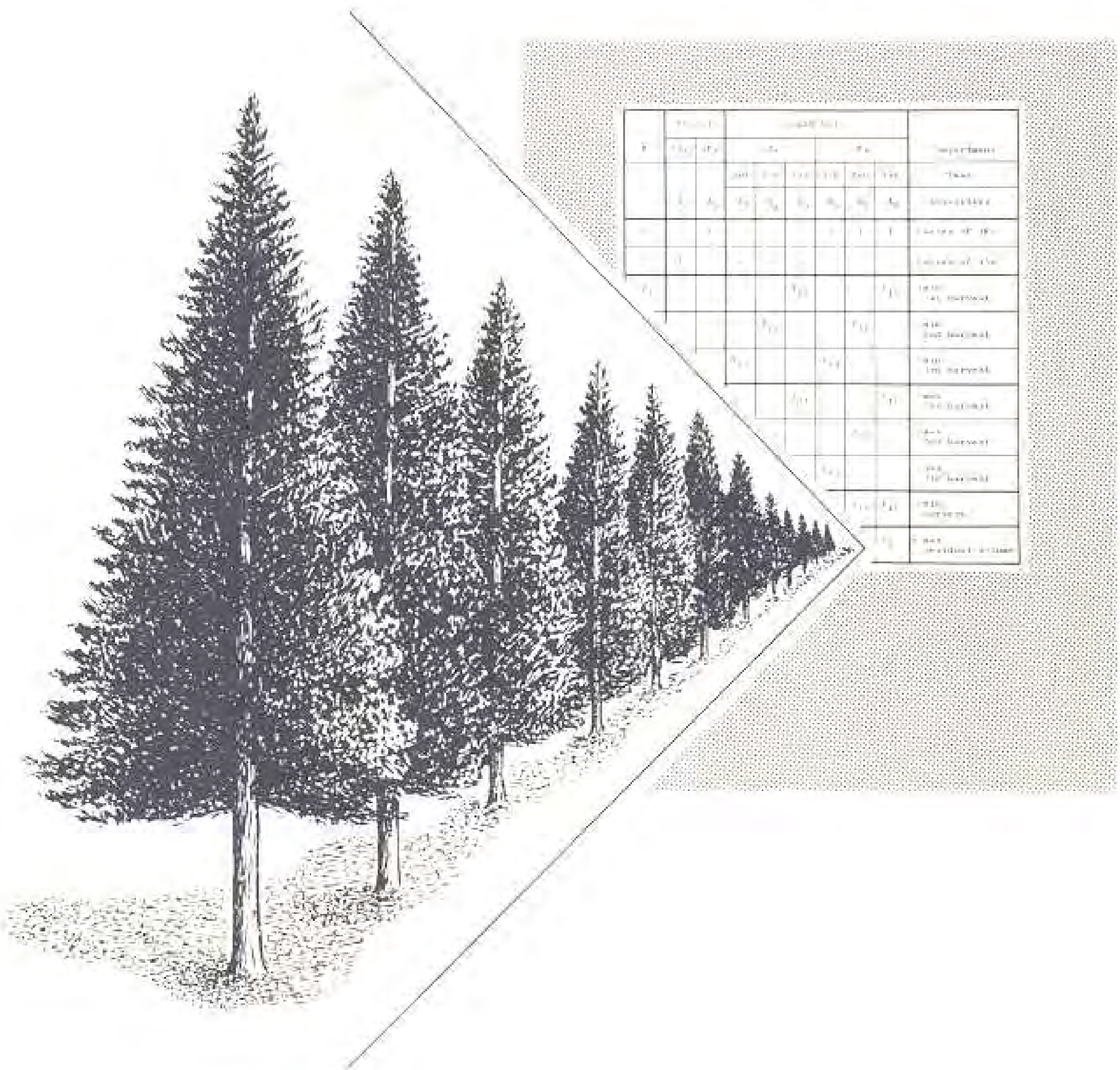
Forest Service - U.S. Department of Agriculture

18 pgs

Evaluating Forest Management Policies by Parametric Linear Programing

Daniel I. Navon   Richard J. McConnen



U.S. FOREST SERVICE RESEARCH PAPER PSW-42   1967

Pacific Southwest Forest and Range Experiment Station
P.O. Box 245, Berkeley, California 94701

Navon, Daniel I., and McConnen, Richard J.

1967. **Evaluating forest management policies by parametric linear programing.** Berkeley, Calif., Pacific SW. Forest & Range Exp. Sta. 13 pp., illus. (U.S. Forest Serv. Res. Paper PSW-42)

An analytical and simulation technique, parametric linear programing explores alternative conditions and devises an optimal management plan for each condition. Its application in solving policy-decision problems in the management of forest lands is illustrated in an example.

Oxford: 624—015.5:U517.948

Retrieval Terms: decision-making; forest management; linear progran ing; management planning; program evaluations; simulation; timber management.

avon, Daniel I., and McConnen, Richard J.

67. **Evaluating forest management policies by parametric linear programing.** Berkeley, Calif., Pacific SW. Forest & Range Exp. Sta. 13 pp., illus. (U.S. Forest Serv. Res. Paper PSW-42)

An analytical and simulation technique, parametric linear programing explores alternative conditions and devises an optimal management plan for each condition. Its application in solving policy-decision problems in the management of forest lands is illustrated in an example.

Oxford: 624—015.5:U517.948

Retrieval Terms: decision-making; forest management; linear programing; management planning; program evaluations; simulation; timber management.

The Authors

DANIEL I. NAVON is an economist studying problems in forest economics and multiple-use, with headquarters in Berkeley. He earned a bachelor's degree in economics at the University of California, Berkeley (1954). Before joining the Station's research staff in 1964, he served as an analyst with the Maritime Cargo Transportation Conference, National Academy of Sciences, San Francisco, Calif. **RICHARD J. McCONNEN** headed forest economics studies at the Berkeley Station from 1961 until 1966, when he became an associate professor of economics at Montana State University, Bozeman. Native of Sidney, Mont., he earned bachelor's (1952) and master's (1953) degrees in agricultural economics at Montana State College, and a doctorate (1958) at the University of California, Berkeley.

A traditional responsibility of management is to evaluate and implement policies. Accounting and mathematical techniques have long been used by managers to perform this task. The development of operations research and electronic computers has ushered in new techniques that make the intensive evaluation of large numbers of alternative policies practical. One of the most powerful of these new techniques is parametric linear programing.

Operations research generally proceeds from analytical or simulation approach. Analytical models are designed to yield an optimal or best solution to a given management situation. Simulation models are simplified versions of management systems which can be "operated" usually with the help of a computer. These models determine the impact of continuously varying conditions on selected components of the system. A simulation model does not yield a solution in the conventional sense, but generates a list of solutions corresponding to different conditions of the system.

Parametric linear programing borrows from both of these approaches. It explores a wide spectrum of alternative conditions, and also devises an optimal management plan for each condition. Thus parametric linear programing qualifies both as an analytical and as a simulation technique.

Several characteristics of parametric linear programing recommend it to managers.

- It is simple to use—requiring only that the users have a grasp of elementary algebra.
- It uses available computer programs.
- It provides information with the rapidity and accuracy typical of computer techniques.
- It generates information which can assist the manager in locating the elements having the greatest impact on the system.

The technique offers additional enticements to natural resource managers, who must often reevaluate policies after unexpected major changes in resources.[1] Computers make it possible to develop quickly new management plans based upon the changed resource inventory. And the revision of the inventory itself can be computerized with the use of existing systems, such as the Map Information Assembly and Display System now in use by land resource managers in the United States and Canada.[2] Management plans, for example, can be revised almost immediately after a fire or storm. Parametric linear programing can then indicate if the previously established goals can still be realized, and then assist the manager in making the necessary adjustments.

This paper describes the parametric linear programing technique, suggests some of its applications to forest management problems, and illustrates its use on a tract of commercial forest land. The primary emphasis has been placed on problems of managing forest land for timber production, but the use of parametric linear programing can be extended to a wide variety of wildland management problems, including the difficult problem of multiple-use management.

---

[1] Navon, D. I. **Computer-oriented systems for wildland management.** J. Forestry 65(7):473-479. 1967.

[2] Amidon, Elliot L. **A computer-oriented system for assembling and displaying land management information.** U.S. Forest Serv. Res. Paper PSW-17, Pacific SW. Forest & Range Exp. Sta., Berkeley, Calif. 34 pp., illus. 1964.

Amidon, Elliot L. **MIADS2—An alphanumeric map information assembly and display system for a large computer.** U.S. Forest Serv. Res. Paper PSW-38, Pacific SW. Forest & Range Exp. Sta., Berkeley, Calif. 12 pp., illus. 1966.

Broido, A., O'Regan, W. G., and McConnen, R. J. **Some operations research applications in the conservation of wildland resources.** Manage. Sci. Ser. A 11(9): 802-814, illus. 1965.

McConnen, R. J., Amidon, E. L., and Navon, D. I. **The use of operations research techniques in determining allowable cut.** Proc. Ann. Mtg. Soc. Amer. Foresters 1966:109-115, illus. 1967.

Linear Programing

Linear programing is an efficient method of selecting the optimum feasible combination of management practices. A combination is feasible if it can be carried out with available resources and does not violate specified policies or goals. These available resources and specified goals are generally referred to as constraints. In forest management, a feasible combination of management practices is optimum if it maximizes profits or some other measure of performance, or in the case of public lands, if it minimizes budgetary expenditures required to achieve a set of specified goals. In many problems, all the goals cannot be incorporated into a single objective function, and some of the goals must be introduced in the program by stating them as constraints.

Problems of resource allocation in which the quantity of the factors of production is given and the policies guiding management have already been determined are often solved by linear programing. The management policies are incorporated in the linear program by setting up a list of constraints, and defining an economic objective and a set of plausible management activities. For example, management policies for a tract of forest land could be formulated as follows: "Maximize the stumpage volume remaining at the end of 10 years subject to constraints on acreage and annual mill deliveries, by carrying out clear- and partial-cutting management activities."

The solution to the linear program accomplishes two aims. First, it specifies the level at which the activities must be operated to maximize—or minimize—the value of the economic objective. In the foregoing example, these activity levels would be the number of acres to be clearcut or partially cut for each of the 10 years.

Second, the solution generates a shadow price for each constraint. The shadow price of a given constraint can be interpreted as the marginal change in the economic objective caused by a unit change in the level of this constraint. The shadow price of the constraint specifying the level of the first-year mill delivery indicates the decrease in the volume of the residual stumpage which would result from a unit increase in this constraint level. This change is referred to as marginal because it is known to be correct only for small changes in the neighborhood of the original constraint level. Therefore only small changes in constraint levels can be evaluated with assurance by these shadow prices. For example, shadow prices could not be used to assess the impact of a 10-percent increase in the first year mill delivery constraint on the volume of the residual stumpage. The obvious solution is to solve the linear program again after adjusting the constraint to the contemplated level. But this procedure rapidly becomes burdensome as the number of contemplated levels increases; unless a special algorithm such as parametric programing is used.

Parametric Linear Programing

Parametric linear programing computes a series of solutions which determine the relationship between the economic objective and any one of the constraints over the entire range of levels which these constraints can assume in the program.

Let us assume that the minimum acceptable delivery at the mill is 10 MMBF per year. The linear program is then solved initially with the annual delivery constraint at the level of 10 MMBF; the initial linear program solution would indicate the maximum residual stumpage corresponding to this delivery schedule. Parametric linear programing could then be used to assess the impact of increasing the level of the "annual delivery constraint" on the volume of the residual stumpage.

For example, consider the results of four hypothetical parametric solutions *(fig. 1)*. As annual deliveries increase, the liquidation volume decreases until it reaches zero. The mathematical structure of parametric linear programing is such that the relation between the liquidation volume and annual deliveries is known to be linear between any two adjacent parametric solutions. Hence, by joining the solution points with straight line segments, the maximum liquidation volume corresponding to an annual delivery level falling between solutions can be read directly from the graph *(fig. 1)*. The relation between the economic objective "maximize the liquidation volume" and the constraint on annual mill delivery has been established over the entire range of levels the constraint can assume in the program. If management prescriptions are required for an annual mill delivery level falling between parametric solutions, the program must be solved independently of the parametric linear program. In most cases the parametric linear program will provide enough solutions to afford a sound basis for management decisions, without requiring additional computations.

Once the parametric linear program has been solved, the policy decision consists of selecting the constraint levels and the corresponding value of the economic objective. This decision may be based on a purely quantitative criterion, but the complexity of most operational situations will require the use of some intuitive judgment.

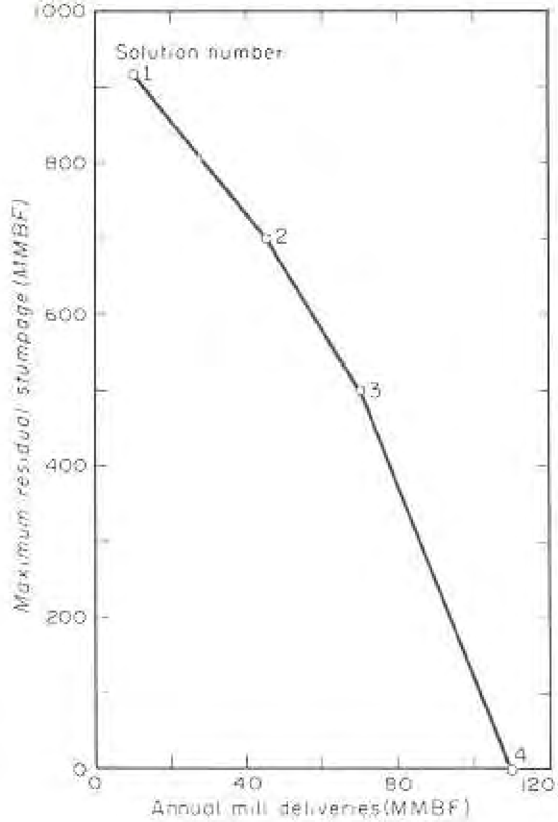


Figure 1—*Parametric relation between volume of annual mill deliveries and maximum residual stumpage.*

A Study in Commercial Forestry

In the following study, parametric linear programing is used to explore increasingly complex policy alternatives in the management of a tract of commercial forest land. The forest manager needs to select a schedule of annual mill deliveries that will leave vigorous and well-stocked stands at the end of 10 years. The greater the annual deliveries, the lower will be the maximum "liquidation" value of the remaining standing timber. The manager must then use his judgment to select the combination of mill delivery schedule and liquidation value which meets the management goals not incorporated explicitly into the program.

Consider a tract of commercial forest covering 11,804 acres. This tract is classified into 17 even-aged stand classes. The timber of each stand class is assumed to be homogeneous with respect to all relevant economic and silvicultural characteristics. Eight of the 17 classes are stocked predominantly with sugar and ponderosa pine; nine with white fir, Douglas-fir, and cedar. The 17 stand classes are grouped into the following four "timber management" categories:

I. Poorly stocked old-growth.
II. Poorly stocked mature-growth.
III. Well stocked old-growth.
IV. Well stocked mature- and young-growth.

One or more management practices can be applied to each category *(table 1)*. Note that for category

Table 1.--*Management practices allowed*

Category	Clear cut	Partial cut	No cut
I. Poorly stocked old-growth	X	--	--
II. Poorly stocked mature-growth	X	--	X
III. Well stocked old-growth	X	X	X
IV. Well stocked mature- and young-growth	--	X	X

I only clearcutting is allowed; hence these stands must be completely clearcut by the end of the 10-year period. Note also that clearcutting is not permitted for Category IV. The broad intent of these prescriptions is to insure that the commercial forest remaining at the end of the 10 years is reasonably vigorous and well stocked. Only a short management period is considered in this illustration. However, the general long-term policy of the company is reflected by the set of management practices allowed by table 1. If, for example, the general long-term policy had been one of "cut out and get out," clearcutting activities would have been permitted in all four timber management categories.

The manager has a broad range of policy alternatives. Clearcutting only the poorly stocked very old stands will produce the desired vigorous forest and result in the highest liquidation value realizable. The annual harvest will necessarily be small. Clearcutting and partial cutting all the timber allowed will still produce the desired type of forest but at the cost of a much lower liquidation value compensated by relatively high annual yields. The range between these extremes is explored systematically by a series of parametric linear programs reflecting increasingly complex policy alternatives.

The first step in developing a policy is to determine the range of available alternatives. In the context of a parametric linear program, this procedure requires the determination of the range of values which the constraints can be made to assume.

In the following illustrations, two groups of constraints are defined:

1. Constraints expressing the number of acres in each stand. In the short run, no change in acreage is contemplated, hence, these resource constraints are treated as fixed.

2. Mill delivery constraints specifying the value —or volume— of timber to be delivered annually over the 10-year management period. It is the range of values which can be assumed by these constraints that is explored by the parametric program.

Simple Policy Alternatives

Let us assume that all annual harvests must be equal. The level of the total harvest will then determine the annual mill delivery schedule.

The management problem can now be defined as a standard linear program, and a related parametric program. The linear program is:

Maximize the liquidation value of the forest at the end of a 10-year period subject to the following constraints:

1. Realize a total harvest of a given level—all annual harvest being equal;

2. Operate within a broadly specified forest improvement policy, i.e., clearcut poorly stocked, old-growth, etc., and

3. Do not exceed the available resources.

And the related parametric program states:

1. Increase continuously the total harvest up to the highest possible level, i.e., until all allowable timber is cut, and

2. Compute the solutions required to define precisely the relation between the total harvest level and the liquidation value of the forest.

To solve the linear program, the initial acreage and the following rates per acre must be available for each stand:

- (A) Yield from clear and partial cutting;
- (B) Annual growth;
- (C) Annual growth after partial cutting; and
- (D) Annual salvage value.

These rates—originally expressed in thousands of board feet (MBF) — were converted to dollar value on the basis of current 1965 stumpage prices of each species and grade. Stumpage prices are likely to change substantially during a 10-year period, but it was assumed that the value relative to grade and species would remain fairly constant.[1] Thus only the absolute amount of the liquidation value would be affected; the management plan itself would remain unchanged.

Fifty-seven parametric solutions were computed, corresponding to total harvest levels ranging from \$0.85 million to \$5.48 million.

A good approximation to the relation between liquidation value and minimum total harvest was obtained by plotting only 5 of the 57 solutions (*fig. 2*). The relation was nearly linear over the range of total harvests likely to be selected—\$0.85 million to \$5.5 million. An increase in total harvest reduced the liquidation value by almost the same amount. The nature of this relationship emerged more precisely when the aggregate timber value—the sum of total harvest and liquidation value—was plotted against the total harvest level (*fig. 3*). Aggregate timber value was highest when the total harvest level was set at \$1.44 million.

A perceptive analyst might have deduced that the maximum aggregate value would not be

[1] The data used are available on request to the Director, Pacific Southwest Forest and Range Experiment Station, P. O. Box 245, Berkeley, California 94701.

Solution number	Liquidation value	Aggregate timber value	Total harvest
	Million dollars		
1	8.97	9.82	.85
27*	8.64	10.08	1.44
44	7.03	10.03	3.0
49	4.76	9.84	5.08
57	4.24	9.73	5.49

Solution summary for figures 2 and 3

*See figure 4.

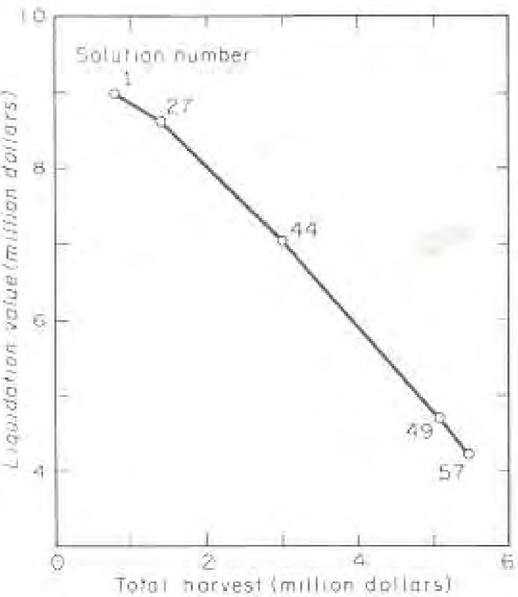


Figure 2—*Parametric relation between liquidation value and total harvest value.*

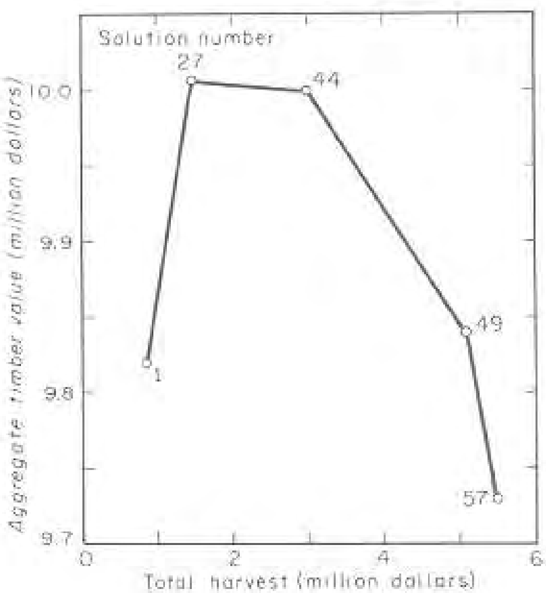


Figure 3—*Parametric relation between aggregate timber value and total harvest value.*

realized at the lowest total harvest level. However, the determination of this optimum without parametric linear programing would require complex and tedious computations. The actual location of the maximum aggregate value is readily explained by analyzing changes in the cutting activity pattern of the parametric solutions. This analysis is based on inspection of the activity levels prescribed by the parametric management plan prepared by the program for each of the 57 parametric solutions. As the annual harvest level is increased, the poorly stocked very old stands cannot meet the demand for timber, and the well stocked young growth and eventually the well stocked old growth must be partially cut. This maximum partial cutting pattern yields the maximum aggregate value. Partial cutting here acts essentially as a timber stand improvement practice. As annual harvest is further increased, clearcutting must be substituted for partial cutting. The loss of annual growth on clearcut acres causes the aggregate value to decline.

Aggregate value can be used as a quantitative criterion to evaluate the policy decision on the level of the total harvest. The decision to harvest a total of $5 million worth (solution 49, *fig. 3*)—rather than $1.44 million (solution 27)—would cost $240,000—the difference in the corresponding aggregate timber values. Lowering total harvest to $3 million (solution 44) would reduce this loss to $50,000. In this particular situation, parametric programing affords the policy maker a simple criterion for evaluating his selection of a minimum acceptable total harvest.

This analysis is based on the assumption that the discount rate is zero. More complex criteria based on nonzero rates of interest are considered below. Even then, the parametric linear program solutions are unlikely to provide a completely adequate basis for decision making. Careful consideration of the assumptions underlying the system and an intuitive assessment of factors not explicitly included in the system must be introduced in the decision process.

Providing Management Prescriptions

Parametric linear programing also provides additional valuable information to the policy maker. Each solution provides a set of detailed management prescriptions required to attain specified annual harvest levels at maximum liquidation value. These prescriptions have been translated by the computer into the management plan illustrated (*fig. 4*). The management plan corresponding to the optimal combination of liquidation value and total harvest may prove to be impractical because of topographical considerations or because the number of acres cut fluctuates too much from year to year. It is likely that some considerations left out in the initial formulation will subsequently appear crucial. Other considerations may be too complex and subtle to be expressed algebraically. The policy-maker can search among the parametric solutions for a management plan which meets the considerations not included in the program, or he can modify the management plan devised by the parametric program. The modified management plan, at best, will yield the same liquidation value. In most cases, deviations from the parametric plan will result in a reduction of the liquidation value.

The program also computes a shadow price corresponding to each of the constraints of the program. The shadow price of each stand acreage constraint indicates the increase in liquidation value which would result from increasing the area of that stand by one acre. Parametric variation of the annual harvest level will induce changes in the resource shadow prices. The "history" of each shadow price for the range of parametric solutions reflects the changes in the value of each stand over the parametric range. For example, as the annual harvest level is increased the shadow price of well stocked old-growth pine may rise above the market price. The purchase of additional acres of old-growth pine would then be profitable. Shadow prices should be used with caution because they are likely to be accurate only for small changes about the constraint level.

The objective in evaluating the range of alternatives is not to have parametric linear programs make decisions but rather, as Novick states:

> . . . to improve the basis for decision making. One step is improvement in the methods of framing the problems and in organizing the available data. A second and perhaps more important step is more effective analysis of the information needed to produce better plans or recommendations. The product of these efforts will facilitate the reason, judgment, experience, and intuition of the decision maker.[2]

Range of Complex Policy Alternatives

In the preceding case, the annual harvests were assumed to be equal. In many cases, the annual harvest level can be allowed to fluctuate within a specified range. The parametric linear program can easily be modified to reflect this requirement.

In the following illustrations, the annual harvest requirements are expressed as a range of allowable harvests. Each year the amount—or the value—of the timber harvested must not exceed some maximum but must be at least as great as some minimum. The basic structure of the model has not been changed; the delivery schedule for a given year, previously defined by a single constraint, is now defined by an upper bound constraint and a lower bound constraint. To focus the analysis on the parametric manipulation of the bounds, the problem was simplified by reducing the number of resource constraints to seven. This was accomplished by aggregating some of the old stands and excluding the young stands. The problem was further simplified by allowing only clearcut activities.

In the next section, parametric programing is used to explore the impact of changes in total harvest on the liquidation value and on the pattern of annual mill deliveries. Then in the following section parametric variations of the bounds themselves are investigated.

Parametric Variation of Total Harvest with Fixed Bounds

The bounds within which annual harvests were allowed to vary were fixed at $340,000 and $570,000 for all 10 annual harvests. The lowest total harvest consistent with this constraint was therefore 10 times the lower annual bound or $3.4 million. The alternatives available to the policy maker were determined by increasing the minimum

[2] Novick, David, ed. **Program budgeting . . . program analysis and the Federal budget.** U.S. Govt. Printing Office, Washington, D.C., 236 pp. illus. 1965.

PARAMETRIC MANAGEMENT PLAN NO. 27

ANNUAL HARVESTS ARE EQUAL. THE LIQUIDATION VALUE OF THE RESIDUAL FOREST IS MAXIMIZED FOR A ZERO DISCOUNT RATE. THE ANNUAL HARVEST LEVEL IS INCREASED PARAMETRICALLY FROM $85,000 TO $549,000.

YEAR	ACTIVITY CODE	NO.	ACRES TO BE CUT	STUMPAGE VALUE/ACRE	ANNUAL HARVEST LESS SALVAGE *
1	PC2S++	50	283	$ 70.	$ 19,810.
1	PC2F++	60	876	$ 36.	$ 31,571.
1	PC 1S+	70	891	$ 91.	$ 81,169.
				ANNUAL TOTAL*	$132,550.
2	PC 2S+	29	1,849	$ 70.	$129,482.
				ANNUAL TOTAL*	$129,482.
3	PC 2S+	28	68	$ 70.	$ 4,776.
3	PC 1S+	68	843	$ 91.	$ 76,715.
3	PC 1F+	78	599	$ 70.	$ 41,999.
3	CC1F-+	168	17	$318.	$ 5,574.
				ANNUAL TOTAL*	$129,064.
4	CC1F-+	167	201	$323.	$ 65,209.
4	CC 1F-	184	217	$279.	$ 60,759.
				ANNUAL TOTAL*	$125,968.
5	CC1S-+	156	24	$733.	$ 17,735.
5	CC 1F-	183	382	$283.	$108,602.
				ANNUAL TOTAL*	$126,337.
6	CC1S-+	155	34	$747.	$ 25,617.
6	CC 1S-	174	159	$634.	$101,191.
				ANNUAL TOTAL*	$126,808.
7	CC 1S-	173	196	$646.	$127,282.
				ANNUAL TOTAL*	$127,282.
8	CC 1S-	172	193	$659.	$127,747.
				ANNUAL TOTAL*	$127,747.
9	CC1S-+	152	161	$790.	$127,690.
				ANNUAL TOTAL*	$127,690.
10	CC1S-+	151	159	$805.	$128,622.
				ANNUAL TOTAL*	$128,622.
				TEN YEAR TOTAL	$1,281,550.

LIQUIDATION VALUE = $8,636,273. ANNUAL HARVEST (INCLUDING SALVAGE) = $144,026.
TOTAL HARVEST = $1,440,260.

ACTIVITY CODE PC = PARTIAL CUT. CC = CLEAR CUT. NC = NO CUT.
1 = VERY OLD GROWTH. 2 = OLD GROWTH. 3 = YOUNG GROWTH.
F = FIR. S = SUGAR PINE. + = WELL STOCKED. ++ = GOOD UNDERSTORY.
- = POORLY STOCKED. -+ = GOOD UNDERSTORY.

* SALVAGE CULLED ANNUALLY ON EACH ACRE NOT CUT DURING THIS YEAR BRINGS THE ANNUAL HARVEST (LESS SALVAGE) TO THE REQUIRED MINIMUM ANNUAL HARVEST OF $144,026.

Figure 4—*Linear programing solution is translated into a management plan in this computer-produced printout.*

total harvest from $3.4 million until the liquidation value for these old-growth stands was forced to zero at $4.8 million. For each parametric solution the program selected the annual harvest schedule which maximized the liquidation value of the forest.

The liquidation value corresponding to the 15 parametric solutions computed by the program decreased from $1.45 million to zero at the maximum total harvest level of $4.8 million. The aggregate timber value also decreased as minimum total harvest increased. These relations are almost inversely proportional *(fig. 5)*.

As could be expected, the 10-year harvest schedule corresponding to a minimum total harvest of $3.4 million requires the clearcutting of the minimum harvest allowed each year *(fig. 6)*. As the minimum total harvest is increased, a consistent pattern of changes emerges in the annual harvest schedules. To take full advantage of growth, annual harvests are kept at the lower bound as long as the total harvest requirements permit. The harvests for the tenth year, the ninth year, the eighth year, etc., are then pushed successively to the upper bound as the minimum total harvest requirement increases *(fig. 6)*. At the highest minimum total harvest, $4.79 million, the annual harvests for years 10 to 5 are at the upper bound ($566,000), the fourth year of harvest has been pushed to $380,000, and the annual harvests for the first 3 years are still at the lower bound *(fig. 6)*.

These results reflect the obvious. If the objective is to maximize the undiscounted liquidation value, cutting must be delayed as long as possible. What is not obvious, except in very simple problems, is what cutting schedule will assure the longest delay. The management plan devised by the parametric program solves this problem.

In the harvest schedules prepared by the parametric program, the shift from the "low" to the "high" harvest schedule occurs occasionally in a single year. Production and market considerations may require that this increase be spread over several years. This additional restriction can easily be introduced into the program by defining special constraints specifying the maximum allowable change in successive harvest levels.

If providing enough timber to keep the mill operating at capacity were required, the annual harvest levels and the economic objective could be expressed in volume units. No changes in the structure of the program would be necessary. The yield coefficients and the constraint levels would simply be converted from dollar values to volume units such as thousand board feet. This conversion was performed for parametric solution No. 12 *(fig. 6)*. The annual and total harvest constraints were divided by 22.66. This is the average value of a thousand board feet of timber weighted by the total volume of each species.

The order in which the timber classes are cut is essentially the same for both the liquidation value and volume maximization programs *(table 2)*. The jump from the lower to the upper bound occurs in both programs from the fifth to the sixth year. However, the number of acres to be clearcut annually varies considerably. The value maximization program required the cutting of 373 acres the first year, but the volume maximization program required only 231 acres. This decrease in the number of acres to be cut reflects the low value of fir compared to that of pine.

Parametric Changes in Bounds

In the two preceding cases, parametric linear programing was used to relate total harvest and liquidation value. In the first case, all annual harvests were equal; in the second case, the annual harvest schedule was selected within specified bounds, as the minimum total harvest was increased parametrically. These bounds defined the range of acceptable values from which the parametric linear program could select, and hence affected the corresponding level of the economic objective being maximized.

The effect of systematic changes in the location of these upper and lower bounds on the value of the economic objective can also be studied. To focus the analysis on changes in the location of the bounds, it will be assumed initially that the minimum total harvest level has already been fixed at 200 MMBF. Then, we will lift this restriction and explore the effects of simultaneous parametric variation in the minimum total harvest and in the location of the bounds.

The location of the upper and lower bounds can be varied in an infinite number of ways. The decision maker must therefore define some patterns of change that are relevant to his problem and that can be investigated with parametric linear programing.[3]

The simplest pattern is obtained by reducing uniformly the range of allowable harvests. The an-

[3] The restrictions on the pattern of changes in the bounds imposed by parametric programing and a discussion of theoretical and technical considerations are included in a separate appendix to this paper. It is available upon request to the Director, Pacific Southwest Forest and Range Experiment Station, P. O. Box 245, Berkeley, California 94701.

Solution summary for figures 5 and 6			
Solution number	Liquidation value	Aggregate timber value	Total harvest
	Million dollars		
1	1.45	4.85	3.40
6	.76	4.83	4.07
12	.28	4.81	4.53
15	0	4.79	4.79

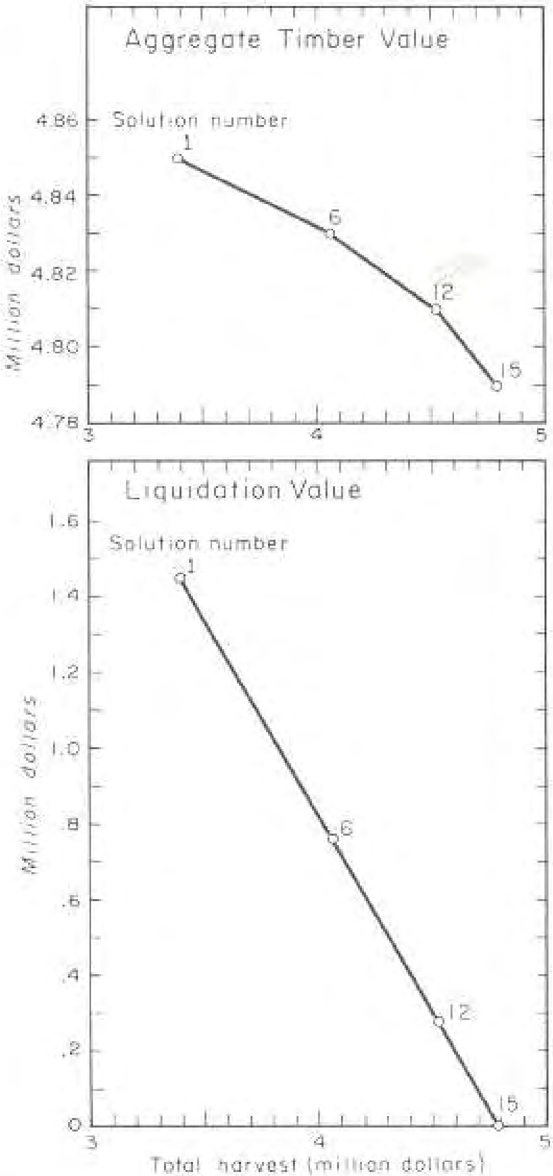


Figure 5—*Parametric relation between aggregate timber value, liquidation value, and total harvest value when annual harvests are allowed to vary between fixed upper and lower bounds.*

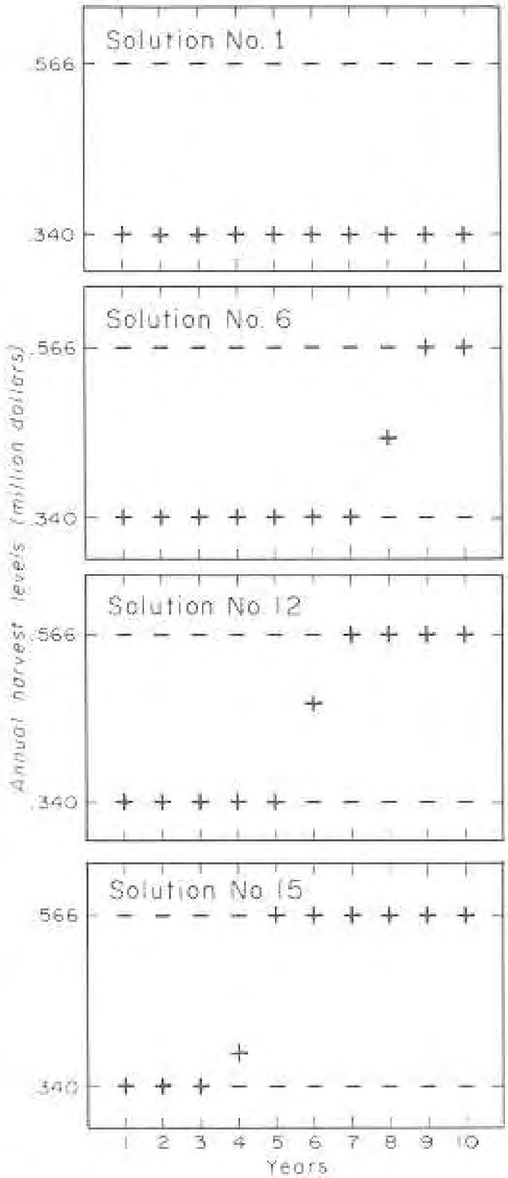


Figure 6—*Alternative schedules for annual harvest.* − *indicates fixed upper and lower bounds between which annual harvests can fluctuate.* + *indicates optimum level for each year of harvest.*

Table 2.--*A comparison of parametric solutions based on stumpage value and stumpage volume data, for similar annual harvest constraint levels*

Year	Stand[1]	Clearcut activities		Annual harvest	
		Value LP	Volume LP	Value LP	Volume LP
		Acres	*Acres*	*Dollars*	*MMBF*
1	1F+	373	231	340,000	15
2	1S++	72	--	--	--
2	1F+	227	229	340,000	15
3	1S+-	183	80	340,000	15
3	1F++	--	141	--	--
4	1S++	181	208	340,000	15
5	1S++	181	206	342,000	15
6	1S++	297	340	566,000	25
7	1S++	294	337	566,000	25
8	1S++	291	334	566,000	25
9	1F-	87	95	--	--
9	1F-+	220	220	--	--
9	1S++	236	230	566,000	25
10	1F-	513	--	--	--
10	1S-	550	550	--	--
Total harvest				4,531,000	200
Residual stumpage				278,000	12.6

[1]Symbols used:
1 = very old growth F = fir S = sugar pine.

nual mill delivery bounds are set initially at 15 and 25 MMBF. The upper and lower bounds are then brought closer together until they meet at the mid-range annual harvest of 20 MMBF (*fig. 7*).

The parametric linear program mapped this pattern of change in three solutions (*fig. 8*). As the range of allowable harvests was narrowed, maximum liquidation volume fell proportionately from 12.6 MMBF to 11.3 MMBF. The loss of liquidation volume amounted to 10 percent of the initial volume or 1.3 MMBF. The cause of this loss can be deduced from *figure* 7. As the bounds approached each other, the loss of growth from the rising lower bound exceeded the gain from the falling upper bound. The net result was a decrease in the liquidation volume. When the range was reduced to 0.8 MMBF the level of annual cut remained at the lower bound for the first 5 years and then jumped to the upper bound for the remaining 5 years.

The maximum liquidation volume corresponding to any range up to 10 MMBF can be computed by simple interpolation. If a management plan is also required, an additional solution can be computed for the desired range.

Some situations may require the investigation of nonuniform changes in the bounds. Parametric programing can be used to perform this analysis subject to the restriction that the changes in the bounds must be proportional. For the preceding example this restriction is roughly equivalent to keeping all the upper and the lower bounds on separate straight lines. These lines can be independently raised, lowered, or rotated about any point on their initial locations. Still more complex patterns of change can be investigated by parametric linear programing.

Because parametric programing can be used to change the slope as well as the level of upper and lower bounds, it can be used to investigate a wide spectrum of planning situations. In the following example, the upper and lower bounds were made to converge over time until the tenth-year range of allowable harvests was forced to zero. This was accomplished in four parametric solutions by tilting the upper bound downward, and the lower bound upward (*fig. 9*). Here again, maximum liquidation volume levels corresponding to intermediate bound locations can be calculated by interpolation. The maximum liquidation volume was

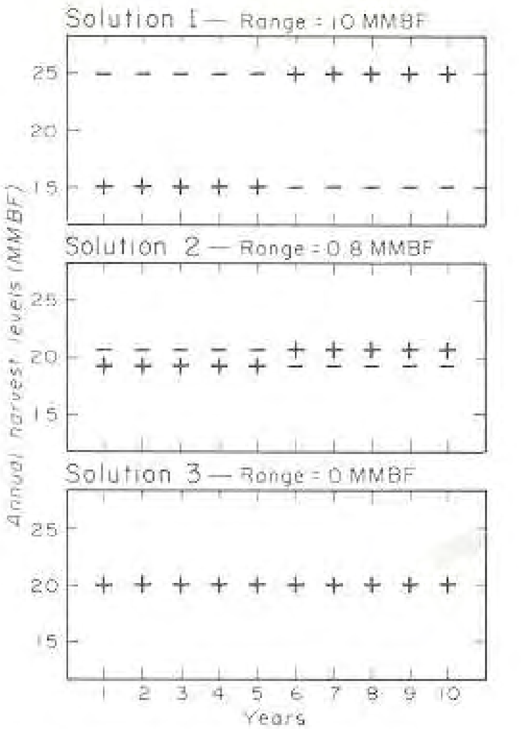


Figure 7—*Alternative schedules for annual harvest. Minimum total harvest is fixed at 200 MMBF. — indicates variable upper and lower bounds between which annual harvests can fluctuate. + indicates optimum level for each year of harvest.*

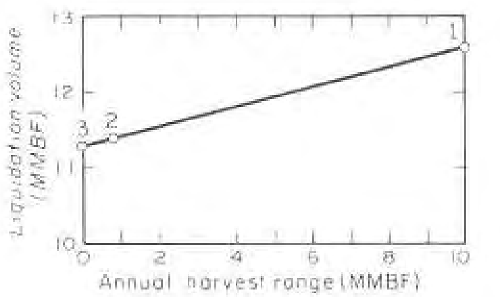


Figure 8—*Parametric relation between the range for annual harvests and the optimal liquidation volume of the forest.*

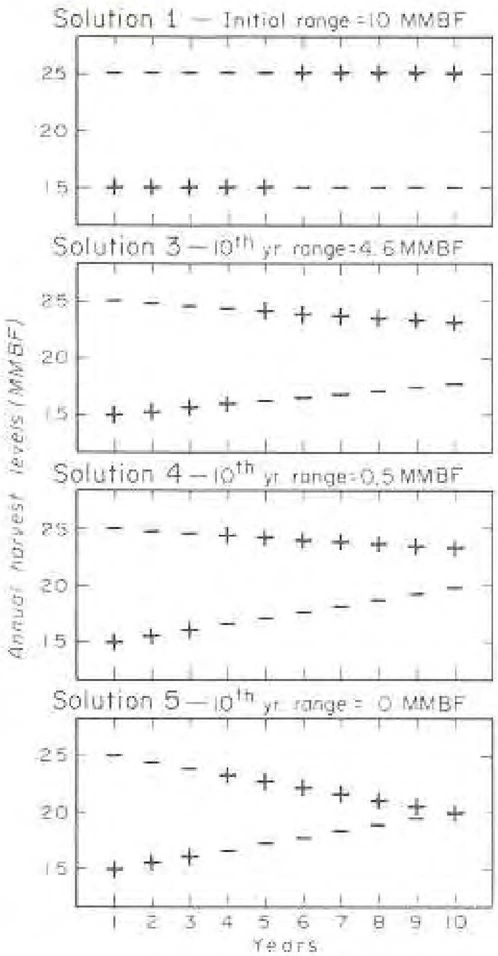


Figure 9—*Alternative schedules for annual harvest. Variable upper and lower bounds converge over time. Minimum total harvest is fixed at 200 MMBF. — indicates bounds between which annual harvests can fluctuate. + indicates optimum level for each year of harvest.*

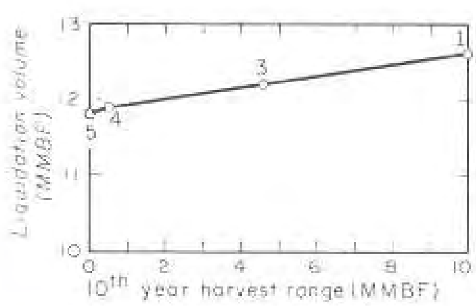


Figure 10—*Parametric relation between alternative pattern of annual harvests and liquidation volume of the forest. Minimum total harvest remains fixed.*

reduced by 0.8 MMBF as the bounds converged on the tenth year *(fig. 10)*. The program was forced to select annual harvests at the upper bound earlier as the bounds converged, causing an accelerated decrease in the maximum liquidation volume. Alternative patterns of convergence may be explored parametrically. For example, only one of the bounds could be pivoted, or the bounds could be pivoted at different rates.

Simultaneous changes in the minimum total harvest and in the bounds can be programed and

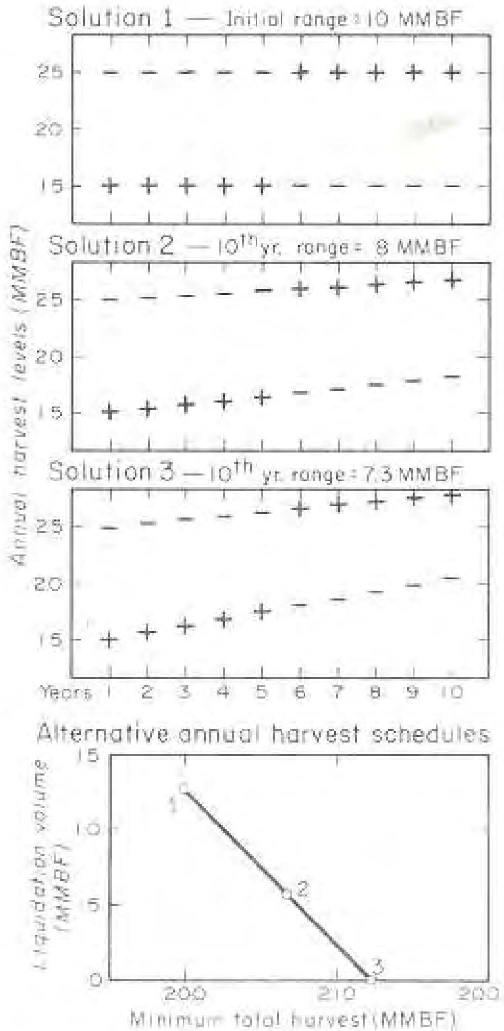


Figure 11—*Alternative schedules for annual harvest. Variable minimum total harvest. Variable upper and lower bounds converge over time.* — *indicates bounds between which levels of annual harvests can fluctuate.* + *indicates optimum level for each year of harvest.*

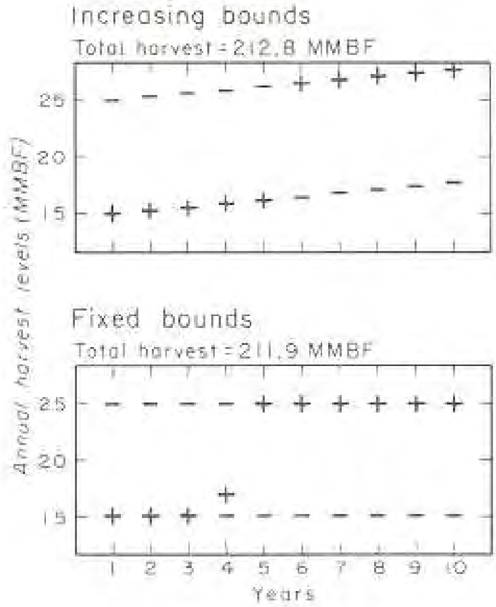


Figure 12—*Alternative schedules for annual harvest at zero liquidation value.*

the consequences on the economic objective assessed. If the upper and lower bounds are raised at different rates *(fig. 11)*, the liquidation volume is reduced to zero at a minimum acceptable total harvest of 212.17 MMBF. With the bounds fixed at the initial levels, the minimum acceptable total harvest corresponding to zero liquidation volume is only 211.92 MMBF. An increase of more than 200 MBF was realized by exploring parametrically changes in the location of the bounds defining allowable cut. This increase is readily explained by the cutting schedules of the two programs corresponding to zero liquidation volume *(fig. 12)*. With the bounds "tilted upward" the program can delay until the sixth year increasing the cut to the upper bound. The resulting additional growth more than compensates for the increase in cut for the beginning years.

In the last cases the economic objective was to maximize the liquidation volume of the forest. This analysis provided the decision maker with information on the production possibilities of the forest. Parametric linear programing can also be used to explore financial possibilities, such as present value of the residual timber and of the intermediate harvests.

To obtain this additional information, it is only necessary to change the coefficients of the objective functions from thousand board feet to dollars.

Contents

To obtain the present value of a stream of goods, the market value at various points in the future must be discounted to the present. Two alternate discount rates were investigated: 5 and 6 percent per annum. Each rate was assumed to endure throughout the planning period. At a minimum total harvest of 200 MMBF, a uniform reduction of the range of allowable harvests from 10 MMBF to zero resulted in a decrease in the present liquidation value of $31,254 for a discount rate of 5 percent, and of $42,466 for a discount rate of 6 percent. The management plans for the two discount rates were similar. Both discount rates exceeded the average rate of increase of the timber value attributable to growth, hence the program consistently selected as many early harvests at the lower bound as possible in order to minimize the net loss imposed by the discount rates.

Summary and Conclusions

Decision-making in commercial forest management is a complex process that seldom focuses exclusively on maximizing profit. Rather, annual harvest levels are more likely to be influenced by consideration of sustained yield than by financial maturity. Accommodating preferred buyers or satisfying the raw material needs of its own processing plants may far outweigh a firm's incentive to maximize its profits.

The realization of profits nevertheless determines the viability of private and—perhaps to a lesser extent—publicly owned enterprises. The decision maker must therefore be able to evaluate the economic implications of the policies that govern his decisions. This evaluation can be efficiently performed by parametric linear programing. The solutions it generates also detail the land management activities necessary to realize the specified policies and economic objective. Parametric linear programing can also determine the precise level at which policies become inconsistent with the fixed resources available. For example, parametric linear programing can readily locate the maximum sustained yield level of a timber holding. It can also determine whether the management policies are consistent. The information provided by parametric linear programing does not diminish the value of sound judgment, but can increase its effectiveness.

The application in this paper of parametric linear programing to a simple but real timber management problem demonstrates its versatility. Policy alternatives of increasing complexity can be realistically evaluated within the linearity restriction of parametric programing.

Many timber management problems can be cast as linear programs. This casting requires a thorough study of the structure of the management problem which will lead to better management even if, in the last analysis, the linear programing formulation is not adopted.

Casamajor, Paul, Teeguarden, Dennis E., and Zivnuska, John A. **Timber marketing and land ownership in Mendocino County.** Calif. Agr. Exp. Sta. Bull. 772, 55 pp., illus. 1960.

Teeguarden, Dennis E., Casamajor, Paul, and Zivnuska, John A. **Timber marketing and land ownership in the central Sierra Nevada region.** Calif. Agr. Exp. Sta. Bull. 774, 71 pp., illus. 1960.

GPO 971-923