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Relationships of Inside and Outside Bark Diameters for Young-Growth Mixed-Conifer Species in the Sierra Nevada

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The linear relationship of inside to outside bark diameter at breast height provides a basis for estimating diameter inside bark from diameter outside bark. Estimates of diameter inside bark and past diameter outside bark are useful in predicting growth and yield. During field seasons 1979-1982, data were obtained from stem analysis of 931 trees in young-growth stands of the mixed-conifer type on the westside Sierra Nevada of California. Species included were coast Douglas-fir, California white fir, incense-cedar, sugar pine, ponderosa pine, and Jeffrey pine. This note provides equations for estimating inside bark diameters, double bark thickness, and past outside bark diameters for each of the species studied.

Retrieval Terms: Pseudotsuga menziesii, Pinus ponderosa, Abies concolor, Pinus lambertiana, Libocedrus decurrens, Pinus jeffreyi, diameter estimation, bark thickness, past diameters, Sierra Nevada, California

As emphasis increases on productivity, forest managers need reliable estimates of inside bark diameters for projecting growth, determining peeled wood volume of trees, and calculating bark volume of individual stems. Because inside bark diameter is linearly related to outside bark diameter, reliable estimates can be derived by linear regression analysis.

Equations relating past outside bark diameter to current inside and outside bark diameters at breast height—applicable only to the Eldorado National Forest in the central Sierra Nevada—were reported earlier.[1] Equations relating bark thickness to past tree diameter for young-growth ponderosa pine in northern California have also been published.[2] But equations specific to young-growth tree species, applicable throughout the range of the mixed-conifer type of the Sierra Nevada in California, have not been reported.

This note describes a method of estimating diameter inside bark (d.i.b.), double bark thickness (Bk), and past diameter outside bark (d.o.b.) from measurements of current d.o.b. at breast height. The equations provided apply only to young-growth (≤80 years), mixed-conifer species of the westside Sierra Nevada and should be used with caution outside this range and geographic area.

METHODS

Young-growth stands throughout the Sierra Nevada mixed-conifer type[3] on the western slope of the Sierra Nevada were randomly sampled as part of a larger growth and yield study serving timber management needs in California. A previous report of this study[4] details the sample design and criteria for selecting sample locations. Sample sites were randomly selected from candidate young-growth, mixed-conifer stands within the study area. Young-growth stands were defined as those in which more than 75 percent of the trees were 80 years old or younger at breast height. A total of 147 sites were examined during field seasons 1979-1982. At each site, from 4 to 10 trees were felled and sectioned for stem analysis. Species were sampled in proportion to their abundance at the site and individual trees of each species were selected randomly.

The six mixed-conifer species sampled included coast Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco var. *menziesii*), California white fir (*Abies concolor* var. *lowiana* [Gord.] Lemm.), incense-cedar (*Libocedrus decurrens* Torr.), sugar pine (*Pinus lambertiana* Dougl.), ponderosa pine (*P. ponderosa* Dougl. ex Laws. var. *ponderosa*), and Jeffrey pine (*P. jeffreyi* Grev. & Balf.). The total sample of 931 trees covered the range of diameters typical of young-growth stands (*table 1*).

Breast height (4.5 ft [1.4 m] above ground on the uphill side of the tree) was marked on each tree before felling. After felling, a cross section was cut at the breast-height mark. Diameter outside bark was measured to the nearest 0.1 inch with a diameter tape. Average bark thickness was determined by averaging eight uniformly spaced measurements of bark thickness, each taken to the nearest 0.05 inch, by the "wood-to-tape" method.[5] Diameter inside bark was then calculated by subtracting twice the average bark thickness from the d.o.b. measurement.

DATA ANALYSIS

Estimating Inside Bark Diameters

Diameter inside bark was plotted over d.o.b., and results indicated a linear relationship for each species. Prediction equations for d.i.b. as a function of d.o.b. were developed by linear regression analysis:

$$\text{d.i.b.} = B_0 + B_1(\text{d.o.b.}) + \epsilon \qquad (1)$$

in which B_0 and B_1 are regression coefficients and ϵ is the error. Residual plots showed the error variance [Var (ϵ)] was not constant but increased with larger tree diameters, such that $\text{Var}(\epsilon) = \sigma^2 \sqrt{\text{d.o.b.}}$ Parameter estimates, therefore, were obtained by weighted least squares analysis (weight $= 1/\sqrt{\text{d.o.b.}}$).

Because of the small sample size (32) of Jeffrey pine, separate equations were not developed for this species. The Jeffrey pine data were combined with that for ponderosa pine for analysis because these species appear most similar in floristic characteristics, growth, and form.

Estimating Bark Thickness

Double bark thickness (Bk) can be estimated directly by transforming the parameters in equation 1 because Bk is the difference between outside and inside bark diameters:

$$\text{Bk} = -b_0 + (1 - b_1)\ \text{d.o.b.} \qquad (2)$$

in which b_0 and b_1 are the estimates of B_0 and B_1.

Estimating Past Outside Bark Diameters

Estimates of breast height d.o.b. for trees of some previous date are used to determine periodic changes in basal areas and tree volumes.[6] The past diameter for a given tree may be found by subtracting total diameter growth from known present diameter. Total diameter growth for the period includes the growth of both wood and bark. Bark growth forms no discernible annual rings and is difficult to measure, however. Total diameter growth, therefore, must be computed indirectly:

d_c = current d.i.b.
d_p = d.i.b. at some specified time in the past
D_c = current d.o.b.
D_p = d.o.b. at some specified time in the past

Then the relationship of equation 1 may be expressed as

$$\hat{d}_c = b_0 + b_1 \cdot D_c \qquad (3)$$

If we assume the relationship of d.i.b. to d.o.b. remains constant for short periods of time, such as a 10-year growth period, equation 1 may also be expressed as

$$\hat{d}_p = b_0 + b_1 \cdot D_p \qquad (4)$$

This assumption seems reasonable not only because of the strong linear relationship between d.i.b. and d.o.b., but also because the thickness of the active phloem (inner bark) may be expected to vary as does the width of the annual increments of the wood in relation to the age of the tree and its growing conditions.[7]

The difference between inside bark diameters from one point in time to another is equal to twice the average radial wood growth (2R) during the period. Radial wood growth can be easily measured from increment cores. Two or more cores per tree should be measured and an average radial growth calculated.[8] Because $2R = \hat{d}_c - \hat{d}_p$, the combined equation of 3 and 4 may be expressed as

$$2R = (b_0 + b_1 \cdot D_c) - (b_0 + b_1 \cdot D_p)$$

and, by rearranging terms, the equation for calculating past d.o.b. becomes

$$D_p = D_c - \frac{2R}{b_1} \qquad (5)$$

Equation 5 expresses past d.o.b. in terms of present d.o.b. and radial wood growth, and accounts for the growth of both wood and bark.

Table 1—*Number, diameter range, and standard error of estimate[1] (SE [$\hat{Y}$]) for the minimum, mean, and maximum diameters of each species*

Species	Trees	Minimum d.o.b.	SE($\hat{Y}$)	Mean d.o.b.	SE($\hat{Y}$)	Maximum d.o.b.	SE($\hat{Y}$)
		Inches		*Inches*		*Inches*	
Douglas-fir	46	4.2	0.068	13.0	0.037	28.5	0.129
Ponderosa and Jeffrey pine	192	4.0	.043	14.7	.024	31.7	.076
Sugar pine	69	3.5	.064	14.2	.035	31.7	.115
White fir	412	2.9	.025	14.0	.012	33.7	.047
Incense-cedar	212	2.9	.034	10.5	.020	34.8	.110
Pooled species: Douglas-fir, sugar pine, white fir	527	2.9	.022	13.9	.011	33.7	.043

[1]Standard error of estimate for the prediction equation $\text{d.i.b.} = b_0 + b_1 \cdot \text{d.o.b.}$

RESULTS AND DISCUSSION

Equations for estimating d.i.b., Bk, and past d.o.b. are given for each of the mixed-conifer species, in which

d.i.b. = diameter inside bark at breast height in inches
d.o.b. = diameter outside bark at breast height in inches
Bk = double bark thickness at breast height in inches
R = average radial wood growth in inches for a specified number of years
r^2 = coefficient of determination

Douglas-fir
d.i.b. = 0.8839 (d.o.b.) − 0.1045
(r^2 = 0.997)
Bk = 0.1045 + 0.1161 (d.o.b.)
Past d.o.b. = present d.o.b. − 2.2627 (R)

Ponderosa and Jeffrey pine
d.i.b. = 0.8967 (d.o.b.) − 0.4448
(r^2 = 0.996)
Bk = 0.4448 + 0.1033 (d.o.b.)
Past d.o.b. = present d.o.b. − 2.2304 (R)

Sugar pine
d.i.b. = 0.8863 (d.o.b.) − 0.1429
(r^2 = 0.997)
Bk = 0.1429 + 0.1137 (d.o.b.)
Past d.o.b. = present d.o.b. − 2.2566 (R)

White fir
d.i.b. = 0.8911 (d.o.b.) − 0.1593
(r^2 = 0.998)
Bk = 0.1593 + 0.1089 (d.o.b.)
Past d.o.b. = present d.o.b. − 2.2444 (R)

Incense-cedar
d.i.b. = 0.8374 (d.o.b.) − 0.0549
(r^2 = 0.995)
Bk = 0.1626 (d.o.b.) − 0.0549
Past d.o.b. = present d.o.b. − 2.3883 (R)

These equations are valid for the size range of diameters sampled for each species (*table 1*). The d.i.b.-d.o.b. relationship should logically show zero d.i.b. for zero d.o.b. When the intercept term in the regression equation is some value other than zero, it suggests that curvilinear regression is required beyond the lower range of the data.

Covariance analysis was used to test the regression equations for species differences. The Bonferroni multiple comparison technique[9] was used to control the overall α level at 0.05. The regression lines for Douglas-fir, white fir, and sugar pine did not differ significantly. For predicting d.i.b. from d.o.b. measurements, a single equation can be used for these three species if the user finds the resulting precision acceptable. The equation

d.i.b. = 0.8899 (d.o.b.) − 0.1522

(r^2 = 0.997) was developed from the pooled data.

Confidence of D.i.b. Prediction Equations

The coefficient of determination (r^2) is shown for each d.i.b. prediction equation. But because weighted regression was used to estimate the regression coefficients, the standard error $[SE(\hat{Y})]$ of a predicted value of d.i.b. (mean value of d.i.b. for a specified d.o.b.) varies with each value of d.o.b. Standard errors are given for the minimum, mean, and maximum d.o.b. for each species to show the range of these error estimates (*table 1*). To calculate the $SE(\hat{Y})$ for any d.o.b. within the diameter range shown for a given species, the formula derived from the covariance matrix[10] and the associated values shown (*table 2*) can be used:

$$SE(\hat{Y}_0) = SD \cdot \sqrt{\frac{A - 2BX_0 + CX_0^2}{D}}$$

in which

$SE(\hat{Y}_0)$ = standard error of the estimate of the mean value of d.i.b. for a given value of d.o.b.
X_0 = observed value of d.o.b.
SD = sample standard deviation from regression, calculated from weighted least squares (estimate of σ).
A, B, C, and D are sums of weighted squares and products (*table 2*).

Once SE $(\hat{Y}_0)$ has been calculated for a given value of d.o.b., confidence

Table 2—*Standard deviation and weighted sums of squares and products of sample data for each species*

Species	SD[1]	Sums of weighted squares and products[2] A	B	C	D
Douglas-fir	0.13399	2263.754	162.3971	13.5878	4386.699
Ponderosa and Jeffrey pine	.16827	11433.600	719.2388	53.9051	99025.500
Sugar pine	.15044	3942.945	254.5168	19.7730	13185.160
White fir	.12510	22782.800	1513.3310	117.0181	375831.200
Incense-cedar	.16514	7824.277	669.4597	70.8707	106335.400
Pooled species: Douglas-fir, sugar pine, white fir	.12939	28989.470	1930.2440	150.3790	633565.900

[1]Sample standard deviation from regression, calculated from weighted least squares (estimate of σ).

[2]Weighted sums of squares and products of sample data, where X_i = the observed value of d.o.b. for the i^{th} tree in the sample:

$$A = \sum \frac{x_i^2}{\sqrt{x_i}} \qquad B = \sum \frac{x_i}{\sqrt{x_i}} \qquad C = \sum \frac{1}{\sqrt{x_i}} \qquad D = \left(\sum \frac{1}{\sqrt{x_i}}\right)\left(\sum \frac{x_i^2}{\sqrt{x_i}}\right) - \left(\sum \frac{x_i}{\sqrt{x_i}}\right)^2$$

and prediction intervals for $(\hat{Y}_0)$ can be calculated from the following formulas:

Confidence interval at X_0 for the *mean value* of d.i.b.

$$\hat{Y}_0 \pm t_{\alpha,n-2}[SE(\hat{Y}_0)]$$

Prediction interval at X_0 for an *individual value* of d.i.b.

$$\hat{Y}_0 \pm t_{\alpha,n-2} \cdot \sqrt{[SE(\hat{Y}_0)]^2 + SD^2\sqrt{X_0}}$$

in which

α = chosen probability level

$t_{\alpha,n-2}$ = percentage point of the t distribution at the chosen level with $n-2$ degrees of freedom; n = number of sample trees for that species (*table 1*).

If multiple intervals are desired, the Bonferroni method[9] can be used to obtain them.

Confidence Limits of Past Diameter Prediction Equations

The variance of the past diameter prediction $(\hat{Y}_p)$ is given by

$$Var(\hat{Y}_p) = (2R)^2 \cdot Var\,\frac{1}{b_1}$$

From the delta method,[11] an approximate value for Var $(\frac{1}{b_1})$ may be expressed as

$$Var\,\frac{1}{b_1} \simeq Var(b_1)/{b_1}^4$$

and the variance of b_1 can be estimated directly from the sample data (*table 2*), as follows:

$$Var(b_1) = SD^2 \cdot \frac{C}{D}$$

Therefore, the standard error of $(\hat{Y}_p)$ for a given d.o.b. and known amount of radial growth is:

$$(\hat{Y}_p) \simeq (2R) \cdot \frac{SD}{{b_1}^2} \cdot \sqrt{C/D}$$

To construct confidence intervals for the mean value of $(\hat{Y}_p)$ for a given current d.o.b. and radial growth, we assume that $1/b_1$ has an asymptotically normal distribution, that is, it approaches normal as the number of sample observations increases. Therefore, the confidence interval for the expected value, E $(\hat{Y}_p)$, may be approximated by

$$\hat{Y}_p \pm Z\left(\frac{\alpha}{2}\right) \cdot SE(\hat{Y}_p)$$

in which

$Z\left(\frac{\alpha}{2}\right)$ = the critical point from the standard normal tables at the desired α level.

The equations provided in this note apply to young-growth trees within the mixed-conifer type of the westside Sierra Nevada, California. These equations should be used with caution outside this age and geographic range. The coefficient estimates for a given species, although useful in one geographic area, may not apply in another because of genetic variation, differences in stand ages, and differences in site conditions.

NOTES

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