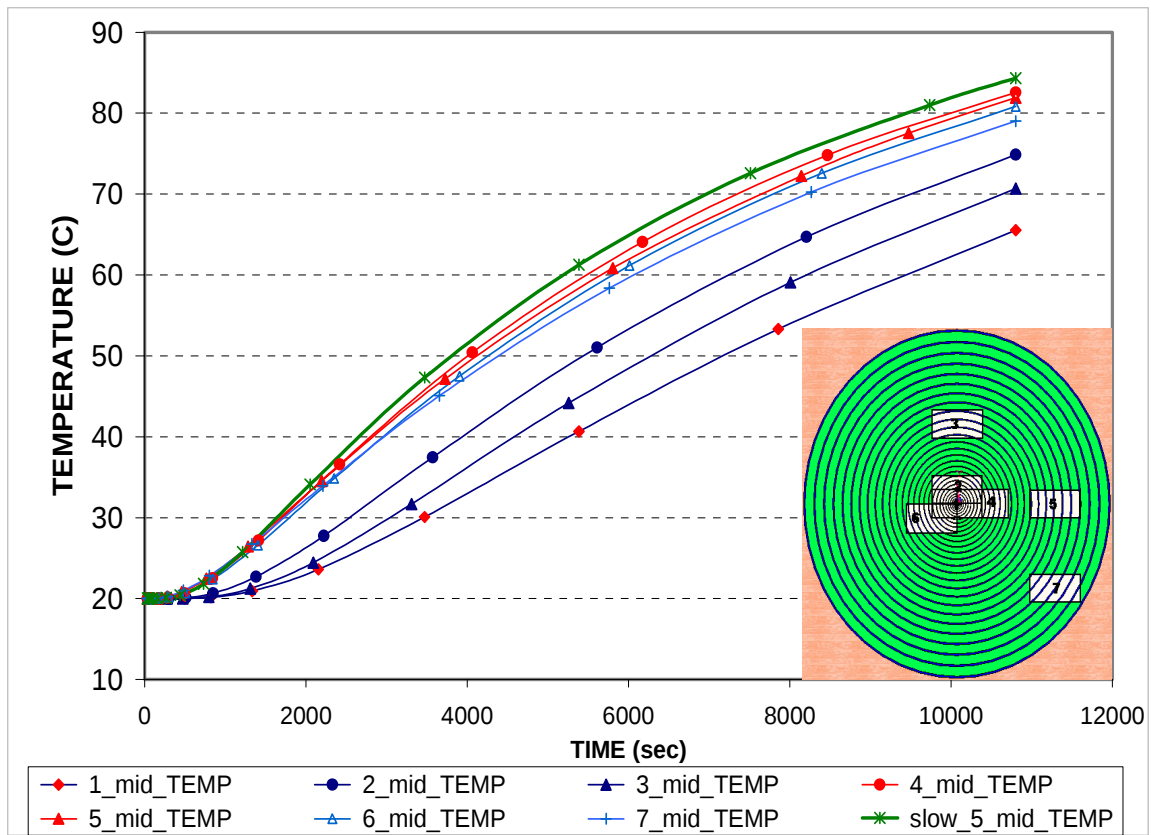
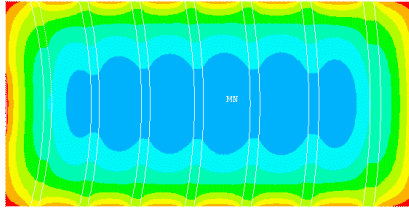


Figure 10. Transient temperature of the center point of each board cut from different locations on a log (small picture in the chart showing the locations of the numbered boards cut from a log).

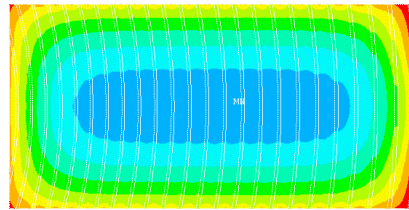
Wood board from fast growth tree

Wood board from slow growth tree

at time 1892 seconds (~30 minutes)

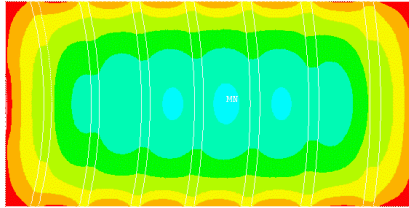


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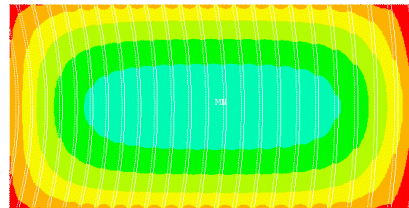


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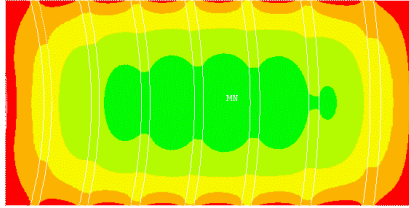


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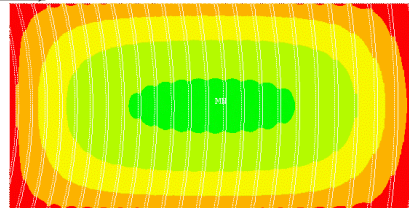


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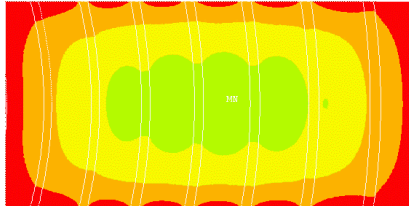


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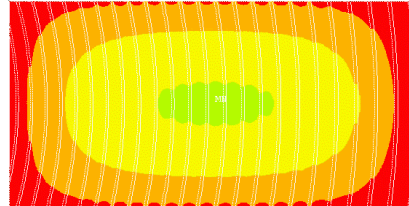


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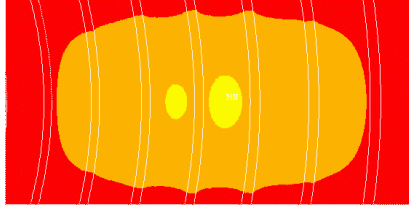


(min=67.573 max=99.256)

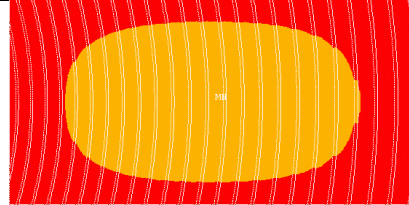


(min=72.270 max=99.638)

at time 10800 seconds (~3 hours)



(min=81.112 max=99.640)



(min=84.321 max=99.826)

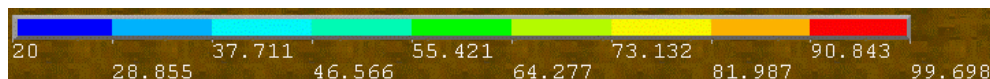


Figure 11. Contour plots of transient heat temperature ($^{\circ}\text{C}$) profiles for two wood boards with different growth rate but cut from the same location, no. 5.