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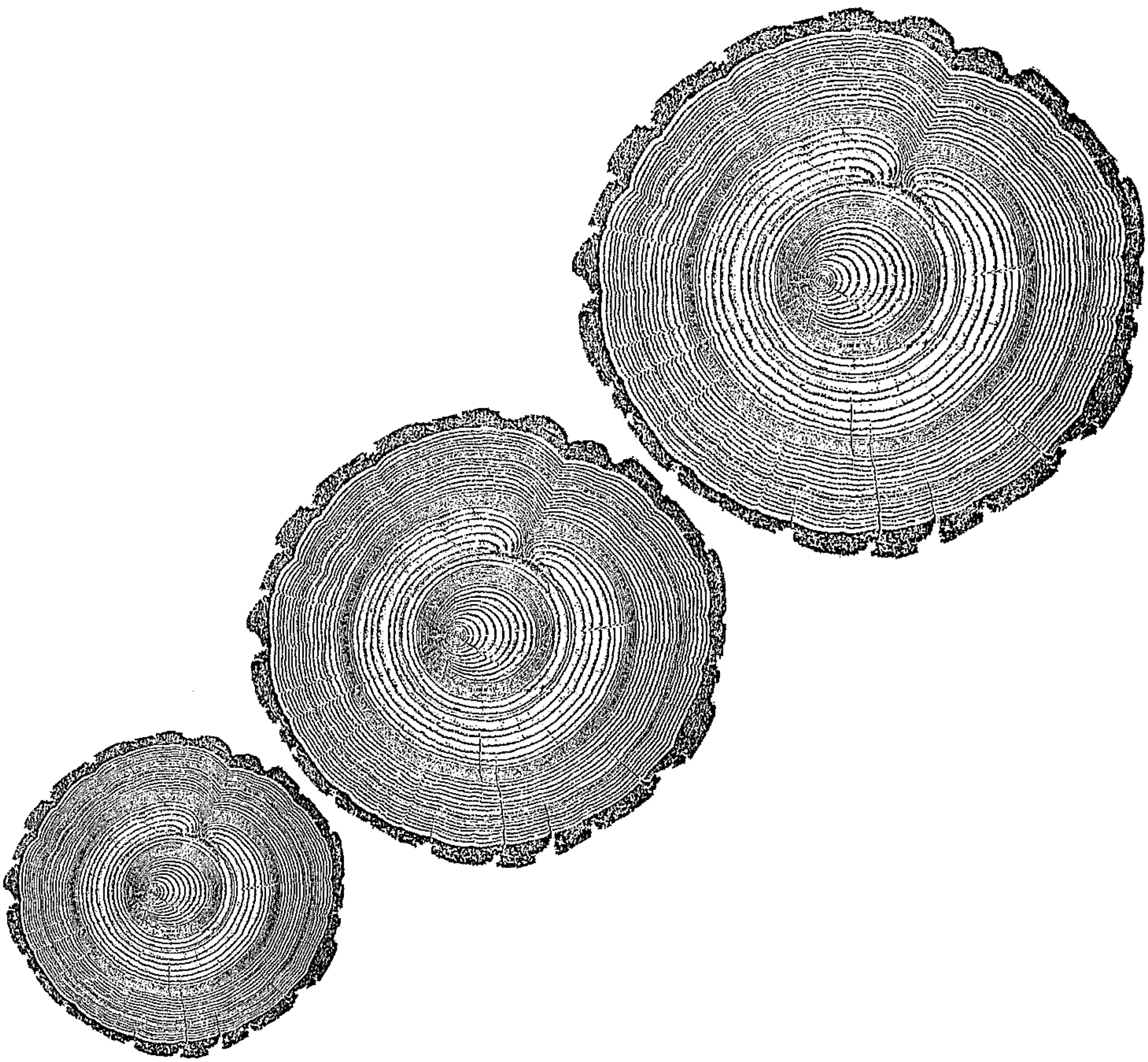
A Diameter Increment Model for Red Fir in California and Southern Oregon

K. Leroy Dolph



Dolph, K. Leroy. 1992. **A diameter increment model for red fir in California and southern Oregon.** Res. Paper PSW-RP-210. Albany, CA: Pacific Southwest Research Station, Forest Service, U.S. Department of Agriculture; 6 p.

Periodic (10-year) diameter increment of individual red fir trees in California and southern Oregon can be predicted from initial diameter and crown ratio of each tree, site index, percent slope, and aspect of the site. The model actually predicts the natural logarithm of the change in squared diameter inside bark between the start and the end of a 10-year growth period. To estimate diameter increment, the predicted value is converted to a change in diameter outside bark. Data used to develop the model came from 1500 tree samples from 56 young-growth stands in the study area. Coefficients for the log-linear model were obtained using least-squares linear regression.

Retrieval Terms: increment (diameter), California red fir, Shasta red fir, California, Oregon

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In Brief . . .

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Red fir forests of California and southern Oregon are becoming more intensively managed for timber production and to enhance production and values of other resources. However, growth and yield information is presently limited in these high-elevation forests. Three geographic variants of the stand prognosis model (PROGNOSIS) (Stage 1973; Wykoff and others 1982) have been developed for growth projection and yield estimates of the major commercial tree species in California, but no growth functions for these variants exist for red fir.

A model was developed to predict periodic (10-year) diameter increment of individual red fir trees in California and southern Oregon. The model actually predicts the natural logarithm of the change in squared diameter inside bark between the start and the end of a 10-year growth period. To estimate diameter increment, the predicted value is converted to a change in diameter outside bark.

Individual tree characteristics and stand and site factor information were recorded in 56 randomly selected young-growth red fir stands throughout the study area. Corresponding diameter growth for the past 10 years was measured from increment cores of the sample trees. Least squares regression techniques were used to relate 10-year diameter growth to the significant tree, stand, and site variables and to obtain estimates of the model parameters.

Information needed to predict diameter increment (for a future 10-year period) of individual trees on a given site includes initial diameter and crown ratio of each tree, site index, percent slope, and aspect of the site. Additional stand variables needed for the prediction, such as stand density (expressed as basal area per acre) and the basal area made up of trees larger than the subject tree, can be calculated directly from the diameters of the individual stems.

The log-linear regression model accounted for 84.6 percent of the variability in log of change in squared diameters observed on the study plots.

Introduction

In recent years, red fir forests have become an increasingly important natural resource in California. Not only is the demand for once "nonpreferred" wood now substantial, these high-elevation forests occupy prime recreational and wildlife habitat areas and much of the snowbelt that furnishes usable surface water in the state. Growth and yield information therefore becomes increasingly important for forest resource management considerations in this forest type. Accurate estimates of both current resource levels and the expected resource changes from implementing various management alternatives are needed for making wise management decisions.

The Prognosis Model for Stand Development (PROGNOSIS), an individual tree growth and yield model, was developed for use in the Inland Empire area of northern Idaho, eastern Washington, and western Montana (Stage 1973). New "variants" of PROGNOSIS result when that model is calibrated for different geographic areas. Three variants of PROGNOSIS have been developed for growth projection and yield estimates of the major commercial tree species in California, but no growth models exist for the red fir type.

This paper describes a log-linear regression model for predicting future 10-year diameter increment for red fir in California and southern Oregon. This model is designed to be used within existing PROGNOSIS variants for silvicultural and management planning for the red fir type. Since California red fir *(Abies magnifica* A. Murr.) and its commonly recognized variety Shasta red fir *(Abies magnifica* var. *shastensis* Lemm.) are considered to be almost identical in silvical characteristics (Hallin 1957), no attempt was made to distinguish between them during the sampling phase of the study. In this paper they are referred to collectively as "red fir."

Methods

Fifty-six natural stands of young-growth red fir were randomly sampled throughout the study area during the field seasons of 1984 - 1988. The study area coincides with the range of the natural distribution of red fir in California, southern Oregon, and extreme western Nevada *(fig. 1)*. This area occupies the high elevations, roughly between 5,000 and 9,000 feet,[1] from Lake County, California, northward through the north Coast and Klamath ranges, and from Kern County, California, northward through the Sierra Nevada and the Cascade ranges to about the latitude of Crater Lake, Oregon (Gordon 1980).

A cluster of five variable-radius plots arranged in an "L" shape, with plot centers 132 feet apart along north and east compass lines, was used to sample each stand. This sample plot layout was selected to be consistent with procedures used

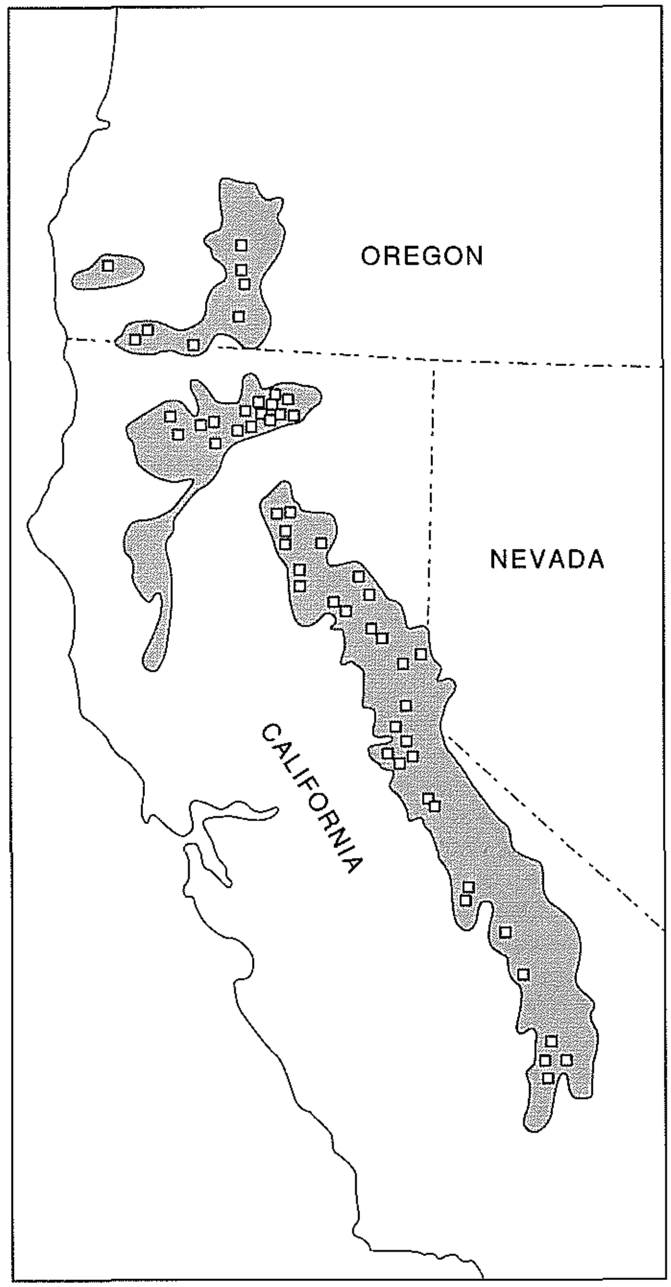

Figure 1—Natural range of red fir and study plot locations.

[1]Metric conversion:
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 square foot = 0.0929 square meters
1 acre = 0.404686 hectares
1 square foot per acre = 0.229568 square meters per hectare

in the Pacific Southwest Region's Compartment Inventory and Analysis (CIA) program. Each plot in the cluster was independently evaluated for study suitability by the following criteria:

- Homogeneous site—No more than one distinct soil type, slope percentage, or aspect was present on a 0.2-acre circular plot around the plot center (radius 52.7 ft).
- Significant amount of red fir—At least 20 percent of the trees 1.0 inch and greater in diameter at breast height (dbh) were red fir.
- Young-growth—No more than 25 percent of the plot trees were older than 120 years at breast height.
- Untreated stand—No silvicultural treatment was evident during the 10 years before measurement.

In stands where at least two of the five plots met these four criteria, all suitable plots were measured. If there were fewer than two suitable plots, no measurements were taken in that stand. The number of measured plots therefore ranged from two to five in each of the clusters; a total of 254 plots were measured in the 56 stands.

Stand and Tree Measurements

Basal area per acre was determined at each plot center by counting all trees that qualified for tallying with a wedge prism. Wedge prisms with basal area factors of 10, 20, 40, or 60 were used, depending upon stand characteristics. All trees 1 inch dbh and larger were listed on a tally sheet. The following characteristics and measurements were recorded for each live tree:

- Species
- Dbh (outside bark)
- Crown position
- Damage, defect, and tree class
- Six-class dwarf mistletoe rating (Hawksworth 1977)
- Total height
- Height to base of live crown
- Past 10-year radial growth

On each conifer tree 3 inches dbh and larger, age and past 10-year radial growth were measured from increment cores extracted at breast height (4.5 ft above the ground on the uphill side of the tree). One core was measured on each tree 3.0 to 5.9 inches dbh. Two cores, taken at right angles, were measured on all trees 6 inches dbh and larger. Radial growth for the past 10-year period was measured to the nearest 0.05 inch, using a hand lens when necessary.

Dead trees (those estimated to have died during the last 10-year period) were recorded on the tally sheet and noted with a mortality code. No measurements were made on these trees other than dbh and an estimate of the year of death.

The 1,537 red fir trees recorded in the sample included 31 dead trees which were estimated to have died during the last ten years. Six of the live trees were deleted from the sample because of severe damage from broken tops, making the calculation of crown ratio impossible. This left 1,500 trees suitable for calibration of the diameter increment model.

Site Factors

Major environmental characteristics of each plot were determined on a 0.2-acre circular plot around the prism plot center. The following information was recorded:

- Slope percent, measured with a clinometer along the line of slope passing through the plot center
- Aspect, measured with a hand compass along the line of slope and recorded as azimuth from true north to the nearest degree
- Site index, recorded as the total height of the best-growing, dominant site tree on the plot at the index age of 50 years at breast height. Site index for most of the plots was determined directly from stem analysis of selected site trees. For plots where no site trees were analyzed, site index estimates were made using height and age data of dominant trees and the site index table for red fir (Dolph 1991).

Site factor information recorded for the entire cluster included:

- Elevation of the vertex plot of the L-shaped cluster, determined from U.S. Geological Survey maps and recorded to the nearest 100 ft
- Latitude of the cluster, determined from U.S.D.A. Forest Service maps, and recorded to the nearest minute.

Analysis

The periodic (10-year) change in squared diameter inside bark (DDS), rather than diameter increment, was selected as the dependent variable as a matter of statistical convenience. As other studies have shown, the trend in ln(DDS) relative to ln(dbh) is linear, and the residuals on this scale have a nearly homogeneous variance (Dolph 1988, Wykoff and others 1982). Also, the increase in DDS over short time periods is often nearly proportional to the length of the time period. This proportionality facilitates predictions for intervals different in length from the growth interval over which the parameters of the model were estimated (Stage 1973).

Tree, stand, and site factor variables (and transformations of these variables) of greatest value in predicting ln(DDS) were identified using the best subsets regression procedure of Minitab (Minitab Inc. 1989). The best subsets of independent variables were defined as those which (1) minimized the residual mean squared error and Mallows' C_p statistic (Hocking 1976), with C_p close to the number of parameters, (2) maximized the coefficient of determination (R^2), and (3) came closest to meeting the assumptions of regression analysis. Of these best subsets, the one chosen for the model was selected on the basis of the number of variables included, biologically meaningful relationships of the dependent and independent variables, and low multicollinearity of the predictor variables. Variance inflation factors (VIF's) were computed for each of the model coefficients to detect collinearity among the selected independent variables. Parameter estimates and VIF's were computed using the multiple linear regression procedure of Minitab.

Results and Discussion

The model that best describes the relationship between 10-year change in inside bark diameter of individual trees and the tree, stand, and site characteristics is expressed as

$$E[\ln(DDS)] = B_0 + B_1 \cdot \ln(D) + B_2 \cdot CRID + B_3 \cdot SI + B_4 \cdot DSQ + B_5 \cdot COS(ASP) \cdot SL + B_6 \cdot BALAR + B7 \cdot \ln(BA) + B8 \cdot SL$$

in which

E[ln(DDS)] = the expected value of the natural logarithm periodic (10-year) change in squared diameter inside bark at breast height, of the subject tree

ln(D) = natural logarithm of the diameter in inches (outside bark) at breast height of the subject tree at the beginning of the 10-year growth period

CRID = $[(CR)^2/\ln(1+D)]/1000$ in which CR = (ratio of live crown length to total tree height) · 100

SI = site index of the plot expressed as the total height in feet of a dominant red fir at a reference age of 50 years breast height

D = diameter outside bark (inches)

DSQ = $D^2/1000$

COS(ASP) = cosine of the plot aspect expressed in degrees from true north

SL = slope of the sample plot (percent/100)

BALAR = basal area in larger trees expressed as $[BL/\ln(1+D)]/100$ in which BL = total basal area (square feet per acre) of all trees on a sample plot that are larger than the subject tree

ln(BA) = natural logarithm of the total basal area (square feet per acre) of the sample plot, at the beginning of the 10-year growth period

$B_0, B_1, \ldots B_8$ = regression parameters estimated from the sample data.

The parameter estimates are:

$\hat{B}_0 = -1.41610$	$\hat{B}_5 = -0.49194$
$\hat{B}_1 = 1.77757$	$\hat{B}_6 = -0.19069$
$\hat{B}_2 = 0.43210$	$\hat{B}_7 = -0.11431$
$\hat{B}_3 = 0.01066$	$\hat{B}_8 = 0.36984$
$\hat{B}_4 = -0.55600$	

The Independent Variables

Independent variables in the model reflect the size, vigor, and competitive stress of the subject tree, and the associated site capability.

Tree diameter at the beginning of the growth period is the most important variable in the model for predicting diameter growth. The logarithm of dbh in a simple linear regression explained 59 percent of the variation in the logarithm of the change in squared diameter during the subsequent 10-year period. Inclusion of the diameter squared term (DSQ) increased the explained variation to 61 percent.

The second most important variable in the equation is the transformed crown ratio term (CRID). Crown ratio indicates the percent of the tree stem which has live foliage, and thus can be considered an expression of tree vigor. Although the model that included crown ratio itself was not selected, the one with the transformed crown ratio term raised the explained variation to 81.3 percent. When the crown ratio squared term is divided by the logarithm of dbh plus 1.0, this transformation gives crown ratio a larger effect for trees with small diameters. Small trees with low crown ratios are usually suppressed and have poor vigor. As trees become larger and mature, low crown ratios do not necessarily indicate reduced rates of diameter growth.

Addition to the equation of the remaining five independent variables, which represent the tree's competitive stress and the site capability, altogether raised the amount of explained variation by another 3.3 percent.

Both total plot basal area (BA) and the transformation of basal area in larger trees (BALAR) reflect the competitive stress on a given tree. The estimated coefficients of the transformed variables ln(BA) and BALAR are negative, indicating reduced growth rates with increased competitive stress at higher levels of density and basal area in larger trees.

Three significant site factor variables are included in the model: site index, slope percent, and the cosine of the aspect times the slope percent. Site index is a rating of the relative site productivity of the sample plot and represents the relationship between height and age of dominant site trees. Since height growth is the indicator of site quality, the trees themselves are considered integrators of many site factor variables such as climate, soils, and physiography.

The slope percent term has a positive coefficient, indicating better diameter growth on steeper slopes. However, the relationship between growth and slope percent above 60 percent is unknown because slopes steeper than 60 percent were not sampled. The interaction of slope and aspect, expressed as the cosine of the aspect times the percent slope, has a negative coefficient. Since the cosine is negative between 90 and 270 degrees, this indicates better growth on the steeper southerly aspects. This seems biologically reasonable because growth of red fir forests at high elevations is probably limited more by length of the growing season than by water availability. When the aspect is level (SL=0), the transformation has a net contribution of zero to the ln(DDS) prediction.

Other independent variables initially tested in the model but not selected by the C_p criterion included plot elevation, latitude, and the 6-class dwarf mistletoe rating. Mistletoe rating was tested both as a continuous variable and as dummy variables representing the six rating levels. Nonsignificance of the mistletoe rating was surprising, but may have been due to the subjectivity involved in the rating system.

Random Cluster Effect

The model explains 84.6 percent of the observed variation in the ln(DDS) with a standard error of estimate of 0.3946. However, in the development of the predictive model, each tree was treated as an independent observation although actually the clusters (which were selected randomly) are the experimental units and tree measurements are subsamples within the clusters. Ignoring this random cluster effect results in a slight underestimate of the variance associated with the regression.

In another study which predicted ln(DDS) of white fir in the Sierra Nevada (Dolph 1988) using the same sampling design, a variance component model was developed to calculate the variance associated with this random cluster effect. In that study, it was found that the random cluster effect caused the standard error of estimate to increase from 0.4158 to 0.4182, an increase of about six tenths of one percent. Because the estimated increase was so small, developing a variance component model separately for the red fir study did not seem warranted. Because the same sampling design was used in both studies and about the same number of trees were measured, increasing the estimate of the standard error by the same percentage to account for the random cluster effect seems reasonable. Thus a more realistic estimate of the standard error, which considers the random cluster effect, would be (0.3946) (1.006) = 0.3970.

Correcting for Log Bias

Using the logarithmic transformation of the dependent variable introduces a problem because the value of interest is the change in squared diameter, not the log of the change in squared diameter. Negative bias (a tendency to underestimate the mean) is introduced when the antilogarithm is used to convert log-normally distributed estimates back to original units, because the antilogarithm yields the median rather than the mean of the skewed arithmetic distribution in original units (Baskerville 1972).

Several different estimators for log-bias correction have been developed, and guidelines for selection of their use have been described (Flewelling and Pienaar 1981). The bias correction proposed by Baskerville (1972) was selected because of its simplicity and relatively widespread usage, even though it may not be the most accurate estimator available. Using Baskerville's estimator, the log bias can be approximately corrected by adding one-half the error variance to the estimate on the log scale, with the assumption that the residuals are normally distributed with respect to the logarithm of DDS. Plots of the residuals—the actual minus the predicted ln(DDS)—and an analysis of their distribution indicated this assumption was reasonable.

Because the standard error of estimate, adjusted for the random cluster effect, is 0.3970, the estimate of the error variance is $(0.3970)^2$ and one-half the error variance equals 0.0788. The appropriate log-bias correction factor is $e^{0.0788} = 1.0820$. The predicted DDS (in the natural scale) would be multiplied by 1.0820 to correct for underprediction.

An Example

To demonstrate how the model works, the predicted ln(DDS) will be calculated for a tree with an initial dbh of 11.3 inches and a crown ratio of 66 percent. Total plot basal area equals 176 ft^2 per acre, and 148 ft^2 of this basal area is made up of trees that are larger than the subject tree. Assuming the plot has a site index of 52 feet, an aspect of 175 degrees, and a slope of 15 percent, the model is evaluated as follows:

$\hat{B}_0 = -1.41610$

$\hat{B}_1 \cdot \ln(D) = (1.77757) \cdot (2.4248) = 4.31025$

$\hat{B}_2 \cdot CRID = (0.43210) \cdot (1.73574) = 0.75001$

$\hat{B}_3 \cdot SI = (0.01066) \cdot (52) = 0.55432$

$\hat{B}_4 \cdot DSQ = (-0.55600) \cdot (0.12769) = -0.07100$

$\hat{B}_5 \cdot COS(ASP) \cdot SL = (-0.49194) \cdot (-0.99619) \cdot (0.15) = 0.07351$

$\hat{B}_6 \cdot BALAR = (-0.19069) \cdot (0.58974) = -0.11246$

$\hat{B}_7 \cdot \ln(BA) = (-0.11431) \cdot (5.17048) = -0.59104$

$\hat{B}_8 \cdot SL = (0.36984) \cdot (0.15) = 0.05548$

The sum of the above effects is equal to the predicted logarithm of the change in squared inside bark diameters during the next 10-year period. Thus, ln(DDS) = 3.55297. In natural units and corrected for log bias, the predicted DDS is

$$(e^{3.55297}) \cdot (1.0820) = 37.78 \text{ in}^2.$$

Confidence Limits

Putting confidence limits on multiple regressions requires computation of the elements of the inverse matrix for sums of squares and cross products as they appear in the normal equations. The inverse matrix is not presented in this paper but is available to anyone wishing to define confidence limits for any combination of the eight independent variables.[2] Confidence limits on the expected mean value of ln(DDS) can be calculated by:

$$\widehat{\ln (DDS)} \pm t \cdot \sqrt{\text{estimated variance of } \widehat{\ln (DDS)}},$$

in which

[2]Data on file at Pacific Southwest Research Station, 2400 Washington Avenue, Redding, CA 96001.

t = 'Students' t value for the desired probability level with 1492 degrees of freedom, and the

estimated variance of $\widehat{\ln(DDS)} = (\sum_{i=o}^{p} \sum_{j=o}^{p} c_{ij} X_i X_j) \cdot (s^2)$,

in which c_{ij} are the elements of the inverse of the cross products matrix for predictors, p equals the total number of independent variables, X_i and X_j are corresponding values of the independent variables (where X_i or $X_j = 1$, if i or j = 0), and s^2 equals 0.1576, the estimated variance about the regression.

For the specified set of independent variables in the described example, the 95 percent confidence interval for the mean ln (DDS) is:

$$3.55297 \pm 1.96 \cdot \sqrt{(0.0022288) \cdot (0.1576)}$$
$$= 3.55297 \pm 0.03673.$$

In natural units (square inches), with correction for log bias, the approximate interval for DDS is

$$(e^{3.51624}) \cdot (1.0820) \text{ to } (e^{3.58970}) \cdot (1.0820)$$
$$= 36.4 \text{ to } 39.2 \text{ in}^2.$$

Thus, we can say the probability is about 0.95 that the true mean of DDS associated with this combination of independent variables will be between 36.4 and 39.2 in^2.

The limits on individual values of ln(DDS) can be obtained by adding one times the estimated variance about the regression to the term under the radical in the formula given for the limits of mean ln(DDS) (Freese 1964). The formula for the limits on an individual value of ln(DDS) would be

$$\ln(DDS) \pm t \cdot \sqrt{(1 + \sum_{i=o}^{p} \sum_{j=o}^{p} c_{ij} X_i X_j) \cdot (s^2)}.$$

The 95 percent confidence interval for an individual value of ln (DDS) for the combination of independent variables shown in the example is

$$3.55297 \pm 1.96 \cdot \sqrt{(1 + 0.0022288) \cdot (0.1576)}$$
$$= 3.55297 \pm 0.77896 = 2.77401 \text{ to } 4.331933.$$

And in natural units the approximate interval would be:

$$e^{2.77401} \text{ to } e^{4.33193}$$
$$= 16.0 \text{ to } 76.1 \text{ in}^2.$$

Thus, the probability is about 0.95 that an individual observed value of DDS will be within this interval for this specific combination of independent variables.

Estimating Future Diameters

The value of primary interest in growth and yield projection is the predicted diameter outside bark at the end of the growth period (dob_e), not the change in squared diameters inside bark. Because

$$DDS = (dib_e)^2 - (dib_s)^2,$$

in which

dib_s = inside bark diameter at the start of the growth period and

dib_e = inside bark diameter at the end of the growth period, upon rearranging terms,

$$dib_e = \sqrt{(dib_s)^2 + DDS}.$$

Taking the square root of an estimated quantity (DDS) introduces a bias in the estimation of dib_e. The amount of this bias is unknown but assumed to be small.

Because diameters of standing trees are measured outside bark instead of inside bark, equations developed for red fir (Dolph 1989) are used to convert diameter outside bark at the start of the growth period (dob_s) to dib_s and to convert dib_e back to dob_e:

$$dib = (0.86951) \cdot (dob^{1.00983})$$
$$dob = (1.17993) \cdot (dib^{0.98027}).$$

The future diameter outside bark of the red fir tree used in the example, with an 11.3-inch dbh and predicted DDS of 37.8 in^2, would be calculated as follows:

$$dib_s = (0.86951) \cdot (11.3)^{1.00983}$$
$$= 10.1 \text{ inches}$$
$$dib_e = \sqrt{(10.1)^2 + 37.8}$$
$$= 11.82 \text{ inches}$$
$$dob_e = (1.17993) \cdot (11.82)^{0.98027}$$
$$= 13.3 \text{ inches.}$$

Conclusions

Future diameters of individual red fir trees in California and southern Oregon can be estimated using data that are normally collected during stand examinations and forest inventories. Information needed from the sample plots include diameter and crown ratio of each tree, site index (base age of 50 years at breast height), aspect, and slope percent. From these data, all independent variables required for the model can be calculated, including total plot basal area, basal area in larger trees, and transformations of the other variables.

The independent variables used in the model express the size and vigor of the subject tree, the competitive stress which it is under, and the quality of the site at which it is growing. Admittedly, many of these independent variables are less than ideal because they were not measured directly but rather estimated. For example, tree diameters were backdated to the start of the growth period, requiring estimates for the change in bark growth during the period. Crown ratios were measured at the end of the growth period and assumed not to have changed significantly during the past 10 years, thus yielding another estimate. Other estimated variables include total plot basal area, basal area in larger trees, and site index. The amount of bias in the regression coefficients due to the error in independent variables is unknown; unfortunately, there are no satisfactory solutions to these potential problems when dealing with this type of biological data.

Even with these potential problems, the model should function well over a wide range of tree size, stand density, and site index within the red fir type of California and southern Oregon.

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