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Height-Diameter Equations for Young-Growth Red Fir in California and Southern Oregon

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Total tree height of young-growth red fir can be estimated from the relation of total tree height to diameter outside bark at breast height (DOB). Total tree heights and corresponding diameters were obtained from stem analyses of 562 trees distributed across 56 sampling locations in the true fir forest type of California and southern Oregon. The resulting equations can predict total tree height as a function of DOB, site index, and stand basal area. Nonlinear regression was used to estimate the equation coefficients. The equations accounted for 70 to 76 percent of the observed variability in total tree heights.

Retrieval Terms: Abies magnifica, height estimation, California, Oregon

Measurement of tree heights is both time-consuming and cumbersome—especially in dense stands. Therefore, the standard sampling procedure for forest inventory is to measure heights of only a few trees and estimate the unmeasured heights from mathematical relationships between height and diameter or between height and diameter and other stand characteristics. Height-diameter equations also can be used to indirectly predict height growth in growth and yield models where actual measurements of height growth are not available for modeling purposes.[1]

This note presents equations for estimating total heights of individual red fir trees in California and southern Oregon. Because California red fir (*Abies magnifica* A. Murr.) and its commonly recognized botanical variety Shasta red fir (*Abies magnifica* var. *shastensis* Lemm.) are considered to be almost identical in silvical characteristics,[2] no attempt was made to distinguish between them during the sampling phase of the study. In this note, they are referred to collectively as red fir. The equations presented here apply only to young-growth trees. No data were collected on trees greater than 120 years old at breast height.

METHODS

Data for this analysis were collected as part of a larger study to develop growth and soil fertility models for red fir within the range of its natural distribution. This range extends from latitude 43°35' N. in the southern Cascades in Oregon, to Lake County, California in the Coast Range, and to the Kern River drainage in the Sierra Nevada, latitude 35°40' N.[2] Elevation ranges from about 5,000 to 9,000 feet.[3]

A total of 56 sample sites, randomly selected from candidate young-growth red fir stands within the study area, were examined during the field seasons 1984-1988. At each site, 2 to 5 (depending on stand composition) variable radius subplots were established for measuring tree growth, stand composition, and site factor variables. At each plot, from 2 to 4 randomly selected trees (except trees with damaged tops or those less than 3.0 inches DOB) were felled and sectioned for collection of stem analysis data. In addition, one dominant red fir "site" tree was felled and sectioned on each plot. Data from these dominant site trees were combined with the other dominant, codominant, intermediate, and suppressed felled trees for this analysis.

Each tree was marked at breast height (4.5 ft above ground on the uphill side of the tree) before felling. Height to the tip of the tree was measured from this reference point to the nearest 0.1 foot. Total tree height was then obtained by adding 4.5 feet. Diameter outside bark (DOB) was measured at the breast-height mark with a diameter tape and recorded to the nearest 0.1 inch.

Total height and DOB were measured on an additional 348 trees that had not been randomly selected for stem analysis but were otherwise suitable (undamaged tops). Heights of these trees were calculated using the tangent method.[4] The angles were measured with a clinometer in conjunction with a telescoping measuring rod. These data were used for model validation (*table 1*).

Site index (total height of dominant trees at 50 years breast-height age) for each plot was determined directly from stem analysis of the site trees. Basal area (ft^2/acre) around each plot center was estimated by counting all trees that qualified for tallying with a basal area factor 20 wedge prism.

Table 1—*Descriptive statistics for the height-diameter modeling and validation data sets*

Data set	Trees	Diameter		Height		Basal area		Site index	
		Mean	Range	Mean	Range	Mean	Range	Mean	Range
		in		*ft*		*ft²/ac*		*ft*	
Modeling	562	16.1	3.0-32.0	71.6	10.7-127.8	286	40-600	55.8	18.0-113.0
Validation	348	14.9	2.8-30.8	70.9	10.0-123.0	296	40-600	56.7	27.0-105.0

DATA ANALYSIS

For predicting heights of young-growth trees, the selected model should give reasonable height estimates for trees with very small diameters (less than 1.0 inch DOB). The linear and log-linear height-diameter equations suggested by Curtis[5] either did not give reasonable estimates for these small trees, or introduced problems with log bias when using the logarithm of height as the dependent variable. Therefore, two nonlinear models were tested. The first model,

$$Ht = 4.5 + \exp(b_0 + b_1 DOB^{b_2}) + e \quad (1)$$

in which

- Ht = total tree height in feet
- DOB = diameter at breast height (outside bark) in inches
- b_i = regression coefficients
- e = a random error component with an expected value of zero and variance of σ^2,

was used by Larsen and Hann[1] for seven conifer species in southwest Oregon. The second model,

$$Ht = 4.5 + \exp(b_0 + b_1/(DOB+1)) + e \quad (2)$$

proposed by Wykoff and others[6] has been used for several species in the western United States.

Larsen and Hann also found a stand's site index and basal area to significantly affect the height-diameter relationship; for a given diameter, trees on high sites were taller than trees on low sites, and trees growing closer together were taller than trees growing more widely spaced. Larsen and Hann fit an expanded form of equation 1 to reflect these differences in site quality and stand density:

$$Ht = 4.5 + \exp(b_0 + b_1 DOB^{b_2} + b_3 X_1 + b_4 X_2) + e \quad (3)$$

in which

X_i = additional independent variables (basal area (BA), natural logarithm of basal area [ln(BA)], site index (SI), or natural logarithm of site index [ln(SI)]).

A corresponding expanded version of equation 2 that included these additional independent variables was also fit to the red fir data:

$$Ht = 4.5 + \exp(b_0 + b_1/(DOB+1) + b_2 X_1 + b_3 X_2) + e \quad (4)$$

Equations 1 to 4 were each fitted to the data by weighted nonlinear regression. Weights of 1/DOB, $1/DOB^2$, $1/DOB^{0.5}$ and 1.0 (unweighted) were tried. The four weighting procedures were compared according to Furnival's index of fit.[7]

RESULTS

The plotted data show the nonlinear relationship of height to DOB over the range of diameters sampled (*fig. 1*). When the four weighting procedures were compared for each model, Furnival's index of fit indicated that the weight of $1/DOB^{0.5}$ provided only a slight improvement over the unweighted procedure. Indices obtained with other weights indicated obviously poorer fits (*table 2*). Inspection of the residual plots from each regression also showed the residuals tended to follow a normal distribution and indicated the variance of total height was fairly homogeneous. The slight difference in fit did not seem to justify the use of weights in calculating the final regression coefficients. These observations agree with those of Curtis,[5] that for stands in the age range of 40 to 100 years, weighting will not usually have any great effect on the results.

Equations 1 and 2, which estimate total height as a function of DOB only, produced fits of the data which were essentially the same (*table 3*). Equation 2, however, requires one less parameter estimate, produces a slightly lower standard error, and a slightly higher adjusted coefficient of determination (r^2). Both equations produce reasonable height estimates for trees with very small diameters.

Equations 3 and 4 likewise produced fits of the data that were nearly the same (*table 3*). The best combination of the additional independent variables in both equations 3 and 4 proved to be site index (SI) and the natural logarithm of basal area [ln(BA)]. Parameters for both of these variables were significant at the 0.05 level.

Each tree was treated as an independent observation for the development of the predictive equations. In actuality, the clusters of plots (which were randomly selected) are the experimental units, and tree measurements are subsamples within the clusters. If all the trees at each cluster had been measured, ignoring this random cluster effect would result in an underestimate of the variance associated with the regressions. However, since the equations were developed from stem analysis data of only a few trees at each cluster, the random cluster effect was not considered significant in calculating the variance estimates. The standard error given for each equation (*table 3*) may be slightly underestimated.

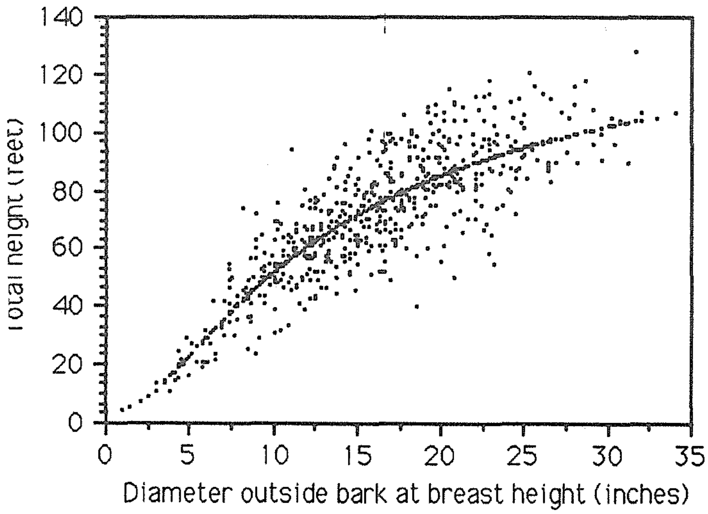

Figure 1—Total tree height plotted over diameter outside bark at breast height for 562 young-growth red fir trees in California and southern Oregon. The solid line represents the nonlinear model (equation 2).

Table 2—*Furnival's index of fit for each equation and combination of independent variables*

Equation number and independent variables[1]	Weights			
	Unweighted	1/DOB	$1/DOB^{0.5}$	$1/DOB^2$
	Index of Fit			
Equation 1				
DOB	11.91	12.34	11.72	17.76
Equation 2				
DOB	11.10	12.34	11.72	17.68
Equation 3				
DOB, SI, BA	10.90	11.40	10.79	16.42
DOB, SI, ln(BA)	10.64	11.13	10.52	16.27
DOB, ln(SI), BA	10.92	11.43	10.81	16.47
DOB, ln(SI), ln(BA)	10.66	11.14	10.54	16.31
Equation 4				
DOB, SI, BA	10.89	11.41	10.78	16.34
DOB, SI, ln(BA)	10.64	11.13	10.51	16.18
DOB, ln(SI), BA	10.92	11.43	10.81	16.39
DOB, ln(SI), ln(BA)	10.66	11.15	10.53	16.23

[1] DOB = diameter outside bark at breast height; BA = basal area; SI = site index; ln(BA) = natural logarithm of basal area; ln(SI) = natural logarithm of site index.

MODEL VALIDATION

Both the model and validation data sets vere divided into five 6-inch diameter lasses to see how well the equations prelict total tree heights for each group. Equation 3, which produced the lowest tandard error of estimate, was applied to he data sets using the coefficients preented in *table 3*. Mean residual, percent ias, standard deviation of the residuals, nd the mean predicted height for each liameter class are presented in *table 4*. The mean residual (mean bias) is the averge of the difference between measured eight and predicted height for all trees in group. Percent bias is the ratio of mean ias to the sample mean expressed as a percent. This is a measure of how far the average prediction misses the average "true" or measured value.

The mean height of all trees in the first diameter group (2- to 7-inch class) for the validation data set is 32.4 feet. The mean bias shows the equation slightly overestimated tree height by an average of 0.99 feet, a percent bias of 3.06. Percent bias is positive for the first four diameter classes and negative for the largest class (26- to 31-inch class). All biases are well below 10 percent, which was considered an acceptable limit. A comparison of the standard deviation of the residuals between the model and validation data sets shows they are each in the same relative range of magnitudes for each class. Differences between the average predicted heights of the model and validation data sets are less than 1 foot in each diameter class.

able 3—*Statistics for the modeling data set with regression coefficients listed for each equation*

quation	Coefficients					Standard error of estimate	Adjusted coefficient of determination
	b_0	b_1	b_2	b_3	b_4		
1	5.14633	-7.43722	-0.76388	-	-	11.9113	0.7010
2	4.97775	-12.27246	-	-	-	11.9087	0.7006
3	4.09279	-7.11737	-0.76425	0.00261	0.15498	10.6368	0.7607
4	3.93290	-11.73318	0.00261	0.15475	-	10.6372	0.7607

Statistics from the validation data indicate equation 3 is accurate and useful for predicting total heights of young-growth red fir trees. Since total heights for the validation data were measured on standing trees using a clinometer, and heights used in model development were measured directly on felled trees, some of the bias and errors may be due to the method used in data collection rather than to the fit of the model.

DISCUSSION AND CONCLUSIONS

Foresters often need estimates of total tree heights when only the diameters of the trees are known. Either equations 1 or 2 can be used to obtain these estimates for young-growth red fir trees. However, if

Table 4—*Comparison of statistics for the modeling data set (MDS) and the validation data set (VDS) using equation 3*

Diameter class (inches)	Trees		Mean residual		Percent bias		Standard deviation of residual		Mean height		Mean predicted height	
	MDS	VDS	MDS	VDS	MDS	VDS	MDS	VDS	MDS	VDS	MDS	VDS
			ft				*ft*		*ft*		*ft*	
2 - 7	40	36	-0.66	0.99	2.20	3.06	7.09	4.87	29.9	32.4	30.9	31.4
8 - 13	70	128	0.38	3.60	0.65	5.86	10.88	11.65	58.0	61.4	57.5	57.8
14 - 19	213	118	-0.02	1.57	-0.02	1.99	12.32	11.97	77.0	79.0	77.5	77.4
20 - 25	113	45	0.38	5.23	0.42	5.46	12.44	13.70	90.0	95.8	89.7	90.6
26 - 31	26	21	-0.24	-5.00	-0.24	-5.20	10.24	14.44	100.4	96.2	100.7	101.2

site index and basal area information are available, equations 3 or 4 should be used to obtain the best prediction of total tree height. Also, if heights are to be predicted for more than one plot or stand, the additional variables (SI and BA) need to be incorporated in the model to explain differences between stands or plots. Equations 3 and 4 both include these additional variables and estimate tree heights about equally well. However, they may not be suitable for stands that have been thinned recently. Selection of the equation to use should therefore be based on user preference and the availability of site index and basal area information.

Total tree heights for trees older than 120 years at breast height should not be estimated with these equations. Such estimates would be made with extrapolations of the basic data and their reliability would be questionable. Also, heights of trees with recognizable older imbedded, broken, or dead tops should not be estimated by these equations.

END NOTES AND REFERENCES

[1]Larsen, David R.; Hann, David W. 1987. *Height-diameter equations for seventeen tree species in southwest Oregon.* Res. Paper 49. Corvallis, OR: Forest Research Laboratory, Oregon State University; 16 p.

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[3]British/Metric Conversion
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 square foot = 0.0929 square meters
1 acre = 0.404686 hectares
1 square foot per acre = 0.229568 square meters per hectare.

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