

Moiré strain analysis of paper

ABSTRACT

Efficient use of paper products involves using modern aspects of materials science and engineering mechanics. This implies the ability to determine simultaneously different components of strain at multiple locations and under static or dynamic conditions. Although measuring strains in paper has been a topic of interest for over 40 years, present capability remains limited to recording strain in a single direction and over gauge lengths exceeding 20 mm. Improved full-field methods are needed. Of several possible optical techniques, moiré demonstrates potential for full-field analysis of paperboard.

KEYWORDS

Extensometers
Moiré effect
Optical measurement
Paper board tests
Strains
Strain analysis

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The past 40 years have been marked by several studies on strain measurement (1-10). Even with uniaxial loading, some sort of extensometer is necessary since crosshead motion does not provide a dependable measure of strain (3, 5-7). From instruments such as the Marten's extensometer, a microscope containing a filer micrometer, and the Tuckerman gauge (instruments available in 1939), Carlson selected the latter to measure strains of paper (2). Although the Tuckerman gauge continues to be one of the most accurate of the available extensometers, it uses a long gauge length (25 mm), has insufficient strain range for paper (0.005), and is limited by its weight and bulk. The requirement for manual reading also precludes using the gauge for other than essentially static tests.

Setterholm and Kuenzi (3) introduced the first strain gauge specifically de-

signed for paper in 1957. The device used the optical lever concept of the Marten's gauge but was superior because it was lighter, simpler, and more economical to construct. Setterholm and Gertjejansen subsequently located two such gauges on opposite sides of compressively loaded paper cylinders to record longitudinal strains (8). A gauge length of at least 25 mm was again used with this gauge. The device had good resolution and accuracy but was time consuming to use because of the need to follow the movement of a hairline of light on a scale and to mark the chart after each extensional increment.

The first strain gauge for paper having an electrical output was that by Jewett (7). This instrument consisted of arms having knife edges which attached through clamps to the specimen. Relative motion between the arms was monitored with a differential transformer. Weights were necessary to counterbalance the device. Setterholm and Gertjejansen (8) also used this gauge to measure longitudinal strains in com-

pressively loaded cylinders. Setterholm, Benson, and Kuenzi (9) measured the shear strain of paper tubes under torsion by attaching to the cylinder two rings separated by 25 mm. Strain was determined by observing, with a traveling micrometer, the relative angular orientation between initially parallel rods projecting radially from each of the rings. The gauge is inexpensive and has a strain resolution of 0.0003, but it must be read manually. Now, clip-on electrical extensometers having adequate range and sensitivity are available commercially for measuring uniform uniaxial tensile and compressive strains.

While important advances have been achieved using these devices, size and weight inhibit their use for recording different components of strain at neighboring locations or under dynamic conditions. With actual structural applications of paper, stress and strain distributions are seldom uniaxial and certainly are not uniform. Many situations involve complex dynamic stress fields and gradients. Measuring average

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PAPER PHYSICS

strains in such cases provides little practical information.

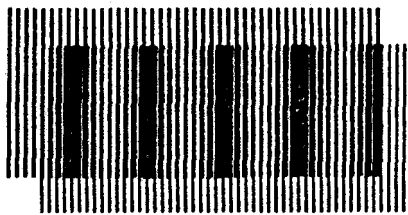
Efficient use of paper involves modern technologies of materials science and engineering mechanics. This requires knowing fully the mechanical behavior of paper—which implies the ability to simultaneously record the three in-plane strains either full-field or at least at multiple locations. Other desired features of a strain measuring method for paper include: (a) no alteration of paper properties, (b) short gauge length, (c) simplicity of equipment and ease of use, (d) low inertia, (e) real-time recording and analysis, (f) minimal data reduction, and (g) simple and infrequent calibration. Because ultimate tensile and compressive strains in paper are typically in the neighborhood of 0.05 and 0.005, respectively, the method should have a strain sensitivity down to 2×10^{-4} and a range of 0.07.

Motivated by these considerations, we recently assessed experimentally numerous alternative optical means for measuring strains in paper.¹ Of the methods evaluated, moiré is the most attractive with excellent potential for routine use.

Moiré

General comments

Moiré measures material deformation by means of interference patterns (fringes) resulting from the spatial mismatch of two optical rulings, one integral to the structural material and the other a separate analyzing grid. If the structural and analyzer grids are originally of equal spatial density (pitch) and properly aligned with each other, there is no optical interference between the two grids, and consequently no fringes are observed. As the structural material deforms, however, the pitch and/or orientation of the structural grid changes and interference fringes, bear-



1. Mechanism of formation of moiré fringes.

ing a known relationship to deformation, are observed directly.

The formation and interpretation of moiré fringes is illustrated in Figs. 1 and 2. Figure 1 shows the mechanism of fringe formation by superimposing two rulings of slightly different spatial densities. Maximum light transmission occurs when the spacings of the two rulings coincide, and a light fringe is produced. Minimum light transmission occurs when the opaque lines of one ruling lie between those of the other ruling, thereby producing a dark fringe.

Fringe interpretation is described in Fig. 2. The fringes of Fig. 2A result from the linear mismatch in pitch between two superimposed vertical rulings.² If the 19.57-line/mm ruling of Fig. 2A represented an analyzer and the 19.38-line/mm ruling represented the deformed condition of an initially identical and aligned 19.57-line/mm vertical ruling, then the fringe pattern of Fig. 2A would correspond to that obtained by subjecting the indicated component to uniaxial tension.

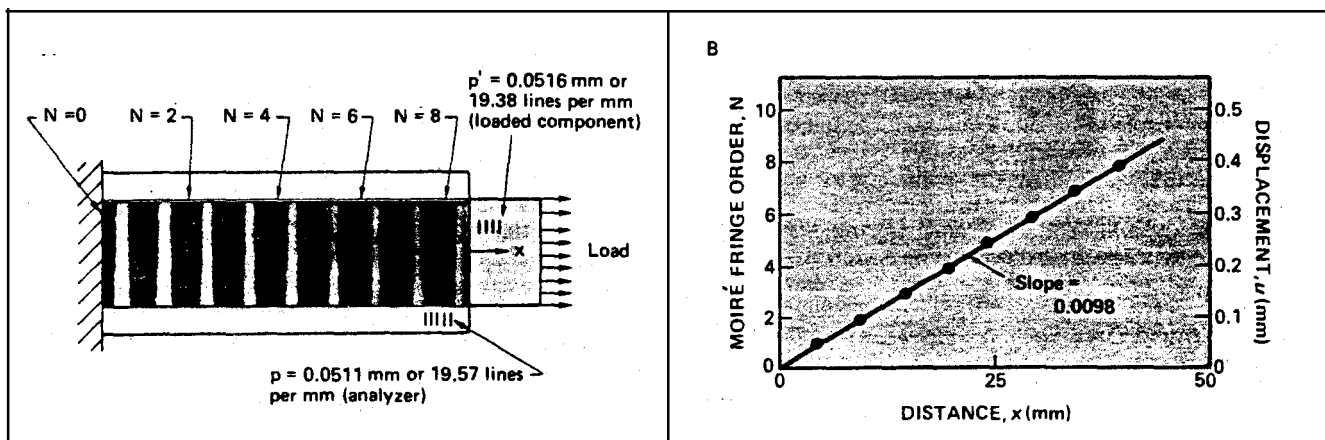
Because moiré fringes are associated with motion normal to the lines of the analyzing ruling, the horizontal displacements are as manifested by the vertical fringes of Fig. 2A. A new moiré fringe order N occurs each time a material point of the structural component moves a distance equal to one analyzing pitch p ($p = 0.0511$ mm).

² $p = 0.0511$ mm, which corresponds to a ruling density of 19.57 lines/mm, vs. $p' = 0.0516$ mm, which corresponds to 19.38 lines/mm.

Since the left end of the component of Fig. 2A is held stationary, the dark fringe at the location is $N = 0$. Material motion to the right increases proportionally as one moves toward the right. Each fringe order N noted along the top of Fig. 2A indicates the number N of pitch distances p that the point has moved, i.e., displacement $u = N \times p$. The distribution of both the fringe order N and the displacement u along the component are plotted in Fig. 2B. Since strain ϵ is equal to the spatial derivative (slope) of the displacement, $\epsilon_x = \partial u / \partial x = 0.0098$ is the strain along the component. Although the strain is uniform throughout the tensile component shown in Fig. 2A, strains in more general engineering members typically vary from point to point.

Moiré fringe patterns may be recorded using photography or video recording and are interpreted easily to yield deformation and strain over the entire field of interest. If cross-grid rulings or orthogonal arrays of dots are used, two families of fringes are observed (associated with the displacements in the two directions) from which the three components of in-plane strain may be determined. Resolution (sensitivity to strain) is directly related to the spatial density of the rulings. Moiré has been used widely and effectively for strain analysis of metals, composites, plastics, and wood (11-14). Rulings having spatial densities of up to 40 lines (or dots) per mm are commonly used.

Once a suitable line or dot pattern has been provided on the paper, the method is simple and convenient. Minimal equipment is necessary: ordinary illumination and films, an analyzer, and perhaps a camera. The advantages of moiré for analyzing strains in paper include: (a) two or three components of in-plane displacement can be measured to readily evaluate all necessary strains; (b) strains are measured at virtually zero gauge length; (c) the paper is not contacted during loading; (d) since no bulk or inertia is involved, the technique is



2. Moiré fringes and associated displacements.

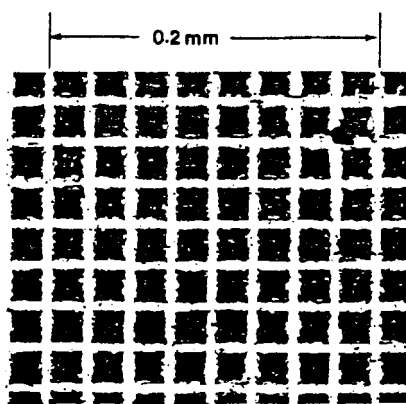
equally suitable for static or dynamic events; (e) measurements are full-field and real-time; (f) no separate calibration is needed; and (g) unlike some other optical techniques, moiré is unaffected by rigid-body motions of the structural material.

To our knowledge, the only prior attempt to use moiré with paper materials is that of Lyne and Hazall (15). The 8-line/mm (200-line/in.) moiré rulings of the study did not provide adequate sensitivity. Maximum uniaxial tensile and compressive strains for paper are on the order of 0.05 and 0.005, respectively. The moiré dot densities of 80 per mm used here are sufficient under tension, but 200 per mm would be necessary with compression.

Other potential optical methods for analyzing strains in paper include holography, speckle, Young's fringes, diffraction, and brittle and photoelastic coatings. Holography and speckle have more than adequate sensitivity and do not require any grid on the surface of the paper. However, both methods need extensive equipment, are bothered by rigid-body motion, and are inconvenient for determining the different strain components. Neither technique is real-time. Strain measurements using coarse grids or circles are insufficiently sensitive for paper. Birefringent coatings tend to reinforce the paper, and brittle coatings are not sufficiently reliable. The point-by-point nature of Young's fringes, and the fact that fringe density rather than the fringes themselves represents the displacements, limits the practical usefulness of this method. Diffraction offers no advantage over moiré.

Moiré technique and results

The principal difficulty in applying the moiré technique to paper lies in obtaining a sufficiently dense dot pattern on the paper specimen. Of the several methods attempted, vacuum deposition of carbon, lead, or aluminum arrays through a perforated mesh was most satisfactory. Good quality grids of 80 dots/mm (2000 dots/in.) were obtained on paper. This concept was motivated by prior success reported for vacuum depositing grids on plastics and metals



3. Photomicrograph of 40-dot/mm carbon cross-grid vacuum deposited onto bleached linerboard.

(16-18). The grid shown in Fig. 3 was deposited through a nickel mesh having an open area of 58%. The mesh was maintained in contact during deposition with a collodion solution and subsequently stripped. Although now limited to 80 dots/mm by available perforated meshes, plans are underway to further enhance the method by using 200 dots/mm.

Our attempts to provide adequate moiré arrays on paper by printing or photographic methods were unsuccessful. While excellent fringe patterns can be obtained by adhering stripping film containing printed rulings to linerboard, serious slippage occurred between the film and the loaded paper. Nevertheless, this concept warrants further pursuit if a sufficiently compliant film and a suitable adhesive can be found.

Figures 4 and 5 are photographs of moiré fringe patterns obtained using vacuum-deposited grids on a smooth, white, printing stock. In Fig. 4 the longitudinal and transverse displacements of a uniaxial-compressed paper sheet are indicated by the vertical and horizontal fringes, respectively. Both active and analyzing cross-grids had a spatial density of 80 dots/mm, so each fringe represents a relative displacement of 0.013 mm (0.0005 in.).

From this information, the uniform longitudinal compressive strain is 0.005.

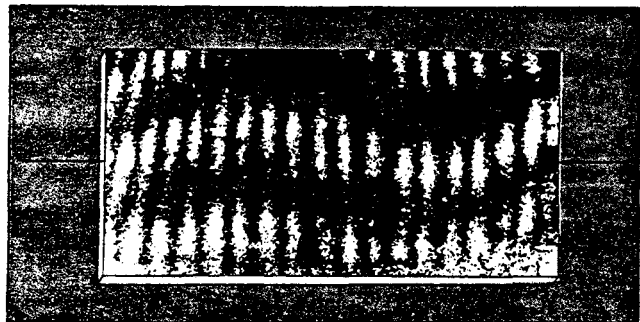
The ratio of the fringe density in the vertical direction to that in the horizontal direction of Fig. 4 is the Poisson's ratio. The paperboard specimen of Fig. 5 was loaded in in-plane bending plus horizontal tension. In this example, vertical rulings of 40 lines/mm (1000 lines/in.) were used, so each fringe in Fig. 5 represents a relative horizontal movement of 0.025 mm (0.001 in.). The top portion of this displacement fringe pattern is in compression, while the bottom is in tension. The left and right horizontal branches of the cross fringe at the center are neutral axes. By spatially differentiating this fringe data, the longitudinal compressive strain at the middle of the top edge of the paper strip of Fig. 5 was calculated to be 0.009 while the tensile strain at the middle of the bottom edge is 0.0133.³ The specimens of Figs. 4 and 5 were loaded using the compression test apparatus described by Gunderson (19). Determination of strains from moiré fringe patterns is discussed fully by Theocaris (11), Durelli and Parks (12), and in many other texts on experimental stress analysis.

Conclusion

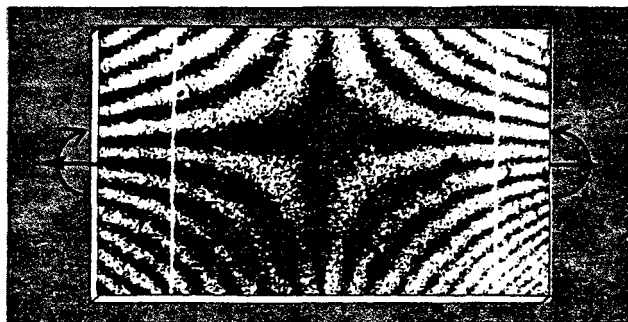
The moiré technique has been evaluated experimentally and found to be advantageous for routine full-field strain analysis of paper. The specimen is not contacted during testing, and the absence of bulk and inertia makes the method equally convenient for slow or impact situations. Moiré is also easy to use, employs short gauge lengths, is real-time, and enables all three in-plane strains to be recorded simultaneously. The 80-dot/mm array used here is completely adequate for tensile loading, although 200 dots/mm are required under compression. Extension of the technique with 200-dot/mm arrays is being pursued.

The presently available but limited capabilities for measuring uniaxial average strains in paper are the combined developments of many investiga-

³ The ultimately desired strain sensitivity of 2×10^{-4} can be achieved by using a 200-line/mm moiré ruling and a specimen format of 3-5 cm in size.



4. Moiré recorded longitudinal and transverse displacements in a horizontally compressed paper sheet (80 lines/mm; 2000 lines/in. grids).



5. Moiré recorded longitudinal displacements in a paper sheet subjected to in-plane tension and bending (40 lines/mm; 1000 lines/in. rulings).

tors over more than 40 years. The moiré results presented here represent the status after only the first serious effort mown to us. Additional research can be expected to enhance the method. We hope that the current success with full-field moiré strain measurements in paper will stimulate interest and accelerate further developments on the topic.

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