THE SINE METHOD AS A MORE ACCURATE HEIGHT PREDICTOR FOR HARDWOODS

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Abstract—Most hypsometers apply a mathematical technique that utilizes the tangent of angles and a horizontal distance to deliver the exact height of a tree under idealized circumstances. Unfortunately, these conditions are rarely met for hardwoods in the field. A "new" predictor based on sine and slope distance and discussed here does not require the same assumptions for accurate height determination. Case studies using a sycamore (*Platanus occidentalis* L.), a water oak (*Quercus nigra* L.), and a southern red oak (*Q. falcata* Michx.) from southern Arkansas are presented to emphasize the sensitivity of the tangent method to erroneous measurement procedures. When heights were measured properly and under favorable circumstances, the results obtained by the tangent and sine methods differed only by about 2 percent. Under more challenging conditions, however, errors ranged from 8 to 42 percent. These examples also highlight a number of distinct advantages of using the sine method, especially when exact tree height is required.

INTRODUCTION

Tree height is one of the most conventional attributes of forest mensuration. Equipment and methods specifically designed to enumerate the vertical dimension of individual trees have been available since the earliest years of forestry (for example, Noyes 1916; Schlich 1911). Few people have questioned the application of these techniques because they are based on fundamental geometric or trigonometric principles. Given textbook definitions of tree height measurement under idealized circumstances, there seemed little need for criticism.

Unfortunately, the accuracy of height measurement has for too long been taken for granted. If carelessly applied, the conventional method of height determination is prone to significant errors. Even subtle violations of the assumptions of this technique (for example, an almost imperceptible lean in a tall tree) can produce noticeable departures from the exact height. In all fairness, it has only been in recent years that technology has caught up to the science behind tree height measurement, making it possible to control or eliminate this error (Blozan, W. 2004. Tree measuring guidelines of the Eastern Native Tree Society. Unpublished report. http://www.uark.edu/misc/ents/measure/tree_measuring_guidelines.htm. [Date accessed: August 20, 2005].

This paper briefly reviews the basic assumptions behind the traditional height measurement technique, including some that can lead to significant errors in height estimation. I will also describe a "new" estimator of height that uses a set of trigonometric relationships that is not sensitive to the same assumptions. Differences between the tangent and sine methods are illustrated in case studies of height measurements of hardwoods in southern Arkansas.

METHODS

Basic Height Measuring Principles

Mathematically speaking, hypsometers typically apply a technique that utilizes the tangents of angles and a horizontal distance to determine tree height. Figure 1 illustrates the basic principles of height determination. With accurate distance and angle measurements, tangent-based hypsometers determine total tree height (TanHT) as follows:

$$\text{TanHT} = [\tan(A) \times b] + [\tan(A') \times b] \qquad (1)$$

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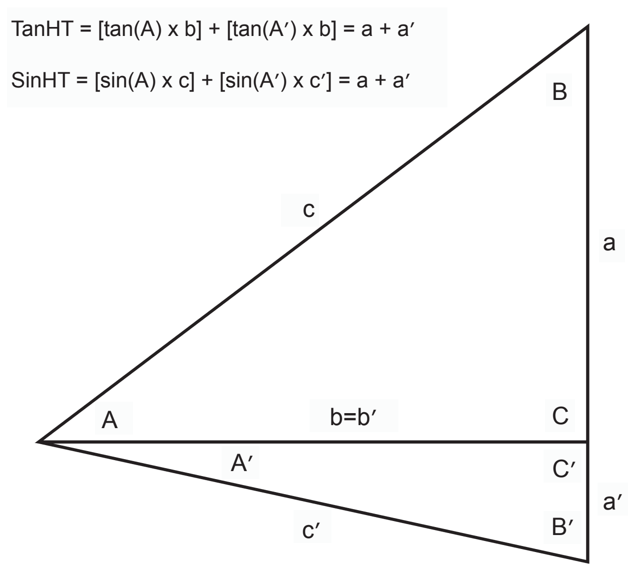

Figure 1—The trigonometric basis for height determination, using both the tangent method (TanHT equation) and the sine method (SinHT equation). Triangles ACB and A′C′B′ are both right triangles. On perfectly flat ground with a truly vertical tree, a + a′ = true tree height, b = horizontal distance, and c and c′ are slope distances.

where the angles A and A′ are measured in degrees, and the distance b is the true horizontal distance from the observer to the stem. Note that under ideal conditions (a tree with no significant diameter on a flat surface with a perfectly vertical stem and its highest living point centered over the bole (fig. 2), the tangent method produces an exactly correct standing height (Husch and others 2003).

However, these conditions are rarely met, especially with hardwoods. A very large proportion of trees lean, have bends or angles in their upper boles, or are found on sloping ground, and these departures from the ideal make it necessary to take corrective actions to predict true height using the tangent method (Falconer 1931, Krauch 1918). Crown asymmetry and shape can also cause problems (Husch and others 2003). For instance, many if not most large hardwoods (especially those growing in relatively open conditions) develop a widely spreading crown with no obvious apex. Under these circumstances, height measurements will almost invariably be taken from a point associated with an edge of the crown (rather than over the bole), biasing tree height estimates using the tangent method upward (fig. 3).

Few people actually adjust for ground slope, tree lean, or skewed crown apex in the field, and failure to make such corrections results in at least some degree of error. When corrections are applied, they are often more ad hoc than mathematically based. For instance, techniques to adjust for the unseen apex of a tree crown include measuring the highest visible limb or projecting through the crown towards an assumed crown peak, even though neither method ensures an actual representation of a real tree top. Some workers average multiple tangent measurements to estimate total height, but have no means to determine if their original numbers are reliable. Others willingly accept solitary tangent heights, knowing that they only need an approximate value.

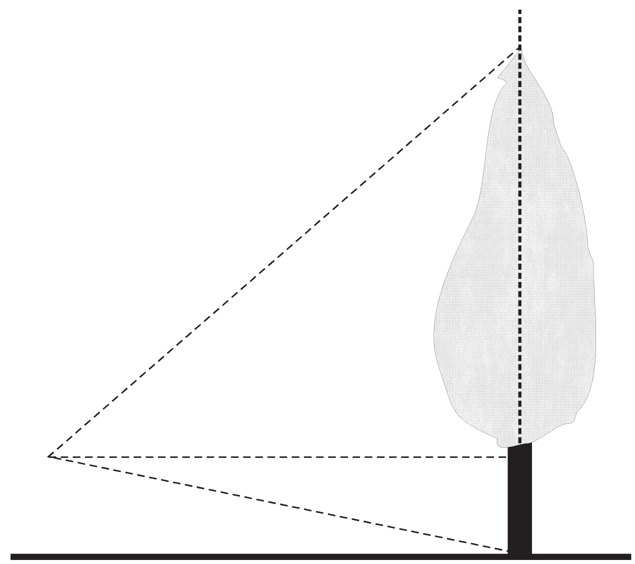

Figure 2—Idealized tree height measurement, where a perfectly vertical tree has its highest live crown directly over its stem. Under these specific circumstances, the tangent method will give exact tree height (without correction).

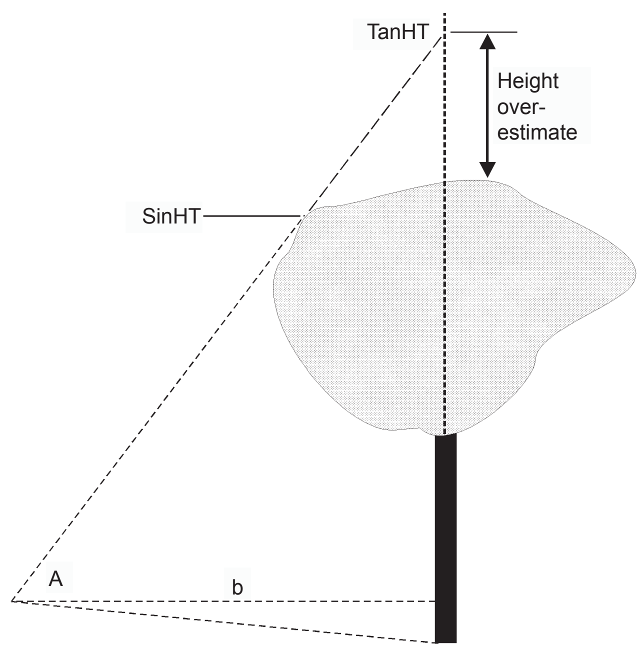

Figure 3—Overestimation bias from the tangent method of height measurement applied to a diffuse crown typical of most hardwoods. The tangent height (TanHT) relies on the angle A and horizontal distance b and projects a non-existent crown apex, which, without correction, overestimates tree height. A direct measurement of the crown intersection, the sine height method slightly underestimates true tree height.

Fortunately, there is a different technique based on slope distances and the sines of the angles capable of good height estimates under real-world field conditions. This estimator resembles the tangent-based approach, with some notable exceptions:

$$SinHT = [\sin(A) \times c] + [\sin(A') \times c'] \tag{2}$$

In this equation, c and c' are slope (not horizontal) distances, and the sine of angles A and A' is used (fig. 1). This technique is possible because accurate and inexpensive laser distance measuring equipment is now available and can be used to directly measure the slope distance to the highest and lowest points of the tree.

Under perfect conditions, TanHT = SinHT = exact standing tree height. However, the sine method is more reliable under less-than-ideal circumstances than the tangent method because it is based on an actual measurement to a real point on a crown, and does not involve projecting a hypothetical crown top based on an angle and a distance measurement (fig. 3). So long as the angles and slope distances are accurate, the trigonometry behind the sine method also ensures that only the true vertical height component is estimated, making the technique insensitive to the slope of the land, the lean of the tree, or the width of the crown (fig. 4).

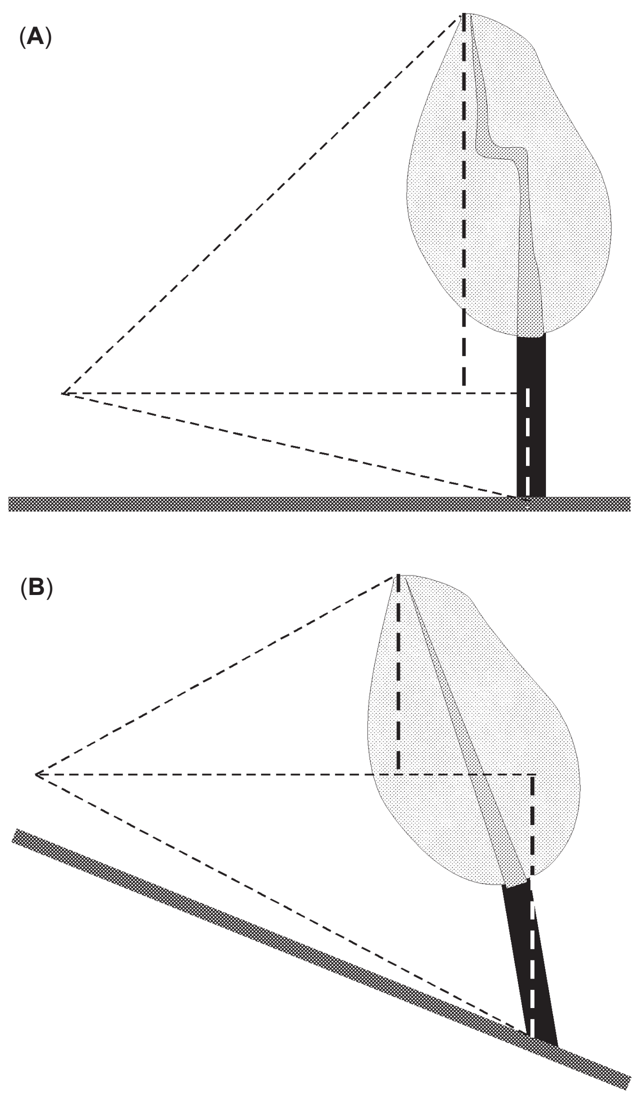

Figure 4—Diagram of how the sine method calculates true tree height for (A) an offset hardwood crown and (B) for a leaning hardwood with an offset crown on sloping ground. Uncorrected tangent-based measurements would overestimate height under both conditions.

Study Implementation

To illustrate the implications of different hardwood crown attributes for height estimates, sample trees were measured using a Laser Technology Impulse 200LR™ laser rangefinder. The 200LR was chosen for its high degree of distance (± 0.2 feet at 1885 feet) and angular (± 0.1 degree) accuracy (Carr 1996). The default height function incorporated in the 200LR calculates the exact horizontal distance to the stem, measures the angles to the top and bottom of the tree, and then uses the tangent method to derive a height estimate (to the nearest 0.1 foot). The 200LR can also be used to determine upper and lower slope distances (and their corresponding angles) using separate functions. Thus, the same laser rangefinder can also be used to provide the sine method height, thereby eliminating potential errors caused by using different technologies.

Three hardwood trees on the grounds of the University of Arkansas-Monticello and the Crossett Experimental Forest (table 1) were selected for measurement. Each was selected to highlight a particular attribute that may influence hardwood height measurement accuracy. A sycamore (*Platanus occidentalis* L.) was chosen because of its lack of obvious lean, while an open-grown water oak (*Quercus nigra* L.) was selected because of its broadly spreading crown, and the southern red oak (*Q. falcata* Michx.) was picked because of its pronounced lean.

It should be noted that this limited sample was used specifically to show the risks of inappropriately applying the tangent method. Even though this may seem to be a "stacked deck" approach to evaluating the techniques, my intention was to highlight the relative insensitivity of the sine method to even gross misapplications of height measurement techniques. After all, even the best trained field crews are not likely to spend much time carefully determining the extent of lean or the skew of crowns unless the trees are obviously affected. Rather, they are more likely to assume that modest departures produce very minor differences in the predicted heights. Indeed, this is often part of their instruction: Avery and Burkhart (1994) state that clinometers can accurately (within 2 to 5 percent) predict height for trees leaning up to 5 degrees. With small trees, or for those conducting a large-scale inventory, this magnitude of error is rarely problematic. For those requiring greater accuracy, even this is unacceptable.

RESULTS AND DISCUSSION

Favorable Conditions

I first tested the 200LR under conditions that approached the ideal. When the two methods were used to estimate the height of a 32 foot tall vertical light pole from horizontal distances ranging from 66 to 107 feet, they produced height estimates that differed by less than 0.2 feet (< 0.6 percent). Thus, there was no

Table 1—Basic attributes of southern Arkansas hardwoods used to highlight differences in height measurement techniques

Tree	Diameter at breast height	Horizontal distance[a]	Defining attribute[b]
	inches	*feet*	
Sycamore	18.8	124	No visible lean
Water oak	46.4	79–255[c]	Wide crown
Southern red oak	40.1	116–182[c]	20 degree lean

[a] Horizontal distance between the measuring station and the tree.
[b] The reason why the tree was chosen for this comparison.
[c] Multiple stations were used at varying distances from the tree.

meaningful difference between the estimates produced by the techniques, given the stated accuracy of the instrument.

In the case of the sycamore with good apical dominance (fig. 5), the tangent method produced a height estimate (72.3 feet) only slightly (2.0 percent) lower than that produced by the sine method (73.8 feet). This small difference arose because the sycamore leaned almost imperceptibly away from the measuring device, so that its highest live leader was not positioned directly over the point to which the horizontal distance was measured.

Wide Crowns

When the height of the water oak was measured from over 250 feet away, the difference between the height estimate produced by the tangent technique (66.8 feet) and that produced by the sine technique (69.1 feet) was moderate (3.3 percent). At this distance, it was possible to view the top of the entire crown (fig. 6), and selection of the crown apex was greatly facilitated. However, this does not change the tangent method assumption that the highest point is located over the point to which the horizontal distance is measured. Thus, the 3.3-percent difference between the height estimates indicates that there is still an obvious offset when the tangent method is used.

When measured from up close (78.5 feet away, horizontal distance), the tangent method yielded a height estimate of 62.4 feet and the sine method one of 67.8 feet (an 8.0-percent difference) for exactly the same leader mentioned in the previous paragraph. At this close proximity, it is virtually impossible to detect

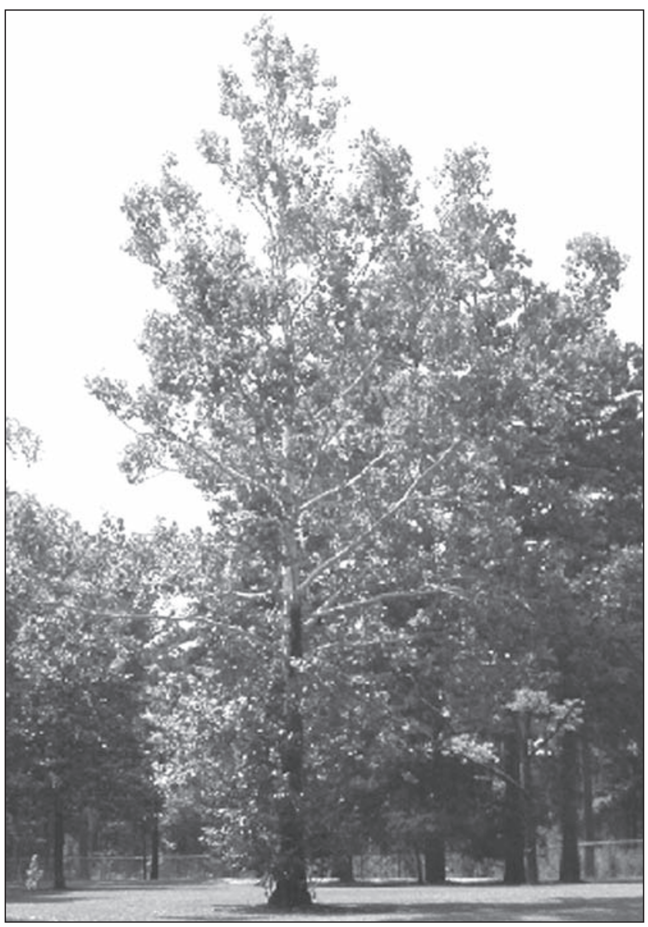

Figure 5—A sycamore with little apparent lean. Picture was taken approximately 125 feet from the stem.

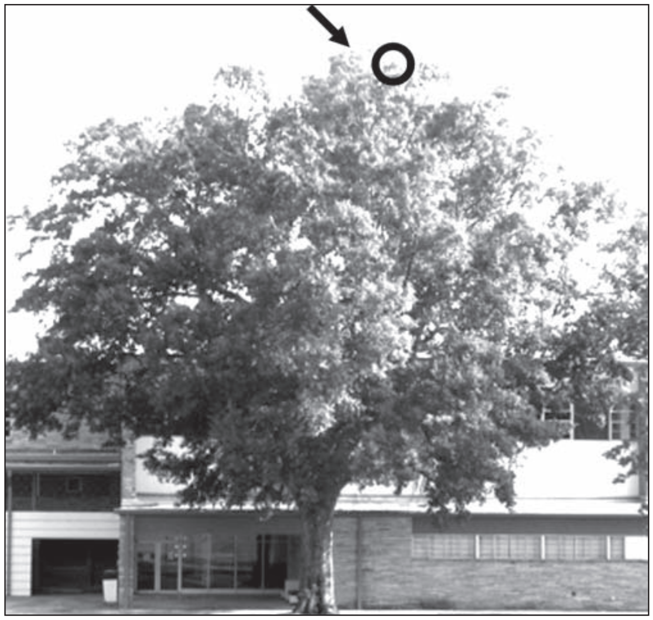

Figure 6—The water oak with a broadly spreading crown. Picture was taken approximately 250 feet from the stem. The arrow identifies the highest visible leader at 250 feet, compared to the circled leader that appeared highest at 106 feet.

the highest point on the crown without first having spotted it at a distance (which is not always possible, especially in dense stands). The nature of how tangent heights are determined (indirect placement of crown top by using horizontal distance and angle, often to an approximated apex) creates a much greater potential for error than the sine method.

To further illustrate this, I selected another viewing point 106 feet from the water oak. At this location, it appeared that a different leader was the highest point on the tree (fig. 6). This new high point produced a tangent height of 73.4 feet. When the sine method was used, however, it produced a height estimate of only 62.4 feet, and it was clear that this leader was in a subordinate crown position. Even though the tangent method predicted a height closer to the true height of the tree, it arrived at this value through compensating errors rather than as a consequence of the validity of the technique. In other words, without strict controls, the observer cannot account for the accuracy of a lone measurement under the tangent method.

Pronounced Lean

A leaning southern red oak provided a classic example of the potential for serious errors in height estimation using the tangent method. This tree was selected for its prominent inclination (20 degrees from vertical) (fig. 7a) specifically to emphasize the effect of lean on height determination. When the tree leaned away from the observer, the tangent method yielded a height estimate of 77.4 feet and the sine method one of 79.3 feet. When this red oak was measured away from its predominant axis of lean, the highest branch was actually offset slightly behind the vertical axis of the tree, and this resulted in a relatively minor (~2 feet) underestimate of tree height.

At a point perpendicular to the lean of the southern red oak, the tangent method yielded a height estimate of 80.2 feet and the sine method one of 80.5 feet. The close correspondence between estimates based on the two methods and a perpendicular perspective indicates that the techniques give very similar results if they are applied properly.

(**A**) (**B**)

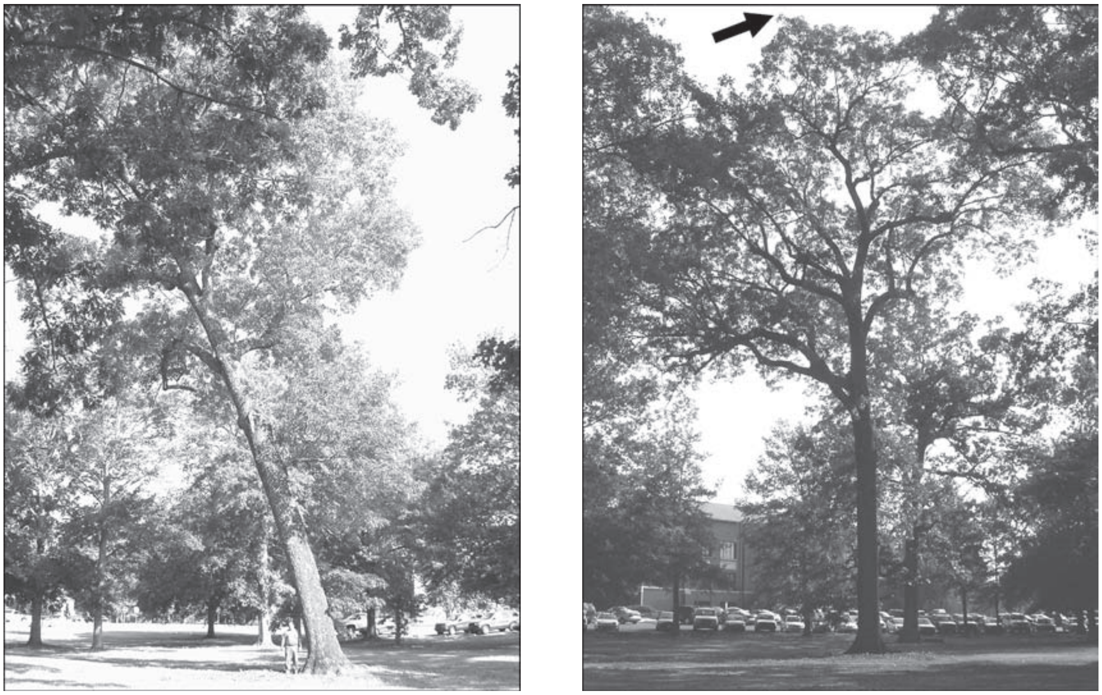

Figure 7—Strongly (20 degrees) leaning southern red oak, with pictures taken from about 100 feet away for the (A) perpendicular to the lean and (B) into the lean perspectives. The arrow in (B) indicates the apparent top of the crown, and indicates the lack of visible lean from this direction at this distance.

However, when the southern red oak was measured with the lean toward the observer, a new branch of the crown (identified by an arrow in figure 7b) appeared to be tallest. This branch was located significantly closer to the measurement station than the vertical bole axis, and therefore the tangent method (if not corrected for horizontal distance) would project a height much greater than the true value. Not surprisingly, the unadjusted tangent method produced a height estimate of 110.8 feet, 33 feet (42 percent) higher than the 77.8- foot estimate obtained by using sines.

Though this last trial violated accepted height measuring procedures, from exactly this vantage point (fig. 7b) the oak did not have an obvious lean, and thus could have misled some observers. Technology is increasingly making the direct measurement of sample trees less necessary. Laser-based dendrometers already on the market are capable of accurate diameter measurement from a distance, so remotely sensed stem measurements will probably become commonplace before long. If this happens, workers may never actually walk up to trees to measure them and large errors may result if the tangent method is used to determine height and tree lean goes undetected.

Tangent Versus Sine Tree Height Determination

As shown in the previous discussion, the tangent method is very sensitive to the point of the crown chosen to represent tree height, especially in wide, skewed, or flat-topped hardwoods. It is possible to avoid large errors in height estimates using the tangent method, but to do this careful measurement of true horizontal distance must be made. In practice, this means identifying the point on the ground directly below the highest point of the tree, a difficult if not impossible prospect under most circumstances.

The sine method avoids inappropriately determined horizontal distances by measuring a real point of the crown, not the projected or assumed apex as with the tangent method. This means that the sine method will never overestimate tree height, which is possible when the tangent method is used incorrectly.

Therefore, to accurately estimate height using the sine method, all one has to do is take appropriate distance and angle readings and correctly identify the highest point on the tree.

However, under some circumstances it can be difficult to find an adequate opening through the canopy to determine slope distance and make a height determination with the sine method. The tangent method does allow for the user to approximate where the top of the crown is (assuming enough can be seen), but this “advantage” is also the flaw that makes inappropriate estimates of tree height possible. Fortunately, since the sine method does not require that a specific viewing distance or direction be used in order to estimate height, it is possible to maneuver around the subject tree until the crown apex becomes visible. The effort expended searching for a good (clear) shot at the highest point of a tree when the sine method is employed is not likely to be greater than the amount of time spent collecting multiple height estimates to average for a more accurate tangent height.

Finally, since it makes use of an actual point on the crown (and does not presume to project one), the sine method is also not as prone to close- proximity errors as the tangent method. This is very advantageous in forests with dense canopies, especially when the trees are all of approximately the same size, since the tangent method should be measured using an angle of 45 degrees or less to help minimize error (fig. 8).

Although considerably more expensive and cumbersome to use, laser technology and sophisticated electronics can substantially improve hardwood height estimates, regardless of the technique. Errors in height prediction can be even more pronounced if older technologies (for example, using clinometers with cloth measuring tapes or pacing) are combined without regard to the degree of error these imprecise techniques impart.

It is also important to remember that any measurement technique requires proper application and the measurement of a consistent standard. It will, for example, always behoove the observer to correctly identify the highest live point of the tree, regardless of the height measurement technique. Accurate

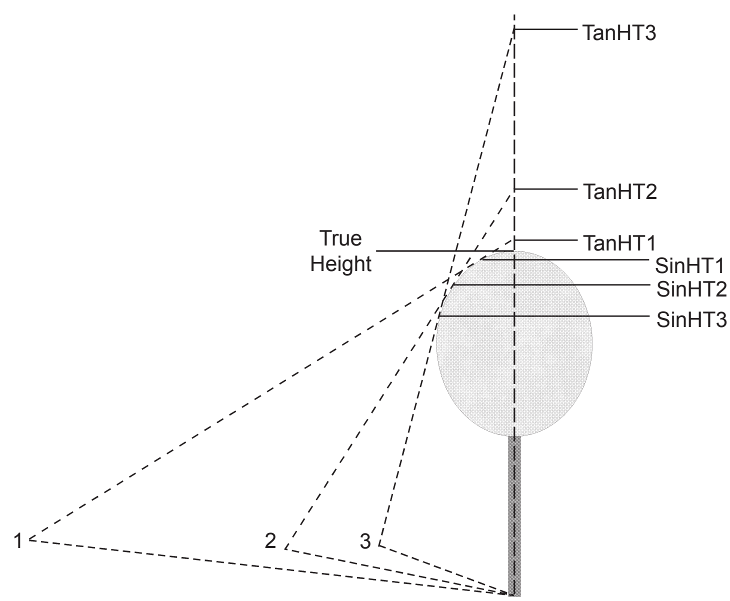

Figure 8—The relative impacts of closeness on height errors for both the tangent and sine methods, given an idealized and opaque hardwood crown. As one approaches the outer edge of the crown (gets closer to the stem), the tangent method provides increasingly greater overestimates, while the sine method underestimates height (although at a lower rate than the tangent).

horizontal distance measurements and reliable angle readings are just as critical in the sine method as they are for the tangent approach. The sine method may be less sensitive to most of the assumptions of the tangent method, but it still requires appropriate implementation to ensure that the highest accuracy is achieved.

CONCLUSIONS

Under typical circumstances, the sine method is the most reliable means currently available to determine standing tree height, largely because it is relatively insensitive to some of the underlying assumptions of the tangent method. Unfortunately, only recently has technology permitted the use of the sine method, whereas the tangent method has been ingrained into procedures and instrumentation for many decades. However, a growing number of individuals and organizations (such as the Eastern Native Tree Society) have begun to tout the advantages of the sine method (ENTS 2005).

Hopefully, the need for consistent and accurate height determination, especially in an era of remotely-sensed and modeled measurements, will encourage more people to use the sine approach. Certainly, in cases where high accuracy is called for (for example, the measurement of champion trees) or conditions exist that would seriously bias height measurement (for example, broad or offset crowns, leaning trees, or steeply sloping terrain), any extra time spent correctly determining height is well worth the effort.

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