Evaluating Growth Models: A Case Study Using Prognosis^{BC}

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Abstract—The ability of the Prognosis^{BC} (Version 3.0) growth model to predict tree and stand growth was assessed against a series of remeasured permanent sample plots, including some which had been precommercially thinned. In addition, the model was evaluated for logical consistency across a variety of stand structures using simulation. By the end of the evaluation process, we were pleased with the performance of the model. Some of the less obvious benefits of growth model evaluation and the value of using multiple approaches when evaluating growth models are discussed.

Introduction

Prognosis^{BC} is a growth and yield simulator adapted from the North Idaho version of the Forest Vegetation Simulator—FVS (Dixon 2002; Stage 1973; Wykoff and others 1982). Prognosis^{BC} is designed to forecast future stand conditions in mixed-species and/or multi-aged (complex) stands found in southeast and central British Columbia (BC). It retains much of the architecture of the original model; however, many of the internal equations have been reformulated and the remainder have been recalibrated. The habitat types required in the original model have been replaced by appropriate units within BC's Biogeoclimatic Ecosystem Classification (BEC) system and inputs and outputs have been converted to metric units (Snowdon 1997; Zumrawi and others 2002). Several different versions of Prognosis^{BC} applicable to various BEC zones or subzones have been developed.[1] This paper addresses Version 3.0 (released in 2003). Version 3.0 is applicable to the Interior Douglas-fir (IDF) BEC zone.

The accuracy of growth and yield model projections affects the quality of forest management decisions. Model validation (evaluation) is an integral part of model development (Scholten and Udink ten Cate 1995). It is aimed at determining the degree to which projections from the model are accurate representations of the real world. Rykiel (1996) identified three types of model validation: operational validation, conceptual validation and data-based validation. In empirical growth and yield modeling, these three validation types could be grouped into two categories: conceptual validation and data-based validation.

In this paper, we present both data-based and conceptual evaluations of Prognosis^{BC} (Version 3.0). The data-based validation examined the accuracy of the model in projecting re-measured data from two permanent sample plot installations not employed in the calibration of the model. The conceptual validation involved conducting a sensitivity analysis of the model relative to different initial stand structures. Sensitivity analysis (for example, Frey and Patil 2002; Kleijnen 2005) occupies a prominent place among forest model evaluation methods (Huang and others 2003; Vanclay and Skovsgaard 1997), since it facilitates assessing model behaviour under a broad range of conditions. Sensitivity analyses have been widely used to evaluate forest growth and yield models (for example, Gertner 1987; Mowrer 1991; Peng and others 2002). Recently, Lacerte and others (2004) used this technique to assess the Lakes States variant of FVS in Ontario under various levels of site index, stand density, and age.

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[1] Additional information on Prognosis^{BC} may be found at: http://www.for.gov.bc.ca/hre/gymodels/progbc/.

Methods

Evaluation Using Independent Data

Independent data were obtained from two permanent sample plot installations located in uneven-aged interior Douglas-fir (*Pseudotsuga menziesii* var. *glauca* (Mirb.) Franco) stands in the Knife Creek Block of the Alex Fraser Research Forest, approximately 20 km southeast of Williams Lake, BC (52°05'N, 121°50W) (fig. 1). This area is located in the dk3 subzone of the Interior Douglas-Fir biogeoclimatic (BEC) zone (Hope and others 1991) in a gently rolling landscape at an elevation of approximately 1000 m.

The first installation, consisting of six plots, was established to follow stand dynamics under three different structural conditions: (1) predominance of large older trees (dbh > 30 cm)—two 0.1-ha plots; (2) predominance of pole-sized trees (dbh 15-30 cm)—two 0.1-ha plots; and (3) predominance of saplings (dbh < 15 cm)—two 0.05-ha plots. The trees on these plots were measured following the 1987, 1992, 1996, and 2003 growing seasons. The second installation, consisting of 24 0.05 ha plots, was set up as a pre-commercial thinning experiment in stands which were diameter-limit logged in the 1960s. Three blocks (replicates) were established, each consisting of three thinning treatments and a control, with two plots located in each block/treatment combination. Measurements of the trees on these plots were made following the 1992, 1996, and 2003 growing seasons. An 11 year growth period (1993 to 2003, inclusive) was used for this study since this period most closely matches the 10-year projection period used in Prognosis[BC].

Most of the plots are located on zonal (mesic) sites, with some plots on slightly drier sites. However, within-site variation is minimal and does not warrant any changes in site classification. Douglas-fir is by far the most prevalent species, accounting for approximately 90 percent of the trees in the plots. Other tree species present are lodgepole pine (*Pinus contorta* var. *latifolia* (Engel.)), spruce (*Picea glauca* (Moench), *Picea engelmanni* (Parry) and their crosses), white birch (*Betula papyrifera* (Marsh.)) and trembling aspen (*Populus tremuloides* (Michx.)). More detail on these installations is given in Marshall (1996) and Marshall and Wang (1996).

Tree variables used in the validation exercise included species, diameter at breast height (dbh) in cm, total tree height in m, height to base of live crown in m, and crown width in m. From these basic measurements, other tree variables such as crown ratio, basal area in trees larger than the subject tree, and crown competition factor were computed. Attributes projected were stems per ha, basal area per ha, total stand volume

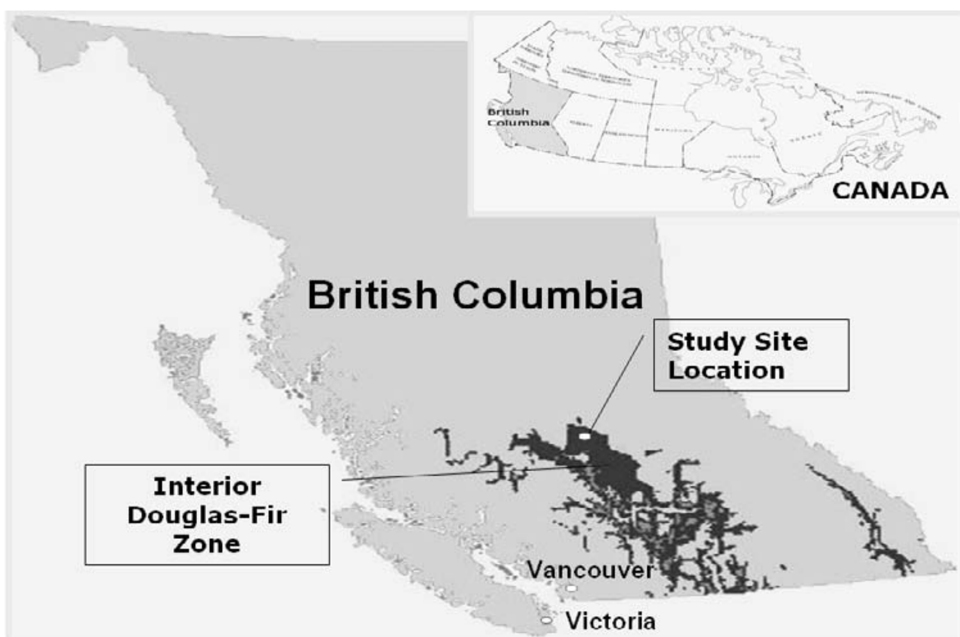


Figure 1—Location of the permanent sample plot installations.

per ha, merchantable stand volume per ha, tree dbh, and tree height. In the interests of space, only projections of stems per ha and dbh growth for trees larger than 7.5 cm dbh are reported here. Evaluations are presented in tabular form and via graphical presentations of regression-based equivalency tests (Robinson and others 2005)

Evaluation Using Simulation

Partial harvesting to produce and/or maintain uneven-aged stand structures has been successfully applied to many drier western interior North American forests (O'Hara 2002), including stands in the IDF zone. The rationale for this type of harvesting in the IDF zone includes maintaining and enhancing mule deer (*Odocoileus hemionus* Raf.) winter range, as well as meeting timber and other forest management goals (Armleder and Dawson 1992; Armleder and others 1986; BC Ministry of Forests 1992).

The BDq approach to specifying target structures for partially harvested stands has been implemented in numerous forest types (Fiedler 1995), including interior Douglas-fir forests (Day 1998). BDq is an acronym which represents target basal area (B), maximum dbh (D), and diminution quotient[2] (q, the tree-frequency ratio between successive diameter classes). These three components together characterize a target structure for a stand.

In this study, we produced a wide range of initial stand structures for pure Douglas-fir stands. These structures resulted from a factorial combination of the three components of the BDq approach. Each component was assigned four levels: basal area (B): 10, 30, 50, 70 m^2/ha; maximum dbh (D): 20, 40, 60, 80 cm; and diminution quotient (q): 1.5, 2.0, 2.5, 3.0. The resulting total number of combinations equalled 64 (4 × 4 × 4). Using 5 cm dbh classes and a minimum class midpoint of 5 cm, we generated dbh distributions (trees/ha per dbh class) corresponding to each BDq combination, as described by Fiedler (1995). Tree class frequencies were initially allocated to the dbh class midpoints. Any combination that generated a frequency of less than one tree/ha for the largest dbh class resulted in the entire combination being considered infeasible and dropped from the analysis. The combinations included in the analysis are shown in table 1.

To allow for a smoother growth simulation among successive dbh classes, we divided each class frequency among the five 1-cm dbh values within each class. To insure that class frequency and basal area would be the same as having allocated the entire frequency only to the class midpoint, we assigned a decreasing tree frequency to each successive larger dbh within each class. This frequency allocation was accomplished as follows: (1) the dbh matching the class midpoint received one-fifth of the class frequency; (2) the dbhs 1 cm and 2 cm larger than the midpoint received frequencies that were 10% and 20% lower than the midpoint frequency, respectively; and 3) the dbhs 1 cm and 2 cm smaller than the midpoint received frequencies that were 10 and 20 percent higher than the midpoint frequency, respectively.

A computer program was written to generate the dbh distributions corresponding to each feasible BDq combination. Those dbh distributions were formatted as Prognosis^{BC} input tree list files. We ran each dbh distribution for a 50-year period, assuming no interventions. For each simulated BDq combination, we forecasted stand-level variables such as basal area (ba—m^2/ha) and quadratic mean diameter (qmd—cm), as well as the following variables by dbh class: live trees/ha, ba/ha (m^2), dbh growth (cm), and mortality

Table 1—Feasible (✓) and infeasible (×) combinations of BDq for the simulation analysis.

B[a]	10				30				50				70			
q[b]	1.5	2.0	2.5	3.0	1.5	2.0	2.5	3.0	1.5	2.0	2.5	3.0	1.5	2.0	2.5	3.0
D[c] = 20	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
D = 40	✓	✓	✓	×	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
D = 60	✓	×	×	×	✓	×	×	×	✓	✓	×	×	✓	✓	×	×
D = 80	×	×	×	×	×	×	×	×	✓	×	×	×	✓	×	×	×

[a] Initial basal area per ha in m^2.
[b] Diminution quotient (tree-frequency ratio between successive dbh classes).
[c] Initial maximum dbh class in cm.

[2] This coefficient is sometimes called de Liocourt's coefficient. Meyer (1952) identified de Liocourt (1898) as the first individual to publish a numerical study of growing stock distribution in uneven-aged forests.

(dead trees/ha). To report sensitivity analysis results, we first specified the level for B, then D, and finally q. For example, the combination B = 10 m^2/ha, D = 20 cm, and q = 1.5 is identified as 10-20-1.5.

Results

Evaluation Using Independent Data

Overall, the mean differences between predicted and observed dbh over the projection period were relatively small, except for the few trees greater than 60 cm dbh where the model overestimated 11-year dbh growth by an average of 0.9 cm (table 2). The mean differences for the thinning treatments (table 3) were also relatively small (<0.2 cm), indicating little bias in predicted dbh for any spacing treatment over the 11-year projection period.

These observations are supported by the graphical representations of the equivalence tests (figs. 2 and 3). In these figures, the solid line is the regression of observed on model-predicted dbh, the grey horizontal bar represents the equivalence region for the intercept, and the diagonal dotted lines represent the equivalence region for the slope. Both figures show that the two independent one-sided confidence intervals for the slope and intercept fall entirely within their specified equivalence regions (±10 percent for the intercept and ±20 percent for the slope), at a significance level (α) of 0.05. Thus, there was strong evidence to reject the null hypotheses of the dissimilarity of the observed and predicted dbh.

However, Prognosis^{BC} predicts periodic dbh growth, not future dbh. Future dbh is calculated as the original dbh plus the dbh growth. The equivalence test on dbh growth of trees in the thinning study indicated some overestimation of dbh growth by Prognosis^{BC} following thinning (fig. 4). This result is not surprising given that the dbh growth function was calibrated using data that did not incorporate recent cutting. Trees of a given dbh growing for a number of years at lower densities would tend to have larger crowns than would be found in trees of a similar size immediately following thinning to that density.

Future stems per ha (or its corollary, mortality level) appeared to be acceptably predicted overall (fig. 5). There was a slight underestimate of future stems per ha, but the regression line and most of the observations fall well within the equivalence region. There were too few plots at each thinning level (6) to perform separate equivalence tests on predicted stems per ha for each of the thinned plots.

Table 2—Differences (cm) between actual and predicted dbh by dbh class (cm).

dbh Class	All	10	15	20	25	30	40	50	60	>60
Stems	3078	1467	969	398	113	41	49	28	9	4
Mean difference	–0.009	–0.154	0.126	0.211	0.048	0.066	–0.055	–0.036	–0.978	–0.925

Table 3—Differences (cm) between actual and predicted dbh by thinning treatment.

	C1[a]	C2[b]	STD[c]	CTRL[d]	N/A[e]
Stems	626	335	729	801	587
Mean difference	–0.135	0.064	–0.069	–0.018	0.168

[a] 3-m clumped thinning treatment. (See Marshall 1996 for a description of this treatment.)
[b] 5-m clumped thinning treatment. (See Marshall 1996 for a description of this treatment.)
[c] Standard thinning treatment. (See Marshall 1996 for a description of this treatment.)
[d] Trees growing in the control plots for the thinning installation.
[e] Trees growing in the 6 plots in the stand structure installation.

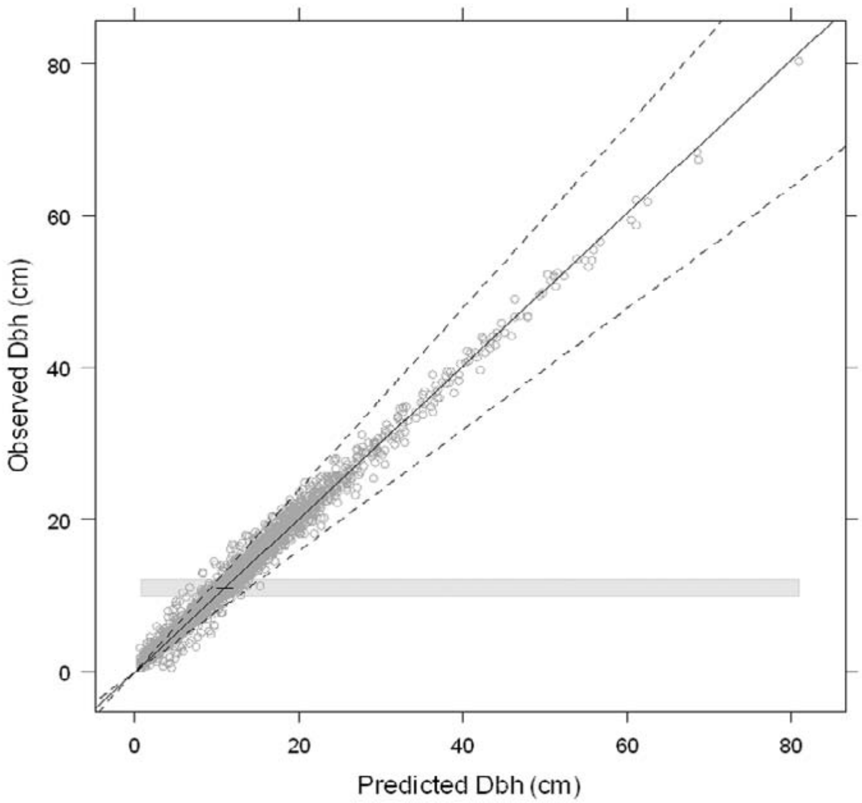


Figure 2—Observed dbh versus predicted dbh for all trees.

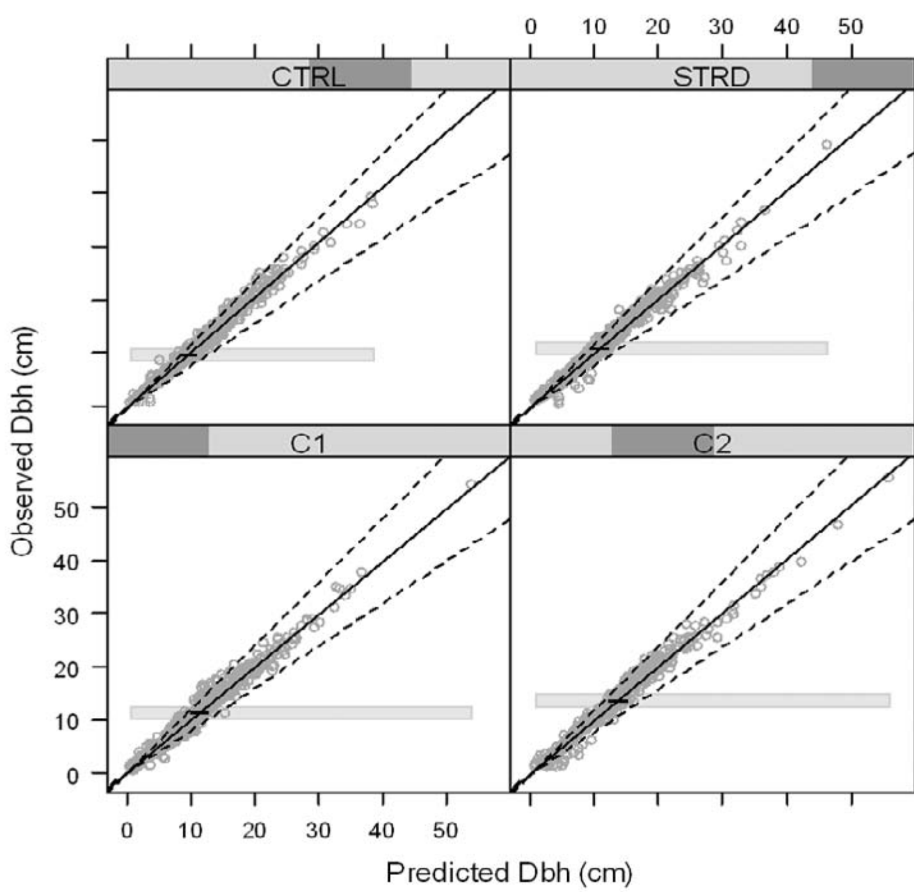


Figure 3—Observed dbh versus predicted dbh for trees in the pre-commercial thinning installation. CTRL represents trees in the control plots; STRD represents trees in the plots that received a standard thinning; C1 represents trees in the plots that received a 3 m clumped thinning; and C2 represents trees in the plots that received a 5 m clumped thinning.

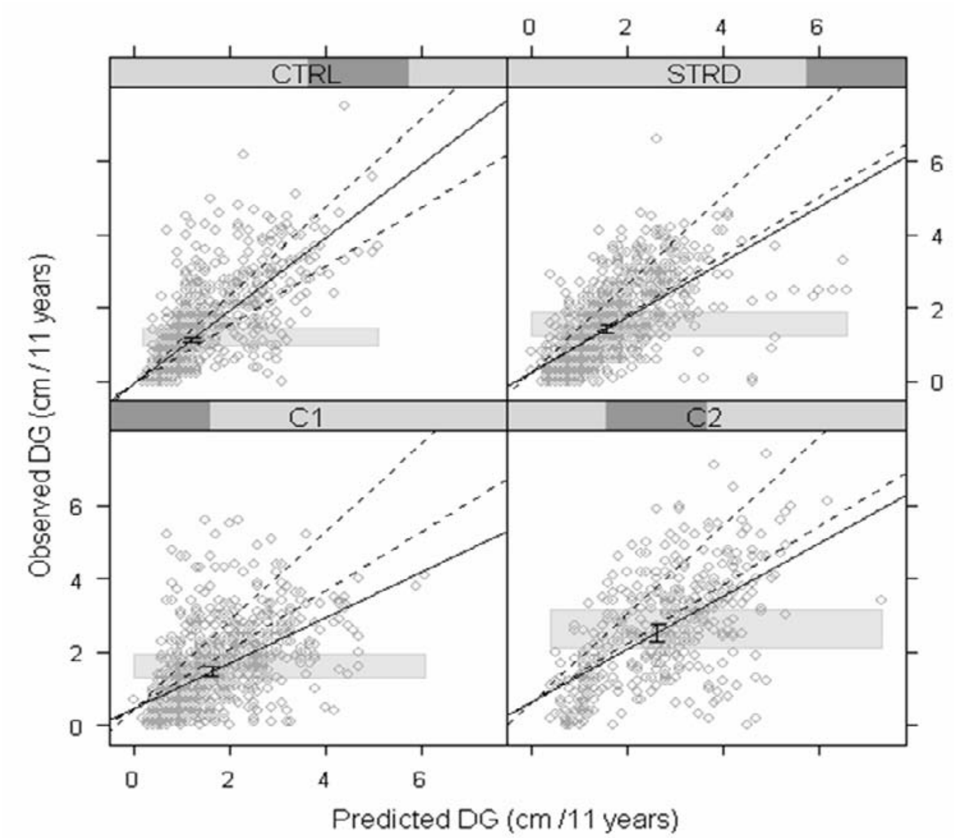


Figure 4—Observed 11-year dbh growth versus predicted 11-year dbh growth for trees in the precommercial thinning installation. CTRL represents trees in the control plots; STRD represents trees in the plots that received a standard thinning; C1 represents trees in the plots that received a 3 m clumped thinning; and C2 represents trees in the plots that received a 5 m clumped thinning.

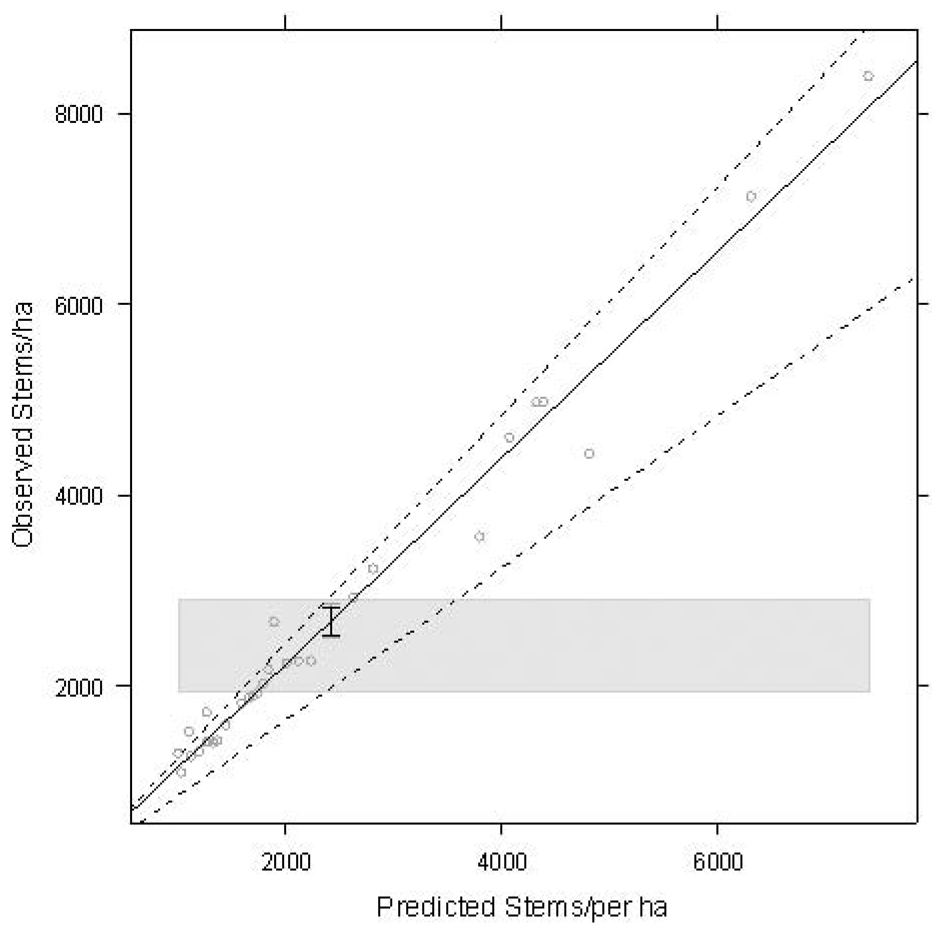


Figure 5—Observed stems per ha versus predicted stems per ha for all 30 plots used in this study.

Evaluation Using Simulation

Out of the 64 original BDq combinations, 25 resulted in a frequency of less than one tree/ha for the largest dbh class and were consequently considered infeasible. Therefore, 39 BDq combinations were analyzed in this study (table 1). Stand structures with a D of 80 cm were feasible only when B was at least 50 m^2/ha, and q was 1.5. Stand structures with a D of 60 cm were feasible for all levels of B only when q was 1.5. When q was 2.0, stand structures with a D of 60 cm were feasible only when B was at least 50 m^2/ha. Stand structures with a D of 60 cm were not feasible when q was 2.5 or 3.0. Stand structures with a D of 40 cm were feasible for all levels of B and q, except for a B of 10 m^2/ha and a q of 3.0. All of the stand structure combinations examined with a D of 20 cm were feasible.

Figure 6 displays forecasted basal area per ha under varying levels of BDq over the simulation period for a q of 1.5. The patterns for q values of 2, 2.5 and 3 were similar. Prognosis^{BC} has a limit on maximum basal area in this region of approximately 53 m^2/ha. Consequently, the projected basal area approached this limit from above (B = 70 m^2/ha) or below (B = 10 or 30 m^2/ha). The trend line for a B of 50 m^2/ha was essentially level.

The projected changes in qmd were almost straight lines, beginning at a particular value that was related to the initial maximum dbh (D) and the value of q. (See fig. 7 for an example using a q of 1.5). The starting qmd was independent of B. The higher the D for a given value of q, the higher the initial qmd. Higher values of q reflect more rapid decreases in tree numbers with increasing dbh class (i.e., relatively more small trees for a given initial basal area), and consequently lower initial values for qmd. As expected, those with lower initial basal areas showed more rapid increases in qmd than those with more dense initial basal areas. The highest level of qmd obtained was almost 35 cm for the 10-60-1.5 scenario (fig. 7c), while the lowest was just under 12 cm for the 70-20-2.5 scenario (fig. 7d). These scenarios also produced the greatest and least changes, respectively, in qmd over the simulation period.

Since the recruitment (regeneration) function of Prognosis^{BC} was not activated for this assessment, all scenarios showed a decrease in stems per ha with time (see fig. 8 for an example using a q of 1.5). Not unexpectedly, the amount of decrease increased as B increased. In both absolute and relative terms, the largest decrease in stems per ha occurred for the smallest D examined (fig. 8a). Increasing q for a given level of B and D, increased the number of stems initially (more small dbh stems required), and resulted in higher levels of mortality over the projection period.

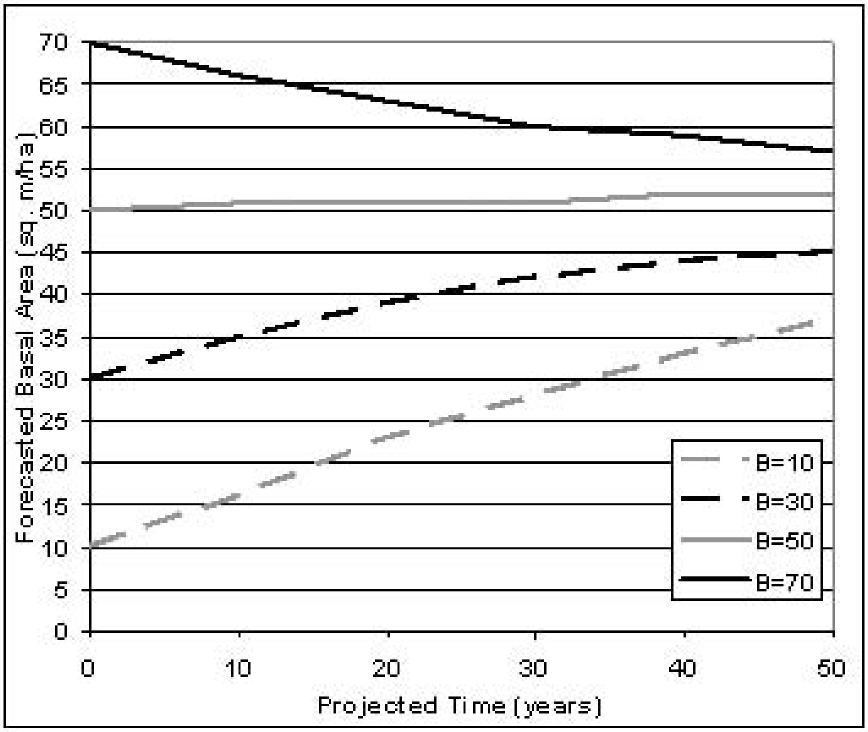
As expected, overall dbh growth was higher for stand structures with lower initial basal areas (fig. 9 vs. fig. 10). Dbh growth was similar when D was 20 and 40 cm, but decreased for a D of 60 cm (figs. 9a and b verses c, figs. 10a and b verses c). Increasing q for a given level of B and D had a slightly negative impact on the overall dbh growth (figs. 9a and b verses d and e, figs. 10a and b verses d and e).

A summary of the changes in stand dynamics associated with different stand structures predicted by the Prognosis^{BC} simulations is given in table 4. These were consistent and align with present biological understanding.

Discussion

Evaluation of any growth model provides several benefits. Documenting the performance of a model under a particular set of conditions provides benchmark information to potential model users as to the degree of trust that they ought to place in model outputs under similar conditions. The evaluation process also can bring specific components of the model under close scrutiny. It is usually beneficial if this scrutiny can be performed at more than one scale. In the case of Prognosis^{BC} (Version 3.0), we were able to identify anomalous behaviour in predictions of single tree attributes for a small subset of trees. This behaviour was traced back to a programming error which was then easily repaired. The impact of this error on stand level projections (for example, basal area per ha growth and future stems per ha) was sufficiently small that it was not previously identified.

We believe that model evaluation is more effective if more than one approach is used. In this study we combined a performance assessment conducted against independent data with a simulation study designed to assess the impact of changes in stand structure on model projections. The former approach allowed us to benchmark performance and the later provided an assessment of the consistency of model behaviour across a wide range



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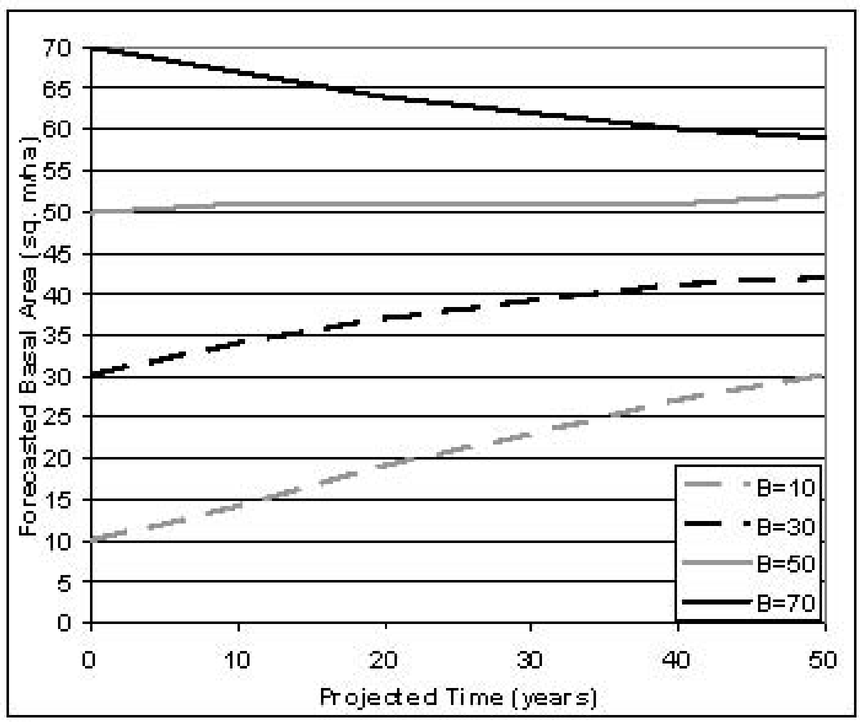
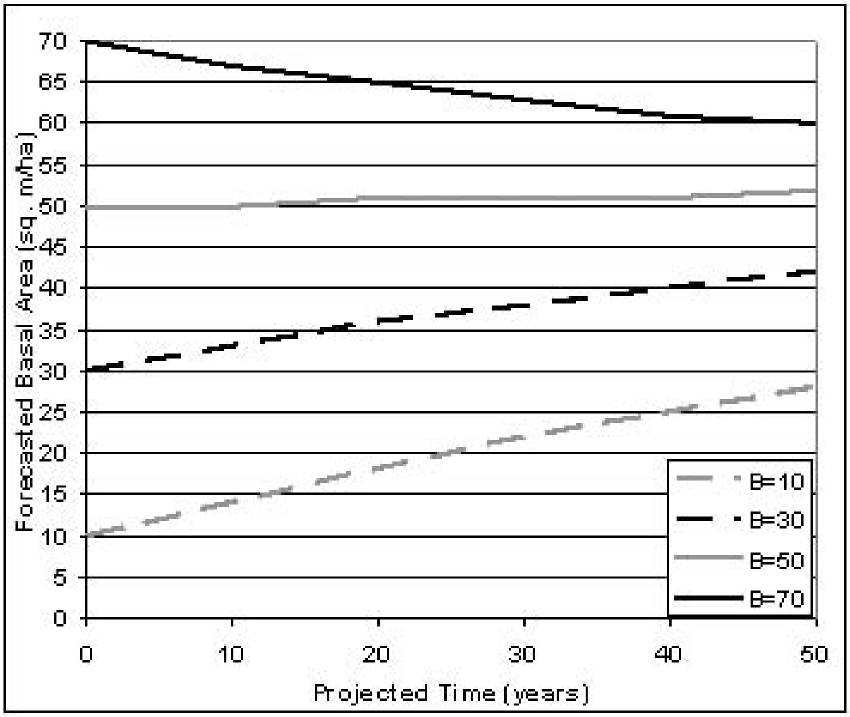


(b)

Figure 6 —Projected basal area for various initial levels of initial basal area (B) with q = 1.5 and maximum diameter (D) at (a) 20 cm; (b) 40 cm; (c) 60 cm; and (d) 80 cm.



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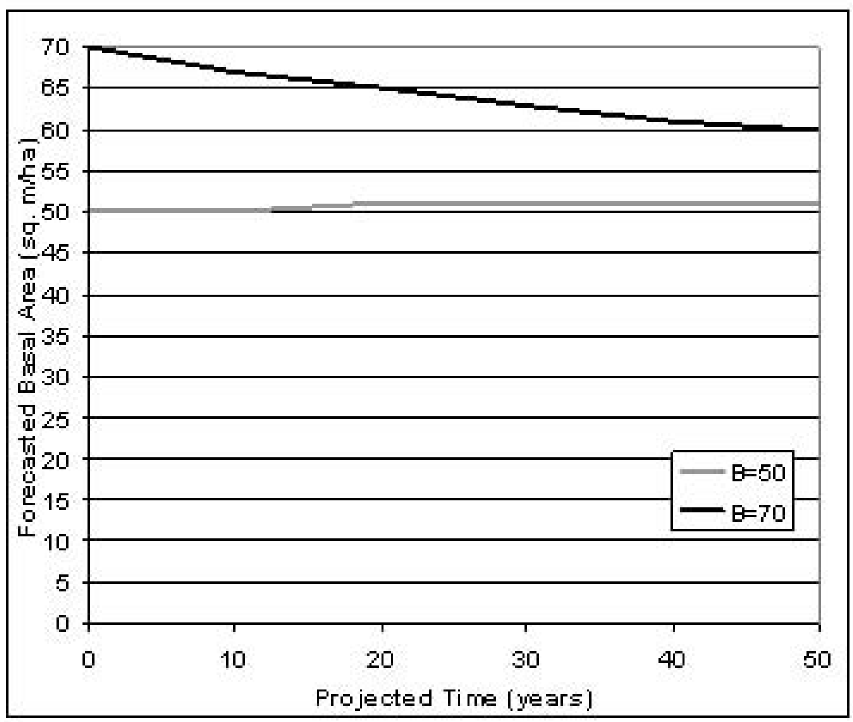

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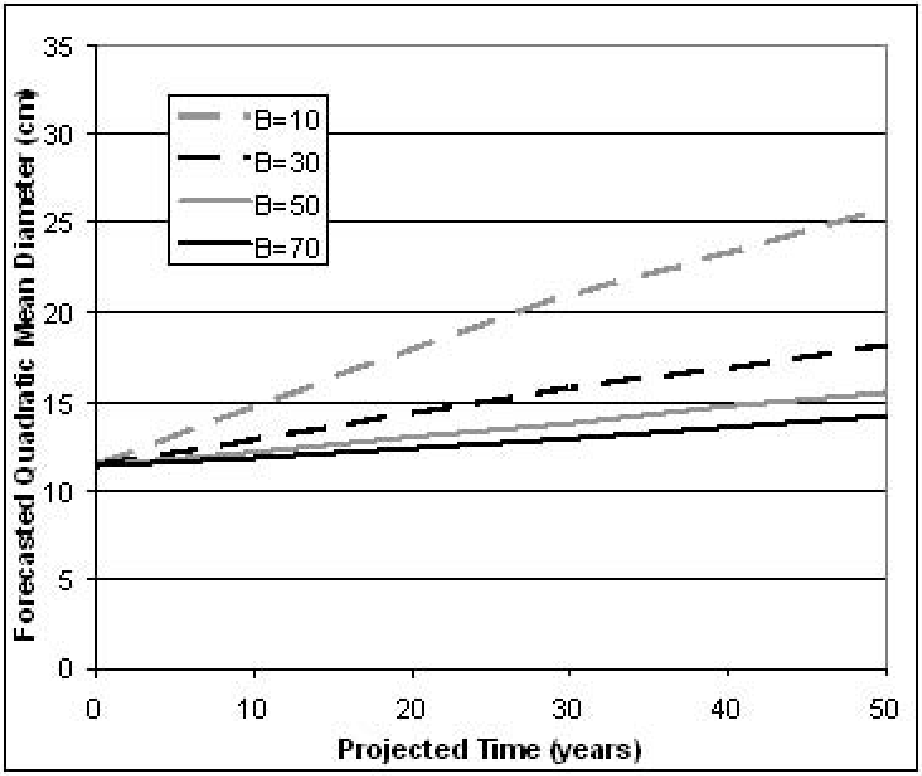

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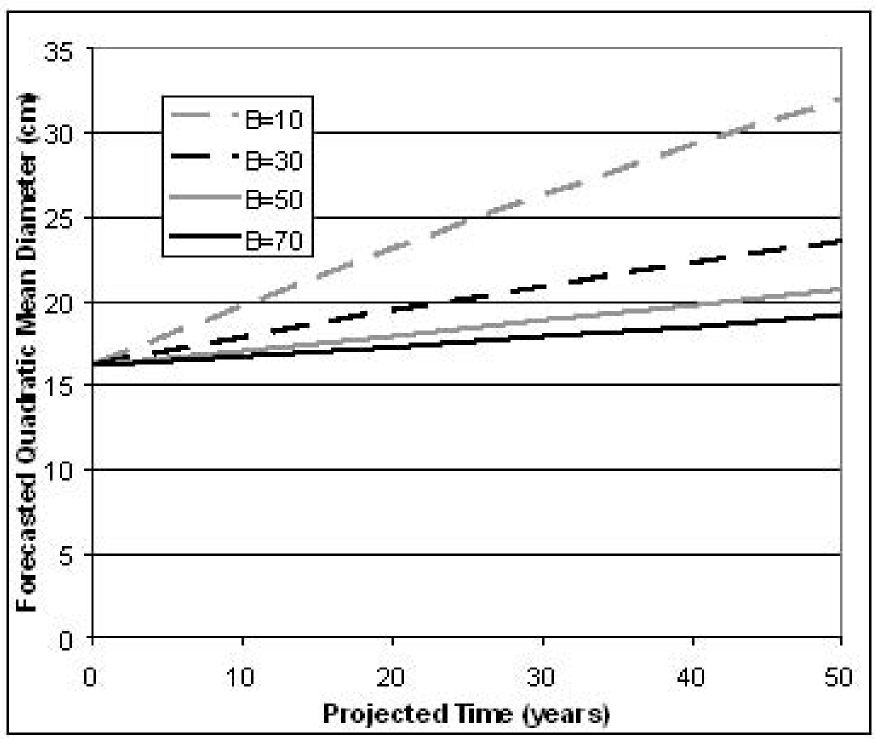
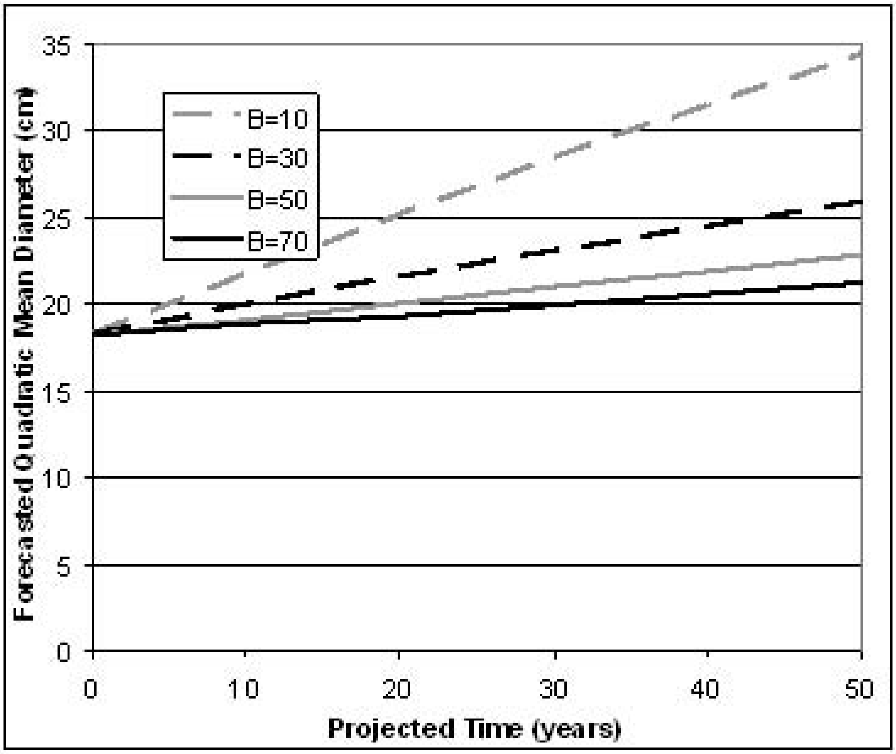


(b)

Figure 7—Projected quadratic mean diameter (qmd) for various initial levels of basal area (B) with q = 1.5 and maximum diameter (D) at (a) 20 cm; (b) 40 cm; (c) 60 cm; and (d) 80 cm.



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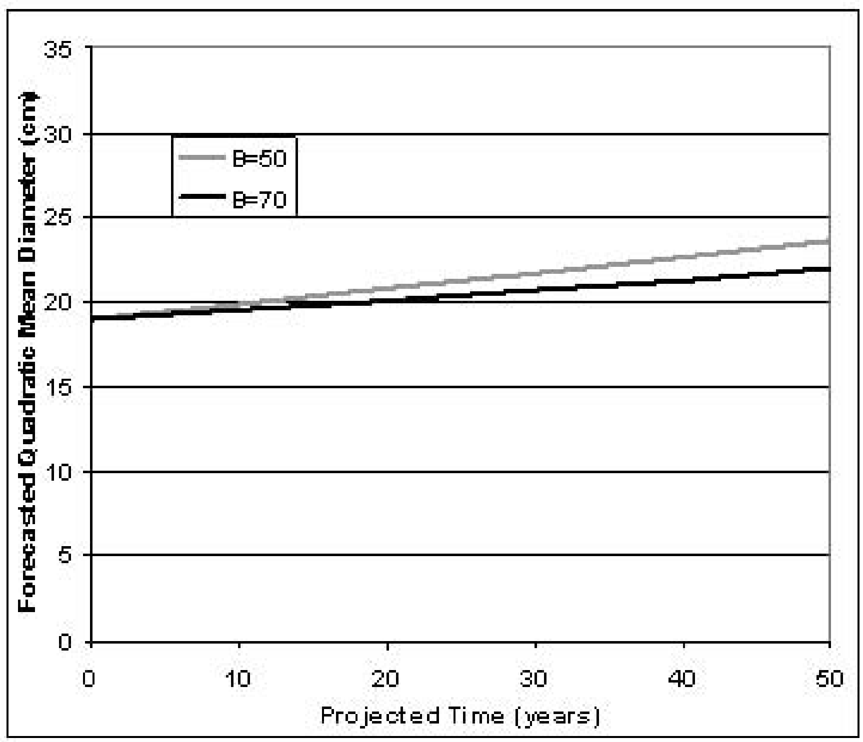

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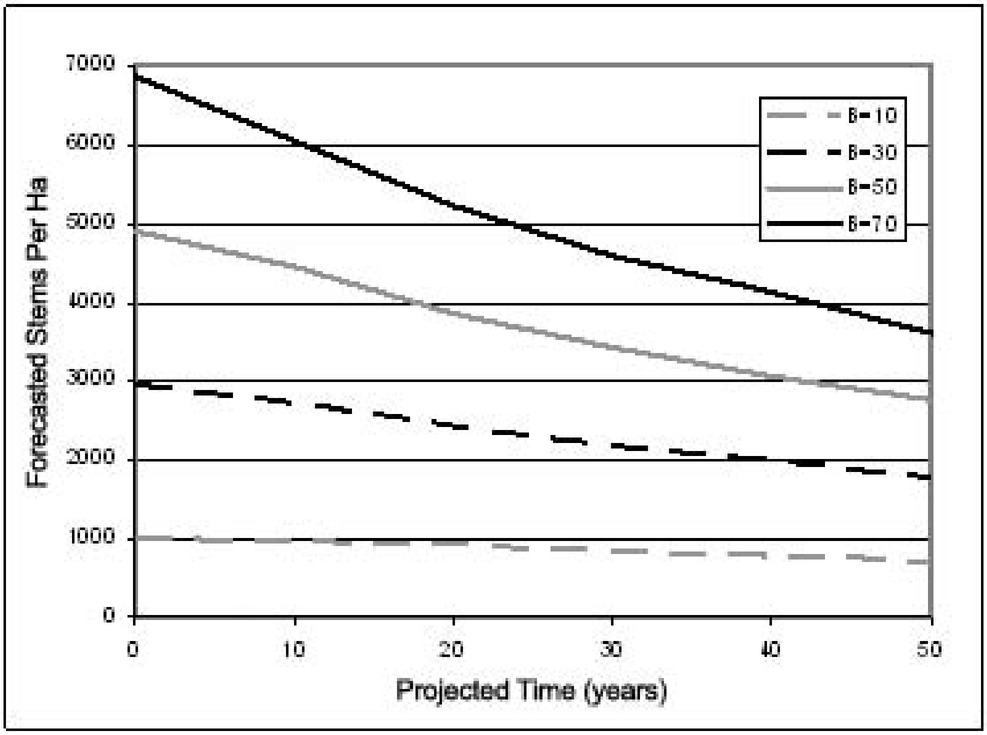

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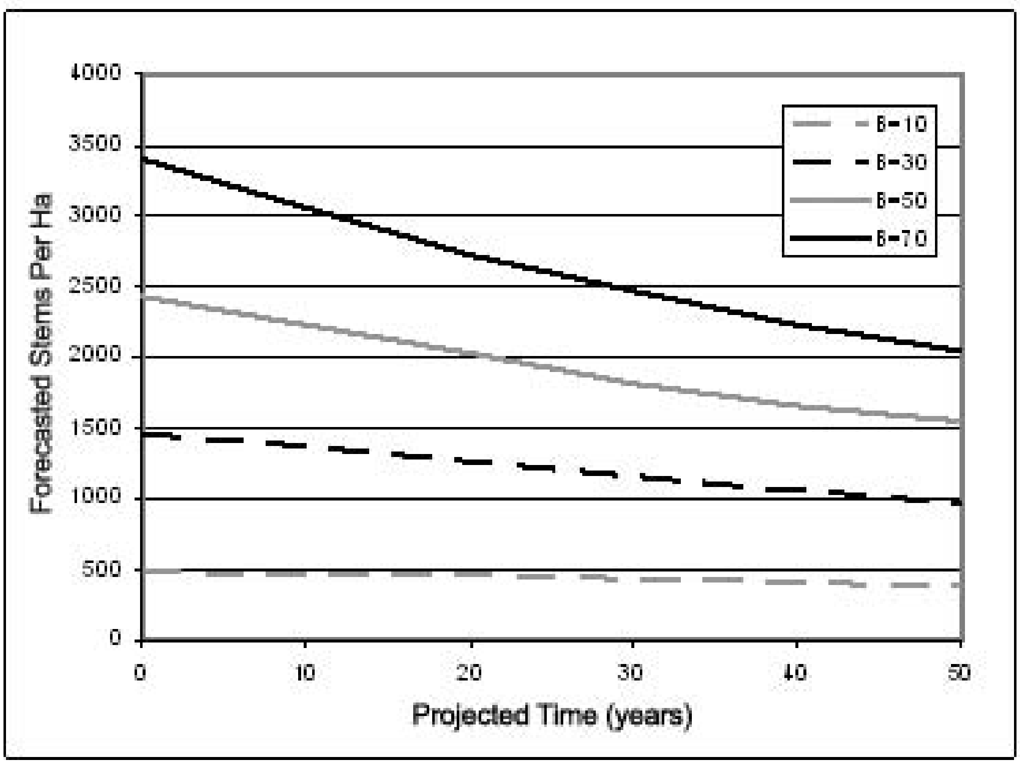
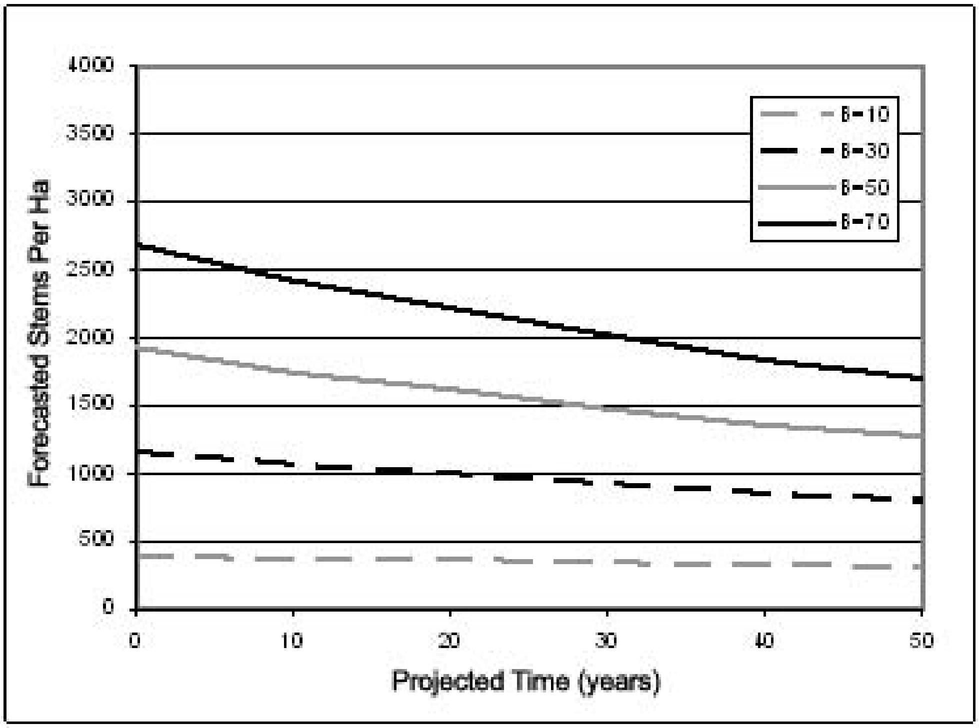


(b)

Figure 8—Projected change in stems per ha for various initial levels basal area (B) with q = 1.5 and maximum diameter (D) at (a) 20 cm; (b) 40 cm; (c) 60 cm; and (d) 80 cm.



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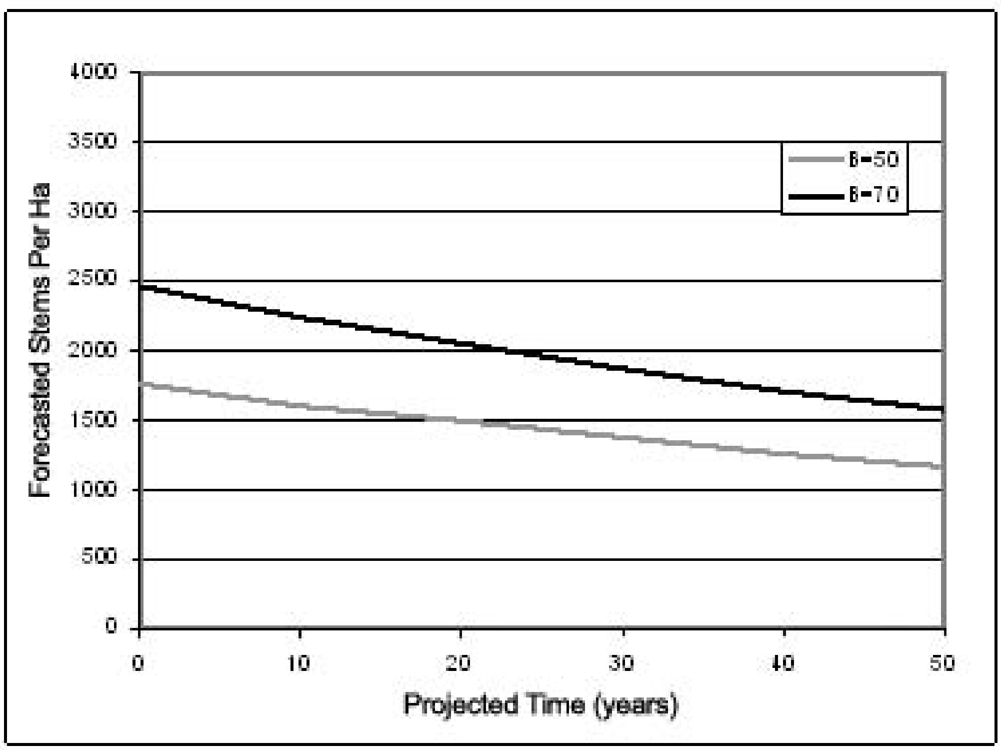

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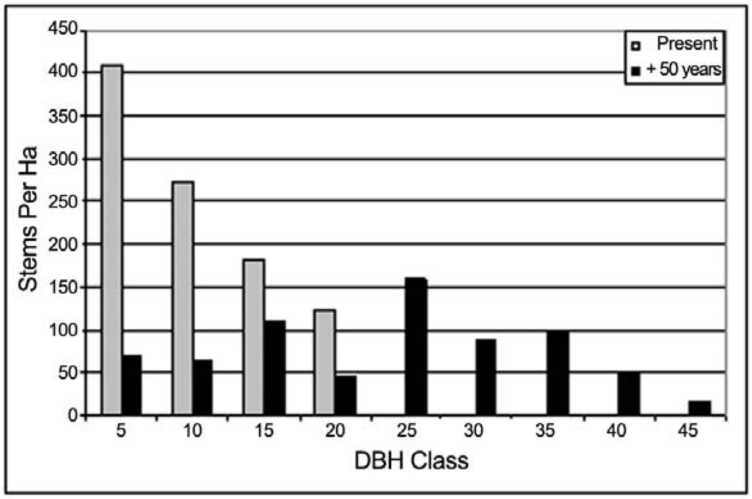

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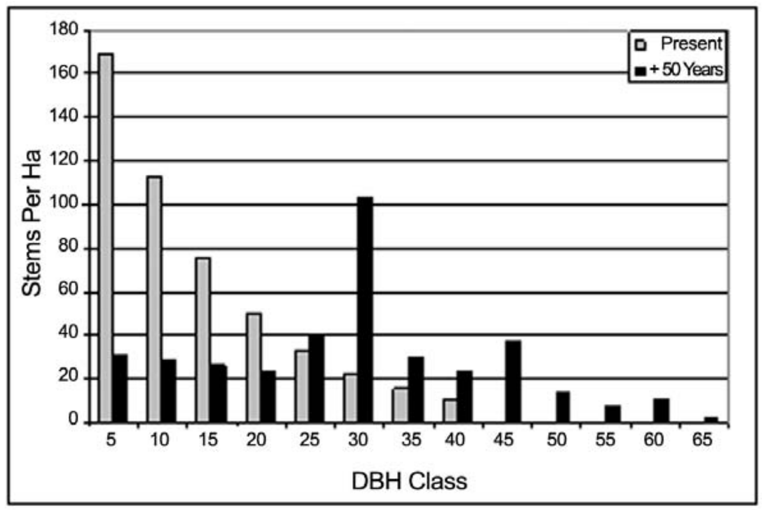

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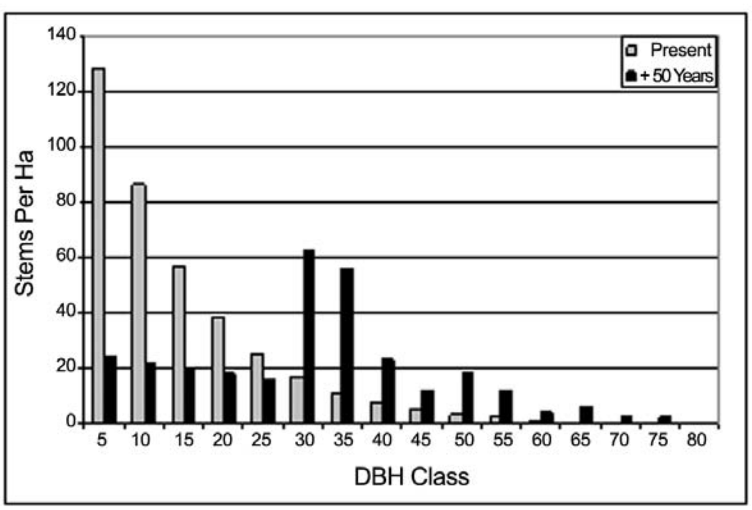
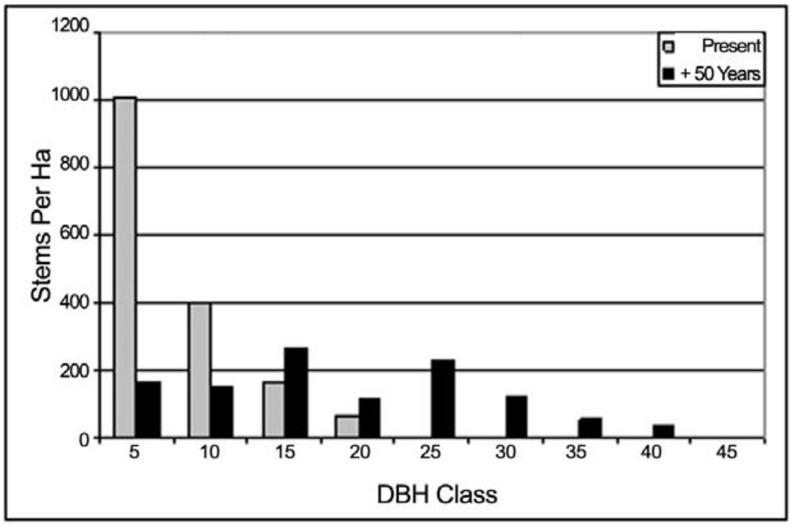


(c)

Figure 9—Dbh distributions at the onset of the simulation and after 50 years for an initial basal area (B) of 10 m2/ha at: (a) D = 20 cm and q = 1.5; (b) D = 40 cm and q = 1.5; (c) D = 60 cm and q = 1.5; (d) D = 20 cm and q = 2.5; and (e) D = 40 cm and q = 2.5.



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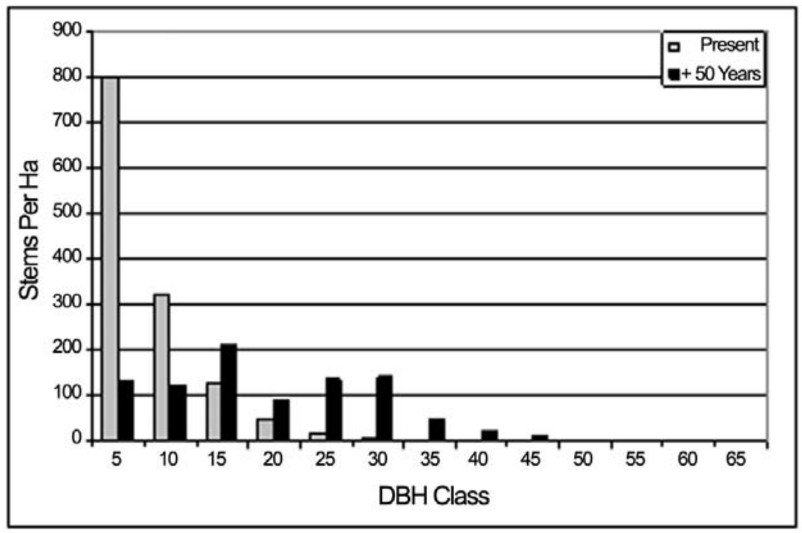

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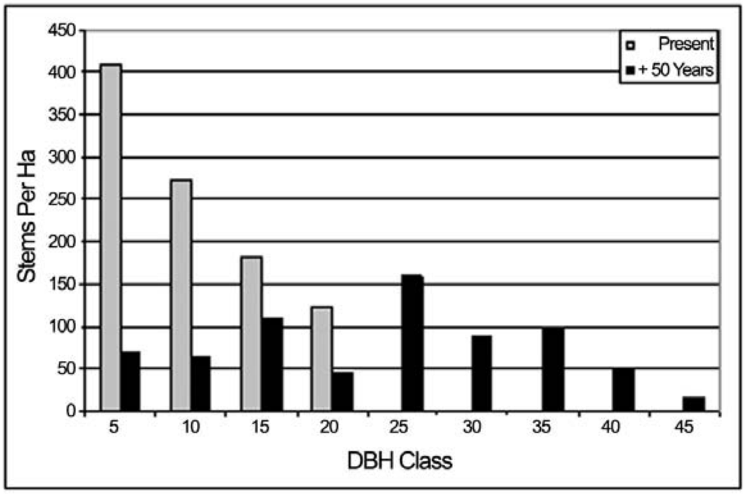
Figure 9—(Continued.)



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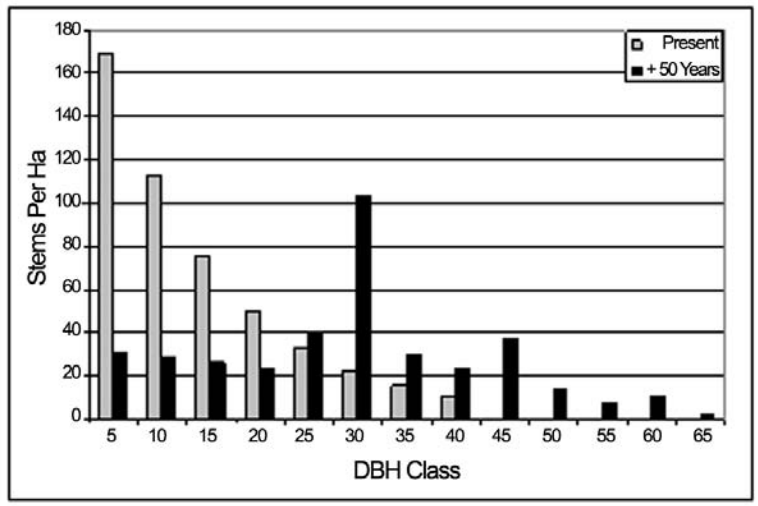

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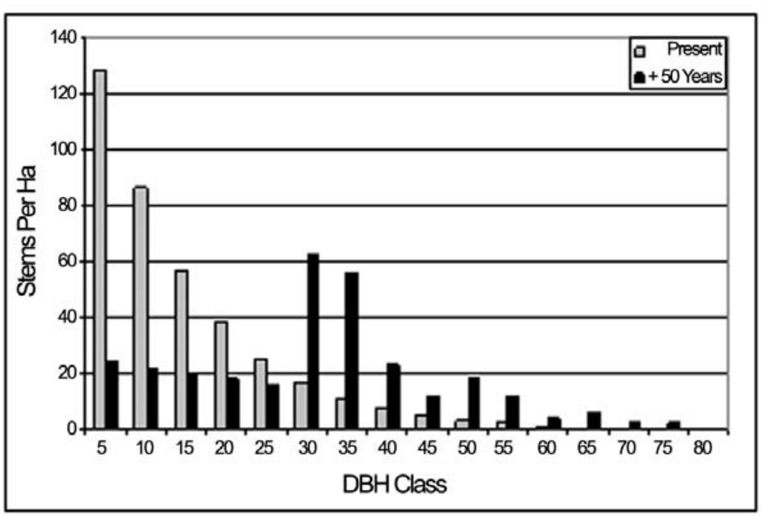
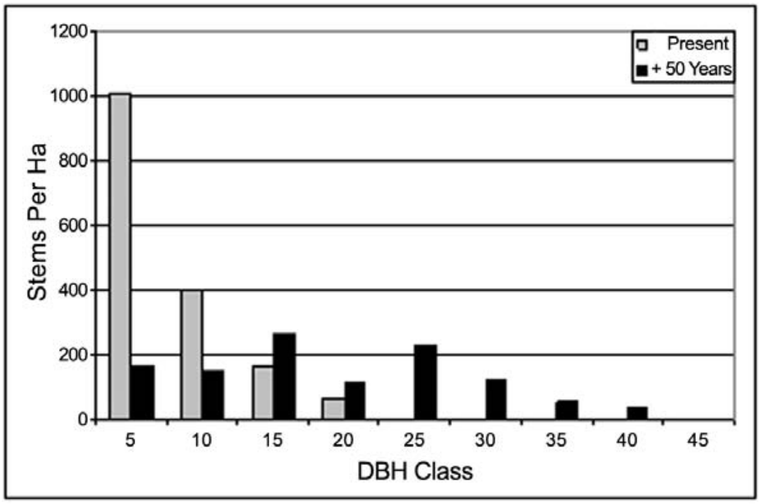
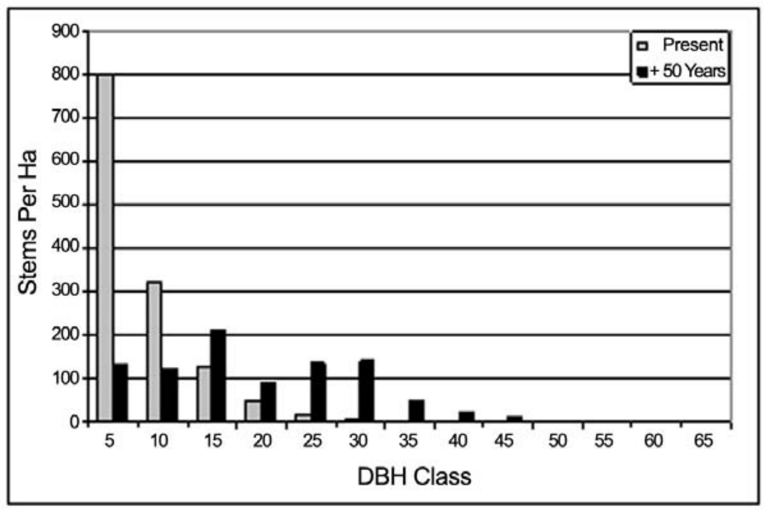


(c)

Figure 10—Dbh distributions at the onset of the simulation and after 50 years for an initial basal area (B) of 50 m2/ha at: (a) D = 20 cm and q = 1.5; (b) D = 40 cm and q = 1.5; (c) D = 60 cm and q = 1.5; (d) D = 20 cm and q = 2.5; and (e) D = 40 cm and q = 2.5.



(d)



(e)

Figure 10—(Continued.)

Table 4—Impact of initial stand structure on various stand attributes after 50 years of projected growth.

Attribute	Increase in stand structure component[a]		
	B[b]	**D**[c]	**q**[d]
Basal area per ha	Movement towards "carrying capacity." If B is ~53 or below, approaches from below; if B is >53 approaches from above.	Slightly slower increase (for lower values of B) or slightly slower decrease (for higher values of B).	Very slight increase (forr lower values of B) or very slight decrease (for higher values of B).
Quadratic mean dbh	Decrease in the rate of increase.	Higher initial value and higher final value after 50 years.	Lower initial and lower final value after 50 years.
Stems per ha	Higher initial value and final value after 50 years. Increase in the mortality rate.	Lower initial value and final value after 50 years. Decrease in the mortality rate.	Higher initial value and final value after 50 years. Increase in the mortality rate.
Dbh growth	Decrease in overall growth rate.	Slight decrease in growth rate.	Slight decrease in growth rate.

[a] The other stand structure components are assumed to remain constant.
[b] Initial basal area per ha (m^2).
[c] Largest dbh class present in the initial stand (cm).
[d] Diminution quotient (tree-frequency ratio between successive dbh classes).

of initial conditions. Both approaches provided information to potential model users on how well the model may perform under certain conditions.

It is important in conducting model evaluations and in interpreting the results of model evaluations to be clear on what is being tested and presented. For example, one could be led to entirely different conclusions about the ability of Prognosis^{BC} to make projections following thinning from viewing only figure 3 or only figure 4. The former shows the accuracy of predicting future dbh values over an 11-year period while the later shows the accuracy of predicting dbh growth over the 11-year period. From a modeller's perspective, it is dbh growth which is of primary interest since that is what is actually being predicted within Prognosis^{BC}. From the perspective of a practitioner interested in predicting future stand structures, it could well be the accuracy of future dbh values that is of interest.

It would have been useful if we could have provided a comparison of the data used in our evaluation with the data used in calibration of the model. There were obvious differences geographically, with the data used in the evaluation coming from a single local area and the calibration data coming from a much wider geographical region. Given this difference, the fact that the model projected tree and stand growth which was close to that observed in the unthinned plots was reassuring. More substantive differences between model projections and observed growth were apparent in the thinned plots. Given the scarcity of thinning in this region historically, it is likely that there was little to no data from thinned stands used in calibrating the model.

Conclusions

In the process of evaluating Prognosis^{BC} (Version 3.0) we discovered a few minor errors in the coding that affected individual tree projections. Once those errors were fixed, the model generally performed well against the test data at both the single tree and the stand level. Although predicted dbh values for individual trees appeared to match closely with actual values in an 11-year period following precommercial thinning, growth in dbh following thinning was clearly overestimated. The simulations performed produced results under a wide variety of stand structures which were consistent with our understanding of stand dynamics. Overall, we felt that Prognosis^{BC} proved to be acceptably robust for the range of conditions examined.

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