THREE-PEE SAMPLING THEORY
and program 'THRP'
for computer generation of selection criteria

L. R. Grosenbaugh

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NOTICE

THIS COMPUTER-PRODUCED PUBLICATION IS AN EXPERIMENTAL EFFORT TO PUBLISH (MORE RAPIDLY AND MORE EFFICIENTLY) INFORMATION ON COMPUTER-ORIENTED THEORIES AND TECHNIQUES.

AT THE SAME TIME, WE ARE TRYING TO IMPROVE SUSCEPTIBILITY OF THE INFORMATION TO AUTOMATED SEARCH AND RETRIEVAL. THE INITIAL SUMMARY AND THE ENTIRE TEXT OF THE PAPER ARE IMMEDIATELY SUITABLE FOR COMPUTER SEARCH BY VIRTUE OF ALREADY BEING ON PUNCHED CARDS. ON A MORE ELEMENTARY LEVEL, H. P. LUHN'S INGENIOUS 11-CHARACTER DOCUMENT IDENTIFIER (FAMILIAR TO USERS OF PERMUTED 'KWIC' INDICES) PROVIDES AN EXCELLENT TOOL FOR VISUAL, MECHANICAL, OR ELECTRONIC RETRIEVAL BY AUTHOR, YEAR, OR INITIALS OF THE FIRST THREE NON-TRIVIAL WORDS IN THE TITLE. THIS IDENTIFIER ALSO SERVES TO MATCH ANY PAGE WITH ITS APPROPRIATE DOCUMENT IN THE EVENT OF SEPARATION OR MIXUPS. SUCH A SYSTEM IS MUCH BETTER ADAPTED TO CUMULATIVE UPDATING OF INDICES THAN IS ANY ARBITRARY LOCAL SERIES OF NUMBERS THAT MUST BE RECYCLED PERIODICALLY.

FINALLY, COMPUTER-PROCESSED TEXT IS EASILY REVISED AND REPUBLISHED. THIS IS AN IMPORTANT CONSIDERATION IN FIELDS WHERE CHANGES AND NEW DEVELOPMENTS ARE OCCURRING SO RAPIDLY.

THE COMPUTER PROGRAM 'PRN', WRITTEN BY THE AUTHOR IN FORTRAN-4 LANGUAGE, WAS USED TO PRINT THIS RESEARCH PAPER AS WELL AS AN EARLIER ONE (PSW-13).

Grosenbaugh, L. R.
1965. *THREE-PEE SAMPLING THEORY and program 'THRP' for computer generation of selection criteria.* Berkeley, Calif.
Pacific SW. Forest & Range Expt. Sta. 53 pp., illus.
(U.S. Forest Service Res. Paper PSW-21)

Theory necessary for sampling with probability proportional to prediction ('three-pee,' or '3P,' sampling) is first developed and then exemplified by numerical comparisons of several estimators. Program 'T RP' for computer generation of appropriate 3P-sample-selection criteria is described, and convenient random integer dispensers are illustrated.

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PACIFIC SOUTHWEST FOREST AND RANGE EXPERIMENT STATION, BERKELEY, CALIFORNIA
FOREST SERVICE, U.S.DEPARTMENT OF AGRICULTURE

THREE-PEE SAMPLING THEORY AND PROGRAM 'THRP' FOR COMPUTER GENERATION OF SELECTION CRITERIA.

L.R.GROSENBAUGH

==================================== SUMMARY ====================================

SAMPLING WITH PROBABILITY PROPORTIONAL TO PREDICTION--3P SAMPLING--IS A NEW TECHNIQUE INVOLVING VARIABLE PROBABILITY. BASIC THEORY AND DERIVED FORMULAE ARE EXPLAINED IN THE CONTEXT OF SAMPLE-TREE-MEASUREMENT FOR APPRAISAL AND SALE OF STANDING TIMBER. A RELATIVE PROBABILITY IS ARBITRARILY ASSIGNED TO EACH TREE IN THE POPULATION AT THE TIME THAT IT IS MARKED FOR HARVEST. EACH PROBABILITY IS PAIRED WITH SOME NUMBER SUBSEQUENTLY DRAWN AT RANDOM FROM (AND REPLACED IN) AN APPROPRIATELY CONSTRUCTED POPULATION OF INTEGERS. THOSE TREES WITH PROBABILITY AS GREAT AS OR LARGER THAN ASSOCIATED RANDOM NUMBER SHOULD BE MEASURED AS SAMPLES. PROBABILITY RECORDS ARE NEEDED ONLY FOR MEASURED TREES, ALTHOUGH COMPLETE RECORDS IMPROVE PRECISION. NUMBER OF SAMPLE TREES TO BE MEASURED CANNOT BE RIGIDLY SPECIFIED IN ADVANCE.

TO PROVIDE THE ARTIFICIAL POPULATION OF NUMBERS APPROPRIATE TO A GIVEN 3P-SAMPLE DESIGN, COMPUTER PROGRAM 'THRP' HAS BEEN WRITTEN IN FORTRAN-4, WITH SUBROUTINE 'IRDM' IN 7090-MAP. A NOVEL FEATURE INVOLVES USE OF THE BOOLEAN LOGICAL OPERATOR 'EXCLUSIVE OR' TO OBTAIN A SINGLE NUMBER AS THE RANDOM RESULTANT OF TWO INDEPENDENT CYCLICAL PROCESSES. CONVENIENT FIELD DISPENSERS FOR COMPUTER-GENERATED INTEGERS ARE ILLUSTRATED.

==================================INTRODUCTION==================================

CONSIDERABLE IMPROVEMENT IN SAMPLING EFFICIENCY OVER THE PAST 25 YEARS HAS BEEN BASED ON NEW THEORY CONCERNING THE SELECTION OF SAMPLES WITH VARYING PROBABILITY. HANSEN AND HURWITZ DEVELOPED THE CONCEPT OF SAMPLING WITH PROBABILITY PROPORTIONAL TO SIZE (‘P.P.S’ SAMPLING), AND TOGETHER WITH MADOW, WROTE A VERY USEFUL TEXT (*4) THAT SUMMARIZES MUCH OF THE THEORY APPROPRIATE TO P.P.S. SAMPLING.

FORESTERS HAVE APPLIED P.P.S. SAMPLING TO TREE POPULATIONS IN A NUMBER OF WAYS--LIST SAMPLING, CLUSTER SAMPLING OF AREA BY CONCENTRIC PLOTS OF DIFFERENT RADII FOR TREES OF DIFFERENT SIZE-CLASS, AND CLUSTER SAMPLING OF AREA BY CRITICAL DISTANCE FROM LINE OR POINT EXPRESSED AS A CONTINUOUS FUNCTION OF TREE DIAMETER OR HEIGHT. SPECIALLY GRADUATED TAPES OR OPTICAL GAUGES ESTABLISHING APPROPRIATE CRITICAL ANGLES (*1) FACILITATE DISCRIMINATION BETWEEN QUALIFYING AND NON-QUALIFYING TREES IN THE AREA SAMPLES.

HOWEVER, SUITABLE LISTS OF TREE POPULATIONS OR EVEN SIMPLE TREE ENUMERATIONS ARE RARELY AVAILABLE IN ADVANCE OF TREE SAMPLING BECAUSE OF EXPENSE, RAPID CHANGE, AND THE ALMOST INSUPERABLE DIFFICULTY OF ESTABLISHING CORRESPONDENCE BETWEEN LISTED OR ENUMERATED TREES AND INDIVIDUAL STANDING TREES IN EXTENSIVE FOREST POPULATIONS. RELATIVELY SMALL NUMBERS OF SAMPLE TREES PERMANENTLY RELOCATABLE FOR RESEARCH PURPOSES OR FOR ESTIMATING GROWTH AND MORTALITY OF WHOLE FORESTS ARE NOTABLE EXCEPTIONS.

IN CLUSTER SAMPLES OF AREA USING P.P.S. (PLOTS, LINES, POINTS), THE ONE OR MORE ACTUAL TREE DIMENSIONS CONSTITUTING 'SIZE' (OR RELATIVE PROBABILITY)

MUST BE MEASURED ON ALL OBVIOUSLY QUALIFYING TREES, IN ADDITION TO ALL VARIABLES OF INTEREST THAT DO NOT COINCIDE WITH 'SIZE'. EVEN DOUBTFULLY QUALIFYING TREES REQUIRE THE LINEAR MEASUREMENT OF BOTH TREE SIZE (PROBABILITY DIMENSION) AND DISTANCE BETWEEN TREE AND REFERENCE POINT OR LINE.

ALTHOUGH P.P.S. SAMPLING USING LISTS OR GEOMETRIC CLUSTERS HAS IMPROVED EFFICIENCY GREATLY OVER EARLIER TYPES OF FOREST SAMPLING WITH ESSENTIALLY EQUAL PROBABILITIES, MANY SITUATIONS OCCUR WHERE THE VARIABLE OF INTEREST (VOLUME, PRODUCT YIELD, VALUE, ETC.) IS ONLY WEAKLY CORRELATED WITH GEOMETRIC PROBABILITY DIMENSIONS CONVENIENTLY MEASURABLE ON STANDING TREES. ALSO, WHERE THE SAMPLE INVOLVES AN APPRECIABLE PORTION OF THE TREE POPULATION, TIME SPENT IN MEASUREMENTS NEEDED ONLY TO INCLUDE OR EXCLUDE INDIVIDUALS OR TO ESTABLISH PROBABILITY DIMENSIONS OF INCLUDED INDIVIDUALS OFTEN BECOMES EXCESSIVE.

SAMPLING WITH PROBABILITY PROPORTIONAL TO PREDICTION ('THREE-PEE', OR '3P', SAMPLING) WAS CONCEIVED BY THE AUTHOR TO MORE EFFICIENTLY HANDLE SUCH SITUATIONS IN THE ABSENCE OF SUITABLE LISTS. IT TAKES INTO ACCOUNT THE FOLLOWING FACTS--(1) SPEEDY OCULAR ASSESSMENT OF SOME QUANTITY ASSOCIATED WITH AN INDIVIDUAL TREE AT THE TIME IT IS VISITED AND MARKED FOR HARVEST IS CHEAP AND NEVER IMPOSES THE BURDEN OF PHYSICAL MEASUREMENT OF EITHER RELATIVE PROBABILITY DIMENSION OR ANY CRITICAL INCLUSION-EXCLUSION DISTANCE, (2) TALENTED ASSESSORS CAN QUICKLY MAKE BIASED OCULAR JUDGMENTS AT THE TIME OF MARKING THAT WILL BE HIGHLY CORRELATED WITH INDIVIDUAL TREE QUANTITIES IMPOSSIBLE TO MEASURE IN THE STANDING TREE (VOLUME AFFECTED BY DEFECT, END-PRODUCT YIELD, DOLLAR VALUE, ETC.), (3) CORRECTION OF SUCH BIASED JUDGMENT BY EQUAL-PROBABILITY TECHNIQUES INVOLVING REGRESSION OR RATIO ESTIMATES HAS BEEN UNSATISFACTORY BECAUSE OF ERRATIC VARIATION AND UNKNOWN FORM OF UNDERLYING FUNCTIONAL RELATIONSHIPS.

3P SAMPLING DIFFERS FROM PUBLISHED FORMS OF VARIABLE-PROBABILITY SAMPLING IN THE FOLLOWING WAYS --(1) UNLIKE LIST SAMPLING, 3P SAMPLING EMPLOYS A RANDOMLY VARYING NUMBER OF SAMPLES AND REQUIRES NO PRIOR ENUMERATION (LIST) OF THE POPULATION --(2) UNLIKE GEOMETRIC P.P.S. SAMPLING, 3P SAMPLING DOES NOT REQUIRE OBJECTIVE (AND EXPENSIVE) DETERMINATION OF INDIVIDUAL TREE DIMENSION AND/OR DISTANCE FROM SAMPLE LOCUS AS A PERQUISITE TO SAMPLE SELECTION --(3) COMPLETE AGGREGATION OF VARIABLE PROBABILITIES SUBSEQUENT TO SAMPLING A PREVIOUSLY UNENUMERATED, UNLISTED, UNSTRATIFIED POPULATION IS MORE FEASIBLE THAN BY ANY OTHER METHOD. THOUGH SUCH COMPLETE AGGREGATION IS NOT NECESSARY, IT GREATLY IMPROVES EFFICIENCY.

A CAREFUL REVIEW OF (*4) TOGETHER WITH MURTHY'S LATER SUMMARY (*8) OF MORE RECENT VARIABLE-PROBABILITY TECHNIQUES SUGGESTS THAT (*2) WAS THE FIRST PUBLICATION TO DEAL WITH THE UNIQUE ASPECTS AND ADVANTAGES OF 3P SAMPLING.

============================ THEORY OF 3P SAMPLING ============================

FOR ALGEBRAIC PURPOSES, AN ASTERISK (*) DENOTES MULTIPLICATION, A DOUBLE ASTERISK (**) DENOTES EXPONENTIATION, AND A SLANT (/) DENOTES DIVISION. THESE OPERATORS ARE EXECUTED IN THE FOLLOWING ORDER--EXPONENTIATION PRECEDES MULTIPLICATION OR DIVISION WHICH PRECEDE ADDITION OR SUBTRACTION WITHIN ANY GIVEN SET OF PARENTHESES, AND THE SEQUENCE FOR EVALUATION OF PARENTHETICAL EXPRESSIONS IS FROM INNERMOST TO OUTERMOST. TO AVOID EXCESSIVE PARENTHESES, VARIABLES ENDING IN 'I' ARE ASSUMED TO BE SUBSCRIPTED VARIABLES HAVING INDIVIDUAL VALUES PECULIAR TO THE 'I'TH TREE. NO SPECIAL NOTATION WILL BE EMPLOYED WHERE SUBSCRIPT IMPLICATION IS OBVIOUS.

IN DEVELOPING 3P-SAMPLING THEORY, IT WILL BE CONVENIENT TO ADOPT A STANDARD LEGEND. ASSUME A FOREST POPULATION CONSISTING OF 'M' TREES, ASSOCIATED WITH EACH OF WHICH IS A PARTICULAR VARIABLE QUANTITY 'YI' THAT CAN BE OBJECTIVELY MEASURED. A RELATIVE PROBABILITY 'KPI' ESTABLISHED IN SOME ARBITRARY MANNER IS ALSO ASSOCIATED WITH EACH TREE. 'KPI' SHOULD BE AN INTEGER, HOPEFULLY CORRELATED WITH OR PROPORTIONAL TO 'YI', THE TREE VARIABLE OF MAXIMUM INTEREST. IF EACH TREE IN THE POPULATION IS VISITED, 'KPI' NEED NOT BE ESTABLISHED FOR A GIVEN TREE UNTIL AFTER SUBJECTIVELY GUESSING 'YI' JUST PRIOR TO APPLICATION OF THE SELECTION PROCESS AT THE PARTICULAR TREE. THERE IS NEVER ANY NEED TO ESTABLISH CORRESPONDENCE BETWEEN A STANDING TREE AND AN ENTRY ON A PREVIOUS RECORD OR LIST.

3P SAMPLING REQUIRES A RECTANGULAR PROBABILITY SET (EQUAL LIKELIHOOD), COMPOSED OF AN ARITHMETIC PROGRESSION OF INTEGERS WITH UNIT INTERVAL FROM ONE

TO 'KZ', AND 'KZ' MUST EXCEED THE LARGEST 'KPI' ASSIGNED TO ANY TREE IN THE POPULATION. LATER, IT WILL BE CONVENIENT TO CONSIDER 'KZ' AS THE SUM OF TWO COMPONENTS 'K' AND 'Z', SO THAT (K+Z)=KZ.

THE 3P-SELECTION PROCESS INVOLVES VISITING EACH OF THE IM' TREES, CONCURRENTLY ESTABLISHING ITS 'KPI' AS AN INTEGER IN THE RANGE FROM ONE THROUGH (KZ-1), THEN DRAWING AN INTEGER AT RANDOM FROM THE SET OF 'KZ' ELEMENTS (FOLLOWED BY REPLACEMENT OF THE SELECTED INTEGER). THE INTEGER THUS DRAWN WILL BE DENOTED AS 'KP'. IF 'KP' EXCEEDS 'KPI', THE TREE IS REJECTED AND ITS 'YI' IS ARBITRARILY ASSUMED TO BE ZERO. OTHERWISE, THE TREE IS SELECTED AND ITS 'YI' IS ACTUALLY MEASURED. THE PROBABILITY THAT A GIVEN TREE WILL HAVE ITS 'YI' MEASURED IS THUS KPI/KZ.

NOW SUPPOSE ALL 'M' TREES IN THE POPULATION WERE GIVEN AN EQUAL OPPORTUNITY OF BEING CHOSEN TO BE THE ONLY TREE WHOSE 'YI' WAS TO BE MEASURED, WITHOUT REGARD TO THE OUTCOME OF THE FIRST SAMPLING. THE PROBABILITY THAT A PARTICULAR TREE WOULD BE SELECTED BY SUCH A SAMPLE IS 1/M, WHILE THE PROBABILITY THAT ANY TREE THUS SELECTED WOULD ALSO BE A TREE SELECTED FOR MEASUREMENT BY THE EARLIER DESCRIBED SAMPLING PROCESS IS KPI/(M*KZ), HENCEFORTH CALLED 'PI'. BY ITS NATURE, 'PI' MUST ALWAYS BE LESS THAN UNITY IF 'KZ' EXCEEDS 'KPI', AND MORE THAN ZERO IF 'KPI' IS AT LEAST ONE.

THE NUMBER OF MEASURED TREES 'N' COULD VARY FROM ZERO TO 'M' IF ONLY THE EARLIER SAMPLE WERE CONSIDERED (WITHOUT REPLACEMENT). HOWEVER, 'N' COULD ASSUME ONLY THE VALUES ZERO OR ONE IF SUCCESS IN BOTH SAMPLES WERE REQUIRED.

SINCE THE LATTER SITUATION IS SIMPLEST, CONSIDER THE EXPECTED VALUE OF YI/PI IN SUCH A SAMPLING SCHEME. REMEMBER THAT IF N=O, YI/PI IS ARBITRARILY SPECIFIED TO HAVE ZERO VALUE. OTHERWISE, N=1 AND YI/PI WILL BE DETERMINED BY MEASUREMENT OF 'YI'. THE EXPECTED VALUE OF YI/PI IN SUCH A MODEL IS

$$\sum^{M}(PI * (YI / PI) + (1 - PI) * ZERO = \sum^{M} YI$$

BUT YI/PI = M*KZ*YI/KPI, WHICH THUS PROVIDES AN UNBIASED ESTIMATE OF AGGREGATE 'YI' IN THE TREE POPULATION.

NOW SUPPOSE THE INITIAL 3P SAMPLE HAD NEVER BEEN REDUCED TO ZERO OR ONE SAMPLE TREE BY THE INDEPENDENT EQUAL-PROBABILITY SELECTION. HOWEVER, 'PI' AS ALREADY DEFINED STILL HAS MEANING. THERE WOULD NOW BE A SUM OF 'M' VALUES OF YI/PI AVAILABLE (SOME ARE ARBITRARILY ZERO. SINCE EACH HAS EXPECTED VALUE EQUAL TO $\sum^{M}$YI THE SUM OF ALL 'M' VALUES HAS EXPECTED VALUE EQUAL TO M*$\sum^{M}$YI.

HENCE, (1/M)* $\sum^{M}$YI/PI HAS AN EXPECTED VALUE OF $\sum^{M}$YI,THE AGGREGATE 'YI' IN THE POPULATION. BUT (1/M)* $\sum^{M}$YI/PI = KZ* $\sum^{M}$YI/KPI, SO THIS EXPRESSION PROVIDES AN UNBIASED ESTIMATE OF AGGREGATE 'YI' IN THE POPULATION. SINCE ONLY THE 'N' VALUES OF MEASURED 'YI' ARE OF CONCERN (THE OTHERS ARE ARBITRARILY ASSUMED TO BE ZERO), THE AGGREGATE 'YI' IN THE POPULATION CAN BE ESTIMATED BY

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3PFIRST = KZ * $\sum^{N}$ YI/KPI IF N=NONZERO

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= ZERO IF N=ZERO

INSTEAD OF MAKING AN ESTIMATE OF ZERO WHENEVER N=O, IT COULD BE SPECIFIED THAT THE ENTIRE PROCESS BE REPEATED WHENEVER AN OUTCOME OF ZERO SAMPLE TREES WAS SECURED. IF THE PROBABILITY OF OBTAINING AN OUTCOME OF N=0 IS CALCULATED AS PZ=(1-KPI/KZ)*(1-KPI/KZ)*...*(1-KPI/KZ) UNTIL ALL 'M' TREES HAVE BEEN ACCOUNTED FOR, THEN THE UNBIASED ESTIMATOR APPROPRIATE TO NONZERO 'N' (WITH REDRAW WHEN N=0) IS

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3PSECOND = (1-PZ) * KZ * $\sum^{N}$ YI/KPI WITH REDRAW IF N=0

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'PZ' IS USUALLY SO NEARLY ZERO AS TO BE NEGLIGIBLE WHEN THE EXPECTED NUMBER OF SAMPLES IS 5 OR MORE. HENCE, THE MUCH MORE CONVENIENT '3PFIRST' ESTIMATOR

CAN BE USED IN MOST PRACTICAL SITUATIONS EVEN THOUGH A REDRAWING IS SPECIFIED FOR A ZERO OUTCOME.

THE TWO ESTIMATORS ABOVE ARE SIMPLE AND UNBIASED, BUT VARIATION IN 'N' IS DIRECTLY REFLECTED BY VARIATION AMONG THE ESTIMATES. NOTE THAT A SINGLE SET OF SAMPLE TREES IS SECURED WITHOUT REPLACEMENT, THAT NO A PRIORI OR A POSTERIORI LIST OR ENUMERATION OF INDIVIDUALS IN THE POPULATION IS REQUIRED (ONLY THAT EACH INDIVIDUAL BE GIVEN A KNOWN OPPORTUNITY TO BE INCLUDED IN THE SAMPLE).

AS LUNG AS 'KZ' EXCEEDS THE LARGEST 'KPI' IN THE POPULATION, THE EXPECTED VALUES OF THE ESTIMATORS ARE EQUAL TO POPULATION VALUES. THIS IS EQUIVALENT TO SAYING THAT THE EXPECTED NUMBER OF SAMPLES TIMES THE LARGEST 'KPI' IN THE POPULATION SHOULD BE SMALLER THAN $\sum^{M}$ KPI . MOREOVER, IT IS DESIRABLE THAT THE PRODUCT (N * (MAX KPI)) BE SMALLER THAN $\sum^{M}$ KPI, SINCE OTHERWISE SAMPLE BLOW-UP FACTORS SMALLER THAN UNITY CAN OCCUR.

THE DISTRIBUTION OF 'N' IN A 3P SAMPLE CAN BE PREDICTED FROM STUDY OF THE WELL KNOWN POISSON SERIES (NOT THE POISSON EXPONENTIAL LIMIT). THE BERNOUILLI OR BINOMIAL SERIES IS A SPECIAL CASE OF THE POISSON SERIES IN WHICH ALL 'KPI' ARE THE SAME. THE EXPECTED NUMBER OF MEASURED TREES IN A 3P SAMPLE IS EQUAL TO THE PRODUCT OF 'M' TIMES THE SINGLE-DRAW EXPECTATION FOR THE ENTIRE POPULATION (OR ESN = M*$\sum^{M}$ KPI), WHICH CAN BE EXPRESSED AS

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3PTHIRD = ESN = (1/KZ) * $\sum^{M}$ KPI

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IF A REDRAW IS SPECIFIED WHEN N=O, THE EXPECTED NUMBER OF NONZERO SAMPLES 'ENZSN' IS

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3PFOURTH = ENZSN = ESN/(1-PZ)

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AS WAS PREVIOUSLY NOTED, IN PRACTICE ,PZ, IS USUALLY SO CLOSE TO ZERO THAT THE TWO FORMULAE MAY BE USED INTERCHANGEABLY.

THE VARIANCE OF 'N' ABOUT 'ESN' FOR A POISSON SERIES IS DERIVED ON PAGE 149 OF RIETZ ('9), AND THE EXPRESSION GIVEN THERE CAN BE SHOWN TO BE ALGEBRAICALLY EQUIVALENT TO

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3PFIFTH = ESN - (1+C**2) * (ESN**2)/M
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WHERE 'C' IS THE COEFFICIENT OF VARIATION (EXPRESSED AS A DECIMAL FRACTION) OF 'KPI' FOR THE ENTIRE POPULATION.

IN A BINOMIAL SERIES, (1+C**2) = 1, SO ESN - (ESN**2)/M MAY BE USED AS A SAFE UPPER BOUND FOR THE VARIANCE OF 'N' WHERE NOTHING IS KNOWN ABOUT THE VARIATION AMONG 'KPI'. IF ONLY THE 'N' SAMPLE VALUES OF 'KPI' ARE KNOWN, (1+C**2) CAN BE APPROXIMATED BY (1/N**2) * ($\sum^{N}$KPI)*$\sum^{N}$(1/KPI). WHERE ALL 'M' VALUES OF 'KPI' ARE KNOWN, (1+C**2) = M*$\sum^{M}$(KPI**2)/($\sum^{M}$/KPI)**2.

IF IT BE SPECIFIED THAT REDRAWING WILL OCCUR WHEN AN OUTCOME OF N=0 IS SECURED, THE VARIANCE IS ABOUT 'ENZSN' RATHER THAN 'ESN', AND N=0 IS REJECTED. SUCH VARIANCE CAN BE COMPUTED AS 3PFIFTH/(1-PZ), MINUS PZ*ENZSN**2. AS NOTED EARLIER, 'PZ, IS USUALLY SO NEARLY ZERO AS TO MAKE THIS THE SAME AS '3PFIFTH'.

IN VIEW OF THE SPEED WITH WHICH OCULAR JUDGMENTS CAN ESTABLISH A MEANINGFUL 'KPI' FOR ANY TREE, IT MAY WELL BE FOUND FEASIBLE TO OBTAIN ,KPI, FOR ALL INDIVIDUALS IN A FOREST POPULATION, INSTEAD OF MERELY FOR SOME PORTION OF THE POPULATION. WHERE THIS IS DONE, MORE EFFICIENT ESTIMATORS OF AGGREGATE in, FOR A POPULATION ARE AVAILABLE, SINCE THE FLUCTUATION CAUSED BY VARIATION IN 'N' ABOUT 'ESN' CAN BE LARGELY REMOVED.

FOR CONVENIENCE, THESE MORE EFFICIENT ESTIMATORS WILL BE DERIVED FROM '3PSECOND' EXPRESSIONS, ALTHOUGH ACCEPTANCE OF ZERO ESTIMATES WHEN N=0 WOULD

LEAD TO THE SAME END RESULT.

FROM '3PTHIRD' AND '3PFOURTH' IT IS OBVIOUS THAT $ENZSN = (1/KZ)*\sum^{M} KPI/(1-PZ)$, SO $KZ = (1/ENZSN)*\sum^{M} KPI/(1-PZ)$. SUBSTITUTING FOR 'KZ' IN '3PSECOND' GIVES $(\sum^{M} KPI/ENZSN) * \sum^{N} YI/KPI$, AND MULTIPLYING BY N/N AND REARRANGING GIVES A DIFFERENT FORM OF THE UNBIASED '3PSECOND' ESTIMATOR FOR AGGREGATE 'YI' IN THE POPULATION, THUS

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$$3PSIXTH = (N/ENZSN) * (\sum^{M} KPI/N) * \sum^{N} YI/KPI$$

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THE EXPECTED VALUE OF N/ENZSN IS UNITY, SO IF 'N' IS UNCORRELATED WITH THE REMAINDER OF THE EXPRESSION, OR IF 'N' COINCIDES WITH ENZSN, THE EXPECTED VALUE OF '3PSIXTH' IS

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$$3PSEVENTH = (\sum^{M} PI/N) * \sum^{N} YI/KPI$$

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UNLESS COVARIANCE BETWEEN 'N' AND '3PSEVENTH' IS NEGATIVE AND LARGE, THE LATTER ESTIMATES WILL HAVE MUCH SMALLER DEVIATIONS FROM POPULATION VALUES THAN WILL CORRESPONDING '3PSECOND' ESTIMATES. THE ONLY POSSIBLE WAY LARGE NEGATIVE COVARIANCE BETWEEN 'N' AND '3PSEVENTH' COULD ARISE WOULD BE FOR 'KPI' TO BE STRONGLY AND NEGATIVELY CORRELATED WITH 'YI', A CIRCUMSTANCE THAT IS DIFFICULT TO IMAGINE WHEN THERE IS ANY REAL BASIS FOR OCULAR ASSESSMENT.

IT IS APPARENT THAT '3PSEVENTH' ESTIMATES ARE MERELY '3PSECOND' ESTIMATES ADJUSTED BY MULTIPLICATION BY ENZSN/N. IN COMBINING 2 OR MORE '3PSEVENTH' ESTIMATES OF THE SAME POPULATION, EACH SHOULD BE WEIGHTED BY ITS OWN 'N' AND DIVIDED BY AGGREGATE 'N' FOR ALL SAMPLES INVOLVED IN THE COMBINATION. SUCH A WEIGHTING PROCEDURE LEADS TO CUMULATIVE ESTIMATES APPROACHING POPULATION VALUES AS AGGREGATE 'N' GROWS LARGE, YET WHEN AGGREGATE 'N' IS SMALL AND AVERAGE 'N' DEVIATES EXCESSIVELY FROM 'ENZSN', THE ERRORS ARE USUALLY SMALLER

THAN IS THE CASE WITH '3PFIRST' OR '3PSECOND'.

SOME OBVIOUS IMPLICATIONS OF '3PSIXTH' AND '3PSEVENTH' FORMULAE FOLLOW. THE STRONGER THE POSITIVE CORRELATION BETWEEN LOG 'KPI' AND LOG 'YI', THE LESS VARIATION AMONG '3PSEVENTH' ESTIMATES. WHEN 'KPI' IS CONSTANTLY PROPORTIONAL TO 'YI', '3PSEVENTH' NEVER VARIES, REGARDLESS OF VARIATION IN 'N'. THUS, VARIATION IN 'N' HAS LITTLE EFFECT ON '3PSEVENTH' ESTIMATES WITH GOOD CORRELATION BETWEEN 'YI' AND 'KPI'.

THE EFFECT OF VARIATION IN 'N' ON '3PSEVENTH' ESTIMATES IS MUCH GREATER WHEN THERE IS ZERO CORRELATION BETWEEN 'YI' AND 'KPI', AS IN THE CASE OF FREQUENCY. HERE, EQUAL-PROBABILITY SAMPLES WOULD BE MUCH MORE EFFICIENT. IT IS INFORMATIVE TO CALCULATE SUCH 3P ESTIMATES OF FREQUENCY, EVEN THOUGH TRUE POPULATION VALUES OF FREQUENCY ARE ACTUALLY AVAILABLE, SINCE THE PERCENTAGE DEVIATION OF A FREQUENCY ESTIMATE FROM POPULATION VALUE CONSTITUTES AN OUTER BOUND FOR THE MUCH SMALLER DEVIATIONS APT TO OCCUR AMONG ESTIMATES OF VARIABLES BETTER CORRELATED WITH 'KPI'.

THE ADVANTAGE OBTAINED IN 3P SAMPLING WITH VERY SMALL OR ZERO VARIATION IN THE RELATIONSHIP BETWEEN 'YI' AND 'KPI' CAN BE SEEN EVEN MORE CLEARLY IN P.P.S. CLUSTER SAMPLING OF AREA, WHERE HORIZONTAL POINT-SAMPLES ARE MOST EFFICIENT FOR TOTAL BASAL AREA ESTIMATES, HORIZONTAL LINE-SAMPLES FOR SUM-OF-DIAMETER ESTIMATES, VERTICAL LINE-SAMPLES FOR AGGREGATE LINEAL FOOTAGE ESTIMATES, PLOT-SAMPLES FOR FREQUENCY ESTIMATES, ETC. ALL OF THE VARIATION IN SUCH P.P.S. SAMPLES IS ATTRIBUTABLE TO UNCONTROLLABLE VARIATION IN 'N'.

TO OBTAIN AN UNBIASED ESTIMATE OF THE VARIANCE OF ANY 3P ESTIMATE REQUIRES TWO OR MORE INDEPENDENT SETS OF SAMPLES (WITH REPLACEMENT). THE MOST EFFICIENT WAY TO OBTAIN MORE THAN ONE SET OF 3P SAMPLES IS TO COMPARE 'KPI' OF EACH TREE VISITED WITH 2 OR MORE RANDOM INTEGERS INSTEAD OF ONE. REGARDLESS OF HOW MANY

TIMES THE TREE QUALIFIES, IT NEED ONLY BE MEASURED ONCE, THOUGH ITS MEASURED 'YI' WILL BELONG TO MORE THAN ONE SET OF SAMPLES IF IT QUALIFIES MORE THAN ONCE. THIS TECHNIQUE CREATES INTERPENETRATING SETS OF 3P SAMPLES.

CALCULATION OF VARIANCE FOR SUCH SIMULTANEOUSLY ACQUIRED INDEPENDENT SETS OF SAMPLES IS STRAIGHTFORWARD--DEVIATIONS OF '3PFIRST' OR '3PSECOND' ESTIMATES ARE MEASURED FROM AN UNWEIGHTED MEAN, WHILE DEVIATIONS OF '3PSEVENTH' ESTIMATES SHOULD BE MEASURED FROM A WEIGHTED MEAN, AS EXPLAINED EARLIER. THE ORDINAL OF EACH SET OR SETS TO WHICH A GIVEN SAMPLE TREE BELONGS MUST, OF COURSE, BE RECORDED AT THE TIME THAT THE TREE QUALIFIES.

INDEPENDENT SETS OF SAMPLES AFFORD AN ADDITIONAL ADVANTAGE IN THAT THEY ALLOW ESTIMATION OF THE CORRELATION BETWEEN 'N' AND ANY OF THE 3P ESTIMATORS, WITH SUBSEQUENT ADJUSTMENT OF THE ESTIMATES IF CORRELATION IS STRONG AND AT THE SAME TIME DEVIATION OF MEAN 'N' FROM 'ENZSN' IS LARGE AND KNOWN. ORDINARILY, HOWEVER, '3PFIRST' OR '3PSECOND' ESTIMATORS SHOULD NOT BE EMPLOYED WHEN 'ENZSN' IS KNOWN, WHILE ADJUSTMENT NEEDED BY '3PSEVENTH' ESTIMATES OF ANY VARIABLE (EXCEPT POSSIBLY FREQUENCY) IS APT TO BE SO SMALL AS TO BE NEGLIGIBLE. WHERE ADJUSTMENTS ARE FOUND DESIRABLE, A LINEAR FUNCTION WILL USUALLY BE QUITE SATISFACTORY IN THE ZONE BETWEEN MEAN 'N' AND 'ENZSN', EVEN THOUGH THE RELATIONSHIP BETWEEN ,'YI', AND 'KPI' IS NONLINEAR, ERRATIC, OR DIFFICULT TO FORMALIZE.

INDEPENDENT SETS OF SAMPLES AFFORD THE ONLY ESTIMATE OF VARIANCE AMONG '3PFIRST' AND '3PSECOND' ESTIMATES, BUT MUCH SIMPLER APPROXIMATE VARIANCES CAN BE CALCULATED FOR '3PSEVENTH' ESTIMATES. THE APPROXIMATIONS CAN BE RATIONALIZED AS FOLLOWS.

IF A FIXED NUMBER OF TREES 'N' BE SELECTED FROM 'M' TREES (WITH REPLACEMENT) FOR MEASUREMENT OF 'YI', EACH WITH PARTICULAR SINGLE-DRAW

PROBABILITY PI = KPI/$\sum^{M}$KPI, THEN THE EXPECTED VALUE OF EACH TI = YI/PI IS $\sum^{M}$YI, AND THE EXPECTED VALUE OF THE VARIANCE OF THE MEAN OF THESE 'N' ESTIMATES OF A TOTAL IS

========

3PEIGHTH = (1/N) * $\sum^{M}$PI*(TI-$\sum^{M}$YI)**2

========

NOW SUBSTITUTE SAMPLE ESTIMATE $\sum^{N}$TI/N FOR $\sum^{M}$YI, AND RECOGNIZE THAT 'PI' BECOMES THE CONSTANT 1/N IN A SELF-WEIGHTING PROBABILITY SAMPLE OF FIXED SIZE 'N'. TO CONVERT FROM ABSOLUTE TO RELATIVE VARIANCE, DIVIDE BY ($\sum^{N}$TI/N)**2. FINALLY, MULTIPLY BY N/(N-1) TO COMPENSATE FOR USING DEVIATIONS FROM THE SAMPLE MEAN INSTEAD OF DEVIATIONS FROM POPULATION MEAN. THE SAMPLE ESTIMATE OF THE RELATIVE VARIANCE OF A TOTAL BASED ON 'N' SAMPLES THEN BECOMES

=======

3PNINTH = ($\sum^{N}$TI**2)/($\sum^{N}$TI)**2 - 1/N, ALL MULTIPLIED BY N/(N-1)

=======

THIS IS AN UNBIASED ESTIMATE OF VARIANCE WHEN SAMPLING WITH 'N' FIXED AND WITH REPLACEMENT. MULTIPLICATION BY ($\sum^{N}$TI/N)**2 CONVERTS FROM RELATIVE TO ABSOLUTE UNITS. 3PNINTH IS A GOOD APPROXIMATION WHEN SAMPLING WITH 'N' FIXED BUT WITHOUT REPLACEMENT IF N/M AND MOST 'PI' ARE SMALL RELATIVE TO UNITY (*4). THE AUTHOR DEMONSTRATES IN THE NEXT SECTION THAT 3PNINTH IS A USEFUL APPROXIMATION EVEN WHERE 'N' IS VARIABLE AND WHERE N/M AND 'PI' ARE MUCH LARGER THAN WOULD USUALLY OCCUR IN PRACTICE.

IN MORE FAVORABLE SITUATIONS WHERE COMPUTERIZED TREE POPULATIONS WERE REPEATEDLY 3P-SAMPLED, SHARPNACK (*10) FOUND THAT, AS EXPECTED, 2/3 OF THE DEVIATIONS FROM POPULATION VALUES FELL BETWEEN FIDUCIAL LIMITS BASED ON THE ABOVE VARIANCE APPROXIMATIONS, ALTHOUGH DISTRIBUTIONS OF THE SAMPLE ESTIMATES WERE SOMEWHAT SKEWED.

=============== EXPECTED VALUES FOR SOME 3P-SAMPLE ESTIMATES ===================

THE MORE IMPORTANT FORMULAE DERIVED IN THE PRECEDING SECTION WERE STATED WITHOUT PROOF IN AN EARLIER PAPER (*2), ALONG WITH A SMALL NUMERICAL EXAMPLE ILLUSTRATING COMPUTATIONAL PROCEDURE FOR OBTAINING ESTIMATES FROM A SINGLE SET OF 2 MEASURED 3P-SAMPLE TREES DRAWN FROM A 9-TREE POPULATION.

IN ORDER TO INVESTIGATE PATTERNS OF VARIATION EXHIBITED BY 3P-SAMPLE ESTIMATES UNDER A VARIETY OF CIRCUMSTANCES, THE PRESENT PAPER COMPUTES 3P-SAMPLE ESTIMATES AND PROBABILITIES OF OCCURRENCE FOR ALL POSSIBLE OUTCOMES DRAWN FROM THE SAME 9-TREE POPULATION BY 3 DIFFERENT DESIGNS OR SAMPLING RATES. SEVERAL DIFFERENT TYPES OF 3P ESTIMATORS ARE COMPARED FOR EACH DESIGN. ANY TRIVIAL DIFFERENCE BETWEEN VOLUME ESTIMATES IN THE PRESENT PAPER AND THE EARLIER ONE CAN BE EXPLAINED BY THE FACT THAT VOLUMES OF THE 9 INDIVIDUAL TREES HAVE NOW BEEN ROUNDED TO THE NEAREST BOARD FOOT TO SIMPLIFY THE COMPUTATIONS.

IN THE 9-TREE POPULATION, 'KPI' IS OCULARLY ASSESSED VALUE TO THE NEAREST DOLLAR (BASED ON GUESSED BOARD-FOOT VOLUME AT 2 CENTS PER BOARD FOOT). TABLE 1 SHOWS ALL POSSIBLE SAMPLES THAT CAN BE DRAWN FROM THE 9-TREE POPULATION BELOW.

GUESSED KPI	ACTUAL FREQUENCY	ACTUALLY MEASURED BOARD-FOOT VOLUME
2	1	106
2	1	106
2	1	106
2	1	106
2	1	106
2	1	106
5	1	248
5	1	248
10	1	526
$32 = \sum^{M} KPI$	$9 = M$	$1658 = \sum^{M} YI$

TABLE 1. 3P-SAMPLE OCCURRENCES AND '3PSEVENTH' ESTIMATES FOR 9-TREE POPULATION

P=PROBABILITIES OF OCCURRENCE IF KZ EQUAL TO 32	20	11	SAMPLE SIZE =N	3PSEVENTH T=FREQ. (NUMBER)	ESTIMATE T=VOL. (80.FT.)	COMPONENT ELEMENT REL. LIKELIHOODS =KPI
.33229816	.14946778	.00811378	0	REDRAW	REDRAW	NONE
.15104461	.14946778	.08113783	1	3.20	1683.20	10
.13291926	.09964519	.01081838	1	16.00	1696.00	2
.12307340	.09964519	.01352297	1	6.40	1587.20	5
.06041784	.09964519	.10818377	2	9.60	1689.60	2, 10
.05594245	.09964519	.13522972	2	4.80	1635.20	5, 10
.04922936	.06643012	.61803063	2	11.20	1641.60	2,5
.02215321	.02767522	.00601021	2	16.00	1696.00	2,2
.01139569	.01660753	.60563457	2	6.40	1587.20	5,5
.02237698	.06643012	.18030629	3	8.53	1655.47	2,5, 10
.01006964	.02767922	.06010210	3	11.73	1691.73	2,2, 10
.00820490	.01845281	.01001701	3	12.80	1659.73	2,2,5
.00517986	.01660753	.05634572	3	5.33	1619.20	5,5,10
.00455827	.01107169	.00751276	3	9.60	1623.47	2,5,5
.00196917	.00410062	.00178080	3	16.00	1696.00	2,2,2
.00372950	.01845281	.10017016	4	10.40	1665.60	2,2,5, 10
.00207194	.01107169	.07512762	4	8.00	1638.40	2,5,5,10
.00089508	.00410062	.01780803	4	12.80	1692.80	2,2,2, 10
.00075971	.00307541	.00417376	4	11.20	1641.60	2,2,5,5
.00072932	.00273375	.00296800	4	13.60	1668.80	2,2,2,5
.00009846	.00034172	.00029680	4	16.00	1696.00	2,2,2,2
.00034532	.00307547	.04173757	5	9.60	1649.92	2,2,5,5,10
.00033151	.00273375	.02968005	5	11.52	1671.68	2,2,2,5, 10
.00006753	.00045563	.00123667	5	12.16	1652.48	2,2,2,5,5
.00004475	.00034172	.00296800	5	13.44	1693.44	2,2,2,2, 10
.00003647	.00022781	.00049467	5	14.08	1674.24	2,2,2,2,5
.00000262	.00001519	.00002638	5	16.00	1696.00	2,2,2,2,2
.00003071	.00045563	.01236669	6	10.67	1657.60	2,2,2,5,5,10
.00001658	.00022781	.00494667	6	12.27	1675.73	2,2,2,2,5, 10
.00000338	.00003797	.00020611	6	12.80	1659.73	2,2,2,2,5,5
.00000119	.00001519	.00026382	6	13.87	1693.87	2,2,2,2,2, 10
.00000097	.00001012	.00004391	6	14.40	1677.87	2,2,2,2,2,5
.00000003	.00000028	.00000098	6	16.00	1696.00	2,2,2,2,2,2
.00000153	.00003797	.00206112	7	11.43	1663.09	2,2,2,2,5,5,10
.00000044	.00001012	.00043971	7	12.80	1678.63	2,2,2,2,2,5, 10
.00000009	.00000169	.00001832	7	13.26	1664.91	2,2,2,2,2,5,5
.00000001	.00000028	.00000977	7	14.17	1694.17	2,2,2,2,2,2, 10
.00000001	.06000019	.00000163	7	14.63	1680.46	2,2,2,2,2,2,5
.00000004	.00000169	.00018321	8	12.00	1667.20	2,2,2,2,2,5,5,10
.00000001	.00000019	.00001628	8	13.20	1680.80	2,2,2,2,2,2,5, 10
.0000000+	.00000003	.00000068	8	13.60	1668.80	2,2,2,2,2,2,5,5
.0000000+	.00000003	.00000679	9	12.44	1670.40	2,2,2,2,2,2,5,5,10

UNITY = UNITY = UNITY = TOTAL $\sum^{42} P$

EXPECTED NUMBER OF SAMPLE TREES

1.0000 1.6000 2.9091

$\sum^{42} p*N=\sum^{9} KPI/KZ=ESN$

POPULATION VALUES

9.0000 1658.00

$\sum^{41} (P*N*T)/ \sum^{41} P*N=\sum^{9} YI$

SYMBOLS AND LEGEND FOR ALL TABLES ARE THE SAME AS IN PRECEDING SECTION.

THE 3 DIFFERENT RATES OF 3P SAMPLING RESULT FROM USING VALUES OF 'KZ' EQUAL TO 32, 20, 11. FOR THE LISTED POPULATION THIS MAKES 'ESN' EQUAL TO 1.0, 1.6, 2.9091, AND 'ENZSN' EQUAL TO 1.4977, 1.8812, 2.9329. EXPECTED VALUE OF VARIANCE ABOUT 'ESN' IS .8301, 1.1650, 1.4611, WHILE EXPECTED VALUE OF VARIANCE ABOUT 'ENZSN' IS .4979, .8408, 1.4033. THE EXAMPLE EMPLOYED TO ILLUSTRATE THE EARLIER PAPER HAD 'ESN' EQUAL TO 1.6, WHILE THE MAXIMUM 'ESN' POSSIBLE WITHOUT BIAS IS 2.9091 UNLESS THE SCALE USED IN ESTABLISHING 'KPI' IS MADE LARGER.

THE TOTAL NUMBER OF POSSIBLE COMBINATIONS INVOLVING 9 THINGS (OF WHICH 6 ARE IDENTICAL, 2 IDENTICAL, AND 1 UNIQUE) TAKEN 0, 1, 2, ... 9 AT A TIME IS (6+1)*(2+1)*(1+1) = 42 COMBINATIONS. THE BINOMIAL PROBABILITIES FOR EACH OF THE 7 CELLS IN THE FIRST VECTOR, THE 3 IN THE SECOND, AND THE 2 IN THE THIRD MUST BE CALCULATED PRIOR TO OBTAINING THE 3-VECTOR KRONECKER PRODUCT, WHOSE 42 TERMS ARE THE DESIRED INDIVIDUAL PROBABILITIES FUR THE 42 POSSIBLE OUTCOMES. HAD EACH OF THE 9 TREES DIFFERED FROM THE OTHERS, THERE WOULD HAVE BEEN 2**9 = 512 PROBABILITIES IN THE KRONECKER PRODUCT. OBVIOUSLY, ONLY VERY SMALL POPULATIONS CAN BE ANALYZED IN THIS MANNER, EVEN ON HIGH-SPEED COMPUTERS.

TABLE 1 SHOWS THE 3 BASIC SETS OF PROBABILITIES, AND THE 42 POSSIBLE OUTCOMES FOR '3PSEVENTH' FREQUENCY AND VOLUME ESTIMATES, AS WELL AS THE 'KPI' OF INDIVIDUAL SAMPLE TREES COMPRISING EACH OUTCOME. FOR CONVENIENCE, OUTCOMES HAVE BEEN ARRAYED BY NUMBER OF SAMPLE TREES ('N'). ALL SUBSEQUENT TABLES CAN BE DERIVED FROM TABLE 1.

TABLE 2 SUMMARIZES PROBABILITIES FOR EACH 'N' WITH CORRESPONDING EXPECTED VALUES FOR VARIOUS FREQUENCY ESTIMATORS ('3PFIRST', '3PSECOND', '3PSEVENTH'). THE ACCUMULATED CROSS PRODUCT OF THE LAST TWO COLUMNS DEMONSTRATES THAT WEIGHTS OF N/(AGGREGATE N) ARE APPROPRIATE FOR COMBINING SEVERAL SETS OF '3PSEVENTH' ESTIMATES INTO A SINGLE ESTIMATE WHICH WILL TEND TOWARD POPULATION VALUE AS NUMBER OF SETS APPROACHES INFINITY.

TABLE 3 DOES FOR VOLUME WHAT TABLE 2 DID FOR FREQUENCY.

IT IS APPARENT THAT ALTHOUGH '3PFIRST' AND '3PSECOND' ESTIMATES ARE UNBIASED, VARIATION IS MUCH GREATER THAN FOR '3PSEVENTH'. IT IS ALSO APPARENT THAT FREQUENCY ESTIMATES ARE ALWAYS POSITIVELY CORRELATED WITH 'N', EVEN IN THE CASE OF '3PSEVENTH' ESTIMATES. THIS IS NECESSARILY SO BECAUSE VARIABLE 'KPI' CAN HAVE ONLY ZERO CORRELATION WITH CONSTANT UNIT FREQUENCY. HOWEVER, THE RELATIONSHIP BETWEEN 'N' AND '3PSEVENTH' ESTIMATES WILL ALWAYS BE SLIGHT WHEN CORRELATION EXISTS BETWEEN 'KPI' AND 'YI', AS IS USUALLY THE CASE WITH VOLUME, VALUE, ETC. IN SUCH SITUATIONS THE SLOPE OF THE RELATION IN A PARTICULAR ZONE MAY BE POSITIVE OR NEGATIVE DEPENDING ON ACCIDENTAL CIRCUMSTANCES NOT INFERRABLE FROM ANY SINGLE SET OF SAMPLES.

TABLE 4 REARRAYS TABLE 1 BY MAGNITUDE OF '3PSEVENTH' FREQUENCY ESTIMATES AND SQUARES THE DEVIATIONS FROM POPULATION VALUE. THIS SQUARED ERROR (WEIGHTED BY PROBABILITY) IS SUBSEQUENTLY SUMMED TO OBTAIN EXPECTED VALUE OF VARIANCE AMONG '3PSEVENTH' ESTIMATES, ALTHOUGH THE VALUE THUS OBTAINED ALSO INCLUDES ANY BIAS PRESENT. THE LAST COLUMN GIVES THE INTERNAL APPROXIMATION FOR '3PSEVENTH' VARIANCE BASED ON VARIANCE WITHIN A SINGLE SAMPLE ('3PNINTH').

TABLE 5 REARRAYS TABLE 1 BY MAGNITUDE OF '3PSEVENTH' VOLUME ESTIMATES AND DOES FOR VOLUME WHAT TABLE 4 DID FOR FREQUENCY.

TABLE 2. RELATION BETWEEN 'N' AND EXPECTED VALUES OF FREQUENCY ESTIMATES
USING SEVERAL DIFFERENT 3P ESTIMATORS AND RATES OF SAMPLING

```
=================================================================================
   3P    I SAMPLE I  PROB. OF   I  3PFIRST  I  3PSECOND I 3PSEVENTH I 3PSEVENTH
SAMPLING I  SIZE  I OCCURRENCE  I ESTIMATED I ESTIMATED I ESTIMATED I  WEIGHTS
 RATE=KZ I   =N   I     =P      I  FREQ.=T  I  FREQ.=T  I  FREQ.=T  I   =N*P
=========I========I============I===========I===========I===========I===========
   32          0      .33229816        .00      REDRAW      REDRAW     .00000000
               1      .40703727       8.35        5.57        8.35     .40703727
               2      .19913855      18.35       12.25        9.18     .39827710
               3      .05235882      29.62       19.78        9.87     .15707646
               4      .00828401      41.92       27.99       10.48     .03313604
               5      .00082820      55.01       36.73       11.00     .00414100
               6      .00005286      68.71       45.88       11.45     .00031716
               7      .00000208      82.79       55.28       11.83     .00001456
               8      .00000005      97.32       64.98       12.17     .00000041
               9      .0000000+     112.00       74.78       12.44     .0000000+
                  I------------I-----------I-----------I-----------I-----------
 EXPECTED VALUES =     UNITY          9.00        9.00        8.75   1.0000=ESN
 WEIGHTED VALUE OF Σ(P*N*3PSEVENTH)/Σ(P*N)          =          9.00
=========I========I============I===========I===========I===========I===========
   20          0      .14946778        .00      REDRAW      REDRAW     .00000000
               1      .34875816       4.86        4.13        7.77     .34875816
               2      .31000725      11.00        9.36        8.80     .62001450
               3      .14434199      18.03       15.33        9.62     .43302596
               4      .03977606      25.77       21.92       10.31     .15910423
               5      .00684957      34.04       28.95       10.89     .03424785
               6      .00074700      42.69       36.31       11.38     .00448200
               7      .00005025      51.60       43.89       11.79     .00035175
               8      .00000191      60.72       51.65       12.14     .00001528
               9      .00000003      70.00       59.54       12.44     .00000027
                  I------------I-----------I-----------I-----------I-----------
 EXPECTED VALUES =     UNITY          9.00        9.00        8.61   1.6000=ESN
 WEIGHTED VALUE OF Σ(P*N*3PSEVENTH)/Σ(P*N)          =          9.00
=========I========I============I===========I===========I===========I===========
   11          0      .00811378        .00      REDRAW      REDRAW     .00000000
               1      .10547918       1.69        1.68        4.92     .10547918
               2      .27308890       5.09        5.05        7.40     .54617780
               3      .31606468       9.06        8.98        8.77     .94819404
               4      .20054437      13.46       13.35        9.79     .80217748
               5      .07614334      18.17       18.02       10.57     .38071670
               6      .01782824      23.09       22.90       11.20     .10696940
               7      .00253055      28.14       27.91       11.69     .01771385
               8      .00020017      33.28       33.01       12.10     .00160135
               9      .00000679      38.50       38.19       12.44     .00006111
                  I------------I-----------I-----------I-----------I-----------
 EXPECTED VALUES =     UNITY          9.00        9.00        8.38   2.9091=ESN
 WEIGHTED VALUE OF Σ(P*N*3PSEVENTH)/Σ(P*N)          =          9.00
=========I========I============I===========I===========I===========I===========
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TABLE 3. RELATION BETWEEN 'N' AND EXPECTED VALUES OF VOLUME ESTIMATES USING
SEVERAL DIFFERENT 3P ESTIMATORS AND RATES OF SAMPLING

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==============================================================================
   3P    I SAMPLE I   PROB. OF   I   3PFIRST   I  3PSECOND  I 3PSEVENTH  I 3PSEVENTH
SAMPLING I  SIZE  I OCCURRENCE I ESTIMATEC  I ESTIMATED I ESTIMATED I  WEIGHTS
 RATE=KZ I   =N   I     =P       I   VOL.=T    I   VOL.=T   I   VOL.=T   I   =N*P
=========I========I============I===========I===========I===========I===========
   32         C      .33229816          .0      REDRAW       REDRAW    .00000000
              1      .40703727      1658.4      1107.3       1658.4    .40703727
              2      .19913855      3314.6      2213.2       1657.3    .39827710
              3      .05235882      4974.8      3321.7       1658.3    .15707646
              4      .00828401      6640.7      4434.0       1660.2    .03313604
              5      .00082820      8312.0      5550.0       1662.4    .00414100
              6      .00005286      9987.8      6668.9       1664.6    .00031716
              7      .00000208     11666.8      7790.0       1666.7    .00001456
              8      .00000005     13349.7      8913.6       1668.7    .00000041
              9      .0000000+     15033.6     10038.0       1670.4    .0000000+
                  I------------I-----------I-----------I-----------I-----------
 EXPECTED VALUES =     UNITY       1658.0      1658.0       1658.1    1.0000=ESN
 WEIGHTED VALUE OF Σ(P*N*3PSEVENTH)/Σ(P*N)       =          1658.0
=========I========I============I===========I===========I===========I===========
   20         0      .14946778          .0      REDRAW       REDRAW    .00000000
              1      .34875816      1037.1       882.1       1659.4    .34875816
              2      .31000725      2071.1      1761.6       1656.9    .62001450
              3      .14434199      3107.8      2643.3       1657.5    .43302596
              4      .03977606      4148.6      3528.6       1659.5    .15910423
              5      .00684957      5193.3      4417.1       1661.9    .03424785
              6      .00074700      6241.0      5308.2       1664.3    .00448200
              7      .00005025      7291.0      6201.3       1666.5    .00035175
              8      .00000191      8342.9      7095.9       1668.6    .00001528
              9      .00000003      9396.0      7991.6       1670.4    .00000027
                  I------------I-----------I-----------I-----------I-----------
 EXPECTED VALUES =     UNITY       1658.0      1658.0       1658.2    1.6000=ESN
 WEIGHTED VALUE OF Σ(P*N*3PSEVENTH)/Σ(P*N)       =          1658.0
=========I========I============I===========I===========I===========I===========
   11         C      .00811378          .0      REDRAW       REDRAW    .00000000
              1      .10547918       574.8       570.2       1672.2    .10547918
              2      .27308890      1139.5      1130.3       1657.5    .54617780
              3      .31606468      1707.2      1693.4       1655.5    .94819404
              4      .20054437      2279.0      2260.5       1657.4    .80217748
              5      .07614334      2853.7      2830.5       1660.3    .38071670
              6      .01782824      3430.4      3402.6       1663.2    .10696940
              7      .00253055      4008.6      3976.1       1665.9    .01771385
              8      .00020017      4587.8      4550.6       1668.3    .00160135
              9      .00000679      5167.8      5125.9       1670.4    .00006111
                  I------------I-----------I-----------I-----------I-----------
 EXPECTED VALUES =     UNITY       1658.0      1658.0       1658.7    2.9091=ESN
 WEIGHTED VALUE OF Σ(P*N*3PSEVENTH)/Σ(P*N)       =          1658.0
=========I========I============I===========I===========I===========I===========
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TABLE 4. SQUARED DEVIATIONS OF '3PSEVENTH' FREQUENCY ESTIMATES FROM POPULATION VALUE, WITH '3PNINTH' APPROXIMATIONS USING VARIATION WITHIN SAMPLE

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=================================================================================
P=PROBABILITIES OF OCCURRENCE  I SAMPLE I 3PSEVENTH I  SQUARED   I APPROX.VAR.
      IF KZ EQUAL TO           I  SIZE  I ESTIMATED I DEVIATION  I  (3PNINTH
   32          20         11   I   =N   I  T=FREQ.  I (T-POP T)  I   ABSOLUTE)
===============================I========I===========I===========I============
 .33229816 .14946778 .00811378     0        REDRAW      REDRAW       REDRAW
 .15104461 .14946778 .08113783     1          3.20     33.6400         NONE
 .05594245 .09964519 .13522972     2          4.80     17.6400       2.5600
 .00517986 .01660753 .05634572     3          5.33     13.4689       1.0378
 .12307340 .09964519 .01352297     1          6.40      6.7600         NONE
 .01139569 .01660753 .00563457     2          6.40      6.7600        .0000
 .00207194 .01107169 .07512762     4          8.00      1.0000       7.6800
 .02237698 .06643012 .18030629     3          8.53       .2209      14.7911
 .06041784 .09964519 .10818377     2          9.60       .3600      40.9600
 .00455827 .01107169 .00751276     3          9.60       .3600      10.2400
 .00034532 .00307547 .04173757     5          9.60       .3600       7.1680
 .00372950 .01845281 .10017016     4         10.40      1.9600      10.8800
 .00003071 .00045563 .01236669     6         10.67      2.7889       5.9164
 .04922936 .06643012 .01803063     2         11.20      4.8400      23.0400
 .00075971 .00307547 .00417376     4         11.20      4.8400       7.6800
 .00000153 .00003797 .00206112     7         11.43      5.9049       4.8065
 .00033151 .00273375 .02968005     5         11.52      6.3504       7.7824
 .01006964 .02767922 .06010210     3         11.73      7.4529      18.2044
 .00000004 .00000169 .00018321     8         12.00      9.0000       3.9314
 .00006753 .00045563 .00123667     5         12.16      9.9856       5.5296
 .00001658 .00022781 .00494667     6         12.27     10.6929       5.7458
 .0000000+ .00000003 .00000679     9         12.44     11.8336       3.2553
 .00820490 .01845281 .01001701     3         12.80     14.4400      10.2400
 .00089508 .00410062 .01780803     4         12.80     14.4400      10.2400
 .00000338 .00003797 .00020611     6         12.80     14.4400       4.0960
 .00000044 .00001012 .00043971     7         12.80     14.4400       4.3886
 .00000001 .00000019 .00001628     8         13.20     17.6400       3.4514
 .00000009 .00000169 .00001832     7         13.26     18.1476       3.1347
 .00004475 .00034172 .00296800     5         13.44     19.7136       6.5536
 .00072932 .00273375 .00296800     4         13.60     21.1600       5.7600
 .0000000+ .00000003 .00000068     8         13.60     21.1600       2.4686
 .00000119 .00001519 .00026382     6         13.87     23.7169       4.5511
 .00003647 .00022781 .00049467     5         14.08     25.8064       3.6864
 .00000001 .00000028 .00000977     7         14.17     26.7289       3.3437
 .00000097 .00001012 .00004397     6         14.40     29.1600       2.5600
 .00000001 .00000019 .00000163     7         14.63     31.6969       1.8808
 .13291926 .09964519 .01081838     1         16.00     49.0000         NONE
 .02215321 .02767922 .00601021     2         16.00     49.0000        .0000
 .00196917 .00410062 .00178080     3         16.00     49.0000        .0000
 .00009846 .00034172 .00029680     4         16.00     49.0000        .0000
 .00000262 .00001519 .00002638     5         16.00     49.0000        .0000
 .00000003 .00000028 .00000098     6         16.00     49.0000        .0000
========================================I====================================
                                         42
  UNITY  =  UNITY  =  UNITY = TOTAL Σ P  I
  EXPECTED NUMBER OF SAMPLE TREES        I
  1.0000    1.6000    2.9091             I
     42      3
     ΣP*N=ΣKPI/KZ=ESN                    I
```

TABLE 5. SQUARED DEVIATIONS OF '3PSEVENTH' VOLUME ESTIMATES FROM POPULATION VALUE, WITH '3PNINTH' APPROXIMATIONS USING VARIATION WITHIN SAMPLE

P=PROBABILITIES OF OCCURRENCE IF KZ EQUAL TO			SAMPLE SIZE =N	3PSEVENTH ESTIMATED T=VOL.	SQUARED DEVIATION (T-POP T)	APPROX.VAR. (3PNINTH ABSOLUTE)
32	20	11				
.33229816	.14946778	.00811378	0	REDRAW	REDRAW	REDRAW
.12307340	.09964519	.01352297	1	1587.20	5012.64	NONE
.01139569	.01660753	.00563457	2	1587.20	5012.64	.00
.00517986	.01660753	.05634572	3	1619.20	1505.44	1024.00
.00455827	.01107169	.00751276	3	1623.47	1192.32	1315.27
.05594245	.09964519	.13522972	2	1635.20	519.84	2304.00
.00207194	.01107169	.07512762	4	1638.40	384.16	880.64
.04922936	.06643012	.01803063	2	1641.60	268.96	2959.36
.00075971	.00307547	.00417376	4	1641.60	268.96	986.45
.00034532	.00307547	.04173757	5	1649.92	65.29	661.09
.00006753	.00045563	.00123667	5	1652.48	30.47	710.25
.02237698	.06643012	.18030629	3	1655.47	6.40	1178.74
.00003071	.00045563	.01236669	6	1657.60	.16	499.71
.00820490	.01845281	.01001701	3	1659.73	2.99	1315.27
.00000338	.00003797	.00020611	6	1659.73	2.99	526.11
.00000153	.00003797	.00206112	7	1663.09	25.91	387.03
.00000009	.00000169	.00001832	7	1664.91	47.75	402.63
.00372950	.01845281	.10017016	4	1665.60	57.76	692.05
.00000004	.00000169	.00018321	8	1667.20	84.64	307.20
.00072932	.00273375	.00296800	4	1668.80	116.64	739.84
.0000000+	.00000003	.00000068	8	1668.80	116.64	317.07
.0000000+	.00000003	.00000679	9	1670.40	153.76	249.17
.00033151	.00273375	.02968005	5	1671.68	187.14	452.20
.00003647	.00022781	.00049467	5	1674.24	263.74	473.50
.00001658	.00022781	.00494667	6	1675.73	314.35	317.90
.00000097	.00001012	.00004397	6	1677.87	394.82	328.82
.00000044	.00001012	.00043971	7	1678.63	425.60	235.45
.00000001	.00000019	.00000163	7	1680.46	504.45	241.58
.00000001	.00000019	.00001628	8	1680.80	519.84	181.30
.15104461	.14946778	.08113783	1	1683.20	635.04	NONE
.06041784	.09964519	.10818377	2	1689.60	998.56	40.96
.01006964	.02767922	.06010210	3	1691.73	1137.71	18.21
.00089508	.00410062	.01780803	4	1692.80	1211.04	10.24
.00004475	.00034172	.00296800	5	1693.44	1255.99	6.55
.00000119	.00001519	.00026382	6	1693.87	1286.66	4.55
.00000001	.00000028	.00000977	7	1694.17	1308.27	3.34
.13291926	.09964519	.01081838	1	1696.00	1444.00	NONE
.02215321	.02767922	.00601021	2	1696.00	1444.00	.00
.00196917	.00410062	.00178080	3	1696.00	1444.00	.00
.00009846	.00034172	.00029680	4	1696.00	1444.00	.00
.00000262	.00001519	.00002638	5	1696.00	1444.00	.00
.00000003	.00000028	.00000098	6	1696.00	1444.00	.00

UNITY = UNITY = UNITY = TOTAL $\sum^{42}P$

EXPECTED NUMBER OF SAMPLE TREES

1.0000 1.6000 2.9091

$\sum^{42}P*N=\sum^{9}KPI/KZ=ESN$

TABLE 6. EXPECTED VALUES OF VARIANCE AMONG NONZERO '3PSECOND' AND '3PSEVENTH' ESTIMATES, COMPARED WITH EXPECTED ABSOLUTE VALUES OF TWO APPROXIMATIONS FOR VARIANCE AMONG '3PSEVENTH' ESTIMATES. THE FIRST ('3PNINTH') IS DERIVED FROM EXPECTED INTERNAL VARIANCE WITHIN '3PSEVENTH' SAMPLES, WHILE THE SECOND ('3PEIGHTH') IS DERIVED FROM EXPECTED INTERNAL VARIANCE WITHIN THE POPULATION WHEN SAMPLED WITH VARIABLE PROBABILITY AND REPLACEMENT.

SAMPLE SIZE =ENZSN (N VARIES IF 3P)	EXPECTED VARIANCE AMONG 3PSECOND ESTIMATES	EXPECTED VARIANCE AMONG 3PSEVENTH ESTIMATES	VARIANCE APPROXIMATIONS	
			N VARIES,NO REPLACE. (3PNINTH)	N=ENZSN, REPLACE. (3PEIGHTH)
	AMONG FREQUENCY ESTIMATES			
1.4977	41.84	22.84	17.20	20.70
1.8812	39.76	15.97	16.13	16.48
2.9329	28.59	8.78	13.31	10.57
	AMONG VOLUME ESTIMATES			
1.4977	611502.9	1687.8	1273.2	1540.0
1.8812	654005.4	1316.9	1188.9	1226.0
2.9329	452001.3	598.3	966.5	786.4

N.B. HAD FAILURE OF SELECTION PROCESS TO OBTAIN ANY SAMPLES BEEN SPECIFIED AS EQUIVALENT TO ZERO ESTIMATE (WITHOUT REDRAW), EXPECTED VARIANCES AMONG SUCH '3PFIRST' FREQUENCY ESTIMATES WOULD BE 102.98, 60.98, 29.49 FOR 'ESN'=1.0, 1.6, 2.9091 RESPECTIVELY. CORRESPONDING EXPECTED VOLUME VARIANCES WOULD BE 2283921.4, 1252024.3, 478185.7.

TABLE 6 SUMMARIZES THE EXPECTED VALUES OF VARIANCES AMONG '3PSECOND' AND '3PSEVENTH' ESTIMATES (A FOOTNOTE GIVES '3PFIRST' VARIANCES). THEN EXPECTED VALUES FOR 2 APPROXIMATIONS OF '3PSEVENTH' VARIANCE ARE GIVEN. THE FIRST ('3PNINTH') IS CONVENIENT WHEN ONLY INTERNAL EVIDENCE FROM A SINGLE SET OF '3PSEVENTH' SAMPLES IS AVAILABLE. THE SECOND ('3PEIGHTH') IS CONVENIENT WHEN ALL VALUES FOR INDIVIDUAL 'YI' AND 'KPI' IN THE POPULATION ARE KNOWN, BUT WHEN IT IS IMPRACTICAL TO COMPUTE EXPECTED VALUES OF VARIANCE BECAUSE OF THE ASTRONOMICAL NUMBER OF POSSIBLE OUTCOMES.

IT IS INTERESTING TO NOTE FROM TABLES 4, 5, 6 THAT EVEN WITH A RIDICULOUSLY SMALL POPULATION AND RELATIVELY LARGE VALUES FOR KPI/KZ AND ENZSN/M, EXPECTED VALUES OF '3PNINTH' AND '3PEIGHTH' ARE USEFUL APPROXIMATIONS FOR THE EXPECTED VARIANCE AMONG '3PSEVENTH' ESTIMATES. WHEN 'KZ' IS EQUAL TO 32, 20, 11, OBSERVED SQUARED ERRORS OF '3PSEVENTH' FREQUENCY ESTIMATES EXCEED EXPECTED VARIANCES 46, 45, 33 PERCENT OF THE TIME. OBSERVED SQUARED FREQUENCY ERRORS EXCEED '3PNINTH' APPROXIMATIONS 54, 45, 33 PERCENT OF THE TIME. WHEN VOLUME REPLACES FREQUENCY, EXCEEDANCES OF EXPECTED VARIANCES OCCUR 0, 311 38 PERCENT OF THE TIME, WHILE EXCEEDANCES OF '3PNINTH' APPROXIMATIONS OCCUR 44, 33, 29 PERCENT OF THE TIME. IN MOST CASES, '3PEIGHTH' APPROXIMATIONS ARE SLIGHTLY CLOSER TO EXPECTED VALUES THAN ARE '3PNINTH' APPROXIMATIONS.

IT IS ALSO INTERESTING TO NOTE IN TABLES 4 AND 5 THAT BOTH FREQUENCY AND VOLUME ESTIMATES TEND TO BE SOMEWHAT SKEWED. WHEN 'KZ' IS EQUAL TO 32, 20, 11, THEN '3PSEVENTH' FREQUENCY ESTIMATES BELOW POPULATION VALUE OCCUR IN 56, 54, 55 PERCENT OF THE CASES, RESPECTIVELY. '3PSEVENTH' VOLUME ESTIMATES ARE BELOW POPULATION VALUE IN 41, 46, 56 PERCENT OF THE CASES. THE MAGNITUDE AND DIRECTION OF SEPARATION BETWEEN POPULATION VALUE AND MEDIAN (I. E., THE FIFTIETH PERCENTILE) OBVIOUSLY DEPENDS BOTH ON SAMPLING RATE AND RELATION

BETWEEN 'KPI' AND 'YI'. SUCH ASYMMETRY USUALLY DIMINISHES AS 'ENZSN' INCREASES AND SHOULD NOT BE CONFUSED WITH BIAS.

A NUMBER OF INFERENCES BASED ON THE TWO PRECEDING SECTIONS CAN NOW BE SUMMARIZED.

(1) '3PFIRST' AND '3PSECOND' ESTIMATES ARE UNBIASED, BUT TEND TO DEVIATE MUCH MORE FROM POPULATION VALUES THAN DO '3PSEVENTH' ESTIMATES.

(2) '3PSEVENTH' ESTIMATES MAY INVOLVE A SMALL BIAS THAT COMPLETELY DISAPPEARS AS N/ENZSN APPROACHES UNITY OR AS VARIANCE OF YI/KPI APPROACHES ZERO.

(3) THE ONLY PRACTICABLE METHOD FOR OBTAINING UNBIASED ESTIMATES OF THE VARIANCE OF 3P SAMPLES FROM LARGE POPULATIONS IS TO OBTAIN INDEPENDENT, INTERPENETRATING SETS OF SAMPLES.

(4) WHEN SUCH INDEPENDENT SETS OF 3P SAMPLES HAVE BEEN SECURED AND WHEN, IN ADDITION, THE AGGREGATE SUM OF 'KPI' IS KNOWN FOR THE POPULATION, ESTIMATES CAN BE ADJUSTED FOR LARGE DIFFERENCES BETWEEN 'N' AND 'ENZSN' IF STRONG CORRELATION BETWEEN 'N' AND 3P ESTIMATES WARRANTS.

(5) WHEN SUCH INDEPENDENT SETS OF 3P SAMPLES ARE LACKING, '3PNINTH' AFFORDS A USEFUL APPROXIMATION OF THE VARIANCE OF '3PSEVENTH' ESTIMATES.

(6) THE SQUARED DIFFERENCE BETWEEN '3PSEVENTH' ESTIMATES AND POPULATION VALUES APPEARS TO BE SMALLER THAN THE EXPECTED OR THE APPROXIMATE VARIANCES IN ROUGHLY 2/3 OF THE CASES WHEN 'ENZSN' IS AT LEAST 3, THOUGH SOME ASYMMETRY IS APPARENT.

(7) MAGNITUDE AND DIRECTION OF SKEWNESS OF 3P ESTIMATES DEPEND ON 'KZ', THE SUM AND DISTRIBUTION OF 'KPI', AND THE RELATION BETWEEN 'KPI' AND 'YI'. SKEWNESS IS NOT SYNONYMOUS WITH BIAS.

================= SOME CONSIDERATIONS IN 3P-SAMPLE DESIGN ====================

IT WILL BE ASSUMED HENCEFORTH THAT IN MOST PRACTICAL SITUATIONS, 'ESN' AND 'ENZSN' ARE SO NEARLY IDENTICAL AS TO MAKE DISTINCTION UNNECESSARY, AND 'ESN' WILL BE USED EVEN THOUGH REDRAW WOULD USUALLY BE SPECIFIED WHEN N=0.

DESIGN OF 3P SAMPLES MAY BE SLIGHTLY INFLUENCED BY 'C', THE COEFFICIENT OF VARIATION AMONG 'KPI' IN THE POPULATION, AND BY 'M', THE NUMBER OF TREES IN THE POPULATION, BUT ASSUMING 'C' TO BE ZERO AND 'M' TO BE THE NEAREST ROUND NUMBER SUCH AS 1000, 10000, ETC. IS USUALLY SAFE.

THERE ARE, HOWEVER, TWO OTHER POPULATION PARAMETERS THAT MUST BE TENTATIVELY GUESSED IN ADVANCE WITH MORE ACCURACY IF 3P SAMPLING IS TO ACHIEVE DESIRED PRECISION WITH TOLERABLE RISK.

PROBABLY THE MOST IMPORTANT GUESS IS FOR $\sum^{M}$KPI, WHICH SHOULD BE WITHIN 20-30 PERCENT OF ITS ULTIMATE VALUE. IF A PRELIMINARY DIAGNOSTIC. TALLY (*1) FOR EACH SALE AREA HAS BEEN MADE BY HORIZONTAL POINT-SAMPLING, THIS INFORMATION IS READILY AVAILABLE WITH ADEQUATE ACCURACY. SUCH RECONNAISSANCE REQUIRES ONLY ONE OR TWO MAN-DAYS OF EFFORT EVEN FOR SEVERAL THOUSAND ACRES.

THE OTHER IMPORTANT PARAMETER THAT MUST BE GUESSED IS THE COEFFICIENT OF VARIATION 'CV' OF THE PROPERLY WEIGHTED RATIOS YI/KPI APPROPRIATE TO THE PARTICULAR MEN WHO WILL BE ASSESSING 'KPI' WHEN THE TREES ARE MARKED FOR HARVEST. THIS DEPENDS ON THEIR SKILL, TRAINING, AND TALENT, BUT REASONABLY CONSCIENTIOUS ASSESSORS SHOULD HAVE NC DIFFICULTY IN ACHIEVING A 20 TO 30 PERCENT COEFFICIENT, AND TALENTED ASSESSORS HAVE BEEN ABLE TO ACHIEVE 15 PERCENT OR LESS. EXPERIENCE WITH PREVIOUS PERFORMANCE ALLOWS SPECIFYING A

'CV' LIKELY TO BE ACHIEVED. THE MOST CONVENIENT APPROXIMATION INVOLVES TAKING THE SQUARE ROOT OF N*3PNINTH FOR A 3P SAMPLE PREVIOUSLY COMPLETED BY THE PARTICULAR ASSESSORS.

WITH THESE GUESSES OR ASSUMPTIONS, THE FIRST STEP IN DESIGNING A '3PSEVENTH' SAMPLE IS TO COMPUTE 'ESN' THAT IS NEEDED FOR THE MAXIMUM TOLERABLE STANDARD ERROR 'TSE'. IF STANDARD ERROR OF PLUS OR MINUS 2 PERCENT CAN BE TOLERATED AND IF THE COEFFICIENT OF VARIATION FOR YI/KPI IS 20 PERCENT, TENTATIVE ESN = (CV/TSE)**2 = 100 TREES. THIS WILL USUALLY BE INCREASED BY ONE OR TWO TIMES THE STANDARD DEVIATION OF TENTATIVE 'ESN' TO DECREASE POSSIBILITY THAT 'N' LESS THAN TENTATIVE 'ESN' WOULD OCCUR. THUS, TENTATIVE 'ESN' MIGHT BE INCREASED TO 110 BY ADDING THE SQUARE ROOT OF '3PFIFTH'.

NOW THE NEW 'ESN' CAN BE USED IN CONJUNCTION WITH GUESSED $\sum^{M}$KPI TO OBTAIN RATE OF 3P SAMPLING, KZ = $\sum^{M}$KPI/ESN. IF $\sum^{M}$KPI WERE GUESSED AT 11000 (DOLLARS, HUNDREDS OF BOARD FEET, ETC.), AND IF 'ESN' WERE SPECIFIED AS 110 TREES, THEN KZ = 100 WOULD RESULT IN THE DESIRED RATE OF 3P SAMPLING.

HOWEVER, THIS DOES NOT SPECIFY WHAT TO DO IN THE CASE OF TREES ASSIGNED A 'KPI' EQUAL TO CR GREATER THAN 'KZ', NOR DOES IT GUARD AGAINST ANOTHER CONTINGENCY--SECURING SO MANY SAMPLE TREES THAT THE BLOW-UP FACTOR FOR THE LARGEST TREES SAMPLED BECOMES LESS THAN UNITY. THE AUTHOR HAS DEVISED A PRACTICAL SOLUTION TO THESE TWO RELATED PROBLEMS IN DESIGN. JUSTIFICATION FOR THIS SOLUTION PRESUMES AN UNDERSTANDING OF THE UNDERLYING PRINCIPLES DISCUSSED BELOW.

IF A FIXED NUMBER OF SAMPLES 'N' BE DRAWN WITH VARIABLE PROBABILITY AND WITHOUT REPLACEMENT, THE SAMPLE ESTIMATE WILL BE BIASED UNLESS 'PI' (EQUAL TO KPI/$\sum^{M}$KPI) FOR THE LARGEST INDIVIDUAL PROBABILITY IN THE POPULATION IS

SMALLER THAN 1/N. THE REASON IS THAT ESTIMATION FROM PROBABILITY SAMPLES ASSUMES THAT THE 'I'TH INDIVIDUAL WILL BE REPRESENTED BY SAMPLES PROPORTIONATELY TO ITS 'PI', YET IT IS IMPOSSIBLE FOR ONE INDIVIDUAL TO CONSTITUTE MORE THAN (1/N)TH PORTION OF ANY SINGLE SET OF 'N' SAMPLES DRAWN WITHOUT REPLACEMENT. FROM THIS, IT FOLLOWS THAT (N) * (MAX KPI) MUST BE SMALLER THAN $\sum^{M}$KPI FOR THE SAMPLE ESTIMATE TO BE UNBIASED.

IF A VARIABLE NUMBER 'N' OF 3P SAMPLES BE DRAWN WITHOUT REPLACEMENT, THE SAMPLE ESTIMATE WILL BE BIASED UNLESS KPI/KZ FOR THE LARGEST PROBABILITY IN THE POPULATION IS SMALLER THAN UNITY. THE REASON IS THAT ESTIMATION FROM 3P SAMPLES ASSUMES THAT KPI/KZ IS THE EXPECTED VALUE OF THE NUMBER OF APPEARANCES OF THE 'I'TH INDIVIDUAL IN A SINGLE SET OF 3P SAMPLES, YET IT IS IMPOSSIBLE FOR THE 'I'TH INDIVIDUAL TO APPEAR MORE THAN ONCE IN ANY SINGLE SET OF SAMPLES DRAWN WITHOUT REPLACEMENT. FROM THIS AND THE FACT THAT ESN = $\sum^{M}$KPI/KZ, IT FOLLOWS THAT (ESN) * (MAX KPI) MUST BE SMALLER THAN $\sum^{M}$KPI FOR THE SAMPLE TO BE UNBIASED. THIS IS THE 3P ANALOG OF THE FIXED-SAMPLE INEQUALITY DEVELOPED IN THE PRECEDING PARAGRAPH.

IT IS, MOREOVER, DESIRABLE THAT (N) * (MAX KPI) BE SMALLER THAN $\sum^{M}$KPI IN 3P SAMPLING, EVEN THOUGH ESTIMATES WHERE THIS CONDITION IS VIOLATED ARE NOT BIASED IN THE STRICT SENSE OF THE WORD (SINCE INDIVIDUALS UNDER-REPRESENTED WHEN 'N' IS LARGE ARE OVER-REPRESENTED WHEN 'N' IS SMALL).

TO PROVIDE A SAFEGUARD AGAINST THE DESIGNER'S IMPERFECT KNOWLEDGE OF THE DISTRIBUTION OF 'KPI' IN THE POPULATION, AND TO REDUCE THE ERRATIC EFFECTS OF SAMPLING OR REJECTING A FEW LARGE TREES WITH 'KPI' JUST BARELY SMALLER THAN 'KZ', IT IS DESIRABLE TO RESTRICT THE POPULATION SAMPLED TO SOMETHING LESS THAN THE COMPLETE POPULATION. WHEN THIS IS DONE, EVERY TREE IN THE EXCLUDED PORTION MUST BE MEASURED, AND THE ERROR-FREE TOTAL OF THESE (WITHOUT

INFLATION) MUST BE ADDED TO THE ESTIMATE FOR THE SAMPLED PORTION.

IT NOW BECOMES CONVENIENT TO SPLIT 'KZ' INTO TWO ADDITIVE PARTS 'K' AND 'Z' SUCH THAT (K+Z) = KZ. THE PROGRESSION OF INTEGERS STOPS WITH 'K', AND THE PROBABILITY SET IS COMPLETED WITH Z=(KZ-K) NULLS, WHICH CAN BE REPRESENTED BY MINUS ZEROS (-0) OR DOUBLE X'S (XX). ALL TREES ASSIGNED 'KPI' LARGER THAN 'K' MUST BE MEASURED WITHOUT RECOURSE TO ANY RANDOM SAMPLING PROCESS. TREES ASSIGNED 'KPI' EQUAL TO OR SMALLER THAN 'K' WILL BE SUBJECTED TO THE USUAL 3P-RANDOM-INTEGER COMPARISON, BUT WHEN PAIRED WITH NULL ELEMENTS WILL ALWAYS BE REJECTED AS SAMPLES. NO TREE CAN BE ASSIGNED A 'KPI' SMALLER THAN UNITY.

THE USE OF NULLS FACILITATES GUARDING AGAINST 'N' EXCEEDING THRESHOLD NB = $\sum^{M}$KPI/K, EVEN THOUGH SUCH SAMPLES WOULD NOT BE BIASED IN THE STRICT SENSE OF THE WORD. THERE IS A RATIO Z/K THAT SATISFIES THE EQUALITY (NB - ESN)**2 = 3PFIFTH. ASSUME C = 0 AS THE SAFE BINOMIAL UPPER BOUND FOR THE POISSON SERIES. SUBSTITUTING IDENTITIES GIVES (ESN * Z/K)**2 = ESN - (1/M) * ESN**2. IF 4 TIMES THE RIGHT-HAND SIDE OF THE EQUALITY IS USED AS THE LOWER BOUND FOR A SINGLE EXCLUDED TAIL INVOLVING PROBABILITIES AGGREGATING 1/40, AND IF IT BE RECOGNIZED THAT WE WOULD BE STILL HAPPIER IF THE RIGHT-HAND SIDE WERE SMALLER THAN THE LEFT-HAND SIDE, THEN IT CAN BE STATED THAT 'N' WILL EXCEED 'NB' LESS THAN 1 TIME IN 40 IF

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3PTENTH = (Z/K)**2 SHOULD EXCEED 4 * (1/ESN - 1/M)
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THUS, AFTER HAVING DETERMINED 'KZ' THAT GIVES APPROXIMATELY THE DESIRED SAMPLING RATE IF THE ENTIRE POPULATION OF 'M' TREES INVOLVING $\sum^{M}$KPI COULD BE 3P-SAMPLED WITHOUT BIAS, A SMALLER LESS VARIABLE POPULATION OF TREES WITH MAXIMUM KPI=K IS DEFINED, AND ALL LARGER TREES ARE MEASURED. THIS ORDINARILY RESULTS IN MEASURING SOMEWHAT MORE TREES THAN WAS ORIGINALLY ANTICIPATED, BUT

WITH CONSIDERABLY LESS SAMPLING ERROR FOR THE TOTAL, SINCE THE SURE-TO-BE MEASURED TREES HAVE NO SAMPLING ERROR, YET OFTEN INVOLVE CONSIDERABLE VOLUME.

IF THE SAMPLE DESIGN SPECIFIES SEVERAL INDEPENDENT INTERPENETRATING SETS OF SAMPLES INSTEAD OF A SINGLE SET, THE TOTAL NUMBER OF SAMPLES NEEDED TO GET THE DESIRED 'TSE' MUST BE DIVIDED BY THE PLANNED NUMBER OF SETS BEFORE CALCULATION OF 'KZ' AND 'K', SINCE THESE PARAMETERS GOVERN EACH INDIVIDUAL SET.

IT IS OBVIOUS THAT THE TECHNIQUE OUTLINED ABOVE FOR DESIGNING 3P SAMPLES INVOLVES EDUCATED GUESSES AND APPROXIMATE ANSWERS, YET EXPERIENCE HAS SHOWN THAT SPECIFICATION OF 'KZ' AND 'K' FOLLOWING THE PROCEDURES DISCUSSED LEADS TO ESTIMATES THAT ACHIEVE THE DESIRED OBJECTIVE IF THE EDUCATED GUESSES DO NOT ERR BADLY. CUSHIONS AGAINST BAD LUCK AND IMPERFECT KNOWLEDGE HAVE BEEN INCORPORATED INTO THE CALCULATIONS OF BOTH 'KZ' AND 'K'.

THE EXPECTED VALUE OF $\sum^{M}$KPI FOR A 3P SAMPLE CAN BE CALCULATED EXACTLY AS (KZ).(ESN - EXPECTED VARIANCE OF ESN), WHICH HELPS FLAG SAMPLES DEVIATING EXCESSIVELY FROM EXPECTATION EVEN WHEN VARIANCE CAN ONLY BE APPROXIMATED.

THE USE OF 3P SAMPLES WHERE THE $\sum^{N}$KPI IS NOT COMPLETELY ACCUMULATED DURING THE SAMPLING PROCESS WILL NOT BE DISCUSSED AT LENGTH. OPPORTUNITIES OF THIS SORT ARISING IN FORESTRY INVOLVE 3P SUBSAMPLING OF PRIMARY LIST, AREA, OR CLUSTER SAMPLES SELECTED WITH KNOWN PROBABILITIES OFTEN UNRELATED TO THE AGGREGATE OF INDIVIDUALS COMPRISING THE PRIMARY UNIT. SINCE THE SAMPLING VARIANCE CAN NEVER BE LESS THAN VARIANCE AMONG THE PRIMARY UNITS WHEN COMPLETELY MEASURED, THE ADVANTAGE OF 3P SUBSAMPLING WOULD CONSIST ONLY OF A GREAT REDUCTION IN COST WITH POSSIBLY ONLY A MODERATE INCREASE IN VARIATION AMONG ESTIMATES OF PRIMARY SAMPLING UNITS. AN EXAMPLE MIGHT BE DENDROMETRY APPLIED TO A 3P SAMPLE OF TREES ON ONE-ACRE PLOT-SAMPLES.

================== 3P-RANDOM-INTEGER GENERATION BY COMPUTER ====================

THE BASIC PROBABILITY SET FOR A 3P SAMPLE WAS EXEMPLIFIED BY A DECK OF CARDS CONTAINING KZ = (K+Z) ELEMENTS IN THE AUTHOR'S EARLIER PAPER (*2). THIS WAS DONE 10 SIMPLIFY EXPOSITION, ALTHOUGH THE PAPER HINTED AT MORE SUITABLE MECHANISMS.

FUR FIELD USE, ESPECIALLY WHERE (K+Z) MIGHT CONTAIN HUNDREDS OR THOUSANDS OF ELEMENTS, LISTS OF RANDOM INTEGERS ARE THE OBVIOUS ANSWER. HOWEVER, USING SUCH LISTS AS PUBLISHED WOULD BE INTOLERABLE IN 3P SAMPLING, SINCE NUMBERS LARGER THAN 'KZ' WOULD HAVE TO BE MENTALLY SKIPPED, AND SINCE THE USER WOULD HAVE TO REMEMBER THAT NUMBERS BETWEEN 'K' AND 'KZ' REPRESENTED NULLS, NOT ACTUAL NUMBERS.

JUST AS COMPUTATION OF 3P-SAMPLE ESTIMATES WOULD HAVE BEEN IMPRACTICAL BEFORE THE DAY OF HIGH-SPEED COMPUTERS, SO WOULD THE PREPARATION OF INDIVIDUAL TAILOR-MADE LISTS OF RANDOM NUMBERS WITH SPECIFIED MAXIMUM AND WITH NULLS INTERSPERSED WITH NUMBERS IN THE RATIO Z/K. HIGH-SPEED COMPUTER PROGRAM 'STX' ACCOMPLISHING THE FIRST JOB HAS ALREADY BEEN DESCRIBED IN REFERENCE (*3), WHILE PROGRAM 'THRP' FUR ACCOMPLISHING THE SECOND JOB IS DESCRIBED BELOW. NULLS INTERSPERSED WITH INTEGERS WILL BE REPRESENTED BY (-0) INSTEAD OF BY THE EARLIER (X) OR (XX) TO SIMPLIFY PRINTING FORMAT.

MONTE CARLO METHODOLOGY AND GENERATION OF STRINGS OF PSEUDO-RANDOM NUMBERS HAVE BEEN THE SUBJECT OF NUMEROUS PUBLICATIONS. A NUMBER OF COMPUTER PROGRAMS FOR SUCH GENERATORS WERE SCREENED BY THE AUTHOR, AND NONE WERE FOUND SATISFACTORY FOR 3P SAMPLING, WHERE TREES ON THE GROUND ARE USUALLY VISITED

IN A NOTABLY NON-RANDOM SEQUENCE. EXCESSIVELY LONG RUNS (SERIAL CORRELATION) CONSTITUTED THE MOST UNSATISFACTORY CHARACTERISTIC OF THE GENERATORS TRIED.

AFTER THE AUTHOR HAD EMPIRICALLY DISCARDED A NUMBER OF GENERATORS AS UNSUITABLE, HULL AND DOBELL'S EXCELLENT ARTICLE (*6) SUBSTANTIATED THE UNSUITABILITY OF MANY OF THESE, AND INDICATED SOME PROMISING LEADS. SINCE THE AUTHOR INTUITIVELY FELT THAT THE MIXED CONGRUENTIAL METHOD HAD THE MOST PROMISE, HE TRIED THE GENERATOR (2**8+1) * X + F, MODULO 2**35, WHICH GAVE THE LEAST GROUNDS FOR SUSPICION IN (*6). 'X' IS THE PREVIOUSLY GENERATED NUMBER, WHILE 'F' IS A CONSTANT THAT IS RELATIVELY UNIMPORTANT AS LONG AS IT IS ODD. AFTER TRIAL OF SEVERAL CONSTANTS, THE AUTHOR ADOPTED 29 327 845 667. 29 327 871 111 HAS THEORETICAL MERIT AMONG NUMBERS PRIME TO 257.

RESULTS WERE STILL UNSATISFACTORY, SO 2 ADDITIONAL OPERATIONS WERE INCORPORATED INTO THE GENERATOR. FIRST, 23 RANDOM 12-DIGIT OCTAL MASKS WERE STORED AND SINGLY COMBINED WITH GENERATED INTEGERS IN THE BOOLEAN 'EXCLUSIVE OR' OPERATION. 7-BIT CIRCULAR ROTATION OF EACH MASK PROVIDED 828 DIFFERENT MASKS BEFORE RECYCLING. IT SHOULD BE NOTED THAT 'EXCLUSIVE OR' WITH A GIVEN MASK WILL TRANSFORM 2**35 INTEGERS INTO 2**35 DIFFERENT INTEGERS WITHOUT REPETITION.

SECOND, A RANDOM PERMUTATION OF THE SEQUENCE OF NUMBERS 1 THROUGH 500 WAS STORED AND USED ON ALTERNATE PAGES OF OUTPUT EITHER TO SPECIFY ORDER IN THE PRINT ARRAY CORRESPONDING TO THE GENERATED PROGRESSION, OR TO SPECIFY ORDER IN THE GENERATED ARRAY CORRESPONDING TO THE PRINT PROGRESSION. THIS WAS INTENDED TO FURTHER DISPERSE NONRANDOM STRINGS OF NULLS WHICH MIGHT SURVIVE THE 'EXCLUSIVE OR' INTERACTION WITH THE RANDOM MASK.

PRIOR TO THE PERMUTATION, ALL GENERATED INTEGERS (MOD 2**35) WERE RIGHT-ADJUSTED SO THEY RETAINED THE SAME NUMBER OF BIT POSITIONS AS DID (KZ-1),

WHICH NUMBER IS THE LOGARITHM OF 'KZ' TO THE BASE 2. ANY NUMBER AFTER ADJUSTMENT THAT WAS LARGER THAN (KZ-1) WAS REPLACED BY A NEWLY GENERATED INTEGER, UNTIL AN ARRAY OF 500 INTEGERS LESS THAN 'KZ' WAS OBTAINED. NULL (-0) THEN REPLACED (+0) AND ALL INTEGERS LARGER THAN 'K'. OBVIOUSLY, THIS PROCEDURE CAN GENERATE A RECTANGULAR POPULATION FROM ZERO TO 'K' IF 'KZ' IS SET EQUAL TO (K+1). SAMPLING WITH EQUAL PROBABILITY WOULD REQUIRE SETTING K=1.

THIS GENERATOR HAS APPARENTLY PROVIDED SATISFACTORY LISTS FOR A WIDE VARIETY OF TRYING SITUATIONS. ALTHOUGH THE AUTHOR COULD FIND NO EARLIER PUBLICATIONS ADVOCATING USE OF 'EXCLUSIVE OR' AS A SECONDARY GENERATING FUNCTION TOGETHER WITH THE INPUT OF RANDOM NUMBERS AS MASKS AND AS PERMUTED SUBSCRIPTS, HE WAS INTERESTED TO NOTE LATER THAT MCLAREN AND MARSAGLIA (*7) CONCLUDED INDEPENDENTLY THAT NONE CF A LARGE NUMBER OF MIXED CONGRUENTIAL AND MULTIPLICATIVE CONGRUENTIAL GENERATORS THEY SCREENED WERE SATISFACTORY.

THE SOLUTIONS THEY RECOMMENDED WERE--(1) INPUTTING A LIST OF RANDOM NUMBERS AND HAVING EACH SUCH NUMBER GENERATE ITS OWN REPLACEMENT, SERIATIM (2) USING OUTPUT FROM ONE GENERATOR AS INPUT TO A SECOND GENERATOR. THEIR EXPERIENCE SEEMS TO BEAR OUT THE AUTHOR'S, AND THEIR SOLUTIONS, THOUGH DIFFERENT, ARE ALONG THE SAME LINES AS HIS.

HOW TO USE 'THRP' TO GENERATE CRITERIA FOR 3P SAMPLING SPECIFIED BY 'K' AND 'KZ' IS EXPLAINED IN APPENDICES A AND B. HOWEVER, TWO FEATURES MIGHT BE MENTIONED HERE. ON THE BOTTOM RIGHT OF EACH PAGE OF 500 INTEGERS AND NULLS ARE TWO FIGURES--THE SUM OF NONNULL INTEGERS, AND THE NUMBER OF NONNULL INTEGERS. ON THE LAST PAGE, THESE FIGURES ARE TOTALLED FOR ALL PAGES AND LABELLED 'KSUM' AND ALONG WITH THE TOTAL NUMBER OF GENERATED ITEMS, LABELLED 'NKZ'. BELOW THESE TOTALS ARE THE SUM OF INTEGERS EXPECTED FOR 'NK' NONNULL ITEMS, THE NUMBER OF NONNULL ITEMS EXPECTED AMONG 'NKZ' TOTAL ITEMS

(WITH SPECIFIED K/KZ), AND THE TOTAL NUMBER OF ITEMS REQUESTED TO BE GENERATED.

THUS, IT IS POSSIBLE TO SCREEN EITHER INDIVIDUAL PAGES OR SETS OF PAGES BEFORE SAMPLING STARTS. PAGES OR SETS DIFFERING EXCESSIVELY FROM EXPECTATION CAN BE DISCARDED BEFORE USE, IF DESIRED, WITHOUT BIASING THE SAMPLE ESTIMATES.

FIELD USE OF RANDOM CRITERIA IN FOREST SAMPLING HAS SUFFERED GREATLY FROM LACK OF SUITABLE DISPENSERS IN THE PAST. CHEAP BIASED DEVICES INVOLVING MARBLES, DICE, SHORT-PERIOD REPEATING SEQUENCES OF NUMBERS, COUNTERS OR TALLIES WITH FIXED SAMPLING INTERVALS, PROBABILITY SETS EXHAUSTED BEFORE REPLACEMENT OR RESHUFFLE, ETC., HAVE TENDED TO GIVE RANDOM SAMPLING A BLACK EYE.

EVEN WHERE OTHERWISE BLAMELESS LISTS OF RANDOM NUMBERS HAVE BEEN EMPLOYED, FOREKNOWLEDGE OF THE NEXT RANDOM NUMBER HAS OFTEN INFLUENCED THE SEQUENCE IN WHICH TREES WERE VISITED OR THE ACTION TAKEN AT A PARTICULAR TREE. WHEN ENUMERATION SEQUENCE UR INDIVIDUAL ASSESSMENT IS SUBJECTIVE, FOREKNOWLEDGE OF THE FORTHCOMING RANDOM CRITERION CAN BIAS SAMPLE ESTIMATES JUST AS SURELY AS LOADING DICE AGAINST OR IN FAVOR OF PARTICULAR OUTCOMES. THOUGH LISTS OF GENUINELY RANDOM NUMBERS IN THE CUSTODY OF A RECORDER AND UNSEEN BY ENUMERATORS OR ASSESSORS SHOULD RESULT IN NO BIAS, SUCH PROCEDURE IS FREQUENTLY NOT FEASIBLE.

CONSEQUENTLY, THE AUTHOR FELT THAT WIDESPREAD SATISFACTORY APPLICATION OF 3P SAMPLING REQUIRED FINDING OR DEVELOPING SUITABLE DISPENSERS FOR THE TAILOR-MADE LISTS OF COMPUTER-GENERATED CRITERIA. HE ORIGINALLY HOPED TO ADAPT SMALL DOUBLE-ROLLER MAP DISPLAY CASES TO THE JOB, BUT THE MANUFACTURER OF THE ONLY MODEL OF WHICH HE WAS AWARE HAD GONE OUT OF BUSINESS.

TO ILLUSTRATE THE SORT OF THING NEEDED, HE MADE THE CRUDE DEVICE SHOWN IN FIGURE 1A. THE DIMENSIONS WERE DETERMINED BY ODDS AND ENDS AVAILABLE, SUCH AS LINT-PICKER CYLINDERS AND HONEYCOMB FRAME, BUT A DEVICE HANDLING 1, 2, 5,

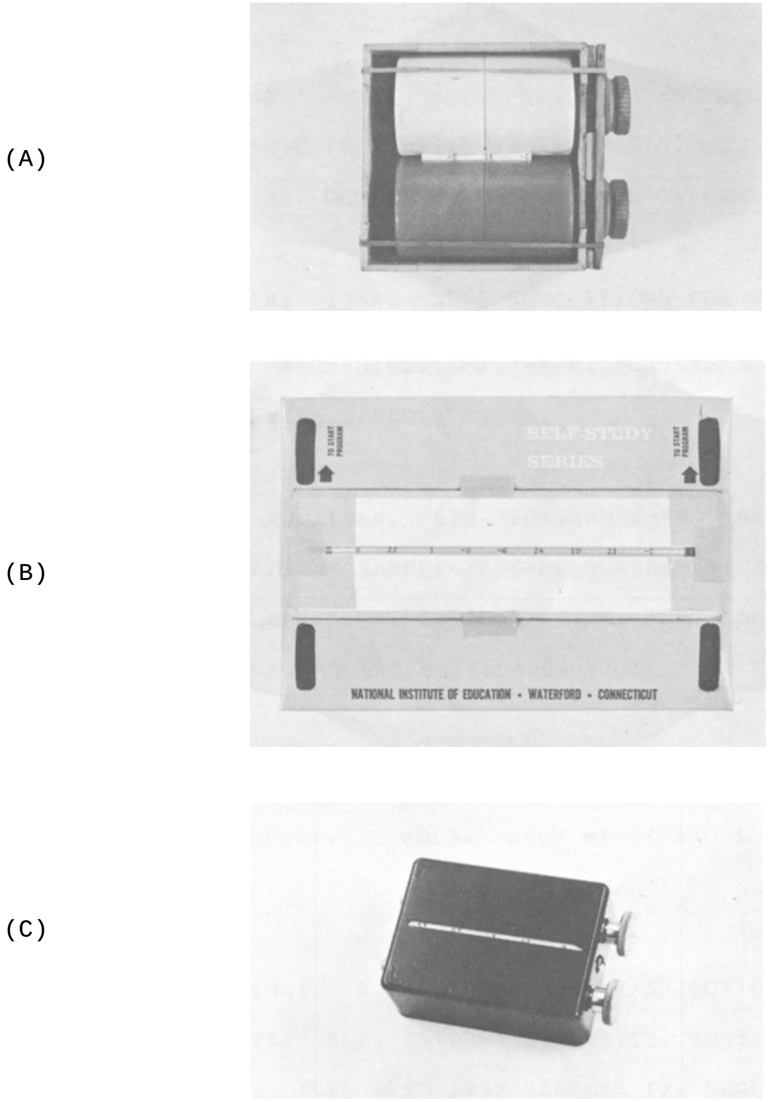
OR 10 COLUMNS WOULD HAVE BEEN PREFERABLE TO THE 3 COLUMNS SHOWN.

THE AUTHOR NEXT LOCATED THE PROGRAMMED LEARNING DEVICE SHOWN IN FIGURE 18. IT IS MADE FOR THE NATIONAL INSTITUTE OF EDUCATION, WATERFORD, CONNECTICUT BY THE EDUCATIONAL AIDS PUBLISHING CORP., CARLE PLACE, L.I., N.Y. ALTHOUGH IT CAN HANDLE THE FULL 10-COLUMN COMPUTER SHEET, A METAL, WOOD, OR PLASTIC OUTER SHELL WOULD BE BETTER THAN ITS PRESENT CARDBOARD CONTAINER. FURTHERMORE, THE CONTAINER COULD BE NARROWER, AND A BETTER BRAKE THAN RUBBER BANDS AROUND BOTH ROLLERS COULD BE DEVISED. HOWEVER, IT IS SUITABLE FOR TEMPORARY USE WHERE THE DISPENSER CAN BE PROTECTED FROM MOISTURE.

FINALLY, CLEMENT MESAVAGE OF THE U.S. FOREST SERVICE'S SOUTHERN FOREST EXPERIMENT STATION BECAME INTERESTED IN THE PROBLEM AND DEVOTED MUCH PERSONAL AS WELL AS OFFICIAL EFFORT TO DEVELOPMENT OF THE PRODUCTION MODEL SHOWN IN FIGURE 10. IT HANDLES 5 COLUMNS (A COMPUTER HALF-PAGE), AND 20 HALF-PAGES CAN BE SPLICED TOGETHER WITH RUBBER CEMENT TO PROVIDE 5000 3P CRITERIA. THE DISPENSER IS MANUFACTURED BY GRANNAN INDUSTRIES, 844 HIGHWAY 62 EAST, MOUNTAIN HOME, ARKANSAS. LISTS ARE PRINTED SO THAT A 4-CHARACTER IDENTIFIER AT BOTTOM OF EACH COLUMN NOT ONLY SERVES AS A BASE AND HELPS ALIGN THE SPLICE, BUT IT ALSO PROVIDES NON-DETACHABLE BRIEF IDENTIFICATION OF THE PARTICULAR JOB OR AREA TO WHICH THE CRITERIA ARE APPROPRIATE.

ALL IN ALL, IT SEEMS TO BE THE MOST SUITABLE AND CONVENIENT DISPENSER OF 3P CRITERIA THAT IS CURRENTLY AVAILABLE. OF THE 5 COLUMNS, ALL CAN BE USED AT EACH TREE WHEN 5 INTERPENETRATING INDEPENDENT SAMPLES ARE DESIRED--AND 5 IS A FAIRLY SATISFACTORY NUMBER FOR MANY PURPOSES--OR FEWER CAN BE USED. WHEN ONLY 1 NUMBER IS NEEDED PER TREE, A NEW CRITERION SHOULD BE ROTATED INTO VIEW AT EACH TREE VISITED UNTIL A COLUMN HAS BEEN EXHAUSTED. THEN THE NEXT COLUMN CAN BE USED BY ROTATING IN THE OPPOSITE DIRECTION, ETC. THE IMPORTANT THING

FIGURE 1. RANDOM INTEGER DISPENSERS THAT DISPLAY ONE LINE OR ONE NUMBER AT A TIME FROM A LIST OF NUMBERS GENERATED BY COMPUTER FOR ANY DESIRED RATE OF 3P SAMPLING.



(A) AUTHOR'S EARLY HOME-MADE DEVICE 13 COLUMNS).
(B) NATIONAL INSTITUTE OF EDUCATION'S PROGRAMMED LEARNING DEVICE (10 COL.).
(C) MESAVAGE'S PRODUCTION MODEL (5 COLUMNS).

IS THAT ROTATION EXPOSING THE FORTHCOMING 'KP' SHOULD NEVER OCCUR UNTIL AFTER 'KPI' FUR THE CORRESPONDING TREE HAS BEEN ASSIGNED AND RECORDED.

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=================================== APPENDIX A ===================================

LISTING OF FORTRAN-4 VERSION OF
PROGRAM THRP
AND 7090-MAP VERSION OF
SUBROUTINE IRDM

```
$IBFTC BLB3     DECK                                                    BLB3   0
C  BLB3----INPUT-OUTPUT OPTIONS AND CONSTANTS FOR THRP (11-11-64)       BLB3   1
           RANDOM NUMBER GENERATOR FOR 3P SAMPLING, BY L.R.GROSENBAUGH.  BLB3   2
C      SPECIFY LOGICAL NUMBER OF DESIRED INPUT TAPE (MRE) AND           BLB3   3
C      OUTPUT TAPE (MPR).   IF REGULAR SYSTEM OUTPUT TAPE IS USED,      BLB3   4
C      'MEOF' SHOULD BE SPECIFIED TO BE ZERO, OTHERWISE TO BE ONE.      BLB3   5
C      'LEFT' AND 'KFIX' ARE CONSTANTS NEEDED BY GENERATING FUNCTION.   BLB3   6
      BLOCK DATA                                                        BLB3   7
      COMMON /B3P/ MRE,MPR,MEOF,LEFT,KFIX,NARG,MASK,L,LIM,K,KZ,M        BLB3   8
      DATA MRE/-1/,MPR/-3/,MEOF/0/,LEFT/8/,KFIX/29327845667/            BLB3   9
      END                                                               BLB3  10


$IBMAP B3P      DECK                                                    B3P    0
*  MAP DECK 'B3P' SHOULD REPLACE FORTRAN-4 DECK 'BLB3' AHEAD OF DECK    B3P    1
*  'THRP' ON 7040-7044 IF BLOCK DATA IS NOT ACCEPTABLE.                 B3P    2
                                                                        B3P    3
       DEC -1,-3,0,8,29327845667                                        B3P    4
       END                                                              B3P    5
```

```
$IBFTC THRP     DECK                                                     THRP    0
C BC THRP--WITH IRDM, GENERATES INTEGERS (WITH REPLACEMENT) SUITABLE FOR THRP    1
C        --SPECIFIED 3P-SAMPLE DESIGN--BY L. R. GROSENBAUGH--(11-11-64).  THRP    2
C   INPUT-OUTPUT TAPES ARE SPECIFIED EARLY IN 'BLB3' OR /B3P/. USER      THRP    3
C       SPECIFIES LOGICAL NUMBER OF DESIRED INPUT TAPE (MRE) AND         THRP    4
C       OUTPUT TAPE (MPR).  IF REGULAR SYSTEM OUTPUT TAPE IS USED,       THRP    5
C       'MEOF' SHOULD BE SPECIFIED TO BE ZERO, OTHERWISE TO BE ONE.      THRP    6
C   FOR EACH SALE, 2 DATA CARDS MUST BE PUNCHED TO CAUSE DRAWING (WITH   THRP    1
C       REPLACEMENT) AND LISTING OF RANDOM INTEGERS APPROPRIATE AS       THRP    8
C       CRITERIA FOR A SPECIFIED 3P-SAMPLE DESIGN. THE 3P-SAMPLE         THRP    9
C       UNIVERSE CONSISTS OF A TOTAL POPULATION OF 'KZ' CARDS, OF WHICH  THRP   10
C       'K' ARE INTEGERS RANGING IN VALUE FROM ONE TO 'K' WITH UNIT      THRP   11
C       INTERVAL, AND 'KZ MINUS K' ARE NULLS REPRESENTED BY -0 (XX'S OR  THRP   12
C       FACE CARDS ARE SYNONYMS FOR NULLS).                              THRP   13
C   SEE 1964. SOC. AMER. FORESTERS, PROC. 1963, PAGES 36- 42,            THRP   14
C       'SOME SUGGESTIONS FUR BETTER SAMPLE-TREE-MEASUREMENT' FOR BASIC  THRP   15
C       THEORY AND FIELD PROCEDURE.                                      THRP   16
C   SEE 1964 U.S. FOREST SERVICE RESEARCH PAPER PSW-13 'STX--FORTRAN 4   THRP   17
C       PROGRAM FOR ESTIMATES OF TREE POPULATIONS FROM 3P SAMPLE-TREE-   THRP   18
C       MEASUREMENTS' FOR SUBSEQUENT PROCESSING OF 3P-SAMPLE DATA.       THRP   19
C   FIRST DATA CARL SHOULD BE BLANK IN COLUMNS 1-4, WITH FULL            THRP   20
C       IDENTIFICATION OF SALE AREA IN COLUMNS 5-64, AND BRIEF CODED     THRP   21
C       IDENTIFICATION OF SALE AREA IN COLUMNS 73-76 (SAME CARD AS FIRST THRP   22
C       CONTROL CARD IN PROGRAM 'STX').                                  THRP   23
C   SECOND DATA CARD SHOULD BE BLANK IN COLUMNS 1-4, WITH ANY 12-DIGIT   THRP   24
C       OCTAL NUMBER (NO 8'S OR 9'S) IN COLUMNS 5-16, DIFFERENT FOR EACH THRP   25
C       SALE.   THIS IS INITIAL ARGUMENT 'L' FOR STARTING THE GENERATOR. THRP   26
C       COLUMNS 18-22 SHOULD CONTAIN 'LIM', THE TOTAL NUMBER OF TREES    THRP   27
C       EXPECTED TO BE MARKED FOR SALE. THIS NUMBER OF TREES OR ITEMS    THRP   28
C       (LIM) SHOULD BE A MULTIPLE OF 500, SINCE EACH PRINTED PAGE OF    THRP   29
C       SAMPLE CRITERIA (INTEGERS) WILL HAVE 10 COLUMNS OF 50 ITEMS EACH. THRP   30
C       COLUMNS 24-27 SHOULD CONTAIN 'K' SPECIFIED BY THE 3P DESIGN.     THRP   31
C       COLUMNS 29-33 SHOULD CONTAIN 'KZ'(ALSO KNOWN AS 'K+Z' OR 'PRBS'). THRP   32
C   PAIRS OF CARDS FOR MANY SALES MAY BE PROCESSED DURING ONE MACHINE    THRP   33
C       RUN, BUT THE PAIR FUR THE LAST SALE SHOULD BE FOLLOWED BY A CARD THRP   34
C       PUNCHED 9959 IN COLUMNS 1-4.                                     THRP   35
C   EACH PAGE OF LISTED INTEGERS WILL HAVE ITS SUM AND NUMBER OF NONNULL THRP   36
C       ITEMS PRINTED AT BOTTOM-RIGHT.                                   THRP   37
C   LAST PAGE OF EACH SALE CONTAINS TOTAL SUM, TOTAL NUMBER OF NONNULLS, THRP   38
C       AND GRAND TOTAL NUMBER OF NONNULLS PLUS NULLS. BELOW APPEARS     THRP   39
C       EXPECTED SUM FOR NUMBER OF NONNULLS ACTUALLY APPEARING, EXPECTED THRP   40
C       NUMBER OF NONNULLS GIVEN 3P DESIGN WITH SPECIFIED 'LIM', AND     THRP   41
C       'LIM' AS SPECIFIED.                                              THRP   42
C   QUICK COMPARISON OF PAGE TOTALS AND GRAND TOTALS WITH EXPECTATIONS   THRP   43
C       AND SPECIFICATIONS ALLOWS REJECTION OF LISTS DEVIATING           THRP   44
        IN AN IMPROBABLE MANNER OR EXCESSIVELY.                          THRP   45
C                                                                        THRP   46
      COMMON /B3P/ MRE,MPR,MEOF,LEFT,KFIX,NARG,MASK,L,LIM,K,KZ,M         THRP   47
      COMMON /LUK/JS,NUM,MNU                                             THRP   48
      DIMENSION ALFATH(10),JS(500),NUM(500),MNU(500)                     THRP   49
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```
  1 FORMAT (72X,18/2X,10(3X,A4),18)                                     THRP    50
  20FORMAT (2H1 ,10A6,6H PAGE .14/2X12HL=1012,6H, LIM=,15,4H, K=04,     THRP    51
  215H, KZ=,15.1H,,A4/2X)                                               THRP    52
  3 FORMAT (2X,1017)                                                    THRP    53
  40FORMAT (2H1 ,1046,6H PAGE 114/2X,2HL=,012,6H, LIM=,I5,4H, K=.I4,    THRP    54
  415H, KZ=,15,IH,,A4/2X/2X,A4,6X,11HKSUM        =,I12,16H,       NK    THRP    55
  42 =,I5.11Ht       NKZ=,15/2X,21HEXPECTED NK*(K+1)/2=.112,16H,        THRP    56
  43NKZ*K/KZ=,15,11H,       LIM=,I5)                                    THRP    57
  5 FORMAT (14,10A6,8X,A4)                                              THRP    58
  6 FORMAT (14,012,1X,15,1X04,1X,15)                                    THRP    59
  7 JPAGE=1                                                             THRP    60
    NK=G                                                                THRP    61
    NKZ=0                                                               THRP    62
    KSUM=0                                                              THRP    63
    KSUB=0                                                              THRP    64
    NKSUB=O                                                             THRP    65
    NONSAM=-0                                                           THRP    66
    READ (MRE,5) KREENO,ALFATH,CDID                                     THRP    67
    IF (KREENO .NE. O) GO TO 33                                         THRP    68
  8 READ (MRE,6) KREEO,L,LIM,K,KZ                                       THRP    69
    IF (KREENO .NE. C) GO TO 33                                         THRP    70
  9 AKZ=FLOAT(KZ)                                                       THRP    71
    AM=37.-1.44269504*ALOG(AKZ)                                         THRP    72
    M=INT(AM)                                                           THRP    73
    CALL MSET                                                           THRP    74
 10 CALL IGEN                                                           THRP    75
 18 WRITE (MPR,2) ALFATH,JPAGE,L,LIM,K,KZ,CDID                          THRP    76
    JPAGE=JPAGE+1                                                       THRP    77
    IFLAG=MOD(JPAGE,2)                                                  THRP    78
    1=0                                                                 THRP    79
    DO 29 1=1,500                                                       THRP    80
    IF (IFLAG .EQ. 0) GO TO 23                                          THRP    81
 22 II=JS(I)                                                            THRP    82
    III=1                                                               THRP    83
    GO TO 24                                                            THRP    84
 23 III=JS(I)                                                           THRP    85
    11=I                                                                THRP    86
 24 NKZ=NKL+1                                                           THRP    87
    IF (NUM(II) .EQ. 0) GO TO 26                                        THRP    88
 25 IF (NUM(II) .LE. K) GO TO 27                                        THRP    89
 26 MNU(III)=NONSAM                                                     THRP    90
    GO TO 29                                                            THRP    91
 27 MNU(III)=NUN(II)                                                    THRP    92
    KSUB=KSUB+NLM(II)                                                   THRP    93
    NKSUB=NKSUB+1                                                       THRP    94
 29 CONTINUE                                                            THRP    95
    WRITE (MPR,3) MNU                                                   THRP    96
 310WRITE (MPR,1) KSUB,CDID,CDID,CDID,CDID,CDID,CDID,CDID,CDID,CDID,    THRP    97
 311CDIDNKSUB                                                           THRP    98
    NK=NK+NKSUB                                                         THRP    99
    KSUM=KSUM+KSUB                                                      THRP   100
    KSU8=0                                                              THRP   101
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      NKSUB=0                                                           THRP 102
      IF (NU .LT. LIM) GO TO 10                                         THRP 103
   32 JEKSUM=INK*(K+1))/2                                               THRP 104
      JENK=INKZ*KUKZ                                                    THRP 105
     OWRITE (MPR14) ALFATH,JPAGE,L,LIM,K,KZ,CDID,CDID,KSUM,NK,NKZ,      THRP 106
     1JEKSUM,JENK,LIM                                                   THRP 107
      GO TO 7                                                           THRP 108
   33 IF (MEOF .EQ. 0) GO TO 35                                         THRP 109
   34 END FILE MPR                                                      THRP 110
   35 RETURN                                                            THRP 111
      END                                                               THRP 112
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SIBMAP IRDM     DECK                                                     IRDM   0
*   IRDM---- CONTINUOUS RANDOM INTEGER GENERATOR FOR THRP ----------------IRDM   1
*         ---BY L. R. GROSENBAUGH (7040-7094 MAP-F4 VERSION OF 11-11-64) IRDM   2
*                                                                        IRDM   3
*                                                                        IRDM   4
*   RANDOM INTEGERS SMALLER THAN A SPECIFIED MAXIMUM 'KZ' ARE GENERATED  IRDM   5
*       WITH REPLACEMENT BY A 4-STAGE PROCESS.                           IRDM   6
*   SEVERAL PARAMETERS MUST BE READ BEFORE 'CALL MSET' CAN READY THE     IRDM   7
*   GENERATING FUNCTION FOR OPERATION. AMONG THESE PARAMETERS            IRDM   8
*       'L' IS OCTAL STARTING ARGUMENT (MOD 2'*35) FOR GENERATOR.        IRDM   9
*       'KZ' IS MAXIMUM DECIMAL INTEGER THAT MUST NOT BE EQUALLED OR     IRDM  10
*        EXCEEDED.                                                       IRDM  11
*       'M' IS 37 MINUS TOTAL DECIMAL NUMBER OF BITS IN (KZ-11.          IRDM  12
*   THEN 'CALL IGEN' INITIATES ACTUAL GENERATION OF 500-ITEM BATCHES OF  IRDM  13
*   RANDOM NUMBERS AND NULLS APPROPRIATE TO THE SPECIFIED PARAMETERS.    IRDM  14
        FIRST-STAGE INTEGERS ARE SUCCESSIVELY GENERATED BY THE MIXED     IRDM  15
*       CONGRUENTIAL METHOD, ALTHOUGH SLIGHT MODIFICATION WOULD ALLOW    IRDM  16
*       USE OF THE MULTIPLICATIVE CONGRUENTIAL METHOD (SEE 1964 JOUR.ACM. IRDM  17
*       11, PAGES 31-40). A LEFT SHIFT ANC ADDITIVE CONSTANT OPTIMUM     IRDM  18
*       FOR MODULUS 2**35 HAVE BEEN EMPLOYED, BUT ARE EASILY CHANGED.    IRDM  19
*   SECOND-STAGE TREATMENT OF 35-BIT FIRST-STAGE INTEGER IS TO 'ERA' IT  IRDM  20
*       WITH AN INDEPENDENT SEQUENCE OF MASKS THAT ARE PROGRESSIVELY     IRDM  21
*       CHANGED BY FIVE 7-BIT ROTATIONS OF EACH OF 23 RANDOM 36-BIT      IRDM  22
*       PATTERNS, RESULTING IN 828 DIFFERENT RANDOM MASKS BEFORE CYCLE   IRDM  23
*       REPEATS.                                                         IRDM  24
*   THIRD-STAGE TREATMENT IS TO TRUNCATE RIGHT-HAND BITS IN EXCESS OF    IRDM  25
*       NUMBER NEEDED TO EXPRESS 'KZ-1', AND TO GENERATE A REPLACEMENT   IRDM  26
*       WHEN THE RIGHT-ADJUSTED RESIDUAL EQUALS OR EXCEEDS 'KZ'.         IRDM  27
*   STAGES 1-3 ARE REPEATED UNTIL A SET OF 500 QUALIFYING INTEGERS ARE   IRDM  28
*       OBTAINED, RANGING FROM 0 THROUGH (KZ MINUS ONE).                 IRDM  29
*   FOURTH-STAGE TREATMENT (IN THRP) IS TO REARRANGE THESE 500 INTEGERS  IRDM  30
*       SO AS TO FURTHER REDUCE ANY OBVIOUS SERIAL CORRELATION, AND TO   IRDM  31
*       TRANSFORM +0 AND ALL NUMBERS LARGER THAN 'K' TO -0,THUS YIELDING IRDM  32
*       A SEE OF CRITERIA APPROPRIATE TO THE 3P-SAMPLE DESIGN SPECIFIED  IRDM  33
*       AS 'K,KZ1.                                                       IRDM  34
*                                                                        IRDM  35
       ENTRY    MSET                                                     IRDM  36
       ENTRY    IGEN                                                     IRDM  37
 MSET SAVE      1,2,4                                                    IRDM  38
       CAL L                                                             IRDM  39
       STO NARG                                                          IRDM  40
       CAL LEFT                                                          IRDM  41
       STA SHIFT                                                         IRDM  42
       CAL M                                                             IRDM  43
       STA MCUT                                                          IRDM  44
       CAL KNUM                                                          IRDM  45
       STA NEWP                                                          IRDM  46
       ADD FIHUN                                                         IRDM  47
       STA STOP                                                          IRDM  48
       STZ INDX                                                          IRDM  49
```

```
       RETURN MSET                                                      IRDM  50
IGEN   SAVE   1,2,4                                                     IRDM  51
       CLA NEWP                                                         IRDM  52
       STA STORE                                                        IRDM  53
       STO RLOD                                                         IRDM  54
LOAD   CLA INDX                                                         IRDM  55
       SUB ONE                                                          IRDM  56
       TPL *+2                                                          IRDM  57
       ADD TWTHR                                                        IRDM  58
       STU INDX                                                         IRDM  59
       ADD KNPA                                                         IRDM  60
       STA NMSK                                                         IRDM  61
       STA MUTA                                                         IRDM  62
       STA FINI                                                         IRDM  63
NMSK   LDQ **                                                           IRDM  64
       CLA RLOD                                                         IRDM  65
       ADD FIVE                                                         IRDM  66
       STO RLOD                                                         IRDM  67
LOOP   RQL 7                                                            IRDM  68
       STQ MASK                                                         IRDM  69
       CAL NARG                                                         IRDM  70
SHIFT  ALS **                                                           IRDM  71
       ADD NARG                                                         IRDM  72
       ADD KFIX                                                         IRDM  73
       STO NARG                                                         IRDM  74
       ORA MASK                                                         IRDM  75
       SLW ORA                                                          IRDM  76
       CAL MASK                                                         IRDM  77
       ANA NARG                                                         IRDM  78
       COM                                                              IRDM  79
       ANA ORA                                                          IRDM  80
       ALS 2                                                            IRDM  81
MCUT   ARS **                                                           IRDM  82
       CAS KZ                                                           IRDM  83
       TRA LOOP                                                         IRDM  84
       TRA LOOP                                                         IRDM  85
*                                                                       IRDM  86
STORE  STO **                                                           IRDM  87
       CLA STORE                                                        IRDM  88
       ADD ONE                                                          IRDM  89
       STA STORE                                                        IRDM  90
       CAS STOP                                                         IRDM  91
       TRA FINI                                                         IRDM  92
       TRA FINI                                                         IRDM  93
       TRA GOON                                                         IRDM  94
GOON   CAS RLOD                                                         IRDM  95
       IRA MUTA                                                         IRDM  96
       TRA MUTA                                                         IRDM  97
       TRA LOOP                                                         IRDM  98
MUTA   STQ **                                                           IRDM  99
       TRA LOAD                                                         IRDM 100
FINI   STQ **                                                           IRDM 101
```

```
       TOV *+1                                                           IRDM 102
       RETURN   IGEN                                                     IRDM 103
*               CONSTANTS                                                IRDM 104
 STOP  STU      **                                                       IRDM 105
 NEWP  STO      **                                                       IRDM 1J6
 KNPA  PZE      NPAT                                                     IRDM 107
 KNUM  PZE      NUM                                                      IRDM 108
 ONE   DEC 1                                                             IRDM 109
 TWO   DEC 2                                                             IRDM 110
 FOUR  DEC 4                                                             IRDM 111
 FIVE  DEC 5                                                             IRDM 112
 SIX   DEC 6                                                             IRDM 113
 TWTHR DEC 23                                                            IRDM 114
 FIHUN DEC 500                                                           IRDM 115
 NPAT  OCT 201450075226,214546337146,322724516102,303667607764           IRDM 116
       OCT 304170563342,221625507015,777210313005,431320677112           IRDM 117
       OCT 504765671151,233717444633,717372135702,375415045242           IRDM 118
       OCT 706156152C34,516724654017,526040760510,543414754156           IRDM 119
       OCT 321113461357,241256504205,505761377112,252344415135           IRDM 126
       OCT 117000633423,404766555015,610746200174                        IRDM 121
*               ERASABLES                                                IRDM 122
 INDX  PZE      0                                                        IRDM 123
 RLOD  PZE      0                                                        IRDM 124
 ORA   PZE      C                                                        IRDM 125
*               LABELLED CONTROL SECTIONS                                IRDM 126
       USE      .DUMY.                                                   IRDM 127
 B3P   CONTRL   B3P                                                      IRDM 128
       USE      B3P                                                      IRDM 129
       EVEN                                                              IRDM 130
 MRE   BSS      1                                                        IRDM 131
 MPR   BSS      1                                                        IRDM 132
 MEOF  BSS      1                                                        IRDM 133
 LEFT  BSS      1                                                        IRDM 134
 KFIX  BSS      1                                                        IRDM 135
 NARG  BSS      1                                                        IRDM 136
 MASK  BSS      1                                                        IRDM 137
 L     BSS      1                                                        IRDM 138
 LIM   BSS      1                                                        IRDM 139
 K     BSS      1                                                        IRDM 140
 KZ    BSS      1                                                        IRDM 141
 M     BSS      1                                                        IRDM 142
       USE      PREVIOUS                                                 IRDM 143
 LUK   CONTRL   LUK                                                      IRDM 144
       USE      LUK                                                      IRDM 145
       EVEN                                                              IRDM 146
 JS    DEC 426,36,333,460,154,308,235,96,110,293,221,361,62,476,109      IRDM 147
       DEC 216,281,420,30,306,445,273,48,475,164,176,437,277,38,353      IRDM 148
       DEC 153,237,465,256,61,287,413,441,369,298,99,428,302,244,116     IRDM 149
       DEC 51,320,463,179,55,483,334,95,234,363,487,253,197,4c9,303      IRDM 150
       DEC 231,435,209,300,18,357,480,143,205,350,255,91,3,131,489       IRDM 151
       DEC 399,174,371,453,118,394,97,346,271,307,31,123,243,384,452     IRDM 152
       DEC 29,343,217,391,102,328,196,142,35,368                         IRDM 153
```

```
       DEC 337,229,493,94,263,317,198,486,75,449,45,279,472,416,245      IRDM 154
       DEC 323,200,57,191,17,370,496,133,260,73,398,240,4,172,339        IRDM 155
       DEC 64,274,187,468,354,165,500,462,137,396,44,121,365,83,289      IRDM 156
       DEC 236,20,152,473,405,92,332,458,406,293,301,212,114,444,356     IRDM 157
       DEC 77,481,351,146,250,52,159,422,349,295,49,359,393,19,265       IRDM 158
       DEC 285,74,314,126,7,167,86,236,372,451,479,68,132,386,103        IRDM 159
       DEC 257,459,10,93,331,26,276,149,203,166                          IRDM 160
       DEC 484,325,98,215,366,259,33,115,192,410,233,58,145,378,454      IRUM 161
       DEC 224,111,139,56,9,266,431,194,319,228,84,141,443,186,129       IRDM 162
       DEC 282,13,490,71,383,78,344,288,85,305,326,402,63,379,429        IRDM 163
       DEC 106,246,442,69,72,381,42,269,291,193,338,244,15,331,81        IRDM 164
       DEC 464,177,299,268,482,340,155,28,313,6,180,264,296,251,144      IRDM 165
       DEC 222,432,469,100,67,446,248,310,352,220,101,440,286,364,400    IRDM 166
       DEC 461,335,185,79,439,8,297,455,50,242                           IRDM 167
       DEC 380,309,188,284,408,128,347,54,423,367,336,411,122,414,11     IRDM 168
       DEC 329,202,447,466,183,488,70,125,283,164,168,438,470,135,24     IRDM 169
       DEC 87,207,434,169,477,112,377,278,404,173,65,395,448,324,206     IRDM 170
       DEC 456,252,390,2,485,151,304,499,12,43,311,412,467,348,225       IRUM 171
       DEC 418,82,261,182,34,201,373,175,424,134,08,497,318,127,178      IRDM 172
       DEC 327,23,388,199,14,105,39,314,111,158,232,53,213,1,189         IRDM 173
       DEC 321,292,219,478,345,474,226,37,214,415                        IRDM 114
       DEC 272,397,430,176,355,249,59,392,124,450,41,294,76,190,258      IRDM 175
       DEC 421,218,494,358,407,136,239,457,90,107,471,387,162,275,254    IRDM 176
       DEC 138,21,376,80,401,315,267,5,150,160,316,66,403,241,22         IRDM 177
       DEC 60,382,130,498,227,280,184,433,40,223,161,385,16,425,322      IRDM 178
       DEC 195,147,113,89,119,342,241,492,216,262,156,32,46,389,417      IRDM 179
       DEC 27,157,211,270,341,88,419,148,312,369,120,375,163,362,495     IRDM 180
       DEC 140,436,47,427,111,25,208,181,491,238                         IRDM 181
NUM    BSS      500                                                      IRDM 182
MNU    BSS      500                                                      IRDM 183
       USE      PREVIOUS                                                 IRDM 184
       END                                                               IRDM 185
```

N.B. INSTRUCTIONS ABOVE ARE COMPATIBLE WITH IBM 704-709-7140-7044-709C-7094, EXCEPT FOR MACROS 'ENTRY', 'SAVE', AND 'RETURN' NOT AVAILABLE TO 704 SAP. LITTLE MODIFICATION IS NEEDED TO ADAPT 'IRDM' TO 704, ALTHOUGH 'THRP' MUST BE CONVERTED FROM FORTRAN-4 TO FORTRAN-2, AND THE 3 MACROS ABOVE MUST EACH BE REPLACED BY APPROPRIATE LINKAGE. 'IRDM' MAKES NO USE OF INDEX REGISTERS, SO IT MAY BE USED WITH MINIMUM 7040, WHICH LACKS SUCH REGISTERS.

INSTRUCTIONS ABOVE ALSO AVOID USE OF INSTRUCTION 'ERA' (BOOLEAN 'EXCLUSIVE OR') NOT AVAILABLE ON 704-7040-7044. HOWEVER, USERS OF 709-7090-7094 CAN ADVANTAGEOUSLY REPLACE 'ORA MASK' ON CARD IRDM 75 WITH 'ERA MASK' AND DELETE CARDS IRDM 76 THROUGH IRDM 80 OR ELSE PUNCH AN ASTERISK (*) IN COLUMN ONE OF EACH OF THEM.

================================== APPENDIX B ==================================

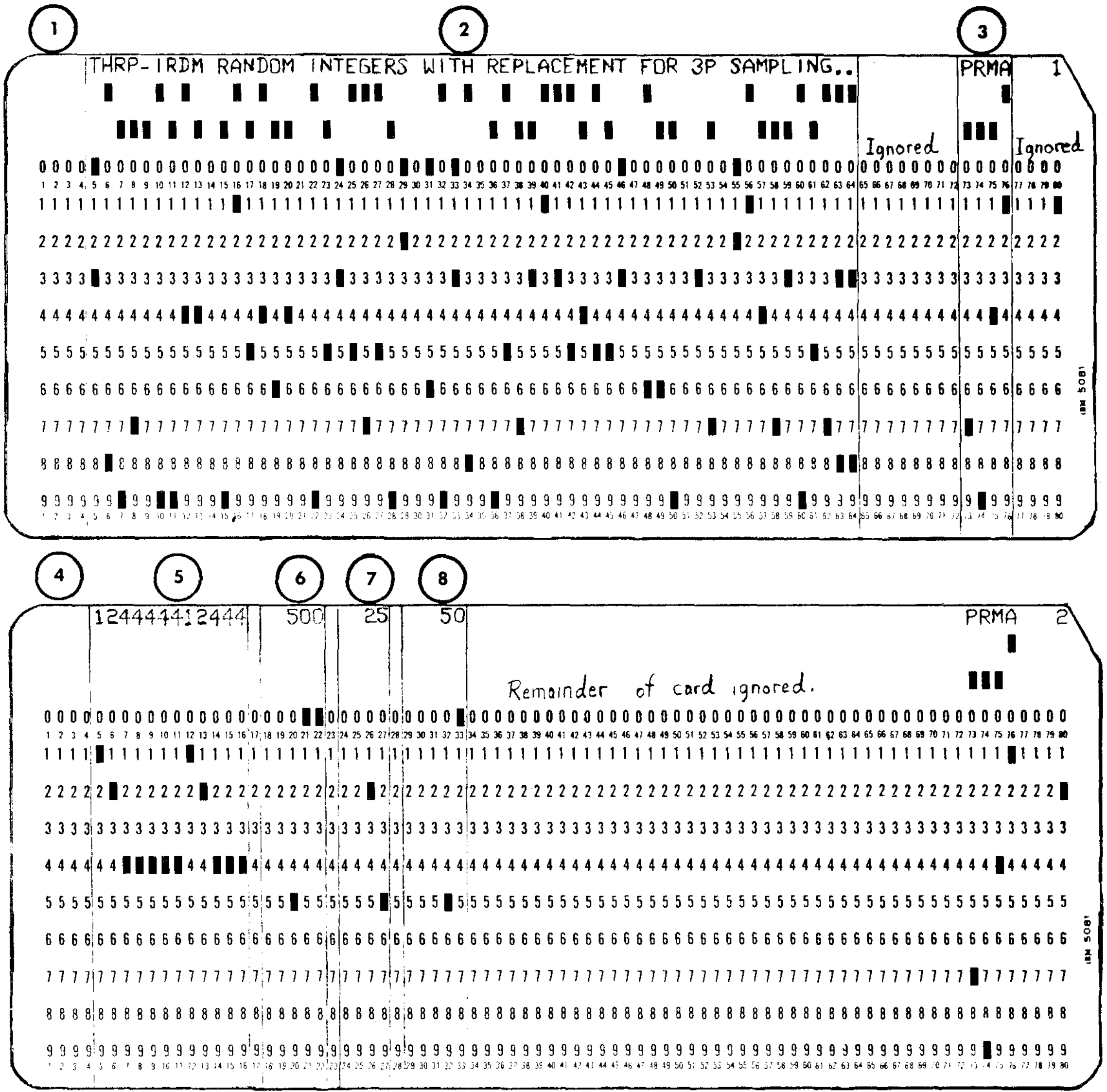
ILLUSTRATION OF INPUT TO PROGRAM 'THRP'

WITH PRINTED OUTPUT RESULTING

INPUT DECK

```
$JOB    (IDENTIFICATION, ETC.)
$IBJOB THRP     MAP
$USE            BLB3(B3P)
$USE            IRDM(LUK)
$IBFTC BL33     DECK                                                     BLB3   0
$IBFTC THRP     DECK                                                     THRP   0
$IBMAP IRDM     DECK                                                     IRDM   0
$ENTRY          THRP
$DATA
    THRP-IRDM RANDOM INTEGERS WITH REPLACEMENT FUR 3P SAMPLING..         PRMA   1
    124444412444   500   25    50                                        PRMA   2
 9999
$EOF
```

FIGURE 2. PARAMETER CARDS CONTROLLING AND DESCRIBING RANDOM INTEGER GENERATION



FORMATS ANC LISTS

```
5 FORMAT (I4.1046,8X,A4)                                       THRP 58
  READ (MRE,5) KREENO,ALFATH,CDID                               THRP 67

6 FORMAT (I4,012,1X,I5,1X,I4.1X,I5)                            THRP 59
8 READ (MRE,6) KREENO,L,LIM,K,KZ                                THRP 69
```

FIGURE 2 (CONTINUED).

LEGEND

1 =KREENO= ALWAYS BLANK.

2 =ALFATH= NAME OF SALE AREA OR JOB.

3 = CDID = BRIEF JOB IDENTIFIER FOR PRINTING AT BOTTOM OF EACH PAGE.

4 =KREENO= ALWAYS BLANK.

5 = L = OCTAL STARTING ARGUMENT FOR GENERATOR--MAY BE ZERO OR ANY 12-DIGIT NUMBER LACKING 'EIGHTS' OR 'NINES'.

6 = LIM = TOTAL NUMBER OF RANDOM INTEGERS DESIRED (INCLUDING NULLS). ENOUGH EXTRAS ARE ALWAYS PRINTED TO FILL THE LAST PAGE.

7 = K = MAXIMUM NONNULL INTEGER INCLUDED IN 3P-PROBABILITY SET.

8 = KZ = (K+Z) OR THE MAXIMUM NONNULL INTEGER PLUS THE NUMBER OF NULLS (-0) COMPRISING THE 3P-PROBABILITY SET.

SEVERAL PAIRS OF CARDS FOR SEVERAL DIFFERENT JOBS CAN BE PROCESSED IN THE THE SAME RUN. THE LAST PAIR OF CARDS ON ANY RUN SHOULD ALWAYS BE IMMEDIATELY FOLLOWED BY A PROGRAM-END CARD PUNCHED 9999 IN COLUMNS 1-4.

THE FIRST PARAMETER CARD OF A 'THRP' JOB IS IDENTICAL WITH THE FIRST CARD USED LATER TO PROCESS THE SAMPLE DATA BY PROGRAM 'STX' (1-3).

=========================

RESULTANT OUTPUT FOLLOWS.

FIGURE 3.   'THRP' PRINTOUT FROM ILLUSTRATORY TEST INPUT

```
THRP-IRDM RANDOM INTEGERS WITH REPLACEMENT FOR 3P SAMPLING.. PAGE      1
L=124444412444,LIM= 500, K= 25, KZ=       50,PRMA

      -0     24     -0     -0     -0     18     -0     19     -0     17
      -0      4     24     -0     18     -0     -0     -0      2     -0
      -0     18     -0     -0     24     16     14     -0     -0     11
       1      6     16     20     -0     14      1     13     -0      8
       6     -0     -0     22      4     -0     -0      3     -0     -0
      20     20      6     -0     -0      6     17     -0     -0     19
       0      2     -0      5     -0      9     -0     -0     -0      8
      18     -0      8     -0     -0      2     -0      2     -0     -0
      12      6     25     -0      9     -0     -0     19     -0     -0
      22     -0     19      8      4     -0     20     -0     -0      9
      -0      7     -0     -0     18      4     -0     -0     18     22
      19     12     -0     25     15     -0     -0     -0     -0      8
      21     -0     -0     13     -0      2     -0     -0     11     -0
      -0     -0     -0     -0      5     -0     -0      5     10     15
      -0      1     23     -0     14     25     -0     25     12     21
      20     18      6     -0     17     25     -0     -0      2      6
      -0      6      5     -0      6     -0     13     -0     12     -0
       5     15     -0      6     15     -0     -0     -0     -0     -0
      -0     -0     -0     -0     -0     -0     -0     11      3      5
       6     -0     24     12      3     -0     20     -0     10     -0
       1     -0     14     -0     -0     -0     24     13     -0     -0
      24     -0     -0     -0     -0      2     18      2      9     -0
      -0     -0     20     -0      8     -0     15     23     16     23
      -0     -0     14     12     -0     -0     19     24     -0     -0
      -0     17     -0     -0     -0      4     -0     -0     -0     22
      -0     -0     25     22     -0     25      5     -0     -0     -0
      -0     10     25     20     -0     -0      4      5     -0      4
      -0     11     18     -0     -0     11     14     18     -0     -0
      14     -0     17     13      4     14     -0     -0     -0     -0
       3     12      1      9     -0     -0     -0     -0     -0     11
      -0     25     -0     23     20     -0     -0     17      3     -0
      -0      7     -0      7     20      1     17     24     -0     -0
      -0     10     19     -0     23     -0     22     23      8     -0
      -0     10     -0      4     -0     25     22     13     18     15
       6     13     25     -0     -0      7     23     -0     -0     23
       3     -0     18     19     -0     -0     18     -0     -0     -0
      -0     23     -0     13     14      6     -0      1     -0     -0
      14     -0     -0     -O     23     14     11     19     -0     -0
      13     -0     -0      8     20     12     -0     20     21     -0
      -0     24     -0     10      1      5     -0     -0     -0     14
      18     -0      2     -0     -0     -0     11      7     -0     15
      -0     -0     -0      6     -0      9     17      8     -0     11
       8     -0     -0     -0     22     -0     -0     -0     -0     10
      -0     -0      9     -0     12      3     17     25     -0     15
      -0     -0     11     -0     15      8      8      8      5     16
       5     -0     -0     -0     21     -0     10     17     -0     -0
      -0      8     23     -0     -0     -0     -0     23     -0     20
      -0     -0     -0     22     -0     -0     10     -0      3     -0
      -0     -0     14     -0     14     -0     16     -0     15     25
      -0     -0     -0      9     25     -0     20     -0     -0     12
                                                                          3304
     PRMA  PRMA   PRMA   PRMA   PRMA   PRMA   PRMA   PRMA   PRMA   PRMA   248
```

FIGURE 3. (CONTINUED)

```
 THRP-IRDM RANDOM INTEGERS WITH REPLACEMENT FOR 3P SAMPLING.. PAGE     2
 L=124444412444, LIM= 500, K= 25, KZ=      50,PRMA

 PRMA     KSUM       =         3304,       NK      = 248,        NKZ= 500
 EXPECTED NK*(K+1)/2=          3224,       NKZ*K/KZ= 250,        LIM= 500
```