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Prediction of Periodic Basal Area Increment for Young-Growth Mixed Conifers in the Sierra Nevada

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A model was developed to predict periodic (10-year) basal area increment for individual trees within the mixed-conifer type of the Sierra Nevada. Two sets of parameter estimates for the model are presented. One set of parameters is used when site index information is available for a particular area and tree species. The other set is applicable when site index data are unavailable.

Tree, stand, and site factor information were recorded at 150 randomly located sample sites. Corresponding diameter growth for the past 10 years was measured from increment cores of the individual trees. Least squares regression techniques were used to obtain estimates of the required parameters.

The model was calibrated for six mixed-conifer species of the westside Sierra Nevada. Species included are Douglas-fir, California white fir, incense-cedar, sugar pine, ponderosa pine, and Jeffrey pine.

Information needed to predict basal area increment of individual trees on a given site includes diameter and crown ratio of each tree, stand density (expressed as basal area per acre), elevation, latitude, and percent slope. Slightly better predictions can be made when site index information is available for a given species on a particular site.

Depending on tree species, and the presence or absence of site index information, the regressions accounted for 77.7 to 87.3 percent of the variability in log of basal area increment observed on the study plots. Variance estimates were obtained from a separate variance component model. This model takes into account that the clusters are the randomly selected experimental units and tree measurements are subsamples within clusters.

INTRODUCTION

Mixed-conifer forests are the largest vegetation type in California, covering more than 13 million acres (Barbour and Major 1977). This type, the Sierra Nevada Mixed Conifer (Society of American Foresters Forest cover type 243, Tappeiner 1980) dominates mid-elevations of the Sierra Nevada's western slopes. The extent of the mixed-conifer type and the amount of timber harvested from the west slopes of the Sierra Nevada emphasize the need for accurate growth and yield prediction methods for this area.

Stand simulation methods currently used for predicting growth of these mixed stands lack the flexibility to allow comparison of different silvicultural operations on subsequent stand development and future yields of the component species. Irregularity in stocking and age structure, species differences in silvical characteristics, and difficulties associated with site quality evaluation of these mixed stands make growth and yield prediction difficult. The most feasible means of reliable prediction for the mixed-conifer type appears to be simulation methods using individual trees as the basic elements.

The stand prognosis model (PROGNOSIS) (Stage 1973; Wykoff and others 1982), originally developed for forests of northern Idaho, eastern Washington, and western Montana, is an individual tree projection system. The framework of PROGNOSIS, with growth functions developed for different species in different geographical areas, is now widely used in the western U.S. for silvicultural and management planning. A geographic variant of PROGNOSIS is being developed for young-growth stands of mixed conifers on the western slopes of the Sierra Nevada.

This paper describes a log-linear regression model for predicting periodic basal area increment of individual trees. This model will be used in the Sierra Nevada PROGNOSIS variant.

Another individual-tree growth projection system, the California conifer timber output simulator (CACTOS) has recently been developed for conifer species of northern California (Wensel and Koehler 1985). No data from the central or southern Sierra Nevada were used in developing this system and only 10 sample sites from the mixed-conifer type in the northern Sierra Nevada were included in the data base. As a result, very little overlap exists in the geographic applicability of the CACTOS and PROGNOSIS growth projection systems for these conifer species.

STUDY AREA

The study area occupies mid-elevations on the western slope of the Sierra Nevada, extending approximately 350 miles north to south from 40°00' to 35°25' N. latitude. Data were collected within the six National Forests in the area, which extends from the northern boundary of the Plumas National Forest to the southern boundary of the Sequoia National Forest. The other four National Forests included are the Tahoe, Eldorado, Stanislaus, and Sierra. Within this area, elevational limits of the mixed-conifer type vary with changes in latitude and aspect. Most of the type is found between 3000 and 7000 ft (900 and 2130 m) elevation, below the red fir (*Abies magnifica* A. Murr.) and lodgepole pine (*Pinus contorta* Dougl.) types, and above the drier foothill types.

The mixed-conifer type typically occurs on productive sites of the westside Sierra Nevada, where precipitation and seasonal temperatures occur in the most favorable combination. Conifer species in this type are California white fir (*Abies concolor* [Gord. & Glend.] Lindl.), incense-cedar (*Libocedrus decurrens* Torr.), Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco), ponderosa pine (*Pinus ponderosa* Laws.), and sugar pine (*P. lambertiana* Dougl.). Stands occur as variable mixtures of species or as small groups of single species. Jeffrey pine (*P. jeffreyi* Grev. & Balf.) is often an associate at upper elevations and also occurs on serpentine soils. Hardwood associates are California black oak, Pacific madrone, tanoak, golden chinkapin and Pacific dogwood (Helms 1980).

METHODS

Diameter growth data were collected on 3230 trees at 150 randomly selected sample locations throughout the western slope of the Sierra Nevada mixed-conifer zone. The total sample covered the range of species and diameters typical of young-growth mixed-conifer stands (*table 1*). Each sample location consisted of a cluster of five plots arranged in an L shape, with plot centers 132 feet apart along north and east compass lines.

Table 1—*Number of sample trees by species and DBH classes used in development of basal area increment prediction equations*

DBH class (inches)	Sample trees, by species: White fir	Incense-cedar	Ponderosa pine	Sugar pine	Jeffrey pine	Douglas-fir	Total
0- 4.9	160	144	38	17	8	22	389
5.0- 9.9	495	278	148	54	25	55	1055
10.0-14.9	455	165	133	58	32	45	888
15.0-19.9	253	46	122	39	27	9	496
20.0-24.9	109	36	51	17	12	6	231
25.0-29.9	35	9	24	8	6	3	85
≥ 30.0	19	24	23	13	4	3	86
Total	1526	702	539	206	114	143	3230

These sample plot layout and subsequent tree measurement methods were selected to be consistent with procedures used in the Pacific Southwest Region's Compartment Inventory and Analysis (CIA) program. Each plot of a cluster was independently evaluated for study suitability by the following criteria:

• Young growth—No more than 25 percent of the trees tallied were older than 80 years at breast height.

• Mixed-conifer type—No more than 20 percent of the trees tallied were conifer species other than those characteristic of the mixed-conifer type. Evidence of white fir regeneration in the stand or surrounding area was required, also.

• Homogeneous site—No more than one distinct soil type, slope percentage, or aspect was present on a 1/5 acre circular plot around the plot center (radius 52.7 ft).

• Untreated stand—No evidence of silvicultural treatment during the 10 years prior to measurement was present.

A cluster was retained as a suitable sample point if two or more plots met the above criteria.

Stand and Tree Data

Basal area per acre was determined at each plot center by counting all trees that qualified for tallying with a basal area factor (BAF) 20 wedge prism. It was desirable to collect growth information on a reasonable number of trees (approximately 5 to 10) on each plot and to measure growth on each tree tallied with the prism so stand density statistics could be "backdated," that is, tree diameters and stand density were adjusted to correspond to the beginning of the growth period measured on the increment cores. The method is described in *Appendix A*. The tally with the BAF 20 prism frequently resulted in either too many or too few trees to measure on each plot for efficient growth data collection. The obvious solution was to change the BAF until the desired number of trees was included in the sample (a method sometimes called the "constant-tally rule"). However, such a procedure may introduce appreciable bias in the basal area estimation (Wensel and others 1980).

To help reduce the magnitude of this bias, a procedure was adopted which allowed only one switch per plot to either a smaller or larger BAF. Based on the number of tally trees obtained with the BAF 20 prism, a guide was developed to determine which BAF prism to use in selecting growth measurement trees (*table 2*). This procedure may still result in a positive bias of the estimated total basal area. However, an even larger bias results from switching BAF until the desired number of trees is obtained (Schreuder and others 1981). The actual amount and significance of any bias is difficult to determine since it will depend upon the spatial distribution of the trees being sampled. Any resulting systematic bias should not significantly affect the utility of the basal area prediction model, especially for the Forest Service. The Pacific Southwest Region's forest inventory procedures also specify using a BAF that will give an average tree count of five to eight trees per plot for determining stand basal area and recording data on the individual trees (U.S. Dep. Agric., Forest Serv. 1986).

Table 2—*Guide used to determine BAF of prism for selecting growth sample trees*

Trees counted with BAF 20 prism	BAF of prism used to select growth sample trees
1-4	10
5-10	20
11-20	40
>20	60

Using the selected prism, all tally trees 1.0 inch (2.5 cm) d.b.h. and larger were listed on the tally sheet. The following characteristics and measurements were recorded for each live tree.

• Species
• D.b.h. (outside bark)
• Crown position
• Damage, defect, and tree class
• Total height
• Height to base of live crown

On each conifer tree 3.0 inches (7.6 cm) d.b.h. and larger, age and past 10-year radial growth were measured from increment cores extracted at breast height (4.5 ft [1.4 m] above the ground on the uphill side of the tree). One core was measured on each tree 3.0 to 5.9 inches (7.6 to 15.0 cm) d.b.h. Two cores, taken at right angles, were measured on all trees 6 inches (15.2 cm) d.b.h. and larger. Radial growth for the past 10-year period was measured to the nearest 0.05 inch (0.13 cm), using a hand lens when necessary.

Dead trees (those estimated to have died during the last 10-year growth period) were recorded on the tally sheet and noted as such with a mortality code. No measurements were made on these trees, other than d.b.h.

Site Factors

Major environmental characteristics of each plot were determined on a 1/5-acre circular plot around the prism plot center. The following information was recorded:

• Slope percent, measured with a clinometer along the line of slope passing through the plot center

• Aspect, measured with a hand compass along the line of slope and recorded as azimuth from true north to the nearest degree

• Ground cover, as percent estimates classed as live vegetation, litter, rock, and bare ground

• Vegetation composition, including identification of all trees and shrubs on the 1/5-acre plot and estimates of the percent of total cover each contributed

Site factors recorded for the entire cluster included these:

• Elevation of the vertex plot of the L-shaped cluster, determined from U.S. Geological Survey maps and recorded to the nearest 100 ft

• Latitude of the cluster, recorded to the nearest minute

• Soil characteristics, recorded near the center of that plot which appeared most typical of the cluster. Soil pits were dug to a depth of 48 inches, or to bedrock, whichever occurred first. Soil horizons were described and representative samples of each were collected for laboratory analyses. In the laboratory, the particle size distribution of each horizon within the particle size control section (generally 10 to 40 inches [25 cm to 1 m]) was determined by the hydrometer method following standard procedures (Day 1965). Soil chemical properties were not tested because mixed conifer ecosystems are generally more limited by moisture stress (which is related to physical soil properties) than by nutrient status.

ANALYSIS

Multiple linear regression was used to identify the model which best characterized the relationship between basal area growth of the individual trees and the independent variables. The best model was defined as that which minimized the residual mean squared error, came closest to meeting the assumptions of regression analysis, and contained predictor variables with low multicollinearity.

Dependent Variable Selection

The dependent variable of the prediction equations is the natural logarithm of periodic basal area increment. Periodic basal area increment (bai) for each tree is defined here as the change in squared inside-bark diameter from the beginning to the end of the most recent 10-year growth period. The constant term (1/4 π) in the standard basal area formula

$$ba = \frac{1}{4}\pi D^2$$

has not been included in this definition since the primary utility of the prediction equations is to derive diameter increment rather than actual basal area increment (see Estimating Future Diameters). Therefore,

$$bai = (dib_e)^2 - (dib_e - 2R)^2$$

in which

dib_e = diameter inside-bark at the end of the growth period. Since diameter was measured outside bark at the end of the growth period (dob_e), a bark adjustment factor was applied to calculate dib_e:

$dib_e = (1/k).(dob_e)$, where k is the bark adjustment factor determined for each species (Dolph 1984). The k values are given in *Appendix A* for each species.

R = average 10-year radial wood growth in inches, as measured on increment cores taken at breast height.

Basal area increment, rather than diameter increment, was selected as the dependent variable because its increase over short time periods is often nearly proportional to the length of the time period. This proportionality facilitates predictions for intervals different in length from the growth interval over which the parameters of the model were estimated (Stage 1973). The logarithmic transformation of basal area increment was used to linearize the relationship between the independent variables and basal area increment. This allowed for selection of variables with commonly available linear least-squares procedures. Also, as other investigators have found, the distribution of the residuals more closely resembles normality with homogeneous variance for the log of basal area growth than for other dependent variables (Wykoff and others 1982). Hann (1980) also found the log transformation appeared to make the variance more homogeneous.

Independent Variable Selection

Tree, stand, and site characteristics (and transformations of these variables) of greatest value in predicting basal area increment were identified by multiple linear regression. A locally written and executed program (WINNOW) was used to calculate all possible regressions with subsets of these characteristics as predictor variables and log of basal area increment [ln(bai)] as the dependent variable. Since the plot clusters were the experimental units, each cluster was assigned an indicator variable. The random cluster effect was therefore forced into these regressions.

Using Mallows' C_p statistic (Hocking 1976), the best subsets of predictor variables were identified for each species. This procedure was considered an initial screening of the independent variables. Although the variables in each subset were the best predictors from a statistical standpoint, some variables had limited practical application. Variables that were difficult to obtain in the field, contributed little to the coefficient of determination (R^2), and did not appear consistently in the equations for each species were eliminated from the subsets.

Variance inflation factors (VIF's) were computed for each of the remaining coefficients to detect collinearity among the independent variables. Because some site factor and stand variables were highly correlated with many of the cluster indicators (resulting in high VIF's), the predictive model was developed using the selected site factor and stand variables but not including the cluster indicators. Estimates of the parameters and VIF's were computed using Minitab (Ryan and others 1985). After the predictive model was developed, the variance components of cluster and tree effects were estimated using a locally written program (EMS). The variance component model *(Appendix B)* included the cluster indicators.

RESULTS AND DISCUSSION

The basic model for the multiple linear regressions predicts the log of 10-year basal area increments of individual trees, dependent on tree, stand, and site characteristics. The model for the expected ln(bai) is expressed as

$$E[\ln(bai)] = LAT + B_1 \ln(D) + B_2 DSQ + B_3 CRID + B_4 BALAR + B_5 \ln(BA) + B_6 EL + B_7 SL + B_8 SLSQ + B_9 SITE$$

in which

bai = periodic (10-year) basal area increment in square inches, inside bark at breast height, of the subject tree

LAT = intercept term that is dependent on tree species and latitude

D = diameter of the subject tree in inches (outside bark) at breast height at the beginning of the growth period

DSQ = $(D)^2/1000$

CRID = $[(CR)^2/\ln(1+D)]/1000$ in which CR = (ratio of live crown length to total tree height) • 100

BALAR = $[BL/\ln(1+D)]/100$ in which BL = total basal area (square feet per acre) of all trees on a sample plot that are larger than the subject tree

BA = total basal area of the sample plot (square feet per acre), at the beginning of the growth period

EL = elevation in hundreds of feet

SL = slope of the sample plot in percent

SLSQ = $(SL)^2$

SITE = site index for the species of the subject tree expressed as total height in feet of dominant or codominant trees at a reference age of 50 years at breast height. (If site index data are unavailable, the value of this variable is set to zero and parameter estimates given (*table 3*) are applicable.)

$B_1, B_2, B_3, ..., B_9$ = regression parameters.

Although this basic model was used for each regression, all of the parameter estimates were not significant for all species. Therefore, final predictions of bai for each species do not always require the same independent variables.

Since site index estimates are not always available or often cannot be obtained due to the lack of suitable "site" trees on a given plot, two sets of parameter estimates are presented for each species (*tables 3* and *4*). If site index information is not available, the parameters in the first set (*table 3*) are applicable. With site index data, the second set of parameters *(table 4)* will result in slightly better predictions of periodic basal area increment.

In both sets, only those parameters significant at least at the 0.05 level are included.

Independent Variables

The independent variables in the model reflect the size, vigor, and competitive stress of the subject tree, and the associated site capability. The influence of these variables on basal area increment can be summarized as follows:

Tree Size—Tree diameter at the beginning of the growth period is the most important variable in the model for making predictions of bai. The coefficient of ln(D) is large and positive; the coefficient of DSQ is relatively small and negative. This combination results in a peaking function in basal area increment over diameter. The combination of ln(D) and DSQ has been included in other predictive equations for bai (Wykoff and others 1982; Ritchie and Hann 1985). Depending on species, these two variables alone account for 45 to 70 percent of the variability in ln(bai).

Tree Vigor—Crown ratio (CR) indicates the percent of the tree stem which has live foliage, and thus can be considered an expression of tree vigor. Although crown ratio itself is not included in the model, the parameters of the transformed variable CRID were significant for all six species. This transformation gives the CR term a larger effect for trees with small diameters. In mixed stands, small trees with low crown ratios are usually suppressed and have poor vigor. On the other hand, as trees become larger and mature, low crown ratios do not necessarily indicate reduced rates of basal area increment.

Competitive Stress—Total basal area (BA) on a given plot and the transform of basal area in larger trees (BALAR) both reflect the competitive stress on a given tree and its social position. Transformed variables, ln(BA) and BALAR, had better correlations with ln(bai) than did crown competition factor (CCF) and its various transformations. Coefficients of these variables are negative, indicating the increased competitive stress at higher levels of density and basal area in larger trees, and thus, reduced growth rates. Division of BL by ln(1+D) gives more weight to this variable for the smaller trees. In the term (1+D), 1.0 is added to the tree diameter to prevent the logarithm from becoming equal to zero for very small trees.

Site Capability—Even without site index in the model, the only significant site factor variables found were latitude (LAT), elevation (EL), and percent slope (SL). It would be desirable to treat latitude as a continuous variable but the discrete classification of latitude into five classes (each class representing one degree of latitude) gave a better statistical fit than the simple linear function of latitude.

Transformations of slope and aspect as described by Stage (1976) were not significant either with or without site index in the model. Ritchie and Hann (1985) also found these transformations of slope and aspect were not significant for basal area increment predictions of grand fir and Douglas-fir in the east-central Coast Range of Oregon.

Several variables which described soil characteristics at each site were included in the initial screening of the independent variables. These variables, such as total soil depth, coarse fragment content, particle-size class of the control section (weighted by horizon depth), and actual percentages of sand, silt, and clay, were not significant in predicting ln(bai) for any of the six mixed-conifer species.

Since suitable site index curves were not available for all species of the mixed-conifer type, three site index estimating equations were developed from stem analysis data of dominant and the tallest codominant trees on the study plots. Site index curves for ponderosa pine,[1] incense-cedar (Dolph 1983), and California white fir (Dolph 1987), were produced from the estimating equations *(Appendix D)*.

When site index was entered in the model, the regression coefficients were significant for all six species. However, their

[1] Unpublished data on file at Pacific Southwest Forest and Range Experiment Station, 2400 Washington Avenue, Redding, CA 96001.

Table 3—*Coefficients for the log of basal area increment model by species when site index data are not available*

Variables		White fir	Incense-cedar	Ponderosa pine	Sugar pine	Jeffrey pine	Douglas-fir
		Coefficients by species					
Latitude (degrees)							
35-35.9		0.6393	0.7817	0.5176	0.4818	-0.1048	(1)
36-36.9		0.7993	0.3385	0.0611	1.2501	-0.1048	(1)
37-37.9		0.7993	0.7817	0.0611	1.2501	-0.1048	0.1108
38-38.9		0.7993	0.6415	0.0611	1.2501	0.0811	0.1108
≥ 39		0.6347	0.7817	-0.0646	1.2501	-1.2377	0.1108
ln(D)	(b_1)	1.70210	1.62688	1.82363	1.68875	1.59954	1.96186
DSQ	(b_2)	-0.50615	-0.57769	-0.66352	-0.50354	-0.37820	-0.41990
CRID	(b_3)	0.36755	0.36532	0.28509	0.32904	0.53164	0.43357
BALAR	(b_4)	-0.29454		-0.22263	-0.54100		
ln(BA)	(b_5)	-0.16000	-0.29673	-0.20976	-0.12917	-0.28181	-0.14607
EL	(b_6)	-0.01153	-0.00860	-0.00931	-0.02137		-0.02277
SL	(b_7)			0.91820			
SLSQ	(b_8)	-0.85300	0.71260				

[1]No Douglas-fir were sampled within this range of latitude.

Table 4—*Coefficients for the log of basal area increment model by species when site index data are available*

Variables		White fir	Incense-cedar	Ponderosa pine	Sugar pine	Jeffrey pine	Douglas-fir
		Coefficients by species					
Latitude (degrees)							
35-35.9		0.1434	0.0540	0.1035	-0.4297	0.1630	(1)
36-36.9		0.3191	-0.1232	-0.3955	-0.4297	0.1630	(1)
37-37.9		0.1434	0.0540	-0.3955	-0.4297	0.1630	-0.1043
38-38.9		0.22462	0.0540	-0.3955	-0.2777	0.1630	-0.1043
≥39		0.1434	0.0540	-0.3955	-0.4297	0.1630	-0.1043
ln(D)	(b_1)	1.53339	1.41389	1.63568	1.26883	1.52284	1.92426
DSQ	(b_2)	-0.47442	-0.48938	-0.54162	-0.35325	-0.26590	-0.44612
CRID	(b_3)	0.35739	0.32660	0.27516	0.27986	0.51033	0.40047
BALAR	(b_4)	-0.44256	-0.16000	-0.32497	-0.79922		
ln(BA)	(b_5)	-0.12359	-0.25287	-0.20902		-0.35579	-0.24596
EL	(b_6)	-0.00700					-0.02290
SL	(b_7)			-0.80370			
SLSQ	(b_8)	-0.83400				-0.80370	
SITE	(b_9)	0.00734	0.01200	0.00621	0.01401	0.00659	0.01108

[1]No Douglas-fir were sampled within this range of latitude.

contribution to the R^2 for each equation was small compared to the contribution afforded by the factors of tree size, vigor, and competitive status. Wensel and Koehler (1985) also found site index to be a significant independent variable for height and diameter growth predictions for mixed-conifer species in northern California.

Most of the independent variables in the model are admittedly less than ideal because they were not measured directly but rather were estimated. For example, tree diameters were backdated to the start of the growth period, requiring estimations for the change in bark growth during the period. Crown ratios were measured at the end of the growth period and assumed not to have changed significantly during the past 10 years, thus yielding another estimate. Other estimated variables include total plot basal area, basal area in larger trees, and site index. The amount of bias in the regression coefficients due to the error in independent variables is unknown; unfortunately, there are no satisfactory solutions to the potential problems when dealing with this type of biological data.

Statistics quantifying the fit of the equations are shown for each species both with and without site index *(table 5)*. The standard error given for each equation was estimated from the variance component model *(Appendix B)* and calculated as

$$\text{standard error} = \sqrt{\hat{\sigma}_c^2 + \hat{\sigma}^2}$$

in which $\hat{\sigma}_c^2$ is the variance component due to the random effect of clusters and $\hat{\sigma}^2$ is due to the random effect of trees.

BAI Prediction—An Example

Stand information and sample plot data from a mixed-conifer stand on the Sierra National Forest will be used to demonstrate the model. For the example, the 5th tree record at the sample plot is a white fir tree with a 22.7-inch d.b.h. and a crown ratio of 40 percent. It is the third largest tree on the plot and, since a BAF 40 prism was used, BL equals 80 ft² per acre. Total basal area at the plot equals 320 ft² per acre. The site is at 37° 30' N. latitude at an elevation of 5200 ft. The plot has a 13 percent slope. Assuming no site index information was available for white fir, appropriate coefficients *(table 3)* are used to evaluate the model as follows:

- Latitude (= 37° 30')
  b_0 = Lat = 0.7993
- Tree diameter (= 22.7)
  $b_1x_1 + b_2x_2 = b_1 \cdot \ln(D) + b_2 \cdot DSQ$
  $= (1.70210) \cdot \ln(22.7) - (0.50615) \cdot (22.7)^2/1000$
  = 5.05377
- Crown ratio (= 40)
  $b_3x_3 = b_3 \cdot CRID$
  $= (0.36755) \cdot ([(40)^2/\ln(23.7)]/1000)$
  = 0.18578
- Basal area in larger trees (= 80)
  $b_4x_4 = b_4 \cdot BALAR$
  $= (-0.29454) \cdot (80/\ln(23.7))/100$
  = -0.07444
- Total plot basal area (= 320)
  $b_5x_5 = b_5 \cdot (\ln(BA))$
  $= (-0.16000) \cdot (\ln(320))$
  = -0.92293
- Elevation (= 52)
  $b_6x_6 = b_6 \cdot EL$
  $= (-0.01153) \cdot (52)$
  = -0.59956
- Slope (= 13%)
  $b_8x_8 = b_8 \cdot SLSQ$
  $= (-0.8530) \cdot (13/100)^2$
  = -0.0144

The sum of the above effects is equal to the predicted logarithm of basal area increment. Thus, $\widehat{\ln(bai)} = 4.4275$.

Log Bias Correction

The logarithmic transformation of the dependent variable was used to linearize the data and to make parameter estimates obtainable by ordinary linear regression minimizing the sum of squares. This introduces a problem because the value of interest for predictive purposes is basal area increment—not the log of basal area increment. Negative bias (a tendency to underestimate the mean) is introduced when the antilogarithm is used to convert log-normally distributed estimates back to original units. This is because the antilogarithm yields the median rather than the mean of the skewed arithmetic distribution in original units (Baskerville 1972).

Table 5—*Statistics for log of basal area increment equations*

Species	Equation type[1]	Number of observations	Standard error of estimate	R^2
White fir	1	1,526	0.4383	0.816
	2	1,509	.4182	.832
Incense-	1	702	.4970	.777
cedar	2	657	.4684	.801
Ponderosa	1	539	.4875	.814
pine	2	524	.4687	.824
Sugar	1	206	.4284	.837
pine	2	192	.3617	.873
Jeffrey	1	114	.3943	.829
pine	2	111	.3770	.837
Douglas-	1	143	.4829	.806
fir	2	143	.4492	.834

[1]1 = without site index; 2 = with site index in the model.

Several different estimators for log bias correction have been developed. These estimators and guidelines for selection of their use have been described by Flewelling and Pienaar (1981). The bias correction proposed by Baskerville (1972) was selected because of its simplicity and relatively widespread usage, even though it may not be the most accurate estimator available. Using Baskerville's estimator, the log bias can be approximately corrected by adding one-half the error variance to the estimate on the log scale, with the assumption that the residuals are normally distributed with respect to the logarithm of basal area increment. Plots of the residuals—the actual minus the predicted ln(bai)—and an analysis of their distribution indicated this assumption was reasonable for each species *(Appendix C)*.

Log bias correction factors are shown *(table 6)* for each species and for each type of prediction, that is, with or without site index in the model. To apply the correction, the predicted bai (in the natural scale) would be multiplied by the appropriate correction factor. In the above example, the corrected bai prediction would be:

$$(e^{4.4275})\,(1.0960) = 91.8 \text{ in}^2$$

Confidence Limits

Putting confidence limits on multiple regressions requires computation of the Gauss or c-multipliers. The c-multipliers are the elements of the inverse matrix for sums of squares and products as they appear in the normal equations. The inverse matrices for each species and for each type of equation (with and without site index) are not presented in this paper. They are available to anyone wishing to define confidence intervals for any combination of species and independent variables.[2] General formulas for confidence limits and specific examples are presented *(Appendix B)* to illustrate the calculation method and show a typical range of values for the confidence intervals.

Model Validation

Tests of model accuracy have been limited to tests on the data set used to calibrate the model, since an independent data set is not yet available for model testing. The primary objective of PROGNOSIS is to predict future tree volumes. This volume prediction will require both the bai model described in this paper and the corresponding height-growth model which is currently being developed. Model validation at this point will become critical to test the accuracy of volume predictions produced by PROGNOSIS. Growth data from permanent study plots, CFI (continuous forest inventory) plots, and remeasurement data from some relocatable temporary plots will be used for model validation at that time.

[2]Data on file at Pacific Southwest Forest and Range Experiment Station, 2400 Washington Avenue, Redding, CA 96001.

Table 6—*Log bias correction factors for each species and for each type of equation (with and without site index in the model)*

Species	Log bias correction factor	
	Without site index in the model	With site index in the model
White fir	1.0960	1.0874
Incense-cedar	1.1235	1.1097
Ponderosa pine	1.1189	1.1099
Sugar pine	1.0918	1.0654
Jeffrey pine	1.0777	1.0711
Douglas-fir	1.1166	1.1009

Estimating Future Diameters

With PROGNOSIS, stand development is simulated by predicting increments in the dimensions of the trees that make up the stand. Therefore, diameter increment rather than increment in basal area is the primary interest. Since bai is the difference between squared inside-bark diameters at the start and the end of a growth period, periodic diameter increment (DG) can be derived from basal area increment by the relationship

$$bai = (dib_s + DG)^2 - (dib_s)^2$$

and, upon rearranging terms

$$DG = \sqrt{(dib_s)^2 + bai} - dib_s$$

in which

dib_s = inside bark diameter at the beginning of the growth period.

If diameter outside bark (dob_s) is measured at the beginning of the growth period, dib_s can be estimated by

$$dib_s = (1/k) \cdot (dob_s)$$

in which k is the bark growth correction factor shown for each species *(Appendix A)*.

Diameter outside bark at the end of the growth period (dob_e) can then be estimated by the equation

$$dob_e = k \cdot \sqrt{(dib_s)^2 + bai}$$

As an example, the predicted future outside bark diameter of the 22.7-inch white fir tree with a predicted bai of 91.8 in^2 would be calculated as follows:

$$\begin{aligned} dib_s &= (1/k) \cdot (dob_s) \\ &= (1/1.1222) \cdot (22.7) \\ &= 20.23 \text{ inches} \\ dob_e &= 1.1222 \cdot \sqrt{(20.23)^2 + 83.3} \\ &= 25.1 \text{ inches.} \end{aligned}$$

Taking the square root of an estimated quantity (bai) introduces a bias in the estimation of dob_e. The amount of this bias is unknown, but assumed to be small.

Outside-bark diameters at the end of the growth period were calculated for all trees in the data base using the above procedure and the predicted 10-year basal area increment. These predicted values were compared to the actual diameters measured at the end of the period *(table 7)*. On the average for all species, the predicted outside-bark diameter was within 1.0 inch of the actual value about 84 percent of the time and about 97 percent of all observations were within 2.0 inches.

SUMMARY AND CONCLUSIONS

Future basal area increment of young-growth mixed-conifer species on the western slope of the Sierra Nevada can be estimated using data that are normally collected during stand examinations and forest inventories.

When a stand is sampled using a variable-radius prism plot, an estimate of stand density (basal area/acre) is obtained by multiplying the total number of "tally" trees by the prism BAF. If each tally tree is listed by species, d.b.h., and crown ratio, and the plot site factors of elevation, latitude, and percent slope are recorded, all of the required information is available for making basal area increment predictions.

From these data, all independent variables required for the model can be calculated, including basal area in larger trees and various transformations of other variables. If total height and breast-height age of a dominant or the tallest codominant tree of each species present on the plot have been recorded, site index estimates for these species can also be calculated.

Since site index information is not always available, the model was designed to be flexible. With or without site index estimates, the model should function well over a wide range of stand density and tree size within the elevational and latitudinal limits of the western slope Sierra Nevada mixed-conifer type.

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Table 7—*Percentages of observations in the data base showing a difference between the observed and predicted outside-bark diameter at the end of the 10-year growth period*[1]

Difference between actual and predicted diameter (inches, less than)	Cumulative percent of observations, by species					
	White fir	Incense-cedar	Ponderosa pine	Sugar pine	Jeffrey pine	Douglas-fir
0.25	29.2	36.5	30.2	23.3	57.0	27.3
0.50	52.2	63.8	54.0	49.5	79.8	48.3
0.75	69.5	78.5	71.6	61.7	87.7	67.8
1.00	81.8	87.0	83.5	74.8	97.4	80.4
1.25	88.6	93.4	91.8	83.0	98.2	87.4
1.50	92.7	96.0	95.7	88.3	99.1	91.6
1.75	95.0	97.9	96.5	91.7	99.1	97.2
2.00	96.7	98.6	97.2	94.7	99.1	97.9
2.50	98.6	99.1	99.1	96.1	99.1	99.3
3.00	99.3	99.4	99.6	97.1	100.0	100.0
4.00	99.9	99.7	99.8	98.1		
5.00	100.0	100.0	100.0	99.5		
6.00				99.5		
6.50				100.0		

[1]Example: the difference between actual and predicted outside-bark diameter was less than 1.25 inches for 93.4 percent of the incense-cedar trees in the data base.

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APPENDIX A

The "Backdating" Process

For model calibration, tree diameters and stand density were backdated to correspond to the beginning of the growth period measured on the increment cores. Backdating of tree diameters was accomplished by subtracting the most recent 10-year total diameter growth from the current size. Since total diameter growth includes the growth of both wood and bark, but only wood growth was measured on the increment cores, an adjustment to correct for bark growth was applied to properly backdate outside bark diameters. Bark growth adjustment factors have been developed for each species (Dolph 1984).

Past outside bark diameter for each sample tree was therefore calculated by the formula

$$Dob_p = Dob_c - (k \cdot 2R)$$

in which

Dob_p = past diameter at breast height, outside bark
Dob_c = current diameter at breast height, outside bark
R = average radial wood growth for the 10-year period
k = species-specific bark growth correction factor:

Species	k
White fir	1.1222
Incense-cedar	1.1942
Ponderosa and Jeffrey pine	1.1152
Sugar pine	1.1283
Douglas-fir	1.1314

Because distances to each tree from the prism plot center had not been recorded, backdating of plot basal area presented another problem. However, as 10-year radial growth had been determined for each conifer included in the sample, plot basal area could be adequately backdated assuming the number of trees per acre had not changed significantly during the growth period. Using this assumption, plot basal area was backdated by the following steps:

1. Current basal area in square feet (BA_c) was calculated for each tree, where $BA_c = (Dob_c)^2 \cdot 0.00545415$
2. Number of trees per acre (TPA) each prism tree represented was determined by the equation TPA = BAF of prism/BA_c
3. Past basal area in square feet (BA_p) was calculated for each tree, where $BA_p = (Dob_p)^2 \cdot 0.00545415$
4. Past total plot basal area (BA_t) was then calculated as $$BA_t = \Sigma (BA_p)(TPA)$$

Hardwood tree species were not increment-bored during the study. Therefore, average values of 10-year diameter growth were used for backdating basal areas of the hardwood species. The average value used for black oak was based on data described by McDonald (1969). The averages used for tanoak and Pacific madrone were based on data described by McDonald (1980).

Species	Average 10-year diameter growth (wood and bark)
	(Inches)
Black oak	1.83
Tanoak	3.03
Pacific Madrone	1.80

Backdating stand density of plots on which trees had died during the growth period presented an additional problem. To approximate the 10-year diameter growth of these trees, had they been alive throughout the growth period, an average diameter ratio (DR) was first calculated for the living trees of each species, where

$$DR = \frac{\text{past 10-year diameter growth}}{Dob_c}$$

The diameter of each dead tree was then multiplied by the appropriate diameter ratio for that species to estimate what the 10-year diameter growth would have been. Since the exact time of death of each tree was not determined, the number of years during the growth period that each dead tree had influenced the stand density also was unknown. An average of 5 years (1/2 of the growth period) was assumed. Therefore, the diameters of these trees at the beginning of the growth period were calculated as

$$Dob_p = Dob_c - [(1/2)(DR)(Dob_c)]$$

Admittedly, this procedure for backdating mortality is imprecise. However, the resulting estimates of past stand basal area are more accurate than those made when mortality is completely ignored. This backdating procedure for mortality was required on less than one percent of the study plots.

Live crown ratio for each tree was not backdated. Crown ratio was assumed to remain about the same for the growth cycle since no measure of the change in live crown ratio was available.

APPENDIX B

Variance Component Model and Confidence Limits

Variance Component Model

In the development of the predictive model, each tree was treated as an independent observation. Actually however, the clusters (which were selected randomly) are the experimental units and tree measurements are subsamples within the clusters. Ignoring this random cluster effect results in an underestimate of the variance associated with each regression. A larger model, including the random cluster effect, was therefore developed to obtain more realistic variance estimates. The proposed variance component model is the following:

$Y_{j(i)} = \ln(bai)_{j(i)}$ = f(tree factors) + g(plot factors) + h(cluster factors) + $a_i + e_{j(i)}$

in which

- i = index representing cluster i
- j(i) = index representing tree j of cluster i
- f, g, and h represent functions of tree, plot, and cluster factors, respectively.
- a_i = random effect of cluster i. Assumed to be normally distributed with mean zero and variance σ_c^2
- $e_{j(i)}$ = error due to random tree effect. It is assumed to be normally distributed with mean zero and variance σ^2

Indicator variables were assigned to each cluster so the variance component due to the random cluster effect could be estimated. The variance of plot effect was ignored because the estimate of this component was much smaller than the other two. (A model including random plot effects was considered but rejected because tests suggested that plot effect contribution to variance was small compared to random cluster and tree variance.)

Locally written and executed programs (WINNOW and EMS) were used because the large amount of clusters (requiring up to 122 dummy variables in the model) made commercial computer time too expensive.

For this model, the variance for a randomly chosen tree in a randomly chosen cluster is

$$\text{Variance}(Y_{j(i)}) = \sigma_c^2 + \sigma^2$$

The variance of the mean calculated from m randomly chosen clusters with n trees per cluster is

$$\text{Variance}(\overline{Y}) = \sigma_c^2 / m + \sigma^2/(mn)$$

Variance components calculated by the EMS program are shown below for each species and for each type of equation (without and with site index in the model). Total variance is the sum of the variance components: due to random cluster effect (σ_c^2) and due to random tree effect (σ^2).

Estimated variance components for regressions without site index:

Species	$\hat{\sigma}_c^2$	$\hat{\sigma}^2$	Estimated total variance
White fir	0.03999	0.15208	0.19207
Incense-cedar	0.07703	0.17000	0.24703
Ponderosa pine	0.05503	0.18273	0.23776
Sugar pine	0.07317	0.11033	0.18350
Jeffrey pine	0.02516	0.13026	0.15542
Douglas-fir	0.10796	0.12521	0.23317

Estimated variance components for regressions with site index:

Species	$\hat{\sigma}_c^2$	$\hat{\sigma}^2$	Estimated total variance
White fir	0.02410	0.15076	0.17485
Incense-cedar	0.05727	0.16206	0.21933
Ponderosa pine	0.04095	0.17876	0.21971
Sugar pine	0.03332	0.09750	0.13082
Jeffrey pine	0.01042	0.13172	0.14214
Douglas-fir	0.07775	0.12395	0.20170

Confidence Limits

The confidence limits on the expected mean value of ln[bai] over m clusters with n trees per cluster not only change by species but also vary with different values of the independent variables. The confidence interval is given by

$$\widehat{\ln(bai)} \pm t \cdot [\text{estimated variance of } \widehat{\ln(bai)}]^{1/2}$$

in which

t = "Student's"- t value for the desired probability level with $(n_o - n_p)$ degrees of freedom where n_o equals the number of observations and n_p equals the number of estimated parameters.

The variance of ln(bai) is equal to

$$\left(\sum_{i=0}^{p}\sum_{j=0}^{p} c_{ij} x_i x_j\right)\left[\hat{\sigma}_c^2/m + \hat{\sigma}^2/(mn)\right]$$

in which c_{ij} are the c-multipliers, x_i and x_j are values of the independent variables (where x_i or x_j = 1, if i or j = 0), p equals the total number of independent variables, and $\hat{\sigma}c^2$ and $\hat{\sigma}_2$ are the estimates of σ_c^2 and σ_2

For the described example, the 95 percent confidence interval for the mean ln(bai) for the specified set of independent variables over one cluster with one tree per cluster (m=1, n=1) is:

$$4.3310 \pm 1.96 \cdot \sqrt{(0.0035)(0.19207)}$$
$$= 4.4275 \pm 0.0508.$$

In natural units (square inches), with correction for log bias, the approximate interval for bai is

$$(e^{4.3767})(1.0960) \text{ to } (e^{4.4783})(1.0960)$$
$$= 87.2 \text{ to } 96.5 \text{ in}^2$$

Thus, we can say that there is a 0.95 probability that the true mean will be covered by intervals constructed in such a manner.

The limits previously discussed are limits for the regression surface at a specified point, not limits on individual values of bai. Having estimated a value of ln(bai) by means of the regression equation, we would like to have some idea as to the limits which might include most of the individual bai values associated with a specific set of independent variables. These limits can be obtained by adding [$\hat{\sigma}_c^2 / m + \hat{\sigma}^2 / mn$] to the estimated variance of ln(bai) in the equations given for the regression limits (Freese 1964). This would make the formula for the limits on an individual value of ln(bai)

$$\widehat{\ln(\text{bai})} \pm t \cdot \sqrt{\left(1 + \sum_{i=o}^{p} \sum_{j=o}^{p} c_{ij} x_i x_j\right)\left(\hat{\sigma}_c^2 / m + \hat{\sigma}^2 / mn\right)}$$

The 95 percent prediction interval for an individual value of ln(bai) for the combination of independent variables shown in the example is

$$4.3310 \pm 1.96 \cdot \sqrt{(1 + 0.0035)(0.19207)}$$
$$= 4.4275 \pm 0.8605$$

And in natural units the approximate interval for bai would be:

$$e^{3.5670} \text{ to } e^{5.2880}$$
$$= 35.4 \text{ to } 197.9 \text{ in}^2$$

Thus, over the long run, an observed value of bai will be within intervals constructed in this manner 19 times out of 20 for this specific combination of independent variables.

APPENDIX C

Examination of Residuals

For the regression analysis, four assumptions have been made, that is, that the errors are independent, have a zero mean, have a constant variance, and follow a normal distribution. Graphical plots of the residuals are usually very revealing when these assumptions have been violated (Draper and Smith 1966).

Histograms of the residuals, observed ln(bai) minus the predicted value, (*fig. 1*) indicate that the residuals follow a normal distribution for each of the mixed-conifer species. When the residuals are plotted against the predicted value of ln(bai) (*fig. 2*) and against the main independent variables (*fig. 3* and *4*), the overall impression of the horizontal bands of residuals indicates no abnormalities. Therefore the assumption of a constant variance does not appear to be violated. As noted by Draper and Smith (1966), "this does not mean that the assumption is correct; it means that on the basis of the data there is no reason to say it is incorrect."

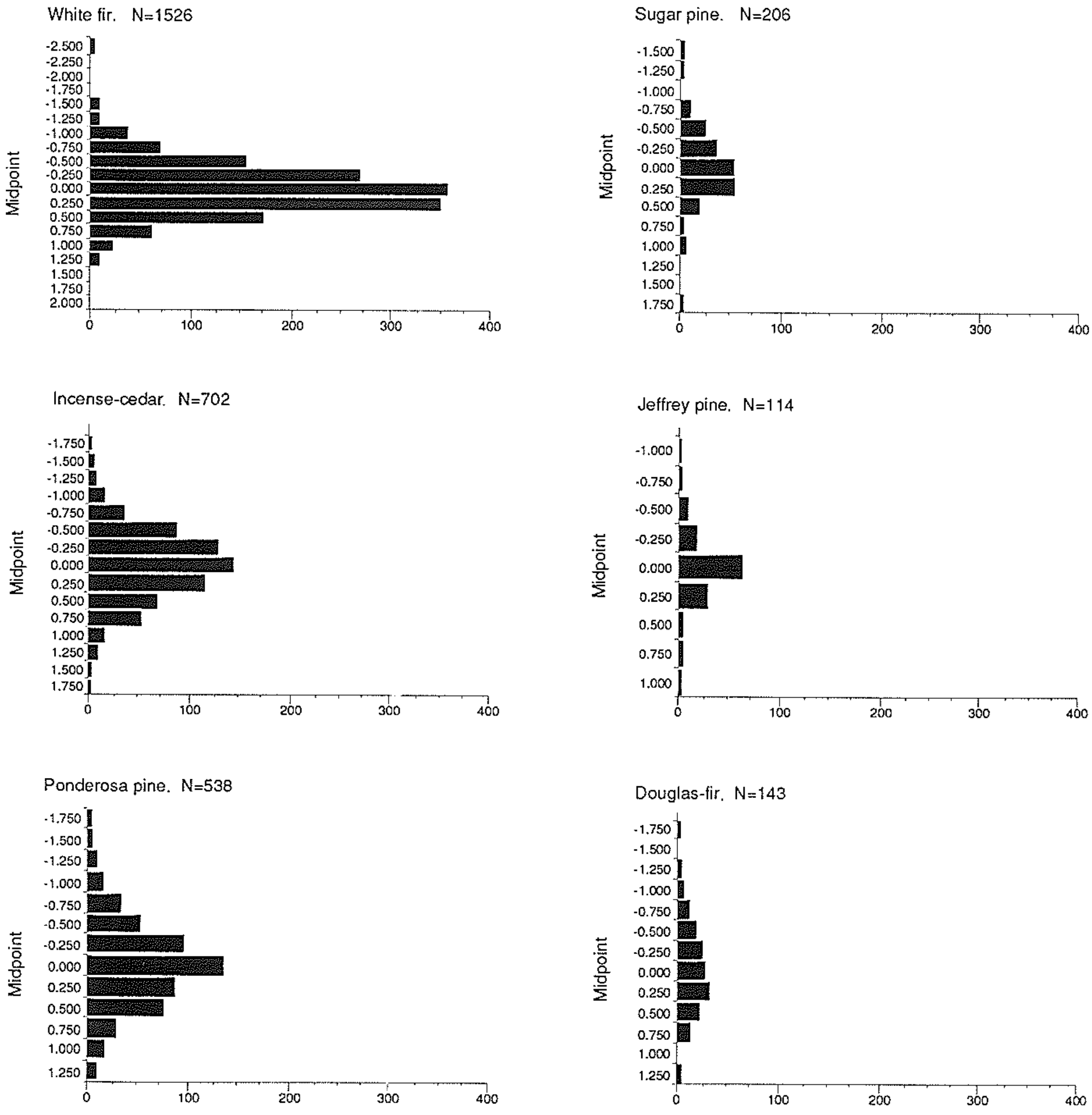


Figure 1—Histograms of frequency of residuals for each of the mixed-conifer species. Midpoints are given for each residual class with increments of 0.25. Residuals are in logarithms of basal area increment.

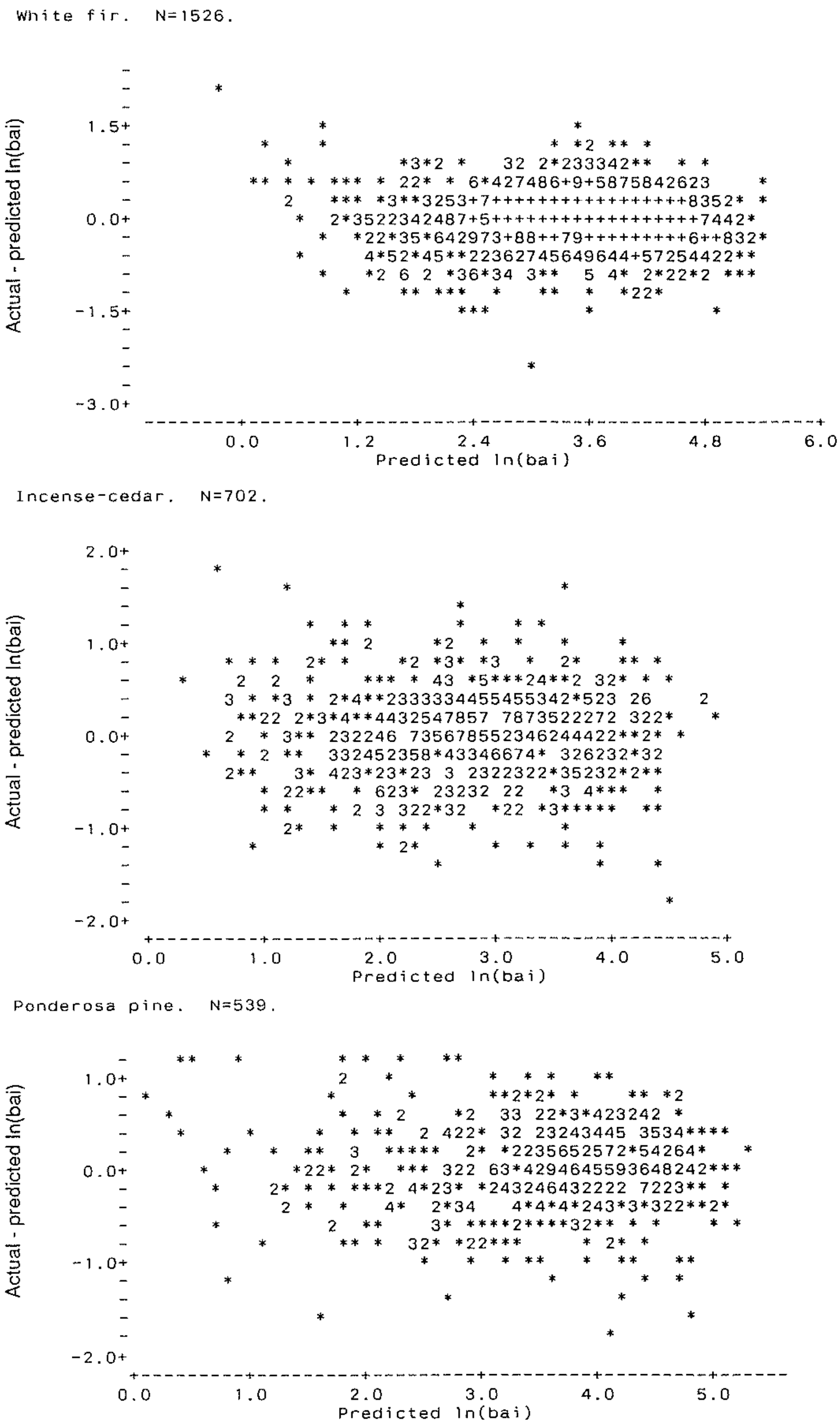
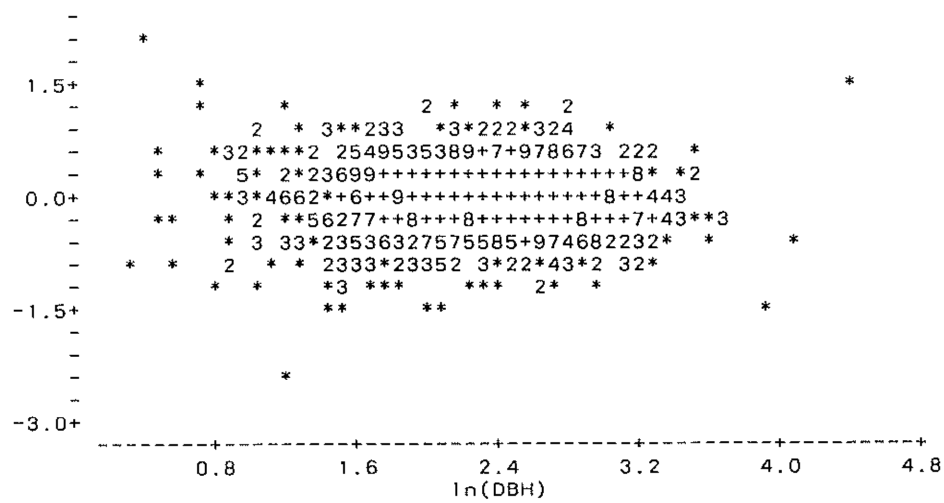
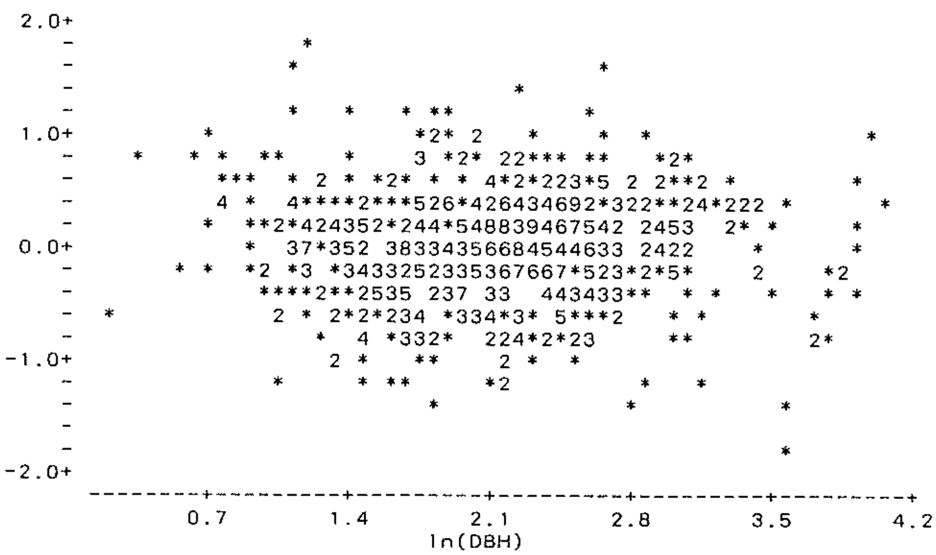


Figure 2—Plots of residuals against the predicted value of 1n(bai) for the three principal mixed-conifer species: white fir, incense-cedar, and ponderosa pine. Each * represents one observation. More than one observation at a given point is represented by the number, or a + if there are more than nine observations at one point.

White fir. N=1526.



Incense-cedar. N=702.



Ponderosa pine. N=539.

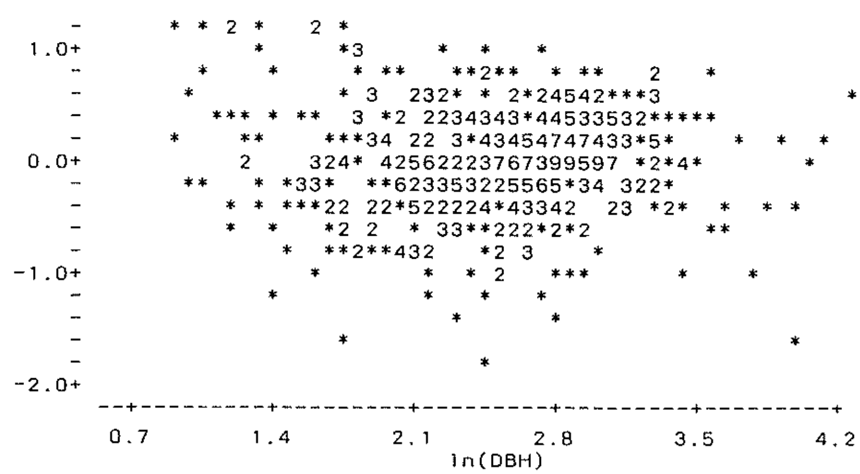
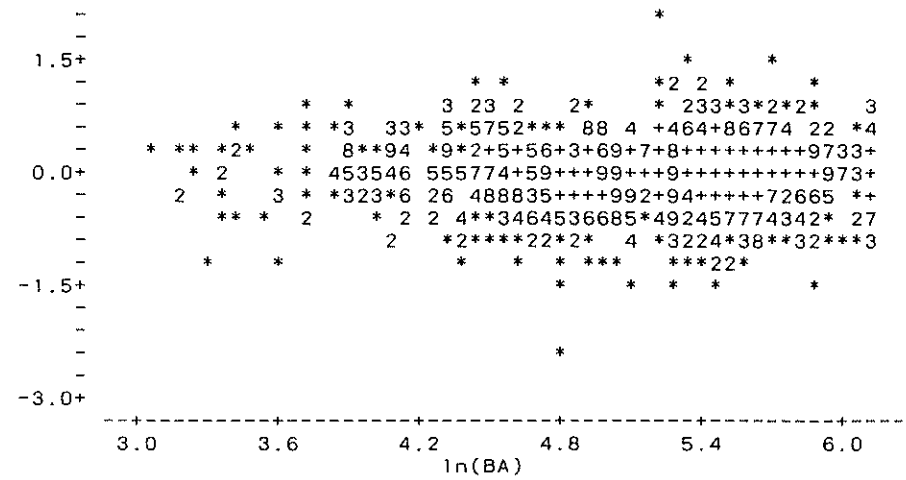
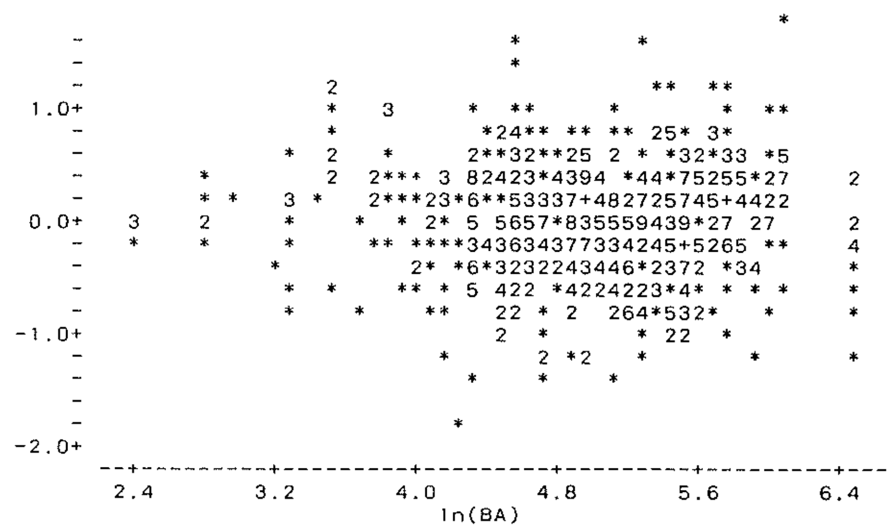


Figure 3—Plots of the residuals against the logarithm of outside-bark diameter at the beginning of the growth period for the three principal mixed-conifer species: white fir, incense-cedar, and ponderosa pine.

White fir. N=1526.



Incense-cedar. N=702.



Ponderosa pine. N=539.

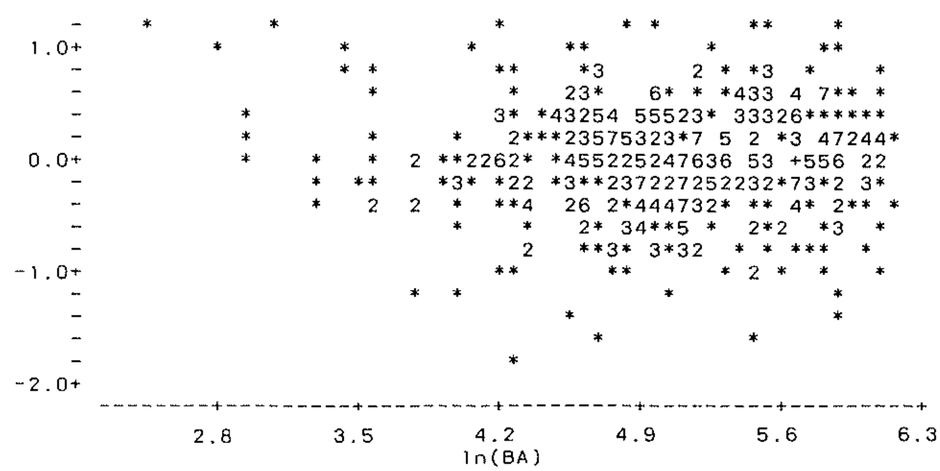


Figure 4—Plots of the residuals against the logarithm of plot basal area [1n(BA)] for the three principal mixed-conifer species: white fir, incense-cedar, and ponderosa pine.

APPENDIX D

Site Index Estimates

Three site index estimating equations were developed to estimate site index as a function of breast-height age and total tree height for the mixed-conifer species. Estimating equations were developed for incense-cedar, California white fir, and ponderosa pine. The white fir equations were also used for Douglas-fir and the ponderosa pine equations were used for site estimates of sugar pine and Jeffrey pine.

Site index (SI) estimates may be obtained from the estimating equations if total height in feet (TH) and breast-height age in years (Age) are known for suitable site trees. A suitable site tree is a dominant or, if no dominants are present, the tallest codominant of a given species which shows no evidence of past suppression.

Site index estimates are expressed as total height in feet at a breast-height age of 50 years. The site index estimating equations are given for computer applications. Site index estimates produced from these equations are also presented in tabular form (*tables 8, 9, 10*) and may be used when total tree height and breast-height age are known.

Site index estimating equation for incense-cedar:

$$\hat{SI} = [e^{-1.051513 + 1.36553 \ln(Age) - 0.0021252 \cdot \ln(Age)^4}]$$

$$\cdot [0.206271 - 0.000617 (Age) + \frac{45.954649}{Age + 5.731439}]$$

$$+ (Th - 4.5)(0.206271 - 0.000617 \; Age + \frac{45.954649}{Age + 5.731439})$$

Site index estimating equation for ponderosa pine:

$$\hat{SI} = 78.53 - [0.710489 - (0.000071)(Age) + \frac{14.1470465}{Age - 1.646774}]$$

$$\cdot [137.770425 (1 - e^{(-0.007593)(Age^{1.182988})})]$$

$$+ [TH - 4.5][0.710489 - 0.00071 (Age) + \frac{14.170456}{Age - 1.646774}$$

Site index estimating equation for white fir:

$$\hat{SI} = 69.91 - [38.020235 (Age^{-1.052133})(e^{(0.009557)(Age)})]$$

$$\cdot [101.842894 (1 - e^{(-0.001442)(Age^{1.679259})})]$$

$$+ [TH - 4.5][38.02035 (Age^{-1.052133})(e^{(0.009557)(Age)}]$$

Table 8—*Total height in feet of dominant and codominant incense-cedar by breast-height age and site index. Reference age is 50 years at breast height*

bh age (yrs)	Incense-cedar site index (ft)																
	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
10				5	6	8	9	11	13	14	16	17	19	21	22	24	25
12				6	7	9	11	13	15	16	18	20	22	24	25	27	29
14			5	7	9	11	13	15	17	19	21	23	25	27	28	30	32
16			6	8	10	12	14	17	19	21	23	25	27	29	32	34	36
18			7	9	11	14	16	18	21	23	25	28	30	32	35	37	39
20		5	8	10	13	15	18	20	23	25	28	30	33	35	38	40	43
22		6	9	11	14	17	19	22	25	28	30	33	36	38	41	44	46
24		7	10	12	15	18	21	24	27	30	33	35	38	41	44	47	50
26		7	10	14	17	20	23	26	29	32	35	38	41	44	47	50	53
28	5	8	11	15	18	21	24	28	31	34	37	40	44	47	50	53	57
30	6	9	12	16	19	23	26	29	33	36	39	43	46	50	53	56	60
32	6	10	13	17	20	24	27	31	35	38	42	45	49	52	56	60	63
34	7	10	14	18	22	25	29	33	36	40	44	48	51	55	59	63	66
36	7	11	15	19	23	27	30	34	38	42	46	50	54	58	62	66	69
38	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72
40	8	12	17	21	25	29	33	38	42	46	50	54	59	63	67	71	76
42	9	13	17	22	26	30	35	39	44	48	52	57	61	65	70	74	79
44	9	13	18	23	27	32	36	41	45	50	54	59	63	68	72	77	81
46	9	14	19	23	28	33	37	42	47	52	56	61	66	70	75	80	84
48	10	15	19	24	29	34	39	44	48	53	58	63	68	73	78	82	87
50	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
52	10	15	21	26	31	36	41	46	52	57	62	67	72	77	82	88	93
54	11	16	21	26	32	37	42	48	53	58	64	69	74	80	85	90	95
56	11	16	22	27	33	38	44	49	54	60	65	71	76	82	87	93	98
58	11	17	22	28	33	39	45	50	56	61	67	73	78	84	90	95	101
60	11	17	23	28	34	40	46	52	57	63	69	75	80	86	92	98	103
62	11	17	23	29	35	41	47	53	59	65	70	76	82	88	94	100	106
64	11	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108
66	12	18	24	30	36	43	49	55	61	67	74	80	86	92	98	105	111
68	12	18	24	31	37	43	50	56	62	69	75	81	88	94	101	107	113
70	12	18	25	31	38	44	51	57	64	70	77	83	90	96	103	109	116
72	12	18	25	32	38	45	52	58	65	71	78	85	91	98	105	111	118
74	12	18	25	32	39	46	52	59	66	73	80	86	93	100	107	113	120
76	12	19	26	32	39	46	53	60	67	74	81	88	95	102	109	116	123
78	12	19	26	33	40	47	54	61	68	75	82	89	96	104	111	118	125
80	12	19	26	33	40	48	55	62	69	76	84	91	98	105	113	120	127

Table 9—*Total height in feet of dominant and codominant ponderosa, sugar, and Jeffrey pine by breast-height age and site index. Reference age is 50 years at breast height*

bh	Ponderosa pine site index (ft)																			
age (yrs)	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
10				6	8	10	12	14	16	18	20	22	24	26	28	31	33	35	37	39	41
12				7	9	12	14	16	19	21	24	26	28	31	33	36	38	40	43	45	48
14			6	8	11	14	16	19	22	24	27	30	32	35	38	41	43	46	49	51	54
16			7	10	13	16	19	22	25	28	31	33	36	39	42	45	48	51	54	57	60
18		5	9	12	15	18	21	24	28	31	34	37	40	43	47	50	53	56	59	63	66
20		7	10	14	17	20	24	27	31	34	37	41	44	47	51	54	58	61	64	68	71
22	5	9	12	16	19	23	26	30	34	37	41	44	48	51	55	58	62	66	69	73	76
24	7	10	14	18	22	25	29	33	36	40	44	48	51	55	59	63	66	70	74	77	81
26	8	12	16	20	24	28	32	36	39	43	47	51	55	59	63	67	70	74	78	82	86
28	10	14	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78	82	86	90
30	12	16	20	24	29	33	37	41	45	49	53	58	62	66	70	74	78	82	86	91	95
32	14	18	22	27	31	35	39	44	48	52	56	61	65	69	73	78	82	86	90	95	99
34	16	20	24	29	33	38	42	46	51	55	59	64	68	72	77	81	86	90	94	99	103
36	18	22	27	31	35	40	44	49	53	58	62	67	71	76	80	85	89	93	98	102	107
38	19	24	29	33	38	42	47	51	56	60	65	70	74	79	83	88	92	97	101	106	111
40	21	26	31	35	40	45	49	54	58	63	68	72	77	82	86	91	96	100	105	110	114
42	23	28	33	37	42	47	52	56	61	66	70	75	80	85	89	94	99	103	108	113	118
44	25	30	35	39	44	49	54	59	63	68	73	78	83	87	92	97	102	107	111	116	121
46	27	32	37	41	46	51	56	61	66	71	76	80	85	90	95	100	105	110	115	119	124
48	29	34	39	43	48	53	58	63	68	73	78	83	88	93	98	103	108	113	117	122	127
50	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
52	32	37	42	47	52	57	62	68	73	78	83	88	93	98	103	108	113	118	123	128	133
54	34	39	44	49	54	59	64	70	75	80	85	90	95	100	105	111	116	121	126	131	136
56	35	41	46	51	56	61	66	72	77	82	87	92	97	103	108	113	118	123	128	134	139
58	37	42	47	53	58	63	68	74	79	84	89	94	100	105	110	115	121	126	131	136	141
60	39	44	49	54	60	65	70	75	81	86	91	97	102	107	112	118	123	128	133	139	144
62	40	45	51	56	61	67	72	77	83	88	93	99	104	109	114	120	125	130	136	141	146
64	42	47	52	58	63	68	74	79	84	90	95	101	106	111	117	122	127	133	138	143	149
66	43	48	54	59	65	70	75	81	86	92	97	102	108	113	119	124	129	135	140	146	151
68	44	50	55	61	66	72	77	82	88	93	99	104	110	115	121	126	131	137	142	148	153
70	46	51	57	62	68	73	79	84	90	95	101	106	111	117	122	128	133	139	144	150	155
72	47	53	58	64	69	75	80	86	91	97	102	108	113	119	124	130	135	141	146	152	157
74	48	54	59	65	71	76	82	87	93	98	104	109	115	120	126	132	137	143	148	154	159
76	50	55	61	66	72	77	83	89	94	100	105	111	117	122	128	133	139	144	150	156	161
78	51	56	62	68	73	79	84	90	96	101	107	113	118	124	129	135	141	146	152	157	163
80	52	58	63	69	74	80	86	91	97	103	108	114	120	125	131	137	142	148	154	159	165

Table 10—*Total height in feet of dominant and codominant white fir and Douglas-fir by breast-height age and site index. Reference age is 50 years at breast height*

bh	White fir site index (ft)																			
age (yrs)	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
10				5	6	7	9	10	11	13	14	15	17	18	19	21	22	23	25	26	27
12				6	7	9	10	12	14	15	17	18	20	22	23	25	26	28	30	31	33
14			5	7	9	11	12	14	16	18	20	22	24	25	27	29	31	33	35	36	38
16			6	8	11	13	15	17	19	21	23	25	27	29	31	33	36	38	40	42	44
18		6	8	10	12	15	17	19	22	24	26	29	31	33	36	38	40	43	45	47	50
20		7	9	12	15	17	20	22	25	27	30	32	35	37	40	42	45	48	50	53	55
22	6	9	11	14	17	20	22	25	28	31	33	36	39	42	44	47	50	53	55	58	61
24	7	10	13	16	19	22	25	28	31	34	37	40	43	46	49	52	55	58	61	64	66
26	9	12	15	18	21	25	28	31	34	37	40	44	47	50	53	56	59	63	66	69	72
28	11	14	17	21	24	27	31	34	37	41	44	47	51	54	57	61	64	67	71	74	78
30	12	16	19	23	26	30	33	37	41	44	48	51	55	58	62	65	69	72	76	79	83
32	14	18	21	25	29	33	36	40	44	47	51	55	59	62	66	70	73	77	81	85	88
34	16	20	24	28	31	35	39	43	47	51	55	59	62	66	70	74	78	82	86	90	94
36	18	22	26	30	34	38	42	46	50	54	58	62	66	70	74	78	82	86	91	95	99
38	20	24	28	32	36	41	45	49	53	57	62	66	70	74	78	83	87	91	95	99	104
40	21	26	30	34	39	43	47	52	56	61	65	69	74	78	82	87	91	95	100	104	108
42	23	28	32	37	41	46	50	55	59	64	68	73	77	82	86	91	95	100	104	109	113
44	25	30	34	39	43	48	53	57	62	67	71	76	81	85	90	94	99	104	108	113	118
46	27	31	36	41	46	50	55	60	65	70	74	79	84	89	93	98	103	108	112	117	122
48	28	33	38	43	48	53	58	63	67	72	77	82	87	92	97	102	106	111	116	121	126
50	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130
52	32	37	42	47	52	57	62	67	72	78	83	88	93	98	103	108	113	118	124	129	134
54	33	38	44	49	54	59	64	70	75	80	85	90	96	101	106	111	117	122	127	132	137
56	35	40	45	50	56	61	66	72	77	82	88	93	98	104	109	114	120	125	130	136	141
58	36	41	47	52	58	63	68	74	79	85	90	95	101	106	112	117	123	128	133	139	144
60	37	43	48	54	59	65	70	76	81	87	92	98	103	109	114	120	125	131	136	142	147
62	38	44	50	55	61	66	72	78	83	89	94	100	106	111	117	122	128	133	139	145	150
64	40	45	51	57	62	68	74	79	85	91	96	102	108	113	119	125	130	136	142	147	153
66	41	46	52	58	64	69	75	81	87	92	98	104	110	115	121	127	133	138	144	150	156
68	42	48	53	59	65	71	77	82	88	94	100	106	112	117	123	129	135	141	146	152	158
70	43	49	54	60	66	72	78	84	90	96	102	107	113	119	125	131	137	143	149	154	160
72	44	50	55	61	67	73	79	85	91	97	103	109	115	121	127	133	139	145	151	157	163
74	44	50	56	62	68	74	80	86	92	98	104	110	116	122	128	135	141	147	153	159	165
76	45	51	57	63	69	76	82	88	94	100	106	112	118	124	130	136	142	148	154	160	166
78	46	52	58	64	70	76	83	89	95	101	107	113	119	125	131	138	144	150	156	162	168
80	47	53	59	65	71	77	84	90	96	102	108	114	120	127	133	139	145	151	157	164	170

Dolph, K. Leroy. 1988. **Prediction of periodic basal area increment for young-growth mixed conifers in the Sierra Nevada.** Res. Paper PSW-190. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 20 p.

Future basal area increment of individual trees of six mixed-conifer species in the Sierra Nevada can be predicted by the log-linear regression model described. Estimates of the model coefficients for California white fir, incense-cedar, ponderosa pine, sugar pine, Jeffrey pine, and Douglas-fir were obtained using least-squares linear regression.

Retrieval Terms: increment (diameter), California white fir, incense-cedar, ponderosa pine, sugar pine, Jeffrey pine, Douglas-fir, Sierra Nevada