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ALLOCATION MODEL FOR FIREFIGHTING RESOURCES... a progress report

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ABSTRACT: A study is underway at the Pacific Southwest Forest and Range Experiment Station to develop computer techniques for planning suppression efforts in large wildfires. A mathematical model for allocation of firefighting resources in a going fire has been developed. Explicit definitions are given for strategic and tactical planning functions. How the model might be used is illustrated by a fictitious but realistic problem. Limitations in the approach and ways in which the model might be improved are described.

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RETRIEVAL TERMS: fire suppression planning; resource allocation models.

Before the planning functions of large-fire suppression can be automated, mathematical models to represent some of the basic operations must be developed. A key planning problem is how best to allocate limited firefighting resources to different tasks in suppressing a going fire. At the Pacific Southwest Forest and Range Experiment Station's Forest Fire Laboratory, Riverside, Calif., a study is underway to develop computer-oriented techniques for solving fire management problems. This note reports progress to date in a study of allocating firefighting resources by computers.

In the study, a solution to the allocation problem is called a "tactical plan." Before such a plan can be devised, however, a "strategic plan" is needed. Broadly speaking, a strategic plan specifies the job to be done; a tactical plan details how best to do it.[1]

STRATEGIC PLANNING

Some definitions of terms are needed before we can be more precise about the planning process:[2]

1. A *fireline segment* is a part of a fireline in which escape of a fire over any point of that part has the same fire damage potential as an escape over any other such point.

2. Two fireline segments are *independent* if, when taken together, they do not form a single fireline segment. "Independent fireline segment" will be abbreviated (IFS).

3. The *type* of a fireline segment is an exact description of the fireline construction-firing-holding procedures specified for that segment and of the physical properties of the fireline in that segment.

4. A *fireline element* is a fireline segment which is of homogeneous type, and which forms an independent construction or holding job or both.

5. The *holding probability* for a fireline element is the conditional probability that the fire will not

escape over that element—given that fireline construction, firing, and holding operations are carried out as specified.

6. A *utility set* for a fire plan is a set of numbers, one for each IFS, which expresses the relative importance to the fire management of stopping the fire on each IFS. An element of a utility set is the *utility*,[3] relative to that set, of the corresponding IFS.

7. The *completion time* for a fireline element is the clock time by which all construction work on that element must be finished.

8. The *plan time* of a suppression plan is the clock time when the plan goes into effect.

Using the above definitions, we can be explicit about what is meant by a strategic plan. A strategic plan, then, has these components: (a) The *plan time;* (b) identification of the *IFS's*, and (c) a *utility set* for the IFS's; and for each *fireline element:* (d) the *completion time,* (e) location, (f) *type,* (g) *holding probability,* and (h) identity of the *element IFS.*

TACTICAL PLANNING

Explication of the tactical planning problem requires a few more definitions of terms:

1. A *resource pool* is a homogeneous collection of firefighting resources, effectively at a single location.

2. *Pool-element travel time* is the elapsed time from plan time until the resources from a particular pool can begin work on a particular fireline element. Thus, "travel time" can include time for resting, eating, machine servicing, or other activities.

3. The *line completion probability* for a fireline element is the probability of completing all construction work called for in the type specification for that element, before the element completion time.

4. The *probability of suppression* for an IFS is the mathematical product of all of the holding probabilities and line completion probabilities for the fireline elements in that IFS.

5. The *expected utility* for an IFS is the product of the utility number and the probability of suppression for that IFS. The *over-all expected utility* for a suppression plan is the sum of the expected utilities for the IFS's.

With these definitions in mind, we can be explicit about the objectives of the tactical planning process: the forces in the resource pools are to be allocated to the fireline elements in such a way as to maximize the over-all expected utility of the suppression plan. The time constraints imposed by completion times and travel times must be included in these calculations.

STRATEGIC PLAN EVALUATION

The allocation process will give the probabilities of suppression for each element and IFS, and for the entire fire. The partial and over-all expected utilities are also calculated. The usefulness of solving the allocation problem lies both in the fact that resources can be used to the best advantage and in that alternative strategic plans can quickly be evaluated and compared. Also, the adequacy of the resources which are assumed to be available is reflected by the suppression probabilities, and resource demands can be adjusted accordingly.

RESOURCE ALLOCATION PROBLEM

Mathematical Statement

We have seen generally how the resource allocation problem is defined and how it fits into the suppression planning process. The problem will be stated mathematically, and then we will look at an example of application as well as a computer solution for that example.

Once the elements of a strategic plan have been defined as outlined above, the following quantities are known:

M = number of IFS's

V_j = the utility number for the j^{th} IFS, $j = 1,2,...,M$

N_j = the number of elements in the j^{th} IFS, $j = 1,2,...,M$

H_{jk} = the holding probability for the k^{th} element of the j^{th} IFS, $k = 1,2,...,N_j$; $j = 1,2,...,M$

Let x_{jk} = the total effective amount of resources used on element (j,k). Then, using the time constraints and line descriptions from the strategic plan, a probability distribution function for completing construction, $P_{jk}(x_{jk})$, is defined for each element.

Let:

R = the number of resource pools

Q_i = the number of resource units in pool (i), $i = 1,2,...,R$

Y_{ijk} = the amount of resources from pool (i) which is used on element (j,k) for all pools and all elements.

Strategy	Probability IFS			Expected Utility IFS			Total Expected Utility
	1	2	3	1	2	3	
Attack all IFS's[1]	([1])	([1])	([1])	--	--	--	--
Attack IFS 2 and 3[1]	([1])	([1])	([1])	--	--	--	--
Attack IFS 1 and 3[1]	([1])	([1])	([1])	--	--	--	--
Attack IFS 1 and 2	0.98	0.98	--	2.8	0.97	--	3.8
Attack IFS 3	--	--	.52	--	--	5.2	5.2

[1]Not feasible.

Figure 4—*Results of first plan.*

IFS	Utility	Element	Completion time	Holding proba-bility	Line type				
					Kind	Length (chains)	Width	Resistance to control	Attack
1	3	1	1130	0.99	Hand	128	6 ft.	Medium	Direct
		2	1130	.97	Bull-dozer	144	1 blade	High	Indirect
2	1	1	1400	.99	Hand	160	4 ft.	Medium	Direct
3	10	1	1300	.95	Hand	120	8 ft.	High	Indirect
		2	1200	.95	Bull-dozer	130	1 blade	High	Indirect

Figure 5—*Fireline characteristics in Plan 2; plan time:0500 hours.*

Type	Amount	Start Work Time (hours) for (IFS, Element)				
		(1, 1)	(1, 2)	(2, 1)	(3, 1)	(3, 2)
Men	100	0845	--	0830	0845	--

Figure 6—*Characteristics of Pool 7.*

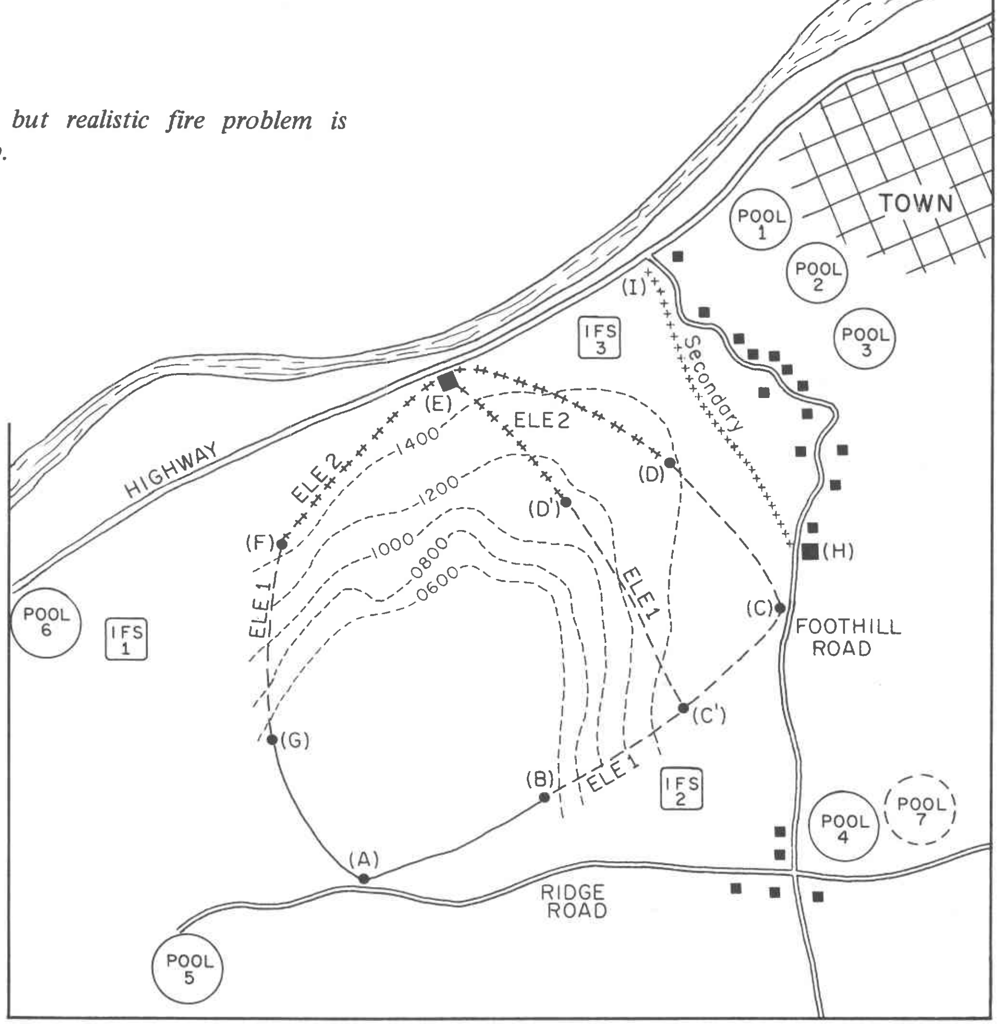
Strategy	Probability IFS			Expected Utility IFS			Total Expected Utility
	1	2	3	1	2	3	
Attack IFS 1, 2, 3	0.77	0.80	0.86	2.3	0.8	8.6	11.7
Attack IFS 2, 3	--	.99	.90	--	.99	9.0	10.0
Attack IFS 1, 3	.90	--	.89	2.7	--	8.9	11.6
Attack IFS 1, 2	.96	.99	--	2.9	.99	--	3.9
Attack IFS 3	--	--	.90	--	--	9.0	9.0

Figure 7—*Results of Plan 2.*

It was apparent that more time and more men were needed. In Plan 2, the proposed line in IFS 3 was moved back to (E)-(D)-(C). This change gave the fireline characteristics shown in fig. 5. An additional 100 crewmen were requested, and the dispatcher said he could have the men at the main fire camp by 0800. These men were designated Pool 7 *(fig. 6).* A quick computer run, using the new strategic plan and the additional manpower, gave the results in fig. 7. The computed allocations are shown in fig. 8.

The calculated probabilities were still lower than desired, but within an acceptable range. It was

Figure 1—*Fictitious but realistic fire problem is illustrated in this map.*



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3	10	1	0900	.95	Hand	144	8 ft.	High	Indirect
		2	1000	.95	Bull-dozer	120	1 blade	High	Indirect

Figure 2—*Fireline characteristics, Plan 1; plan time:0500 hours.*

Pool (No.)	No. of units	Unit type	Start Work Time (hours) for (IFS, Element)				
			(1, 1)	(1, 2)	(2, 1)	(3, 1)	(3, 2)
1	1	Tractor	--	0630	--	--	0630
2	1	Tractor	--	0800	--	--	0800
3	100	Men	0645	--	0630	0645	--
4	30	Men	0545	--	0530	0545	--
5	50	Men	0715	--	0700	0715	--
6	1	Tractor	--	0830	--	--	0830

Figure 3—*Resource Pool characteristics, Plan 1; plan time: 0500 hours.*

Define a pool-element efficiency factor e_{ijk}. This is the fraction of the total available time which resources from pool (i) can spend working on element (j,k). Then the effective amout of resources on element (j,k) is given by,

$$x_{jk} = \sum_{i=1}^{R} e_{ijk} y_{ijk} \qquad (1)$$

Using the notation defined here, the *probability of suppression* of the fire on the j^{th} IFS is:

$$P_j = \prod_{k=1}^{N_j} H_{jk} P_{jk} \left(\sum_{i=1}^{R} e_{ijk} y_{ijk}\right) \qquad (2)$$

Then, the *over-all expected utility*, U, is given by,

$$U = \sum_{j=1}^{M} V_j P_j = \sum_{j=1}^{M} V_j \prod_{k=1}^{N_j} H_{jk} P_{jk} \left(\sum_{i=1}^{R} e_{ijk} y_{ijk}\right) \qquad (3)$$

Our problem is to find resource allocations, y_{ijk}, which will maximize U. At the same time the y's must sum up to the resource quantities which are available in the resource pools. (These equality constraints assume that the resources are committed to the fire):

$$\sum_{j=1}^{M} \sum_{k=1}^{N_j} y_{ijk} = Q_i,\ i = 1,2,\ldots,R \qquad (4)$$

Note that, if the y's always are adjusted to maximize U, then U is a function only of the total resource quantities, Q_i. Let us call this function W(Q). Then:

$$W(Q) = \underset{[y_{ijk}]}{\text{Max}} \sum_{j=1}^{M} V_j \prod_{k=1}^{N_j} H_{jk} P_{jk} \left(\sum_{i=1}^{R} e_{ijk} y_{ijk}\right) \qquad (5)$$

with the $[y_{ijk}]$ subject to (4). We also note that,

$$V_j, H_{jk}, e_{ijk}, y_{ijk}, P_{ijk} \geq 0 \qquad (6)$$

for all permissible values of the subscripts.

Equations (4), (5), and (6), together, define a nonlinear mathematical programming problem with equality constraints. A digital computation scheme and an experimental computer program have been devised for finding numerical solutions to the problem defined here. The basic method is that of Lagrange multipliers[4].

A Fictitious Fire Problem

In this example of a fire suppression planning problem, I assume that the fire boss and his staff have available and are trained to use a computer-aided suppression planning system. It would consist of EDP equipment, housed in a van or trailer, along with the required operating accessories. In addition to doing the allocation job, such a system would store and retrieve map and other types of information needed in planning a suppression effort. It would be able to calculate fire spread automatically. The system would also provide a number of inventory and record-keeping functions.

The scenario for the example is this: a fire started in the evening at point A on Ridge Road *(fig. 1)*.[5] Heavy brush and very steep terrain prevented control by the first attack forces. However, by working through the night, with some reinforcements, the crew held the rear of the fire along the lines shown.

The fuel and weather situations were such that a large fire was expected. So a suitable suppression organization was established, and mobilization of forces was begun.

Fire perimeters were estimated for 2-hour intervals starting at 0600 the next morning. An attempt to meet a 10 a.m. control objective produced a tentative strategic plan, Plan 1. Bulldozers would be started from unloading area (E) on the main highway. Tractor lines would go to points (F) and (D′) where the terrain would stop further safe progress.

Handcrews would start from the ends of the completed lines at (G) and (B) to tie in with the tractor line at (F) and with another segment of handline from (C′) to (D′). The segment starting point (C′) was a trail junction which could be reached easily from Foothill Road.

Three independent fire segments (IFS's) were identified. The left flank of the fire, (A)-(G)-(F)-(E), was designated IFS 1. The segment (B)-(C′) on the right flank was IFS 2. Line (E)-(D′)-(C′), ahead of the fire, was IFS 3.

A plan time of 0500 was established. The fire staff agreed on a *utility set* (3,1,10); that is, the damage threatened by loss of the fire on IFS 3 was judged 10 times as great as on IFS 2, and potential damages on IFS 1 were three times those on IFS 2.

The fireline characteristics in Plan 1 are shown in fig. 2 and the resource pool parameters are given in fig. 3. These data were quickly run through the computer, and the prediction was not favorable. (All of the results in this example were actually computed.) Figure 4 summarizes the results. The flanks could likely be held by abandoning the head of the fire. Or, by putting all available manpower on IFS 3 at the head, there was a low probability (about 50-50) of holding the fire at that line.

Segment	Element	Pool 1	Pool 2	Pool 3	Pool 4	Pool 5	Pool 6	Pool 7
		Bulldozers		————*Men*————			*Bull-dozers*	*Men*
1	1	--	--	57	30	--	--	--
1	2	1	1	--	--	--	--	--
2	1	--	--	--	--	--	--	64
3	1	--	--	43	--	50	--	36
3	2	--	--	--	--	--	1	--

Figure 8—*Resource allocations in Plan 2.*

decided to go ahead and attack all IFS's using this plan, with the additional precaution of ordering out another bulldozer to clear an old truck trail (H)-(I) as a secondary line in IFS 3. The bulldozer would be at unloading area (H) at about 0900, giving plenty of time to make certain the job would be done.

Since we only have probability estimates of fire suppression for this example, we cannot say whether the effort would be successful. We do know that, using the allocation model, the available resources would be applied in the best possible way, consistent with the assumptions which were made.

DISCUSSION

This example gives some idea of the way a computer-aided fire suppression planning system could be used. The system could also be used in essentially the same way, at intervals during the execution phase, using updated information on fire behavior and work progress. As time progressed, the estimates of the outcome would change and improve in reliability, allowing close control of the suppression effort.

The suppression system model proposed here is an abstraction from the real process, as any such model must be. Fire suppression is made equivalent to the construction and holding of fireline. While this is, I believe, the most important aspect, there are others. As an example, air attack with fire retardants is not always used to build fireline; in some instances, a better way to consider retardant dropping is as an effort to buy time. The model described above includes time limitations, but does not include a use of resources (aircraft and retardant) to change these limitations. Similarly, the use of water or chemical retardants from firetrucks to stop or slow fire spread is not included in the model.

The model does not explicitly assign resources (men) to holding or firing operations, though it can be assumed that such manpower would be subtracted from the manpower pools which are defined for construction jobs. These limitations of the model are pointed out, not as errors or fundamental limitations of the approach, but as problems for additional work.

NOTES

[1]Bratten, F. W. *Forest fire suppression system analysis.* 1968. (Unpublished rep. on file at the Pacific SW. Forest & Range Exp. Sta., U.S. Forest Serv., Riverside, Calif.)

[2]I am assuming that the reader is familiar with, or has access to, the common terminology of wildfire suppression. See, for example, the *Glossary of Terms Used in Forest Fire Control.* (U.S. Dep. Agr. Handb. 104, 24 p., 1956.)

[3]See, Luce, R. D., and Raiffa, H. *Games and decisions.* 509 p. New York: John Wiley & Sons, Inc., 1958, (chap. 2) for a discussion of utility in the sense used in this note. A simple example of a suitable utility set would be the dollar values of the resources threatened on each independent fireline segment. Most utility assignments would include other, less tangible, considerations.

[4]Everett, H. *Generalized Lagrange multiplier method for solving problems of optimum allocation of resources.* Operations Res. 11:399-417. 1963.

[5]My apologies to fire control experts who may spot unlikely aspects of this purely fictitious problem.

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