Accuracy and equivalence testing of crown ratio models and assessment of their impact on diameter growth and basal area increment predictions of two variants of the Forest Vegetation Simulator

Laura P. Leites, Andrew P. Robinson, and Nicholas L. Crookston

Abstract: Diameter growth (DG) equations in many existing forest growth and yield models use tree crown ratio (CR) as a predictor variable. Where CR is not measured, it is estimated from other measured variables. We evaluated CR estimation accuracy for the models in two Forest Vegetation Simulator variants: the exponential and the logistic CR models used in the North Idaho (NI) variant, and the Weibull model used in the South Central Oregon and Northeast California (SO) variant. We also assessed the effects of using measured (CR_m) versus predicted (CR_p) crown ratio for predicting 10 year DG and 30 year basal area increment (BAI). Evaluation criteria included equivalence tests, bias, root mean square error, and Spearman's coefficient of rank correlation. Inventory data from the Winema and the Colville National Forests were used. Results showed that the NI variant models overpredicted CR when CR_m was below 40% and underpredicted CR when it was above 60%, whereas the SO variant model overpredicted CR when CR_m was smaller than 60%. Differences between CR_m and CR_p were positively correlated with differences in DG predictions. Using CR_m versus CR_p resulted in 30 year BAI absolute percent differences of 10% or less for more than 50% of the plots.

Résumé : Les équations d'accroissement en diamètre (I_d) qui font partie de nombreux modèles de croissance et de rendement forestiers utilisent le rapport de cime (RC) comme variable indépendante. Lorsque le RC n'est pas mesuré, il est estimé à partir d'autres variables mesurées. Nous avons évalué la précision de l'estimation du RC pour les modèles utilisés dans deux variantes du Simulateur de la végétation forestière : les modèles exponentiel et logistique utilisés dans la variante du nord de l'Idaho (NI) ainsi que le modèle de Weibull utilisé dans la variante du centre sud de l'Oregon et du nord-est de la Californie (SO). Nous avons également évalué les effets de l'utilisation du RC mesuré (RC_m) versus le RC estimé (RC_e) pour prédire I_d sur 10 ans et l'accroissement de la surface terrière (I_g) sur 30 ans. Les critères d'évaluation comprenaient les tests d'équivalence, le biais, l'erreur quadratique moyenne et le coefficient de corrélation de rang de Spearman. Les données d'inventaire des forêts nationales de Winema et de Colville ont été utilisées. Les résultats montrent que les modèles de la variante NI surestiment la croissance lorsque les valeurs du RC_m sont inférieures à 40% et la sous-estiment lorsque les valeurs du RC_m sont supérieures à 60 %, alors que le modèle de la variante SO la surestime lorsque les valeurs du RC_m sont inférieures à 60 %. Les différences entre le RC_m et le RC_e sont corrélées positivement avec les erreurs de prédiction de I_d. L'utilisation du RC_m plutôt que du RC_e pour prédire I_g sur 30 ans produit une différence de 10 % ou moins dans plus de la moitié des placettes.

[Traduit par la Rédaction]

Introduction

Forest growth and yield models project tree growth over time assuming a particular management regime and are commonly used as decision-support tools in forestry management. When such models are accurate in their predictions, they provide a useful evaluation of management alternatives (Wykoff et al. 1982; Korzukhin et al. 1996; Canavan and Ramm 2000).

Empirical equations of diameter growth (DG) are an important component of many individual-tree-based growth and yield models (e.g., FVS, Wykoff et al. 1982; ORGANON, Hann 2006; etc.). These equations project diameter growth based on the initial conditions of a subject tree, including its diameter at breast height (DBH), species, height, and crown ratio, as well as stand attributes. Crown ratio (CR), defined as the percentage of crown length from the base of the live crown to the tree tip to total tree height, is considered to be an indirect measure of the photosynthetic capacity of the tree. Additionally, CR often serves as an indicator of a tree's competitive status in a stand. As such, DG is positively correlated to CR, other factors being equal (Wykoff et al. 1982; Hann and Larsen 1991; Hasenauer and Monserud 1996; Lessard et al. 2001).

Received 1 June 2008. Accepted 11 December 2008. Published on the NRC Research Press Web site at cjfr.nrc.ca on 10 March 2009.

L.P. Leites.[1] Department of Forest Resources, University of Idaho, Moscow, ID 83844-1133, USA.
A.P. Robinson. Department of Mathematics and Statistics, University of Melbourne, VIC 3010, Australia.
N.L. Crookston. Rocky Mountain Research Station, USDA Forest Service, 1221 South Main, Moscow, ID 83843, USA.

[1]Corresponding author (e-mail: lleites@uidaho.edu).

Despite its importance as a predictor in DG equations, CR is often assessed imprecisely. Measuring CR is subject to error, even though stand attributes are commonly assumed to be measured without error (Omule 1980; Gertner 1990). Measurement errors arising from field crew mistakes and (or) faulty instruments can be substantial (Omule 1980). For example, even though CR can be measured with standard height measurement instruments, when the crown is uneven, the field crews subjectively and visually rearrange the crown branches to obtain the CR value. There is ample evidence in the literature about the ambiguity of visual estimations of tree characteristics. Studies such as Nicholas et al. (1991) and Ghosh et al. (1995) highlight the degree of variation that can arise from subjective measurements.

Alternatively, CR can be estimated from tree-level models. Several CR prediction models are found in the literature, each with different advantages and disadvantages in their mathematical form and with different degrees of accuracy. Exponential functions, for example, are simple and may present meaningful parameters (e.g., Holdaway 1986), but they can also predict CR values greater than 100% in the extremes of the data (Zumrawi and Hann 1989). Several studies have used this mathematical form to predict CR (see, e.g., Holdaway 1986; Soares and Tome 2001; Sharma et al. 2002). The use of logistic equations is advantageous because predictions are easily constrained to be between 0 and 1, i.e., 0% to 100% CR (Ritchie and Hann 1987; Zumrawi and Hann 1989; Hasenauer and Monserud 1996; Temesgen et al. 2005). The Weibull probability density function has also been used; for example, Dixon (1985) used the Weibull to model the stand CR distribution, and from it, the tree-level CR was assigned to the tree in relation to its rank in the DBH distribution. Regardless of the model chosen, the use of predicted CR in the DG equations implies the use of a random variable instead of a known predictor variable (Hann and Zumrawi 1991). As a consequence, the accuracy with which CR is predicted will affect the accuracy of the DG predictions.

The Forest Vegetation Simulator (FVS) is a tree-level nonspatial modeling framework based in the United States, with more than 20 different variants. This study is concerned with two of these variants: the North Idaho (NI) and the South Central Oregon and Northeast California (SO). The four CR models used in these variants each predict tree-level CR at a given point in time using species-specific allometric relationships. These models provide estimates when CR is not measured for all or some of the trees; that is, the CR is imputed when it is not measured. CR values (measurements or estimates) are then used to predict DG. The NI and SO variants use different strategies to predict DG depending on the size of the trees; consequently CR affects the DG predictions differently in each variant.

The FVS user's guide strongly recommends measuring CR for all trees, because the use of predicted CR (CR_p) values is expected to yield less-accurate predictions than those obtained by using measured CR (CR_m) values (Wykoff et al. 1982). The initial CR values (CR_m or CR_p) are used only at the beginning of the first prediction cycle. In subsequent cycles, the CR change for every tree is predicted by the models and CR is updated. As a result, the influence of the initial value of CR declines as the number of projections increase.

Considering that measuring CR will present a degree of measurement error, and an additional cost, and that predicting CR may incorporate error to DG predictions, it is useful to assess the accuracy of the CR predictions and the effect of using CR_m versus CR_p on DG and basal area increment (BAI) predictions.

In this study we evaluated three CR models used in the FVS NI and SO variants. These models are (1) Hatch's (1980) exponential equation, (2) the NI logistic equation, and (3) Dixon's (1985) Weibull distribution based model. These models are applied to different tree-size groups (Table 1). Our objectives were (1) to evaluate the models in terms of prediction accuracy and bias by quantifying the differences and assessing equivalence between CR_m and CR_p at the beginning of the prediction cycle, (2) to evaluate the effect of using CR_m versus CR_p at the beginning of a 10 year prediction cycle on the DG predictions, and (3) to evaluate the effect of using CR_m versus CR_p at the beginning of a 30 year prediction cycle on predictions of BAI.

Methods

Data

The data were acquired from the USDA Forest Service Pacific Northwest Region's Current Vegetation Survey project. This survey started in 1993 with the objective of providing timber volume estimates for management planning for the western regions to be used in the 1997 Resource Planning Act Assessment (US Forest Service 2005*a*). These data were later combined with those from other inventories into the Pacific Northwest Forest and Inventory Analysis Integrated Database (US Forest Service 2006). The data for this analysis were retrieved from the latter database but include only the data obtained during the Pacific Northwest Region's Current Vegetation Survey initial installation survey (US Forest Service 2005*a*).

We identified two forests that are within the geographic ranges for which the FVS SO and NI variants were developed (Dixon 2005; US Forest Service 2005*b*). The Winema National Forest (WNF) is within the geographical range of application of the SO variant, and the Colville National Forest (CNF) is within the geographical range of application of the NI variant. The WNF is located in south-central Oregon (42°56′15″N, 121°48′45″W) and comprises approximately 420 000 ha on the eastern slopes of the Cascade Mountain Range. Its elevation ranges from 1219 to 2500 m above sea level. The climate is characterized by cold winters and warm, dry summers. Vegetation ranges from mixed conifer forest (Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco), lodgepole pine (*Pinus contorta* Dougl. ex Loud.), mountain hemlock (*Tsuga mertensiana* (Bong.) Carr.), ponderosa pine (*Pinus ponderosa* Dougl. ex P. & C. Laws.), sugar pine (*Pinus lambertiana* Dougl.), western white pine (*Pinus monticola* Dougl. ex D. Don)) on the west side, to ponderosa pine, lodgepole pine, and western juniper (*Juniperus occidentalis* Hook.) forest on the northern and eastern sides, with marshlands and meadows occurring through the entire forest (US Forest Service 2005*c*).

The CNF is located in northeastern Washington

Table 1. Summary of diameter growth (DG) prediction strategies and crown ratio (CR) models used in the Forest Vegetation Simulator (FVS) NI and SO variants.

DBH threshold for CR model	CR model	DBH threshold for DG prediction	DG prediction
NI variant			
<7.6 cm (3 in.)	Logistic	<7.6 cm (3 in.)	DG predicted from height growth*; predictions not affected by CR, therefore not evaluated
>7.6 cm (3 in.)	Exponential	>7.6 cm (3 in.)	DG predicted by two nonlinear equations; dds uses CR as a predictor variable†
SO variant			
>2.5 cm (1 in.)	Weibull	<7.6 cm (3 in.)	DG predicted from height growth; CR affects at two levels‡
		>7.6 cm (3 in.)	DG predicted by two nonlinear equations; dds uses CR as a predictor variable‡

Note: The SO variant uses a logistic model for trees with DBH <2.5 cm; this model was not evaluated because we did not have measured CR in trees smaller than 2.5 cm DBH. Different tree-size thresholds, based on DBH, are used for the application of CR models and DG models in the SO variant.

*Dixon (2005).

†Wykoff et al. (1982).

‡US Forest Service (2005*b*).

(48°53′00″N, 118°46′00″W) and comprises approximately 445 170 ha. Its elevation ranges from 650 to 2200 m above sea level. Vegetation on the CNF ranges from ponderosa pine and Douglas-fir on the drier western portion, to the western redcedar (*Thuja plicata* Donn ex D. Don) and hemlock (*Tsuga* spp.) forests on the moist eastern side (US Forest Service 2005*d*). Summary statistics for each data set are presented in Tables 2 and 3.

The initial installation survey data were collected between 1993 and 1996 using systematic sampling (see US Forest Service (2005*a*) for a detailed description of the sampling methodology). At the WNF a total of 529 plots were installed. Of these plots, 527 were classified as forest areas and included in this analysis. At the CNF a total of 771 plots were installed. In this analysis, 580 CNF plots were included. Each plot on these inventories comprised five 0.076 ha subplots; those subplots classified as forested areas were chosen as our simulation units (SU). The CNF comprised 2700 SUs and the WNF comprised 2475 SUs.

Measurements in each SU included species identification, DBH (measured to the whole 0.1 in. (1 in. = 2.54 cm)), total tree height (Ht, measured to the nearest foot (1 ft = 30.48 cm)), 10 year radial growth on trees with DBH >3.0 in. (measured in cores to the nearest 0.05 in.), CR (10% classes), and crown width (measured to the last foot). CR was calculated as the percentage of the measured tree height covered by live foliage (measuring the crown base to the nearest foot). Crew members were instructed to compensate for abnormal voids within the crown or missing branches on one side of the crown by visually redistributing branches when the crowns were uneven (US Forest Service 2005*a*). All these variables were used to describe the initial SU conditions at the beginning of the prediction cycle. The 10 year radial growth measurements are used at the beginning of the FVS runs to calibrate the diameter increment model (Wykoff et al. 1982). Variables used to FVS runs were in English units. Simulation results were converted to metric units.

Analysis

Accuracy and bias of CR predictions

The CR models evaluated are of the following form:

FVS NI variant logistic model:

$$[1] \qquad \mathrm{CR}_i = \frac{1}{1 + \mathrm{e}^{k}i}$$

$$k_i = \mathrm{FCR}_i + b_0 + b_1\mathrm{DBH}_i + b_2\mathrm{Ht}_i + b_3\mathrm{BA}_j$$

where i is tree index, j is plot index, CR is tree crown ratio as a proportion of tree height (0.05 to 0.95), FCR is a random error term, DBH is diameter at breast height (inches), Ht is total height (feet), BA is stand basal area (ft²/acre), and b_0 to b_3 are species-specific regression coefficients.

FVS NI Hatch's (1980) exponential model:

$$[2] \qquad \ln(\mathrm{CR}_i) = \mathrm{HAB}_j + b_1\mathrm{BA}_j + b_2\mathrm{BA}_j^2 + b_3\ln(\mathrm{BA}_j) + b_4\mathrm{CCF}_j + b_5\ln(\mathrm{CCF}_j) + b_6\mathrm{DBH}_i + b_7\mathrm{DBH}_i^2 + b_8\ln(\mathrm{DBH}_i) + b_9\mathrm{Ht}_i$$

$$+ b_{10}\mathrm{Ht}_i^2 + b_{11}\ln\,(\mathrm{Ht}_i) + b_{12}\mathrm{PCT}_j + b_{13}\ln(\mathrm{PCT}_j)$$

where i is tree index, j is plot index, ln(CR) is the logarithm of CR, HAB is the intercept dependent on habitat type, BA is stand basal area (ft²/acre), CCF is stand crown competition factor, DBH is diameter at breast height (inches), Ht is total height (feet), PCT is the percentile in the stand basal area distribution, and b_1 to b_{13} are species-specific regression coefficients.

Table 2. Winema National Forest data description.

		DBH (cm)			Total height (m)			CR (%)		
Species	*N*	Mean	SD	Range	Mean	SD	Range	Mean	SD	Range
PICO	14 050	15.7	11.0	2.5–96.2	10.9	6.1	1.8–38.7	42	21	5–85
PILA	720	27.5	25.4	2.5–135.4	14.0	10.0	1.8–43.9	54	19	5–85
PIMO	247	26.5	21.8	2.5–100.3	15.3	10.0	1.8–45.7	49	21	5–85
PIPO	11 896	29.0	23.3	2.5–140.2	14.3	9.9	1.8–52.7	47	18	5–85
PSME	688	43.7	33.1	2.5–173.9	23.4	11.9	1.8–54.6	44	19	5–85
TSME	1 227	31.2	20.4	2.5–127.3	15.7	8.2	1.8–41.8	49	18	5–85
Other	9 368	29.2	23.8	2.5–151.7	15.7	10.3	1.8–54.6	41	19	5–85

Note: *N*, number of trees; DBH, diameter at breast height; CR, crown ratio in percentage. Species code: PICO, lodgepole pine; PILA, sugar pine; PIMO, white pine; PIPO, ponderosa pine; PSME, Douglas-fir; TSME, mountain hemlock.

Table 3. Colville National Forest data description.

		DBH (cm)			Total height (m)			CR (%)		
Species	*N*	Mean	SD	Range	Mean	SD	Range	Mean	SD	Range
PICO	9 707	16.1	8.3	2.5–96.7	15.3	4.9	1.8–39.6	28	17	5–85
PIMO	300	25.2	18.3	2.5–98.7	17.5	10.6	1.8–48.2	46	23	5–85
PIPO	1 130	40.9	23.0	2.5–129.8	22.5	9.4	1.8–54.3	43	19	5–85
PSME	16 796	24.5	16.8	2.5–120.4	17.2	8.4	1.8–49.7	43	21	5–85
THPL	8 487	16.7	16.3	2.5–157.1	11.8	8.2	1.8–45.7	45	21	5–85
TSHE	3 112	14.1	11.4	2.5–96.0	9.6	6.7	1.8–43.3	44	22	5–85
Other	19 388	21.0	15.2	2.5–124.7	17.1	9.4	1.8–52.4	40	21	5–85

Note: *N*, number of trees; DBH, diameter at breast height; CR, crown ratio in percentage. Species code: PICO, lodgepole pine; PIMO, white pine; PIPO, ponderosa pine; PSME, Douglas-fir; THPL, western redcedar; TSHE, western hemlock.

FVS SO Dixon's (1985) CR model:

This model predicts the stand CR distribution using a Weibull probability density function. After specifying the stand CR distribution, it assigns CR values to individual trees based on their basal area percentile in the stand. A summary follows:

(1) Use the three parameter Weibull probability density function to describe plot CR distribution:

$$[3] \qquad f(x) = \frac{c}{b}\left[\frac{x-a}{b}\right]^{(c-1)} e\left[\frac{(x-a)}{b}\right]^{c}$$

where x = CR.

(2) Define Weibull parameters as follows: a and c are species-specific constants, and b is calculated for a given species as follows: (*i*) Calculate mean stand CR (MCR) from relative stand density index (RSDI, ratio between Reineke's (1933) stand density index and maximum stand density index):

$$\text{MCR} = d_0 + d_1\text{RSDI}$$

where d_0 and d_1 are species-specific parameters. (*ii*) Then calculate $b = j_0 + j_1\text{MCR}$, where j_0 and j_1 are species-specific parameters.

(3) Once the stand CR distribution is specified, then CR values are assigned to individual trees based on the tree rank in the stand basal area distribution.

Predictions of the FVS NI CR models (rounded to the nearest 1%) were compared with actual measures (measured to the nearest 10%) using the CNF data, and the predictions of the FVS SO CR models (rounded to the nearest 1%) were compared with actual measures (measured to the nearest 10%) using the WNF data. Only observations with CR_m were included in the evaluation.

Because of the difference in precision between the CR models' predictions and the CR measurements (1% and 10%, respectively), we evaluated the performance of the equations using CR_p both as rounded to the nearest 1% and as a discrete variable by converting it into the same CR_m 10% classes. The results were almost identical between evaluations. We are presenting the results using CR_p as a continuous variable because that is the way it is later used in the diameter growth equations, and our objective was to evaluate these equations in the context of how they are used in FVS.

Because we wanted to evaluate these equations under the conditions applied by model users, we have not omitted the effect of the additional error in the predictor variables caused by the use of different sampling designs when developing the CR models and when collecting the WNF and CNF data. As reported by Hann and Zumrawi (1991), this additional error, which is a consequence of different sampling designs, can be substantial and increases as the difference between sampling designs increases. Model users rarely follow the sampling designs used to develop the models when collecting their data, even though adopting such a practice will avoid unnecessary error. Unfortunately, sample design information for most of these CR models is not readily available to the user.

We calculated root mean square error (RMSE) and bias by species and CR_m classes as follows:

$$[4] \quad \mathrm{RMSE}_j = \sqrt{\frac{\sum_{i=1}^{n_j} (\mathrm{CR}_{mij} - \mathrm{CR}_{pij})}{n_j}}$$

$$[5] \quad \mathrm{BIAS}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} (\mathrm{CR}_{mij} - \mathrm{CR}_{pij})$$

where CR_m is measured CR, CR_p is predicted CR, j is species index, i is tree index, and n_j is the number of trees observed in the jth species.

Additionally we assess bias by constructing equivalence tests (Robinson et al. 2005). Equivalence tests differ from traditional significance tests in that the null hypothesis is specified as the parameter being tested is not equal to a constant. In the case of model validation with equivalence tests, the null hypothesis is $\mu_o \neq \mu_p$, where μ_o is the parameter of interest for the observations, and μ_p is the parameter of interest for the evaluated model predictions. This strategy is advantageous because the error of mistakenly validating the model is a type-1 error with a small, fixed probability (Robinson et al. 2005). Using equivalence tests in this case allows us to test whether CR_m and CR_p values are statistically equivalent for a given established equivalence region (null hypothesis is rejected) or whether there is insufficient evidence to validate the model (null hypothesis accepted).

For this study, we followed the procedure described in Robinson et al. (2005). A linear model was fitted between CR_m and CR_p for each CR model (Table 1); and a two one-sided test strategy (TOST) was used to calculate the confidence intervals (CI) for the slope and intercept parameters. If the CI for the intercept and the CI for the slope of this fitted model are not contained entirely within the defined regions of equivalence, then the null hypothesis is not rejected. Since residuals of the ordinary least squares (OLS) linear model fitted between the two types of CR values were not normally distributed and were heteroscedastic, we constructed the equivalence test by species using a nonparametric bootstrap procedure described in Robinson et al. (2005). The percentage of times the TOST confidence intervals for the slope and intercept estimates were within the region of equivalence determined whether the null hypothesis was rejected (Robinson et al. 2005). Although the data also presented correlation between same-species observations within a plot, we used OLS to fit each linear model in the bootstrap procedure, since the OLS intercept and slope estimates are unbiased estimates even when there is unaccounted correlation in the data. The criterion to define the region of equivalence for the intercept was the CR_p mean ± 10, i.e., if the difference between CR_m and CR_p was larger than one 10% CR class, then CR_m and CR_p were considered dissimilar. The region of equivalence for slope was set to 1 ± 10%. Equivalence tests were set at $\alpha = 0.05$.

Estimated DG using CR_m versus CR_p

To predict DG at the tree level, FVS groups trees according to some species-specific DBH threshold and then uses different approaches for each group. The threshold for NI and SO variant is DBH 7.6 cm (3 in.). In the FVS SO variant, this threshold is different from the one used to determine which CR model is applied (Table 1). In both variants, DG for trees below the 7.6 cm DBH threshold is calculated as follows: first height growth is predicted; then using height–diameter relationships, diameter is updated at the end of the prediction period, and DG is calculated as the difference. The NI variant height growth model for this tree-size group does not include CR as a predictor variable. Consequently, CR does not affect the DG predictions for this group and was not included in the DG evaluation. The DG predictions for trees below the 7.6 cm DBH threshold in the SO variant are affected mainly in the following two ways by the initial CR value. One is through the height growth model, which has three components, one of which uses CR as one of the predictor variables. Additionally, the height–diameter relationships used to update DBH at the end of the prediction period have coefficients that are species and CR class specific (US Forest Service 2005).

DG for trees with DBH above 7.6 cm is predicted through the application of two nonlinear equations, one of which (dds) uses CR as a predictor variable (Wykoff et al. 1982). The dds model used in the NI variant is the same as the base dds model (Wykoff et al. 1982). FVS SO variant dds model uses additional variables but CR is used in the same way (US Forest Service 2005). A summary of the DG prediction strategies and CR models used by both FVS variants is presented in Table 1.

FVS incorporates two stochastic features to model the random variation that characterizes tree growth. Random errors are assigned to the dds predictions in two different ways depending on the number of trees in the sample. If this number of trees is above a threshold, a random error is assigned to each tree prediction. Otherwise each tree observation is augmented by two additional records, each of which keeps the tree's characteristics but differs on the DG predictions and the trees per acre (TPA) each tree represents. The sum of the TPA of these three records equates the TPA represented by the original observation (Stage 1973; Wykoff et al. 1982). In our analysis, the number of trees in every simulation unit was below the mentioned threshold. Consequently, every observation was tripled, and random errors were not added to the tree-level predictions. However, for the evaluation of tree-level DG we used the weighted average of the triples, thereby eliminating this effect.

We ran FVS NI and SO variants using CR_m and again using CR_p, with no changes to any other variable or model input. DG comparisons are between these two runs (DG_m and DG_p, respectively). FVS was run using the default mode, as is commonly done, and we had no information available to do otherwise.

All the observations for the CNF had measurements of the previous 10 year DG, whereas 93% of the observations for the WNF had measurements of the previous 10 year DG. These measurements were used only to calibrate the FVS dds model at the beginning of the prediction cycle (Wykoff et al. 1982). Every observation in both data sets included measured total tree height. Consequently, total tree height at the beginning of the prediction cycle was measured and not estimated for all of the observations.

To assess the importance of the CR on the DG predictions we calculated the correlation between the differences between CR_m and CR_p (CR_d) and the differences between DG_m and DG_p (DG_d). We chose this metric because the ef-

fect of CR on DG predictions for both tree-size groups within each FVS variant is indirect, and error in CR is not directly reflected on the DG predictions. We calculated Spearman's coefficient of rank correlation (ρ) between CR_d and DG_d by FVS variant, tree size, and species. Additionally, we calculated RMSE of the DG prediction differences by CR_m classes and species. RMSE was calculated as in eq. 4 but substituting CR_p and CR_m with DG_p and DG_m, respectively. We assumed DG_m to be the nominal value.

Estimated BAI at the SU level using CR_m versus CR_p

To assess the impacts of CR_m versus CR_p on BAI (BAI_m and BAI_p, respectively) at the SU level, we ran FVS NI and SO variant models for three 10 year cycles to predict 30 year BAI (m^2/ha). This prediction includes not only tree-level DG increment, but also tree-level height increment, stand mortality, and regeneration estimates that often use DG as a predictor variable. As with DG, the only difference between runs was the use of CR_m or CR_p at the beginning of the prediction cycle. In subsequent cycles, CR is updated for every tree using the FVS variant's models. We then calculated the difference and percent difference between the BIA_m and BAI_p estimates, assuming BAI_m to be the nominal value. Additionally, we calculated the proportion of SUs falling within 10% absolute percent difference classes. This provides model users with the possibility of calculating the probability of a certain size difference in BAI occurring when CR_m is used.

All analyses were performed in the statistical environment R (R Development Core Team 2005).

Results

Accuracy and bias of CR predictions

Values of bias and RMSE by CR_m class and species are presented in Tables 4 and 5. Data sets for both national forests reflect the species composition that is covered by each of the FVS variants. The three models are biased when evaluated with the CNF (NI variant) and WNF (SO variant) data. Both the NI variant logistic and Hatch's (1980) exponential models present the same pattern of bias. For every species, when CR_m class was less than 40%, the two models overpredicted CR, and when it was above 60%, they underpredicted CR. When CR_m was between 40% and 60%, the result differed by species but in absolute value the bias was always smaller than for the other two CR_m classes for every species. In terms of accuracy, the CR_m classes below 40% and above 60% presented lower accuracy, with RMSE values doubling the values for the 40%–60% class for many species. Bias and RMSE values for the logistic model, however, were largest for the CR_m class below 40% for most species, whereas for Hatch's (1980) exponential model they were largest for the CR_m class above 60%. Dixon's (1985) model overpredicted CR for CR_m classes below 60% for all species. Both bias and RMSE values decrease as the CR_m class increases (Table 4).

Results from the equivalence tests are presented in Table 6. These tests failed to validate the CR models (the null hypothesis of $\mu_o \neq \mu_p$ was not rejected except in one case). For the three models, the CI for the slope was not contained within the equivalence region (0.9–1.1). Additionally, the CI values for the slope for the three models indicate that mostly these models predicted similar CR values for all the trees evaluated (CI within –0.2 and 0.2 for every model). The CI for the intercept was not contained within the equivalence region (mean $CR_m \pm 10$) for two of the models: the NI logistic model and the SO Weibull model. Looking at the equivalence regions (Table 6), we see that the SO Weibull model had the greatest difference between the equivalence region and the CI for the intercept. The CI for the intercept of the NI exponential model fell within the equivalence region.

Estimated DG using CR_m versus CR_p

As we expected, ρ was positive and high for all classes, i.e., larger CR_d values were associated with larger DG_d values (Table 7). In general, ρ was higher for all species of the NI variant than for the other two tree-size groups in the SO variant. The tree-size group with DBH <7.6 cm of the SO variant presented the lowest, yet positive, ρ values.

RMSE as a percentage of DG_p ranged between 4% and 70% for species of the SO variant and between 13% and 49% for species of the NI variant. Within FVS variants and DBH thresholds, the differences in RMSE as a percentage of DG_p among CR_m classes are larger than differences among species. This result is associated with the RMSE and bias presented by the CR models. The larger the RMSE for a given species and CR_m class in the CR evaluation, the larger the RMSE value for that species and CR_m class in the DG evaluation (Tables 4, 5, 8, and 9).

Estimated BAI at the SU level using CR_m versus CR_p

The mean difference between BAI_m and BAI_p for the FVS NI variant was 0.15 m^2/ha (standard deviation (SD) = 1.41), whereas the mean absolute difference was 0.80 m^2/ha (SD = 1.16). For the FVS SO variant the mean difference was 0.14 m^2/ha (SD = 1.20), and the mean absolute difference was 0.60 m^2/ha (SD = 1.04). The proportion of SUs falling in a given absolute percent difference class are presented in Table 10 for each FVS variant. More than 50% of the evaluated SUs fell within the 0%–10% absolute percent difference class in both cases.

Discussion

Accuracy and bias of CR predictions

Overall the accuracy of the CR models was low if we consider that errors on visual estimation of CR are usually within less than 20%, i.e., two 10% classes (see, e.g., McRoberts et al. 1994). Interestingly, the NI variant logistic model and Hatch's (1980) exponential model present the same type of bias. Half of the variables in theses models are the same, and even though the model forms are different, in the range of the CR values they behave similarly. Both of these models were evaluated using the CNF data, so it could be argued that this forest may be different from the populations used to develop these models. However, the CNF lies within the geographical area for which these CR models were developed and are being used. The results from our evaluation indicate that the predictive ability of these models can be low even when applied to forests within the geographic range of recommended use of these two FVS variants.

Table 4. Bias and root mean square error (RMSE) of crown ratio (CR) predictions (%), by measured CR class (%, CR_m class) and by species for the Forest Vegetation Simulator SO variant evaluated with the Winema National Forest data.

		Bias			RMSE		
		CR_m class			CR_m class		
Species	*N*	<40%	40%–60%	>60%	<40%	40%–60%	>60%
PICO	14 050	–31.4	–9.9	14.0	35.7	20.0	22.8
PILA	720	–36.7	–22.6	0.0	39.0	26.8	15.6
PIMO	247	–42.5	–25.5	–5.4	45.2	30.1	19.1
PIPO	11 896	–35.1	–20.3	–0.1	37.6	24.8	15.5
PSME	688	–30.2	–15.1	7.3	32.4	19.8	17.4
TSME	1 227	–42.7	–28.9	–4.9	45.6	32.4	18.5
Other	9 368	–41.2	–26.6	–4.5	44.8	32.4	20.3

Note: *N*, number of trees. Species code as in Table 2. All trees have DBH larger than 2.5 cm (1 in.); a Weibull-based model is used for this tree size.

Table 5. Bias and root mean square error (RMSE) of crown ratio (CR) predictions (%), by measured CR class (%, CR_m class) and by species for the Forest Vegetation Simulator NI variant CR equations evaluated with the Colville National Forest data.

		Bias			RMSE		
		CR_m class			CR_m class		
Species	*N*	<40%	40%–60%	>60%	<40%	40%–60%	>60%
DBH <7.6 cm; logistic CR equation							
PICO	710	–28.0	4.8	28.3	31.1	11.5	30.9
PIMO	34	–40.4	–8.5	5.5	44.5	13.8	13.7
PIPO	32	–29.0	3.8	22.2	33.7	12.5	25.9
PSME	1 716	–36.4	–8.6	17.9	39.6	16.0	23.3
THPL	2 155	–28.2	–2.9	21.0	31.8	12.4	25.5
TSHE	674	–35.8	–8.4	17.4	39.0	15.0	22.2
Other	2 058	–35.5	–9.6	19.8	38.5	15.6	24.8
DBH >7.6 cm; Hatch's (1980) exponential equation							
PICO	8 997	–9.9	12.1	26.8	15.3	16.4	31.7
PIMO	266	–23.0	–1.6	10.2	27.9	12.8	20.7
PIPO	1 098	–8.5	10.3	32.6	14.1	14.7	35.2
PSME	15 080	–14.3	3.1	21.5	18.4	12.0	26.5
THPL	6 332	–16.9	5.0	29.2	20.1	10.5	31.7
TSHE	2 438	–19.8	2.0	27.0	22.6	8.9	29.4
Other	17 330	–30.7	–8.0	12.7	33.8	14.2	18.3

Note: *N*, number of trees. Species code as in Table 3.

Table 6. Equivalence tests for three crown ratio (CR) models: Forest Vegetation Simulator (FVS) NI variant logistic and exponential models, and FVS SO variant Weibull model.

		β_0			β_1		
CR model	*N*	ER	TOST CI	H_0: not equivalent	ER	TOST CI	H_0: not equivalent
NI logistic	7 379	26.19–46.19	53.66–54.23	Not rejected	0.9–1.1	0.06–0.08	Not rejected
NI exponential	51 541	30.54–50.54	46.69–46.90	Rejected	0.9–1.1	0.17–0.18	Not rejected
SO Weibull	38 196	33.67–53.67	65.18–65.50	Not rejected	0.9–1.1	0.17–0.19	Not rejected

Note: The equivalence region (ER) for the intercept, β_0, was set to the mean measured CR ± 10. The ER for the slope, β_1, was set to 1 ± 10%. A two one-sided test strategy (TOST) was used to calculate the confidence intervals (CI) for the slope and intercept ($\alpha = 0.05$). The null hypothesis (H_0) of equivalence test is equal to the statistic being not equivalent to the defined equivalence region.

In the case of the FVS SO Weibull model, the maximum stand density index (SDI) values, assigned to individual species at the beginning of the runs, have a large effect on the resulting crown distribution, with high maximum SDI values skewing the CR distribution towards higher CR values (eq. 3). The accuracy of the tree-level CR predictions will be affected in turn. According to G. Dixon (USDA Forest Service, personal communication, 2006) many of the FVS SO variant default maximum SDI values are high, and this may explain why we observed predictions above 60% when

Table 7. Spearman's coefficient of rank correlation ($\alpha = 0.001$) between crown ratio (CR) differences (CR_m and CR_p) and diameter growth (DG) prediction differences (DG_m and DG_p) by species.

	NI variant		SO variant			
	DBH >7.6 cm		DBH <7.6 cm		DBH >7.6 cm	
Species	*N*	ρ	*N*	ρ	*N*	ρ
PICO	8 997	0.68	2691	0.43	11 359	0.70
PILA	—	—	92	0.39	628	0.67
PIMO	266	0.84	25	0.62	222	0.81
PIPO	1 098	0.77	1010	0.58	10 886	0.55
PSME	15 080	0.74	26	0.62	662	0.45
THPL	6 332	0.68	—	—	—	—
TSHE	2438	0.78	—	—	—	—
TSME	—	—	74	0.15	1 153	0.71
Other	17 330	0.65	951	0.51	8 417	0.35

Note: *N*, number of trees; ρ, Spearman's coefficient of rank correlation. Species code as in Table 2.

Table 8. Root mean square error (RMSE, cm) of diameter growth (DG) prediction differences with DG_m as the baseline; RMSE is presented in absolute value and as a percentage of the mean DG_p ($\%DG_p$) by CR_m class and by DBH threshold (Table 1) for the Forest Vegetation Simulator SO variant.

		CR_m class					
		<40%		40%–60%		>60%	
Species	*N*	cm	$\%DG_p$	cm	$\%DG_p$	cm	$\%DG_p$
DBH <7.6 cm							
PICO	2 691	1.17	31	0.32	7	0.36	8
PILA	92	1.01	27	0.05	1	0.19	4
PIMO	25	1.42	30	0.03	1	0.37	7
PIPO	1 010	1.41	31	0.11	2	0.27	5
PSME	26	1.38	28	0.18	4	0.28	5
TSME	74	0.47	26	0.54	26	0.64	40
Other	951	1.48	45	0.28	8	0.38	10
DBH >7.6 cm							
PICO	11 359	0.55	37	0.69	33	1.09	38
PILA	628	3.23	67	2.63	41	1.48	22
PIMO	222	1.43	51	0.61	23	0.29	10
PIPO	10 886	0.85	39	0.65	22	0.68	17
PSME	662	1.01	36	0.7	19	0.65	14
TSME	1 153	0.43	33	0.4	25	0.58	36
Other	8 417	1.47	70	2.05	64	2.49	49

Note: *N*, number of trees. Species code as in Table 2.

using this CR model. We did not evaluated whether changing the FVS SO variant's default maximum SDI resulted in improved CR predictions by the Weibull model.

Investigating the causes of the low predictive ability of these CR models was beyond the scope of this evaluation. Given the very different modeling approaches between Hatch's (1980) exponential and the logistic models and Dixon's (1985) Weibull-based model, and the error and coefficients of determination in CR models reported by other studies (see e.g., Holdaway 1986; Hasenauer and Monserud 1996), we may argue that the measurement error in tree CR measurements introduced by subjective determination of crown base and (or) visual rearrangement of uneven crowns to obtain an estimate may complicate the modeling.

Estimated DG using CR_m versus CR_p

The positive and high ρ values between CR_d and DG_d for every species and for both FVS variants highlight the importance of the initial CR value for tree DG prediction in the FVS. Even though the effect of the CR value used at the beginning of the prediction cycle is not directly observable in the DG prediction, large differences in CR resulted in large differences in DG. Interestingly, for the same species under the same tree-size group (DBH >7.6 cm) in both FVS variants, the ρ values for the FVS NI variant (evaluated using the CNF data) were higher than the ρ values for the FVS SO variant (evaluated using the WNF data) with the exception of lodgepole pine. One of the reasons for this result could be the additional predictor variables in the dds model

Table 9. Root mean square error (RMSE, cm) of diameter growth (DG) prediction differences with DG_m as the baseline; RMSE is presented in absolute value and as a percentage of the mean DG_p by CR_m class for trees with DBH >7.6 cm (Table 1) for the Forest Vegetation Simulator NI variant.

		CR_m class					
		<40%		40%–60%		>60%	
Species	*N*	cm	$\%DG_p$	cm	$\%DG_p$	cm	$\%DG_p$
PICO	8 997	0.38	21	0.46	21	1.01	34
PIMO	266	0.9	26	0.49	13	0.94	18
PIPO	1 098	0.42	17	0.48	18	1.57	49
PSME	15 080	0.46	24	0.46	19	0.97	29
THPL	6 332	0.38	20	0.38	15	1.12	37
TSHE	2 438	0.48	25	0.41	17	1.07	37
Other	17 330	0.43	24	0.4	18	0.7	25

Note: *N*, number of trees. Species code as in Table 3.

Table 10. Percentage of stimulation units (SU) within absolute percent difference between basal area increment predicted using measured crown ratio (BAI_m) and BAI predicted using predicted crown ratio (BAI_p) classes by Forest Vegetation Simulator variant (BAI_m as baseline).

		Absolute percent difference class (%)										
FVS variant	*N*	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99	>100
NI	2700	51.0	24.0	12.0	6.0	2.0	2.0	0.4	0.2	0.2	1.0	0.8
SO	2475	64.0	25.0	7.0	1.6	0.6	0.8	0.2	0.2	0.0	0.4	0.4

Note: For the NI variant 0.6% of the SUs had negative BAI_{mCR}; these results were not included in this table — their absolute percent difference ranged from 16% to 100%.

used by the FVS SO variant. Additionally, even though CR is a predictor variable in both variants' dds model, the coefficients used are variant and species specific. For the FVS SO variant the ρ values were lower for the smaller-tree group (DBH <7.6 cm) than for the larger-tree group for every species. This result could be an indication of a smaller effect of the CR value when the FVS predicts DG from height growth (Table 1).

RMSE values given as a percentage of DG_m values highlight the effect of the RMSE and bias of the CR models on the DG predictions. Furthermore, they point to the importance of the CR distribution of the stand being modeled in determining how large the effect of using CR_p on the DG_p could be. For example, if the stand's CR distribution centers around 50% and has a low SD, predictions by the NI variant CR model will present low bias and RMSE for most trees. In consequence the differences between DG_m and DG_p will be small. However, if the stand CR distribution centers around 70% or around 30% and has low SD, then the proportion of trees with over- or underpredicted CR becomes substantial, and the differences between DG_m and DG_p will be large for most trees. The same is true for the SO variant; if the stand CR distribution is centered around 60%, predictions by the CR Weibull model will present low bias and RMSE for most trees.

Estimated BAI at the SU level using CR_m versus CR_p

In both FVS variants, differences between CR_m and CR_p did not result in large differences between BAI_m and BAI_p for most SUs. There are several nonexclusive explanations for this result. First, the CR_m or CR_p values are input only at the beginning of the first prediction cycle. At the end of this cycle, the FVS predicts CR change for every tree and estimates CR. The effect on the DG predictions of using CR_m versus CR_p is largest at the end of the first 10 year cycle and lessens with each subsequent prediction period. Second, the effect of the differences in DG at the end of the first cycle could be lessened at the SU level if the CR_m distribution for that SU contributes to compensation of errors, i.e., similar proportion of trees in each CR_m class in the case of the NI variant, and similar proportion of trees below and above 60% CR_m in the case of the SO variant (Tables 8 and 9). Third, tree-level nonspatial prediction systems such as FVS aggregate tree-level diameter and height growth with mortality and regeneration models to estimate stand BAI. In this study's 30 year projection, the initial value of one of the prediction component's predictor variable (CR) resulted in BAI absolute percent differences below 10% for a majority of SUs. Despite the large bias and RMSE values for the CR_m classes less than 60%, the FVS SO variant presented the lowest proportion of SUs with absolute percent difference greater than 50% (Tables 6 and 9).

Conclusions

The three CR models were inaccurate and biased and failed to be validated with the equivalence tests when evaluated with the CNF and WNF data. Both the logistic and Hatch's (1980) exponential models used in the NI variant overpredicted CR when CR_m class was less than 40% and underpredicted CR when CR class was above 60% for every evaluated species. These models, though different in mathematical form, use similar predictor variables and behave

similarly. Dixon's (1985) model used in the SO variant overpredicted CR for CR_m classes below 60% for all species. Additionally, equivalence tests showed that for all the trees evaluated, the three CR models predicted a small range of CR values.

Large CR_d resulted in large DG_d. The effect of using CR_p as opposed to CR_m on the 10 year DG prediction is strong for every species, as shown by the ρ values and the RMSE expressed as percentage of the DG_m. The effect was larger for NI variant predictions than for SO variant predictions. Within the SO variant predictions, the effect was larger for the larger-tree-size group. Within FVS variants and tree-size group, the magnitude of the effect appeared to be more influenced by the CR_m class of the trees rather than by the species.

SU-level 30 year BAI predictions did not result in large differences when either CR_m or CR_p was used. More than 50% of the evaluated SUs presented an absolute percent difference between BAI_m and BAI_p of less than 10%. The fact that differences in CR values were present only at the beginning of the first prediction period, the possibility for compensating DG differences at the SU level, and that the FVS combines tree-level DG and height growth with SU-level mortality and regeneration calculations to yield the SU-level BAI help explain this result.

Acknowledgements

The authors are very grateful to Dr. David W. Hann, Dr. Valerie LeMay, the Associate Editor, and an anonymous referee for valuable comments on an earlier draft of the manuscript; and to Dr. Gary Dixon for providing valuable suggestions and information about each FVS variant's components. Support from the US Forest Service Grant 04DG11010000037, via Dr. Timothy Link of the University of Idaho, is appreciated.

References

Canavan, S.J., and Ramm, C.W. 2000. Accuracy and precision of 10 year predictions for Forest Vegetation Simulator — Lakes States. North. J. Appl. For. **17**(2): 62–70.

Dixon, G.E. 1985. Crown ratio modeling using stand density index and the Weibull distribution. Internal Report. USDA Forest Service, Forest Management Service Center, Fort Collins, Co.

Dixon, G.E. 2005. Inland Empire and North Idaho Variants of the Forest Vegetation Simulator [online]. Available from http://www.fs.fed.us/fmsc/fvs/documents/variant_overviews.php [accessed 27 April 2006].

Gertner, G.Z. 1990. The sensitivity of measurement error in stand volume estimation. Can. J. For. Res. **20**: 800–804. doi:10.1139/x90-105.

Ghosh, S., Innes, J.L., and Hoffmann, C. 1995. Observer variation as a source of error in assessments of crown condition through time. For. Sci. **41**(2): 235–254.

Hann, D.W. 2006.ORGANON user's manual. Ed. 8.2. Department of Forest Resources, Oregon State University, Corvallis, Ore.

Hann, D.W., and Larsen, D.R. 1991. Diameter growth equations for fourteen tree species in southwest Oregon. For. Res. Lab. Res. Bull. 69. Oregon State University, Forest Research Laboratory, Corvallis, Ore.

Hann, D.W., and Zumrawi, A.A. 1991. Growth model predictions as affected by alternative sampling designs. For. Sci. **37**(6): 1641–1655.

Hasenauer, H., and Monserud, R. 1996. A crown ratio model for Austrian forests. For. Ecol. Manage. **84**: 49–60. doi:10.1016/0378-1127(96)03768-1.

Hatch, C.R. 1980. Modelling crown size using inventory data. Mitt. Forstl. Bundes-Versuchsanst. Wien, **130**: 93–97.

Holdaway, M.R. 1986. Modeling tree crown ratio. For. Chron. **10**: 451–455.

Korzukhin, M.D., Ter-Mikaelian, M.T., and Wagner, R.G. 1996. Process versus empirical models: which approach for forest ecosystem management? Can. J. For. Res. **26**: 879–887. doi:10.1139/x26-096.

Lessard, V.C., McRoberts, R.E., and Holdaway, M.R. 2001. Diameter growth models using Minnesota Forest Inventory and Analysis data. For. Sci. **47**(3): 301–310.

McRoberts, R.E., Hahn, J.T., Hefty, G.J., and Van Cleve, J.R. 1994. Variation in forest inventory field measurements. Can. J. For. Res. **24**: 1766–1770. doi:10.1139/x94-228.

Nicholas, N.S., Gregoire, T.G., and Zedaker, S.M. 1991. The reliability of tree crown classification. Can. J. For. Res. **21**: 698–701. doi:10.1139/x91-095.

Omule, A.Y. 1980. Personal bias in forest measurements. For. Chron. **56**(5): 222–224.

R Development Core Team. 2005. R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria.

Reineke, L.H. 1933. Perfecting a stand-density index for even-aged forests. J. Agric. Res. **46**: 627–638.

Ritchie, M.W., and Hann, D.W. 1987. Equations for predicting height to crown base for fourteen tree species in Southwest Oregon. For. Res. Lab. Res. Pap. 50. Oregon State University, Forest Research Laboratory, Corvallis, Ore.

Robinson, A.P., Duursma, R.A., and Marshall, J.D. 2005. A regression-based equivalence test for model validation: shifting the burden of proof. Tree Physiol. **25**: 903–913. PMID:15870057.

Sharma, M., Burkhart, H., and Amateis, R.L. 2002. Modeling the effect of density on the growth of loblolly pine trees. South. J. Appl. For. **26**(3): 124–133.

Soares, P., and Tome, M. 2001. A tree crown ratio prediction equation for eucalypt plantations. Ann. For. Sci. **58**: 193–202. doi:10.1051/forest:2001118.

Stage, A.R. 1973. Prognosis model for stand development. USDA For. Serv. Res. Paper INT-137.

Temesgen, H., LeMay, V., and Mitchell, S.J. 2005. Tree crown ratio models for multi-species and multi-layered stands of southeastern British Columbia. For. Chron. **81**(1): 133–141.

US Forest Service. 2005*a*.Current Vegetation Survey. Quality Assurance/Quality Control/Accuracy Assessment Report. Pacific Northwest Region [online]. Available from http://www.fs.fed.us/r6/survey/document.htm [accessed 18 August 2005].

US Forest Service 2005*b*. South Central Oregon/ Northeastern California Variant of the Forest Vegetation Simulator [online]. Available from http://www.fs.fed.us/fmsc/fvs/documents/variant_overviews.php [accessed 27 April 2006].

US Forest Service. 2005*c*. Winema National Forest. Forest Plan [online]. Available from http://www.fs.fed.us/r6/winema/management/forestplan/1990plan/index.shtml [accessed 21 February 2006].

US Forest Service. 2005*d*. Colville National Forest. Forest Plan [online]. Available from http://www.fs.fed.us/r6/colville/forest/projects/87-forest-plan/plan87.html [accessed 21 February 2006].

US Forest Service. 2006. Pacific Northwest FIA Integrated Database user guide, version 2.0 [online]. Available from http://

www.fs.fed.us/pnw/fia/publications/data/pdfs/User%20Guide.pdf [accessed 15 February 2006].

Wykoff, W.R., Crookston, N.L., and Stage, A.R. 1982. User's guide to the Stand Prognosis model. USDA For. Serv. Gen. Tech. Rep. INT-133.

Zumrawi, A.A., and Hann, D.W. 1989. Equations for predicting the height to crown base of six tree species in the Central Western Willamette Valley of Oregon. For. Res. Lab. Res. Pap. 52. Oregon State University, Forest Research Laboratory, Corvallis, Ore.