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Nonlinear Equations for Predicting Diameter Inside Bark at Breast Height for Young-Growth Red Fir in California and Southern Oregon

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Inside bark diameters of young-growth red fir can be estimated from the relationship of inside bark diameter to outside bark diameter at breast height. Inside and outside bark diameters were obtained from stem analyses of 562 trees distributed across 56 sampling locations in the true fir forest type of California and southern Oregon. The resulting equation can predict inside bark diameter as a function of outside bark diameter. Because stand basal area and site index also influence the inside-outside bark diameter relationship, an additional equation that includes both of these independent variables is given. Weighted, nonlinear regression was used to estimate the equation coefficients. Application of the prediction equations in calculating past tree diameters is discussed.

Retrieval Terms: Abies magnifica, diameter estimation, bark thickness, past diameters, California, Oregon

The diameter of a tree at breast height (4.5 ft[1] above the ground on the uphill side of the tree) is a basic measurement in forest inventory. This diameter, measured outside bark, is perhaps the only variable closely related to tree volume that can be quickly and accurately measured on large numbers of trees at low cost.[2] However, reliable estimates of diameter inside bark or average bark thickness are needed for wood volume calculations. Intensive studies of growth and yield also require information on bark thickness, rates of bark growth, and the relation between bark growth and wood growth. Indirect estimates of inside bark diameter, bark thickness, and relative rates of bark and wood growth can be derived from the relationship between breast-height diameter inside bark (dib) and breast-height diameter outside bark (dob).

This note presents equations for predicting dib from measurements of dob and for predicting dib as a function of dob, basal area, and site index for young-growth red fir in California and southern Oregon. Procedures for using the predicted dib, in conjunction with radial wood growth as measured from increment cores, to estimate tree diameter as of some past date are described.

Since California red fir (*Abies magnifica* A. Murr.) and its commonly recognized botanical variety Shasta red fir (*Abies magnifica* var. *shastensis* Lemm.) are believed to be almost identical in silvical characteristics,[3] no attempt was made to distinguish between them during the sampling phase of the study. In this note, they will be referred to collectively as red fir.

METHODS

Data for this analysis were collected as part of a larger study to develop growth and soil fertility models for red fir within the range of its natural distribution. This range extends from latitude 43°35' N. in the southern Cascades in Oregon to the Kern River drainage in southern California, latitude 35°40' N.[3] Elevation ranges from about 5,000 to 9,000 feet.

A total of 56 sample sites, randomly selected from candidate young-growth red fir stands within the study area, were examined during the field seasons 1984-1988. Young-growth stands were defined as those in which at least 75 percent of the trees were 120 years old or younger at breast height. At each site, 2 to 5 (depending on stand composition) variable radius subplots were established for measuring tree growth, stand composition, and site factor variables. At each plot, from 2 to 4 randomly selected trees (except trees with damaged tops or those less than 3.0 inches dob) were felled and sectioned for collection of stem analysis data. In addition, one dominant red fir "site tree" was felled and sectioned on each plot. Data from these dominant site trees were combined with the data from the other dominant, codominant, intermediate, and suppressed felled trees for this analysis.

Each tree was marked at breast height before felling. After felling, a cross section was cut at the breast-height mark. Diameter outside bark was measured at the breast-height mark with a diameter tape and recorded to the nearest 0.1 inch. Average bark thickness was determined by averaging four uniformly spaced measurements of bark thickness, each taken to the nearest 0.05 inch, by the "wood-to-tape" method.[4] Diameter inside bark was then calculated by subtracting twice the average bark thickness from the dob measurement.

Site index (total height of dominant trees at 50 years breast-height age) for each plot was determined directly from stem analysis of the site trees. Basal area (ft^2/acre) around each plot center was estimated by counting all trees that qualified for tallying with a basal area factor 20 wedge prism. Descriptive statistics for the data set are presented in *table 1*.

Table 1—*Descriptive statistics for the 562 young-growth red fir trees in the data set*

Characteristic	Range			Standard
	Minimum	Maximum	Mean	deviation
Diameter outside bark (inches)	3.0	32.0	16.1	5.64
Diameter inside bark (inches)	2.5	29.1	14.4	5.13
Site index (feet)	18	113	55.8	15.48
Basal area (ft^2/acre)	40	600	286.2	112.54

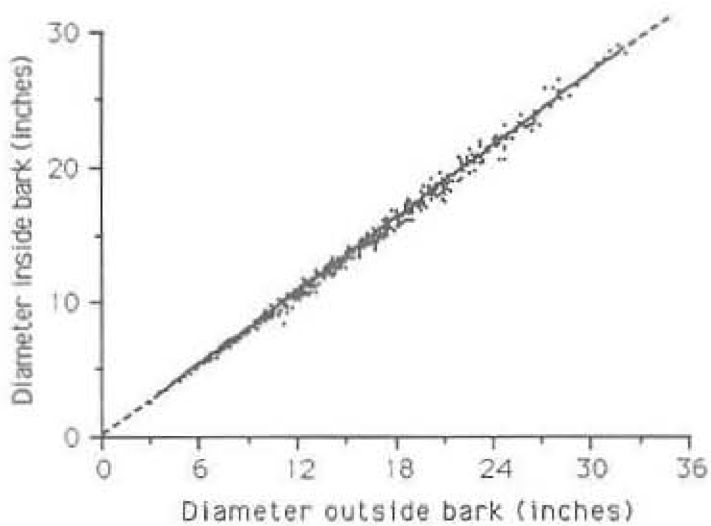

Figure 1—Diameter inside bark plotted over diameter outside bark for 562 observations of young-growth red fir trees in California and southern Oregon. The solid line represents the regression equation 3.

ANALYSIS OF DATA

The data were analyzed with two main objectives in mind: (1) to develop equations for predicting current diameter inside bark when the current diameter outside bark is known, and (2) to estimate outside bark diameter as of some previous date when current dob and the amount of radial wood growth that has occurred since that date are both known.

Predicting Diameter Inside Bark

A plot of the data indicated a strong linear relationship of dib to dob *(fig. 1)*. The linear model,

$$dib = b_0 + b_1 dob + e \qquad (1)$$

in which

dib = diameter inside bark at breast height in inches
dob = diameter outside bark at breast height in inches
b_i = regression coefficients
e = a random error component with an expected mean value of zero and variance of σ^2

was fit to the data by weighted least squares regression. Weights of 1.0/dob, 1.0/dob^2, 1/$dob^{0.5}$, and 1.0 (unweighted) were tried. Furnival's index of fit,[5] computed for each regression, showed that the weight of 1/$dob^{0.5}$ provided the best fit for the data. Over the range of diameters sampled (*table 1*), equation 1 resulted in a good fit of the data; however, it resulted in negative predictions of dib for very small trees (less than 1/4 inch dob). This seemed unacceptable since the models are specifically for young-growth trees.

To avoid the negative dib predictions, the linear model,

$$dib = b_1 dob + e \qquad (2)$$

which forces the regressions through the origin, was fit to the data. Furnival's index of fit indicated the weight of 1/dob provided the best fit of this equation and showed considerable improvement in fit over equation 1. Equation 2 resulted in reasonable dib predictions for very small trees, but a plot of the residuals revealed an unacceptable trend (variance not constant), resulting from forcing the regression through the origin.

Since the linear models did not give satisfactory results, a nonlinear model was fit to the data with weighted, nonlinear regression. The model

$$dib = b_1 dob^{b_2} + e \qquad (3)$$

used by Ritchie and Hann[6] for Douglas-fir in western Oregon and by Larsen and Hann[7] for six conifer species in southwest Oregon resulted in reasonable dib predictions for small trees and eliminated the undesirable trend in the residuals. A plot of equation 3 is superimposed on the original data *(fig. 1)*.

Although Ritchie and Hann[6] and Larsen and Hann[7] used the weight of 1/dob^2 in fitting the model, comparison of Furnival's index of fit indicated the weight of 1/dob resulted in the best fit of the red fir data. The asymptotic estimate of the 95 percent confidence interval about b_2 does not include 1.0, indicating the nonlinear trend in the model is significant.

Based on the assumption that the growth rate of a tree would affect the relative rates of bark growth and wood growth, two additional variables that reflect site quality and stand density were included. Equation 3 was expanded to include site index (SI) as an indicator of site quality, and basal area per acre (BA), which provided an estimate of stand density. The expanded model,

$$dib = b_1 dob^2 + b_3 SI + b_4 BA + e \quad (4)$$

was also fit to the data using the weight of 1/dob. Neither of the asymptotic estimates of the 95 percent confidence intervals about b_3 or b_4 included 0.0, indicating each variable was significant at the 0.05 level. This indication of significance differs from the findings of Monserud[8] who found that these variables did not significantly affect the dib/dob relationships for Douglas-fir in northern Idaho and northwestern Montana.

Each tree was treated as an independent observation for the development of the predictive equations. In actuality, the clusters (which were randomly selected) are the experimental units, and tree measurements are subsamples within the clusters. If all the trees at each cluster had been measured, ignoring this random cluster effect would result in an underestimate of the variance associated with the regressions. However, since the equations were developed from stem analysis data of only a few trees at each cluster, the random cluster effect was not considered significant in calculating the variance estimates. The standard error given for each equation *(table 2)* may be slightly underestimated.

Estimating Past Diameters

Estimates of breast-height diameters outside bark for trees as of some previous date are useful in growth and yield studies. Past diameters of trees on temporary plots are compared to current diameters to determine periodic basal area growth. The increase in diameter during the period is the result of increase in thickness of both wood and bark, and both must be accounted for in converting present diameters outside bark to equivalent past diameter. Two measurements at breast height are needed for each tree: (1) diameter outside bark and (2) average radial growth of wood for the desired period as measured on an increment core.

A method for converting dib back to dob is also needed to obtain an estimate of past dob. The model

$$dob = b_1 dib^{b_2} + e \quad (5)$$

was fitted to the data by weighted regression using weights of 1.0/dib, $1.0/dib^2$, $1.0/dib^{0.5}$, and 1.0 (unweighted). Furnival's index of fit for each regression indicated the weight of $1.0/dib^{0.5}$ provided the best fit for the data. The estimation of past dob is, then, a four-step process:

- Measure current dob and radial wood growth for the desired period.
- Convert current dob to current dib using equation 3 or 4.
- Subtract twice the radial wood growth from current dib to obtain past dib.
- Convert past dib to past dob with equation 5.

When a sequence of measurements and estimates is used in this manner, the propagated error is likely to be substantially greater than that indicated by the indices of fit for either equation.

RESULTS AND DISCUSSION

Table 2 presents the regression coefficients and the associated statistics for the five equations fitted with the data. When stand density and site index information are available, equation 4 is recommended for obtaining the most precise predictions of dib. The negative coefficient for basal area indicates that for trees of the same dob, those growing under higher densities will have thicker bark. The positive coefficient for site index causes an increase in predicted dib; for trees of the same dob, those on higher sites will have thinner bark. In general, the ratio of bark growth to wood growth will be higher on slower growing trees.

If dib predictions are not needed for small trees (less than about 5 inches in diameter at breast height), the linear relationship given by equation 1 will result in suitable predictions of dib for trees in the range of 5 to 30 inches dob. However, when more precise predictions are required for the full range of diameters from 0.0 to 30 inches, use of the nonlinear model (equation 3) is recommended if site index and stand density information are not available.

Table 2—*Regression coefficients, standard error of estimate, and Furnival's index of fit for the prediction equations*

Equation	Coefficient					Standard error of estimate	Furnival's index of fit
	b_0	b_1	b_2	b_3	b_4		
1	-0.205	0.90714				0.4036	0.7595
2		0.89295				0.4121	0.3705
3		0.86951	1.00983			0.4035	0.3672
4		0.83560	1.02073	0.00481	-0.00048	0.3768	0.3457
5		1.17993	0.98027			0.4403	0.4119

END NOTES AND REFERENCES

[1]British/Metric Conversion:
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 square foot = 0.0929 square meters
1 acre = 0.404686 hectares
1 square foot per acre = 0.229568 square meters per hectare.

[2]Spurr, Stephen H. 1952. *Forest inventory*. New York: Ronald Press; 476 p.

[3]Hallin, William E. 1957. *Silvical characteristics of California red fir and Shasta red fir*. Tech. Paper 16. Berkeley, CA: California Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 8 p.

[4]Mesavage, Clement. 1969. *Measuring bark thickness*. Journal of Forestry 67(10): 753-754.

[5]Furnival, George M. 1961. *An index for comparing equations used in constructing volume tables*. Forest Science 7(4): 337-341.

[6]Ritchie, Martin W.; Hann, David W. 1984. *Nonlinear equations for predicting diameter and squared diameter inside bark at breast height for Douglas-fir*. Res. Paper 47. Corvallis, OR: Forest Research Laboratory, Oregon State University; 12 p.

[7]Larsen, David R.; Hann, David W. 1985. *Equations for predicting diameter and squared diameter inside bark at breast height for six major conifers of southwest Oregon.* Res. Note 77. Corvallis, OR: Forest Research Laboratory, Oregon State University; 4 p.

[8]Monserud, Robert A. 1979. *Relations between inside and outside bark diameter at breast height for Douglas-fir in northern Idaho and northwestern Montana.* Res. Note INT-266. Ogden, UT: Intermountain Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 8 p.

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