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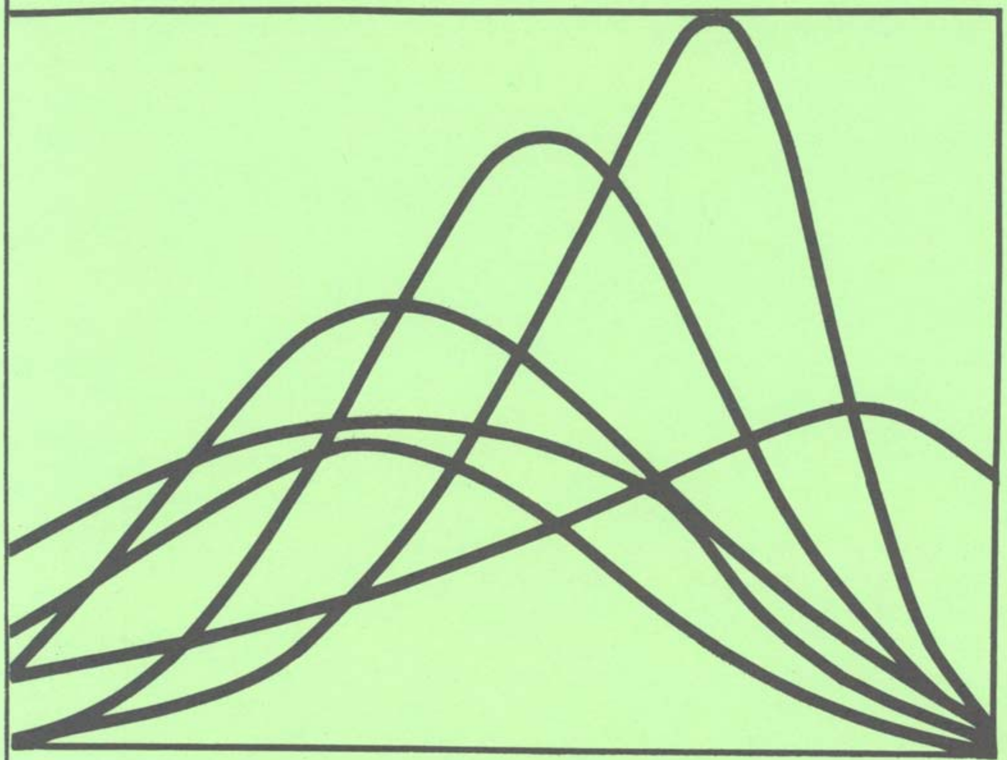

Optimum Target Sizes for a SEQUENTIAL SAWING PROCESS

H. Dean Claxton

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A method for solving a class of problems in random sequential processes is presented. Sawing cedar pencil blocks is used to illustrate the method. Equations are developed for the function representing loss from improper sizing of blocks. A weighted over-all distribution for sawing and drying operations is developed and graphed. Loss minimizing changes in the control variables of saw settings are calculated from the developed equations.

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Retrieval Terms: sawing; operations research; sequential processes; saw setting; mathematical analysis; quality control.

CONTENTS

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Workers in operations research and systems analysis encounter problems based on sequential processes having random components. Raw material conversion based on a series of cutting operations is one example of this class of problems. At each stage in the series, operating characteristics of the saws, together with their random factors, affect succeeding stages. Determining the best size settings for each type of saw in the series is a complex decision process that requires consideration of saw characteristics and the loss caused by having the wrong size piece at the end of the process.

This paper describes a method of solving this problem. The technique is applicable to a wide range of sequential cutting operations. The cutting of cedar pencil blocks is used as an illustration.

Cedar pencil blocks are about 3 by 3 by 7-1/4 inches. They are sawn and air dried before being sent to further cutting processes. This process uses a circular headsaw, a horizontal bandsaw, resaw, and a double arbor edger. Logs are cut into pieces of wood that are essentially square in cross section. The number of blocks contained in each piece varies according to the length of the logs. These log length pieces are then air-dried and cut into 7-1/4-inch blocks. The resulting blocks form several size groups according to the characteristics and size settings of saws that cut them. Each block is subsequently cut (in a radial plane) into 12 pencil slats if it is the proper size. If the block is too small, slats are lost; if it is too large, wood is wasted.

If the final size is = $X°$, the block will produce exactly 12 pencil slats. If the final size is not $X°$, a loss will occur. The final size of a block depends on the value of the saw settings. The problem can be stated by specifying a loss function $L(X)$, with the effect of saw settings as control variables Γ. The aim is then to find the values of Γ that will produce the least expected loss.

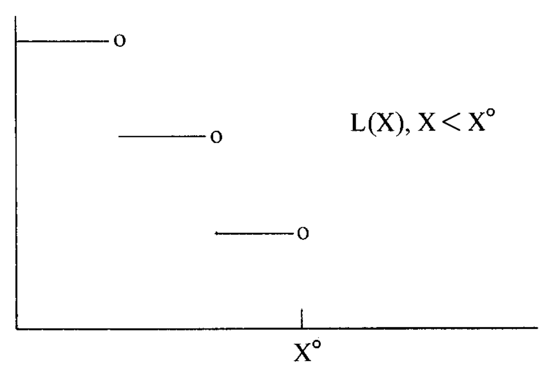
This formulation of the loss function ignores two steps in the actual process. One is the cutting of the 7-1/4-inch length, which does not affect the number slats per block and is therefore excluded from consideration. The second step ignored, the cutting which determines the radial dimension of the blocks, is actually a part of the sawing sequence, but it does not affect the number of slats obtainable. Although the blocks are essentially square in cross section (radial and tangential dimension) only the tangential dimension affects the number of slats obtained. The size requirements for the radial dimension of blocks will be satisfied throughout the range of setting that will be considered. Consequently the cuts which determine the radial dimension of blocks can be ignored.

DECISION PROBLEM

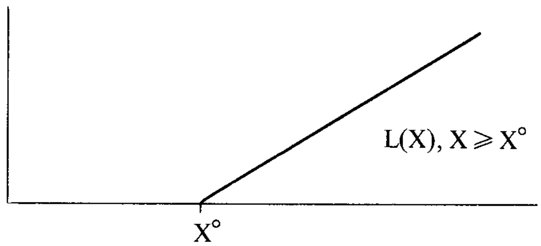
The decision is to choose saw settings that will minimize losses. Losses can be described as a function of block sizes resulting from the control variables of saw setting. Since random factors involved in the process of sawing and drying affect the final block sizes, the loss function can be specified as a random variable with a given probability distribution. To determine such a probability distribution, variability in both sawing and drying must be considered, as it is the final block size that determines how many slats will be obtained.

Loss Function

A loss function can be written, $L(X)$. If blocks are too small and one or more slats are irrecoverably lost, an appropriate loss function could be drawn as shown:



If a block is too large and raw material is wasted in planing, edging, or other sizing operations, an appropriate loss function could be as shown:



Control Variables

The control variables, Γ, are the settings of saws that produce blocks. There are five in this process:

Headsaw	$\Gamma_1, \Gamma_4, \Gamma_5$
Resaw	Γ_2
Double arbor edgers	Γ_3

These can be called target settings, as the sizes resulting from a given setting are not exact, but vary around the setting. The sawing result of a given saw setting can be thought of as a random variable with a probability density function that can be written

$$f(X, \Gamma_1, \ldots, \Gamma_5)$$

in which f is the density function of the variable X(final size).

Expected Loss Function

The expected loss function can be written

$$E[L(X, \Gamma_1, \ldots, \Gamma_5)] =$$

$$\int_{-\infty}^{\infty} L(X) \cdot f(X, \Gamma_1, \ldots, \Gamma_5)dX$$

To minimize $E[L(X,\Gamma_1,\ldots,\Gamma_5)]$ with respect to Γ, it is necessary to set

$$\frac{\partial E}{\partial \Gamma_1} = \frac{\partial E}{\partial \Gamma_2} = \ldots = \frac{\partial E}{\partial \Gamma_5} = 0$$

If we regard Γ as the vector of present target settings and Γ' as the vector of target settings providing minimum loss, we can then solve for the changes in settings $\Gamma' - \Gamma$. If the present settings, Γ, can be changed to settings providing lower losses, nonzero values of $\Gamma' - \Gamma$ will be obtained. For the relevant range of values of Γ, all values other than Γ' will produce a larger loss than the loss from Γ'.

PROBABILITY DENSITY FUNCTION

Our probability density function of final block size is affected by both sawing and drying operations. In this section we will discuss the specifications of the drying function and the sawing function, and the technique for transformation of variable.[1]

The probability density function for sawing and drying, $f(X,\Gamma)$, can be derived from $f^1(Y,\Gamma)$ and $f^2(P)$, in which Y is the wet size as sawn, and P is the ratio of dry size over wet size. The sawing distribution $f^1(Y, \Gamma^1, \ldots, \Gamma_5)$ is assumed independent of the drying distribution $f^2(P)$.

Specification of the density function $f(X,\Gamma)$ can be obtained by using the transformation of variable technique.

Transformation of Variables

We have in effect, for fixed Γ, two random variables Y and P with probability density functions $f^1(Y)$ and $f^2(P)$. We want the probability density of $X = Y \cdot P$. As we have assumed that Y and P are independent, the joint density function of Y and P (by a well-known theorem) is

$$\varphi(Y,P) = f^1(Y) \bullet f^2(P)$$

We define the transformation of the variables Y,P into the variables X,V as

$$Y,P \xrightarrow{T} X,V$$

We assume V = P, which here provides the auxiliary variable V. V is introduced as a mathematical device to allow the transformation between two two-dimensional spaces. The transformation T is defined by a system of two equations:

$$T \begin{cases} X = Y \bullet P \\ V = P \end{cases} \quad \text{or in general,} \quad \begin{matrix} X = X(Y,P) \\ V = V(Y,P) \end{matrix}$$

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[1]Freeman, Harold. *Introduction to statistical inference.* Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., p. 78-82. 1963.

As the transformation is one to one, there exists an inverse transformation,

$$Y,P \overset{T^{-1}}{\leftarrow} X,V$$

which is defined by the following system of equations:

$$T^{-1}\begin{cases} Y = \dfrac{X}{V} & \quad Y(X,V) \\ & \text{or} \\ P = V & \quad P(X,V) \end{cases}$$

The determinant of the Jacobian of a transformation (Y,P)→(X,V) is

$$|J| = \begin{vmatrix} \dfrac{\partial Y}{\partial X} & \dfrac{\partial Y}{\partial V} \\ \\ \dfrac{\partial P}{\partial X} & \dfrac{\partial P}{\partial V} \end{vmatrix}$$

For the case at hand the determinant of the Jacobian of T is

$$|J| = \begin{vmatrix} \dfrac{1}{V} & -\dfrac{X}{V^2} \\ \\ 0 & 1 \end{vmatrix} = \frac{1}{V}$$

The joint density function of X,V can then be expressed as

$$\theta(X,V) = \varphi[Y\ (X,V\ ),P\ (X,V\ )] \bullet |\ J\ |$$

For this case, since

$$Y(X,V) = X \bullet V^{-1}$$
$$P(X,V) = V$$
$$|\ J\ | = V^{-1}$$
$$\theta\ (X\ ,V) = \varphi\ [X \bullet V^{-1}\ ,V] \bullet V^{-1}$$

The density function of X is obtained by integrating θ (X,V) over V.

$$h(X) = \int_{\text{lower limit of V}}^{\text{upper limit of V}} \theta(X,V)\ dV$$

Then, substituting from the previous definition,

$$f(X,\Gamma) = f_v f^1\ (X \bullet V^{-1}\ ,\Gamma) \bullet f^2\ (V) \bullet V^{-1}\ dV$$

We now have the probability density function of the variable X written in terms of the probability density functions of the original variables Y and P; that is f^1 and f^2 . The next step is to specify these functions.

In the example given we were interested in the variable X, which was the product of two variables Y and P. It should be noted that the same technique can be applied to the difference of two variables as follows:

Consider the variable U where U = X — Y

Define $X,Y \overset{T}{\rightarrow} U,V$

$$T\begin{cases} U(X,Y) = X - Y \\ V(X,Y) = Y \end{cases}$$

$$T^{-1}\begin{cases} X(U,V) = U + V \\ Y(U,V) = V \end{cases}$$

$$|\ J\ | = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$$

Assuming X and Y stochastically independent,

$$(X,Y) = f(X) \bullet g(Y)$$

then

$$h(U) = f_v f\ (U+V) \bullet g(V)\ dV$$

Drying Function

The specification of f^2 (P), the drying function, being the least complex, can be considered first. Recall the definition of P as the ratio of dry size over wet size and the probability density function of P as f^2 (P). The cumulative distribution function of P can be written

$$F^2(P) = \int_{\infty}^{P} f^2\ (P)dP$$

One general formulation for F2 (P) can be written

$$F^2(P) = [\ 1+e^{-\alpha_1(P-\alpha_2)}]^{-\alpha_3}$$

Given values for the coefficients α_1 ,α_2 , and α_3 , the cumulative distribution could be computed for given values of P. Note that the distribution is independent of Γ. Values of Γ affect only the wet size distribution, which is the result of sawing.

Sawing Distribution

The distribution, f^1 (Y), of block sizes depends on the five saw settings identified earlier. The distribution is also affected by the combination of saws used to produce a group of blocks and the number of blocks in each group. The sawing distribution can be specified as follows:

$f^1(Y,\Gamma) =$	$W_1 \bullet f_1^1\ (Y,\Gamma_1)$	Headsaw
+	$W_2 \bullet f_2^1\ (Y,\Gamma_2)$	Resaw
+	$W_3 \bullet f_3^1\ (Y,\Gamma_3)$	Double arbor edger
+	$W_4 \bullet f_4^1\ (Y, \Gamma_2)$	Headsaw-resaw; bottom cuts from 6- or 9-inch boards
+	$W_5 \bullet f_5^1\ (Y,\Gamma_4,\Gamma_2)$	Headsaw-resaw; top cuts from 6-inch boards
+	$W_6 \bullet f_6^1\ (Y,\Gamma_5,\Gamma_2)$	Headsaw-resaw; top cuts from 9-inch boards
+	$W_7 \bullet f_7^1\ (Y)$	Headsaw-double arbor edger

in which

W_i is the weight or proportion of blocks sawn by method i

f_i^1 for i = 1, 2, 3 are distributions determined by one saw alone

f_i^1 for i = 4, 5, 6, 7 are distributions created by a combination of saws.

Now we define

$$F_i^1\ (Y) = \int_{\infty}^{Y} f_i^1\ (Z,\Gamma)\ dZ$$

and specify

$$F_i^1\ (Y) = [\ 1+e^{-\beta_{1i}(Y-\beta_{2i})}]^{-\beta_{3i}}$$

The mode of the density function derived from this cumulative distribution function is

$$M_i = \beta_{2i} + \frac{\log \beta_{3i}}{\beta_{1i}}$$

Then let

$$L_i = \frac{\log \beta_{3i}}{\beta_{1i}}$$

and

$$\beta_{2i} = M_i - L_i \qquad i = 1,2,3$$

We further assume the mode to be the sum of the relevant saw setting and some constant. For values of i from 1 to 3 we will denote k=i and

$$\beta_{2i} = (\Gamma_k + c_i) - L_i$$

For values of i from 4 to 7 a slight modification of this formulation is required. There are only three saws, and the last four of the distributions are the results of combinations of the saws. We will consider each of the combinations separately.

Headsaw-Resaw Combination; Bottom Cuts From 6- or 9-inch boards: $f_4^1(Y,\Gamma_2)$

The initial cut made by the headsaw determines the straightness of one side of a square several blocks long. The other side of all the blocks in this square is cut by the resaw. Thus, the size of such blocks is determined by both the headsaw and the resaw in the following manner: For blocks with one side cut by the headsaw and one side by the resaw, when cut from the bottom of a 6- or 9-inch board, the wet size Y is determined by the setting of the resaw and the irregularity of the headsaw cut. The actual thickness setting of the headsaw has nothing to do with the thickness of the bottom piece cut from 6- and 9-inch boards.

As the variations of the saw are assumed fixed, or not subject to control, the difference between the density function of these blocks and those with both sides cut by the resaw is specified by a different function with the same control parameter as the resaw function. The cumulative distribution function is as follows:

$$F_4^1\ (Y) = \int_{\infty}^{Y} f_4^1\ (Z,\Gamma_2)\ dZ$$

then we specify

$$F_4^1\ (y) = [1+e^{\beta_{14}(Y-\beta_{24})}]^{-\beta_{34}}$$

in which

$$\beta_{24} = M_4 - L_4$$

But above we have

$$\beta_{22} = M_2\ L_2$$

We shall assume the difference between M^4 and M^2 to be caused by an effect from the irregular headsaw cut. Then we can write

$$\beta_{24} = M_2 + (M_4 - M_2) - L_4$$

and

$$\beta_{24} = (\Gamma_2 + c_2) + (M_4 - M_2) - L_4$$

Headsaw-Resaw Combination; Top Cuts From 6-Inch Boards: $f_5^1\ (Y, \Gamma_2, \Gamma_4)$

When the blocks are produced from the top cut, the size setting of the headsaw as well as its variability has a direct bearing on sizes of the blocks. We can designate Y_1 as the size of the boards cut by the headsaw (nominally 6 inches) and Y_2 as the size of blocks produced by the resaw. The bottom blocks from the 6-inch board are sawn by the resaw, leaving the size of the top blocks as the difference between the 6-inch headsaw setting and the resaw setting. As before specify

$$F_5^1\ (Y, \Gamma_4, \Gamma_2) = \int^Y_\infty f_5^1\ (Z, \Gamma_4, \Gamma_2)\, dZ$$

$$F_5^1\ (Y, \Gamma_4, \Gamma_2) = [1+e^{-\beta_{15}(Y-\beta_{25})}]^{-\beta_{35}}$$

$$\beta_{25} = M_5 - L_5$$

$$\beta_{25} = [(\Gamma_4 + c_4) - (\Gamma_2, + c_2)] - L_5$$

Headsaw-Resaw Combination; Top Cuts From 9-Inch Boards: $f_6^1 (Y,\ \Gamma_5, \Gamma_2)$

These blocks are produced in much the same manner as the blocks with distribution f_5^1. The exception is that the top cut from the wider board is a residual from two resaw cuts instead of one. Specification is as follows:

$$F_6^1\ (Y, \Gamma_5, , \Gamma_2) = \int^Y_\infty f_6^1\ (Z, \Gamma_5, \Gamma_2)\, dZ$$

$$F_6^1\ (Y, \Gamma_5, \Gamma_2) = [1+e^{-\beta_{16}(Y-\beta_{26})}]^{-\beta_{36}}$$

$$\beta_{26} = M_6 - L_6$$

$$\beta_{26} = [(\Gamma_5 + c_5) - 2(\Gamma_2 + c_2)] - L_6$$

Headsaw-Double Arbor Edger Combination: $f_7^1 (Y)$

Some blocks have one side cut by the headsaw and the other side by the double arbor edger. The probability density function of these blocks can be expected to vary widely from either the headsaw or the double arbor edger blocks. The headsaw cut edge is influenced by the variability of the headsaw, but not its size settings. Similarly the double arbor edger side is influenced by the variation of the saw but the size setting is not relevant. As the size of these pieces is primarily influenced by the way a board is positioned for feeding to the double arbor edger, the wet size can be regarded simply as a random variable with distribution f_7^1 (Y). Specification of the function is as before:

$$F_7^1 (Y) = \int^Y_\infty \quad f_7^1\ (Z)\, dZ$$

$$F_7^1\ (Y) = [1+e^{-\beta_{17}(Y-\beta_{27})}]^{-\beta_{37}}$$

PARAMETERS OF CUMULATIVE DISTRIBUTION FUNCTIONS

Parameters of the cumulative distribution functions

$$F_1^1, \ldots, F_7^1 \text{ and } F^2 (P)$$

can be estimated using the technique of nonlinear least squares.

Block Size Distribution

Estimates of the probability of occurrence of blocks at or below specified wet sizes for each of the sawing methods defined were made as follows:

A sample of blocks from each sawing method was drawn and its critical dimension measured at three places on each block. An average of these three measurements was taken to be the block size. Histograms were constructed by defining intervals of 0.001 inch and counting all blocks whose dimension was smaller than the maximum limit of each interval. Then the number of such blocks in each interval was divided by the total number of blocks of that sawing method to provide observations suitable for fitting the specified functions.

The estimated coefficients and accuracy measures are shown in *table 1,* and the data points and resulting curves in *figure 1.*

From the definition of F^1 (Y,(3) we obtain the probability density function as $f^1(Y,\beta) = \dfrac{dF^1(Y,\beta)}{dY}$

and

$$\frac{dF^1}{dY} = -\beta_3\,[1+e^{-\beta_1(Y-\beta_2)}]^{-\beta_3-1} \bullet [-\beta_1\, e^{-\beta_1(Y-\beta_2)}]$$

Table 1—*Estimated coefficients for block size distribution*

Item	β_1	β_2	β_3	Mean squared deviation
F_1^1	27.0284	3.02729	2.99416	0.01606
F_2^1	46.1520	3.09395	1.00253	.01707
F_3^1	478.9543	3.13276	.34495	.02092
F_4^1	86.6924	3.13536	.33552	.01304
F_5^1	39.5252	3.08650	.33309	.01078
F_6^1	25.3623	3.07697	1.04801	.01589
F_7^1	75.4782	3.17829	.13328	.02962

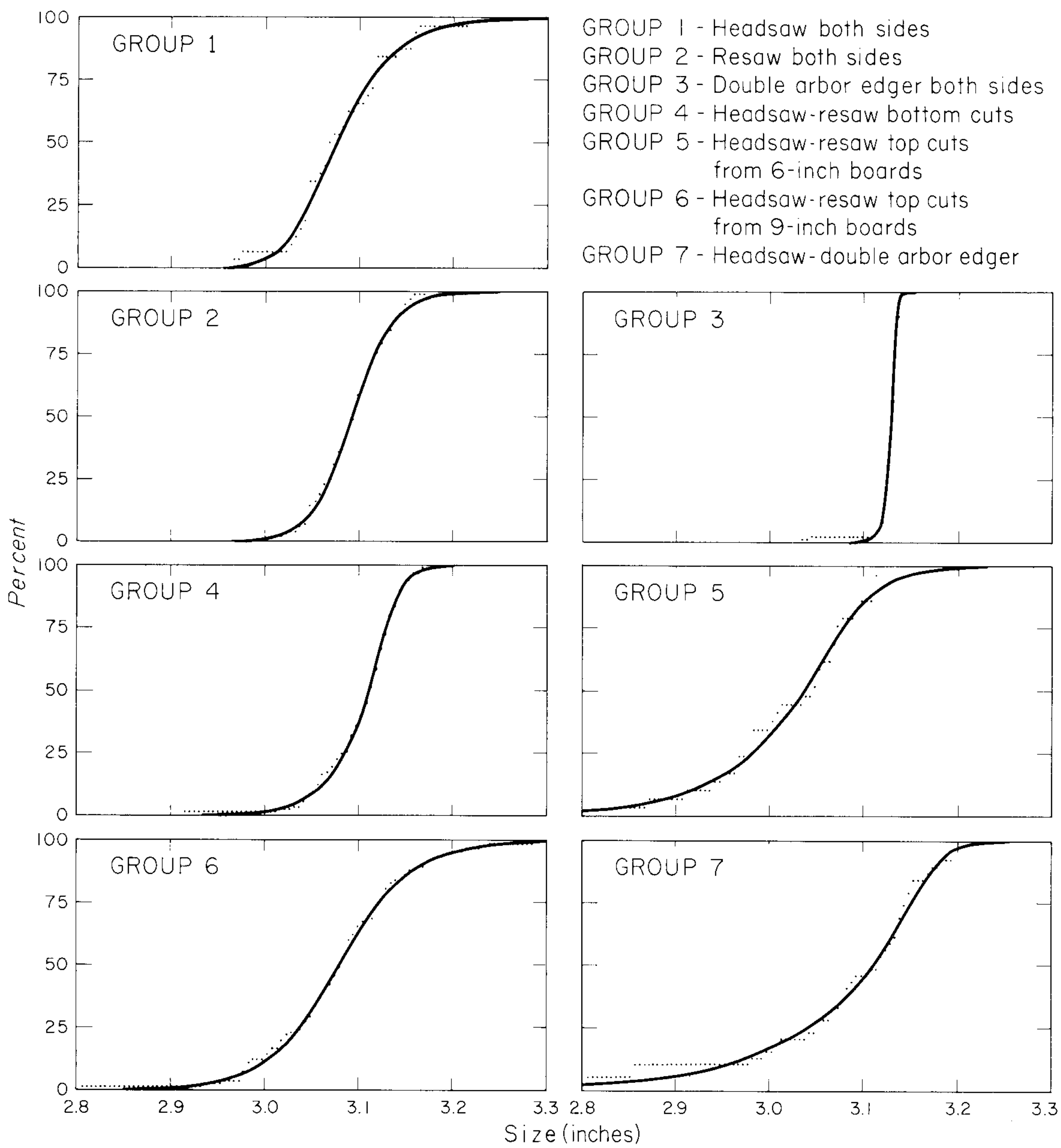


Figure 1—Block size distribution is determined by saw characteristics and settings. The curves show the distribution for three saws and four combinations of saws.

The resulting probability density functions using the estimated values for β are as shown in *figure 2*.

To combine these results into a single distribution, estimates of the proportion of total block production from each sawing method are required as weights. As the sample taken to determine frequency of size occurrence in each group did not contain the information necessary to determine proportion of blocks in each sawing group, another sample was taken. The fraction of blocks in each group is:

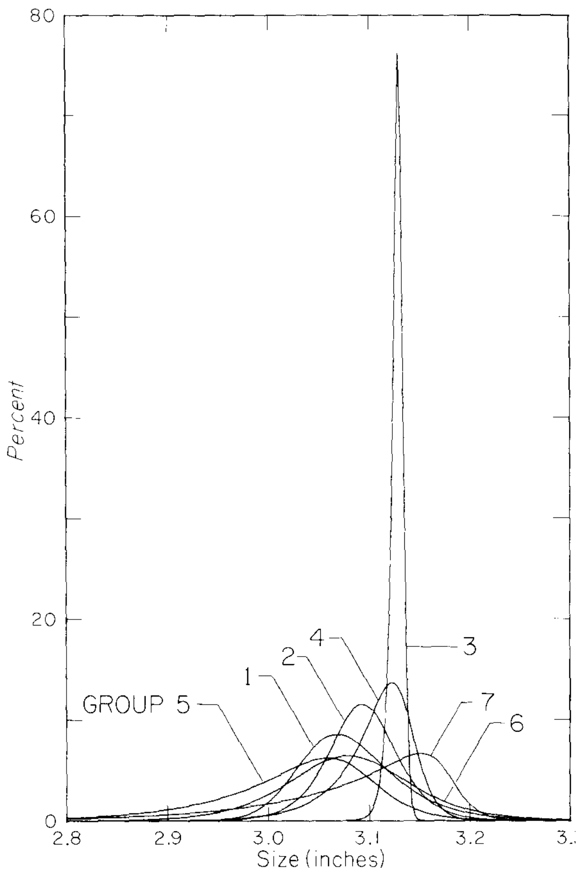


Figure 2—The probability density curves for the sawing operation shown here are for the various saws and combinations. Group 1-Headsaw both sides; 2-Resaw both sides; 3-Double arbor edger both sides; 4-Headsaw-resaw bottom cuts; 5-Headsaw-resaw top cuts from 6-inch boards; 6-Headsaw-resaw top cuts from 9-inch boards; 7-Headsaw-double arbor edger.

$W_1 = .01558$ $W_5 = .00554$
$W_2 = .09254$ $W_6 = .04274$
$W_3 = .58856$ $W_7 = .13071$
$W_4 = .12432$

Combining the previous results with these weights produces the over-all sawing density function.

Drying Distribution

The drying distribution was obtained in much the same manner as the sawing distribution. A sample of blocks was drawn and measured while wet and later remeasured when dry. The ratio of dry size to wet size is the variable for which a probability density function is to be constructed. Like the sawing function, the drying function is a sum of individual drying functions. Here the individual functions are the result of different wood types drying at different rates. The wood types were (1) kiln-dried sapwood, (2) air-dried sapwood, and (3) kiln-dried light heartwood.

A separate cumulative distribution function for each type of wood was estimated using the same algebraic form and the procedures described. For computational convenience the coefficients for these curves were computed using the variable 100 • (1-P) rather than P. The coefficients are given in *table 2* and the graphs of the functions in *figure 3*.

Table 2–*Estimated coefficients for drying distribution*

Item	α_1	α_2	α_3	Mean squared deviation
F_1^2 (P)	1.92096	3.58300	0.69993	0.03136
F_2^2 (P)	2.28439	1.51630	1.72986	.00822
F_3^2 (P)	1.93950	-1.45374	125.94974	.01124

The proportion of blocks occurring in each wood type was obtained from a separate sample and used as weights along with the above given coefficients to produce a weighted drying distribution function. This was then combined with the weighted sawing distribution to obtain the over-all sawing and drying density function given in *figure 4*.

DETERMINATION OF SAW SETTING CHANGES

The final form of the expression is then

$$0 = \frac{\partial E}{\partial \Gamma} = \int_X L(X) \int_V \left[\frac{\partial}{\partial \Gamma} \sum_{i=1}^{M} W_i f_i^1 (X \cdot V^{-1}, \Gamma) \right] \cdot$$

$$\sum_{j=1}^{N} U_j f_j^2 \,(V) V^{-1}\, dV dX \quad M = 6, N = 3$$

where $\frac{\partial E}{\partial \Gamma}$ is the vector of partial derivatives $\frac{\partial E}{\partial \Gamma_k}$

Note that M is equal to 6 because the 7th distribution is independent of Γ, and that the partial with respect to Γ can be brought through the two integrals, their limits and L(X) both being independent of Γ.

Rather than solving for Γ we will solve for another vector of variables, Z. This procedure is somewhat more convenient, and by the composite function rule of differentiation is equivalent to solving with respect to Γ given the linear relationship between Z and Γ.

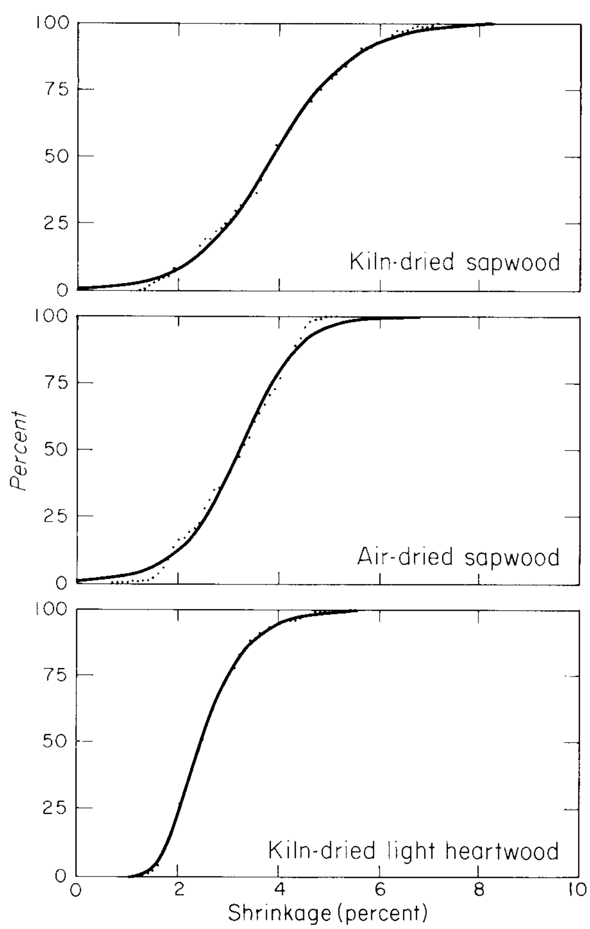


Figure 3—Distribution of shrinkage in the drying operation was determined for three types of wood.

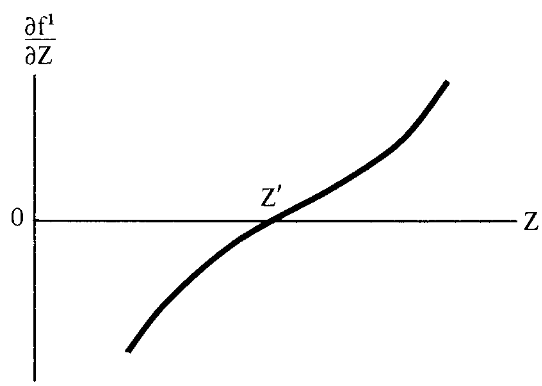
Then the partial for the ith sawing distribution has the form

$$\frac{\partial f_i^1}{\partial Z_i} = W_i \beta_{1i}^2 \beta_{3i} e^{-\beta_{1i}(X/V - Z_i)} \cdot$$

$$[1 + e^{-\beta_{1i}(X/V - Z_i)}]^{-\beta_{3i} - 2} \cdot [\beta_{3i} e^{-\beta_{1i}(X/V - Zi)} - 1]$$

Solving Equations by Approximation

We expect each partial derivative of the expected loss functions with respect to Z_i to have zero value and to be continuous in the range of interest. The partial will then have the shape shown below:



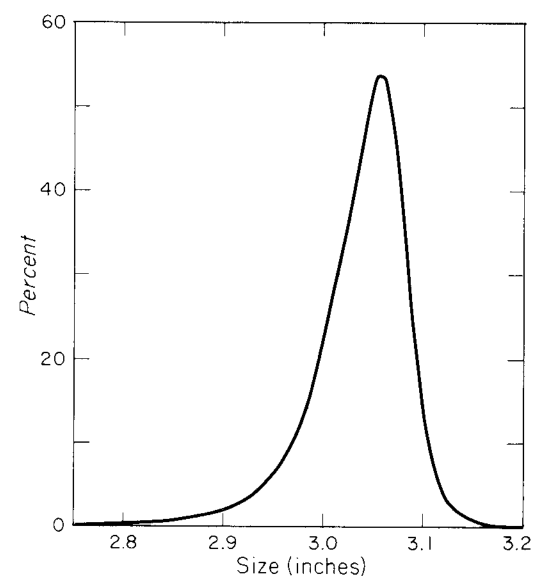


Figure 4—The weighted distribution functions for the drying and sawing operations were combined to produce this over-all curve.

We are given a tolerance e; a lower limit on the accuracy with which we are to find the zero point Z'_i. (In this case we used e = 0.001.) In other words we wish to find Z'_i such that

$$|Z'_i - Z'_i| < \frac{e}{2}, \qquad e > 0$$

To obtain an approximation of Z'_i, we used the Newton iteration procedure.

Calculating Setting Changes

The solution, Z', can be translated into a set of optimum values, one for each saw setting.

For Γ_1, and Γ_3 (Headsaw and Double Arbor Edger)

Each Z_i is a parameter of one density function. For each minimization, we obtain a minimizing Z_i, Z'_i:

$$Z'_i = (\Gamma'_k + c_k) - L_k \qquad k=i = 1,3$$

But we know

$$\beta_{2k} = (\Gamma_k + c_k) - L_k \qquad k = 1,3.$$

So that the difference in setting is given by

$$\Gamma'_k - \Gamma_k = Z'_i - \beta_{2k} \qquad k=i = 1,3.$$

For Γ_2 (Resaw)

Both f_2^1 and f_4^1 are functions of Γ_2 and no other Γ_k. (Recall the earlier "sawing distribution" discussion of the headsaw-resaw combination, bottom cuts.) We take

$$Z_2 = (\Gamma_2 + c_2) — L_2$$

and express Z_4 in terms of Z_2

$$Z_4 = Z_2 + (\beta_{24} - \beta_{22}).$$

We then take the partial with respect to Z_2, then obtain a minimizing

$$Z'_2 = (\Gamma_2 + c_2) - L_2.$$

As

$$\beta_{22} = (\Gamma_2 + c) - L_2$$

then

$$\Gamma_2' - \Gamma_2 = Z_2' - \beta_{22}.$$

For f_5^1 (Headsaw-Resaw; Top Cuts From 6-Inch Boards) and f_6^1 (Headsaw-Resaw; Top Cuts From 9-Inch Boards)

Both f_5^1 and f_6^1 are functions of two saw settings, (Γ_4 and Γ_5). Both are also functions of the resaw setting Γ_2, for which we have already obtained a change in setting.

We can then calculate the changes in Γ_4 and Γ_5 given $\Gamma'_2; - \Gamma_2$ and values for Z_5 and Z_6. We have for Γ_4 a minimizing Z_5, as

$$Z_5' = [(\Gamma_4' + c_4) - (\Gamma_2' + c_2)] - L_5$$

and

$$\beta_{25} = [(\Gamma_4 + c_4) - (\Gamma_2 + c_2)] - L_5.$$

Then

$$\Gamma_4' - \Gamma_4 = (Z_5' - \beta_{25}) + (\Gamma_2' - \Gamma_2)$$

Or

$$\Gamma_4' - \Gamma_4 = (Z_5' - \beta_{25}) + (Z_2' - \beta_{22})$$

We have also a minimizing Z'_6 as

$$Z'_6 = [(\Gamma'_5; + c_5) - 2(\Gamma_2' - c_2)] - L_6$$

and

$$\beta_{26} = [(\Gamma_5 + c_5) - 2(\Gamma_2 + c_2)] - L_6.$$

Then, as above,

$$\Gamma_5' - \Gamma_5 = (Z_6' - \beta_{26}) + 2(Z_2' - \beta_{22}).$$

Numerical Results

This gives us the complete vector $(\Gamma' - \Gamma)$ as follows:

$(\Gamma'_1 - \Gamma_1) = (Z'_1 - \beta_{21})$

Headsaw change for 3-inch setting

$(\Gamma'_2; - \Gamma_2) = (Z'_2; - \beta_{22})$

Resaw setting change

$(\Gamma'_3; - \Gamma_3) = (Z'_3 - \beta_{23})$

Double arbor edger setting change

$(\Gamma'_4 - \Gamma_4) = (Z'_5 - \beta_{25}) + (Z'_2 — \beta_{22})$

Headsaw change for 6-inch setting

$(\Gamma'_5 - \Gamma_5) = (Z'_6 - \beta_{26}) + 2(Z'_2 - \beta_{22})$

Headsaw change for 9-inch setting.

Minimizing values (Z'_i) are tabulated here.

$Z'_1 = 3.092$ inches

$Z'_2 = 3.121$ inches

$Z'_3 = 3.126$ inches

$Z'_5 = 3.299$ inches

$Z'_6 = 3.377$ inches

Minimizing changes in the saw settings calculated using the coefficient estimates given in *table 1* are as follows:

Headsaw, 3-inch cut	3.092 - 3.027	= .065
Resaw	3.121 - 3.094	= .027
Double arbor edger	3.126 - 3.133	= -.007
Headsaw, 6-inch cut	(3.145 - 3.087) + 0.027	= .085
Headsaw, 9-inch cut	(3.129 - 3.077) + .054	= .106

CONCLUSIONS

The method of analysis presented here should be of value to operations researchers and systems analysts engaged in optimizing (or improving) results from random sequential processes. The method was illustrated by solving to determine optimum settings for saws in a sequential process. Of course, the loss functions and data for the illustration are not widely applicable. The method is. Appropriate data and loss functions would be required before a different application could be made.

Problems based on sequential processes of this nature have proven difficult to solve. We hope the method presented here will lead to solutions of a number of similar problems.

SUMMARY

Claxton, H. Dean
1972. **Optimum target sizes for a sequential sawing process.** Pacific Southwest Forest and Range Exp. Stn., Berkeley, Calif., 10 p., illus. (USDA Forest Serv. Res. Paper PSW-87)

Oxford: 852.2-015.6: 832.15.
Retrieval Terms: sawing; operations research; sequential processes; saw setting; mathematical analysis; quality control.

A method of solving problems based on sequential processes is presented and illustrated using the sawing of cedar pencil blocks as an example. Losses resulting from cutting of blocks to improper sizes are described as a function of block sizes resulting from the control variables of saw settings, from drying processes, and from random factors.

From the saw setting variables $\Gamma_1 \ldots \Gamma_5$ an expected loss function is developed. A joint probability density function for sawing and drying is developed from separate functions for sawing and drying. The distribution of block size groups as determined by three saws and four combinations of saws is analyzed.

Parameters of the cumulative distribution functions are estimated and curves for the cumulative distribution functions and density functions are given for the sawing operation. Parameters are estimated and curves drawn for the drying of three wood types. A weighted over-all density function for both sawing and drying is graphed.

An over-all expression for loss minimization is given and equations are derived for the five saw settings. Loss minimizing changes in the saw settings are calculated from these equations.

The method is widely applicable and should be valuable in operations research and systems analysis of random sequential processes.

G. P. O. 794-425/ 3731

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