

Determination of the particulate extinction-coefficient profile and the column-integrated lidar ratios using the backscatter-coefficient and optical-depth profiles

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A new method is considered that can be used for inverting data obtained from a combined elastic-inelastic lidar or a high spectral resolution lidar operating in a one-directional mode, or an elastic lidar operating in a multiangle mode. The particulate extinction coefficient is retrieved from the simultaneously measured profiles of the particulate backscatter coefficient and the particulate optical depth. The stepwise profile of the column-integrated lidar ratio is found that provides best matching of the initial (inverted) profile of the optical depth to that obtained by the inversion of the backscatter-coefficient profile. The retrieval of the extinction coefficient is made without using numerical differentiation. The method reduces the level of random noise in the retrieved extinction coefficient to the level of noise in the inverted backscatter coefficient. Examples of simulated and experimental data are presented. © 2007 Optical Society of America

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1. Introduction

The conventional elastic lidar equation includes the backscatter coefficient in the scattering volume and the two-way attenuation along the path from the lidar to the scattering volume. The elastically scattered signal contains insufficient information to separate the attenuation and the backscatter component. This is generally achieved by using an assumption regarding the profile of the lidar ratio, that is, the ratio of the extinction and backscatter coefficients. The simplest and frequently used method is to assume a range independent lidar ratio over the measurement range. Unfortunately, such an *a priori* assumption can yield a highly inaccurate measurement result.

High spectral resolution lidar, combined elastic-inelastic lidar, or the elastic lidar that operates in a multiangle mode allow separating the profiles of the backscatter component and the two-way attenuation. From the latter, the extinction coefficient, which is

usually the key parameter of interest, can then be retrieved. Unfortunately, the problem of determining the extinction-coefficient profile from the two-way attenuation (or the related optical depth profile) is not satisfactorily solved in such a way. The straightforward extraction of the extinction coefficient from the optical depth profile requires the use of numerical differentiation. Because the real signals are generally corrupted by random and systematic noise, this procedure can yield unacceptably poor measurement accuracy, especially when the lidar signals are measured in a clear atmosphere.

The principal drawback of such a retrieval technique is that only the optical depth profile is used for extracting the extinction coefficient. The valuable information concerning particulate loading contained in the backscatter term is generally not used to put constraints on the extracted extinction coefficient. In the study by Kovalev [1], a technique for the extraction of the extinction coefficient was proposed where both the optical depth and the backscatter coefficient profiles were used in the inversion procedure. However, this technique still requires utilizing numerical differentiation. More

recently, a regularized algorithm for Raman lidar processing was proposed where the inversion was made without numerical differentiation, using explicit relationships between the backscatter and the optical depth, and an *a priori* smoothness constraint [2].

In this study, an alternative technique is introduced that also allows extracting the particulate extinction coefficient without using numerical differentiation. The extinction coefficient is extracted from the particulate backscatter coefficient profile while using an estimate of uncertainty boundaries of the optical depth profile as a constraint. The extinction coefficient is extracted by assuming a stepwise profile of the particulate column-integrated lidar ratio. The optical depth profile and its uncertainty boundaries are only used to determine the best value for the stepwise lidar ratio over extended zones along the searching direction. In other words, the stepwise lidar ratios for different zones are found that provide the best agreement between the initial (inverted) profile of the optical depth and that obtained by the inversion of the backscatter-coefficient profile. Determining the lidar ratio prior to determining the extinction coefficient allows one to decrease the level of noise in the profile of the latter.

Before the inversion is made, one should separate the altitude profiles of the particulate optical depth and the particulate backscatter coefficient from the molecular components. This requires knowledge of the vertical molecular profile and the lidar solution constant. The latter can be determined, for example, by using the assumption of an aerosol-free atmosphere over high altitudes.

2. Method

A. Principle of Using the Stepwise Column-Integrated Lidar Ratio for Determining the Extinction Coefficient from the Profile of the Backscatter Coefficient

Let us consider the general principle of the retrieval of the particulate extinction coefficient, $\kappa_p(h)$, when the profiles of the particulate backscatter coefficient, $\beta_p(h)$, and the particulate optical depth, $\tau_p(h_j, h)$, over some altitude range from the starting point, h_j , to h are available. The basic relationship between $\kappa_p(h)$, $\beta_p(h)$, and the lidar ratio, $S_p(h)$, is

$$\kappa_p(h) = S_p(h)\beta_p(h). \quad (1)$$

Accordingly, the dependence of $\tau_p(h_j, h)$ on $\kappa_p(h)$ can be written in the form:

$$\tau_p(h_j, h) = \int_{h_j}^h \kappa_p(h') dh' = \int_{h_j}^h S_p(h') \beta_p(h') dh'. \quad (2)$$

Equation (2) can be expressed as [3,4],

$$\tau_p(h_j, h) = S_{pj}(h_j, h) \int_{h_j}^h \beta_p(h') dh', \quad (3)$$

where $S_{pj}(h_j, h)$ is the column-integrated lidar ratio over the altitude range from h_j to h and is defined as

$$S_{pj}(h_j, h) = \frac{\int_{h_j}^h S_p(h') \beta_p(h') dh'}{\int_{h_j}^h \beta_p(h') dh'}. \quad (4)$$

If $S_p(h) = \text{const.}$, $S_{pj}(h_j, h) = S_p(h)$. Note also that the column-integrated lidar ratio over any height interval Δh is equal to the mean value of $S_p(h)$ over this interval if either $\beta_p(h) = \text{const.}$, or $S_p(h) = \text{const.}$ In other words, only simultaneous sharp changes in both functions, $S_p(h)$ and $\beta_p(h)$, will cause a significant difference between the mean and column-integrated lidar ratios. As with any mean value, the column-integrated lidar ratio is generally a relatively smooth function that varies significantly less than the initial, not integrated, function $S_p(h)$. If the column-integrated lidar ratio has no sharp change over the height interval from h_j to h , one can apply the assumption, $S_{pj}(h_j, h) = S_{pj} \approx \text{const.}$ within this interval. Accordingly, the actual optical depth profile $\tau_p(h_j, h)$ in Eq. (3) can be approximated by the function $\langle \tau_p(h_j, h) \rangle$, which is defined as

$$\langle \tau_p(h_j, h) \rangle = S_{pj} \int_{h_j}^h \beta_p(h') dh'. \quad (5)$$

As follows from Eqs. (3) and (5), the difference between the two optical depths, $\langle \tau_p(h_j, h) \rangle$ and $\tau_p(h_j, h)$, is

$$\Delta \tau_{pj}(h) = S_{pj} \int_{h_i}^h \beta_p(h') dh' - S_{pj}(h_j, h) \int_{h_i}^h \beta_p(h') dh'. \quad (6)$$

The basic principle of the below method is similar to that proposed in the studies [2,5]. One should find the best value for the lidar ratio that minimizes the difference between the initial (inverted) optical depth (or the mean extinction coefficient) and that obtained after the inversion. However, the extinction coefficient is now extracted from the backscatter component of the signal. Such a specific facilitates the retrieval of the extinction coefficient both in multiangle and one-directional methods, which allow separating the profiles of the backscatter component and the two-way attenuation.

In our method, the constant column-integrated ratio, S_{pj} , should be found that minimizes the difference $\Delta \tau_{pj}(h)$ between the profiles $\tau_p(h_j, h)$ and $\langle \tau_p(h_j, h) \rangle$

[Eqs. (3) and (5)] over some extended altitude range from h_j to $h_{j,\max} = h_j + \Delta h_{j,\max}$. To perform the inversion, the maximal acceptable difference, $\Delta\tau_{p,\max}(h)$, between these two optical depth profiles should be initially established. The ratio $S_{p,j}$ must be selected that makes the inequality

$$|\Delta\tau_{p,j}(h)| \leq \Delta\tau_{p,\max}(h) \quad (7)$$

valid for any point within the above interval, $\Delta h_{j,\max}$. The corresponding extinction coefficient over the above altitude interval can be obtained from the backscatter coefficient by using a simple formula,

$$\kappa_p(h) \approx S_{p,j}\beta_p(h). \quad (8)$$

To utilize such a technique, one should divide the total height interval over which $\beta_p(h)$ and the optical depth from ground level, $\tau_p(0, h)$, are known, into separate height intervals (i.e., from h_j to h_{j+1} , then from h_{j+1} to h_{j+2} , and so on). Within each of these intervals, the actual column-integrated lidar ratios, $S_{p,j}(h_j, h)$, $S_{p,j+1}(h_{j+1}, h)$, $\dots$, are replaced by constant values, $S_{p,j}$, $S_{p,j+1}$, and so on. The length of the intervals is a user-defined parameter; it can be either constant or variable within the total altitude range. The measurement accuracy will depend on the appropriate selection of such intervals; this is a critical aspect of the technique above. Different criteria can be used for the selection of the intervals; the two simplest methods are discussed in [6]. In our study, the interval selection is based on obtaining the maximal possible length for each consecutive stepwise interval of the lidar ratio.

B. Uncertainty Boundaries for the Inverted Particulate Optical Depth

Optical depth profiles are generally corrupted by noise and systematic distortions. This takes place usually over far ranges, but in some cases, near-end distortions of the optical depth may also be significant. Therefore, before extracting the extinction coefficient from the functions $\beta_p(h)$ and $\tau_p(0, h)$, one should establish the minimal and maximal heights, $h_{\min}$ and $h_{\max}$, such that neither near nor far-end distortions significantly impact the retrieved extinction coefficient. The principles and criteria for the selection of the optimal values for $h_{\min}$ and $h_{\max}$ are discussed by Kovalev *et al.* [7]. After such a height interval is established, some mean optical depth and its uncertainty boundaries must be determined. The selection of the uncertainty boundaries of the optical depth, $\tau_p(0, h)$, is the principal issue. Generally, it is assumed that standard statistics provide a proper estimation of the uncertainty of the retrieved aerosol extinction even when using numerical differentiation techniques [2,8–11]. Unfortunately, in many cases, especially in multiangle measurements, systematic errors of an unknown sign and magnitude can have a prevailing influence. These errors do not obey common distri-

butions such as Gaussian or Poisson, and there is no commonly accepted technique for addressing the situation. This quite general issue still requires a thorough investigation, which is beyond the scope of this study. To introduce the principles of our new retrieval technique, the simplest empirical estimate of the optical depth uncertainty is used. It is based on calculating maximal and minimal profiles of the original optical depth, $\tau_p(0, h)$, and determining a smoothed, so-called base optical-depth profile, $\tau_{p,b}(0, h)$, as described by Kovalev [1] and Kovalev *et al.* [6]. Note that in our method, this optical depth is not used to extract the extinction coefficient by its numerical differentiation; it is only used as a reference function, which is compared with the optical depth calculated with Eq. (5). The upper and lower uncertainty boundaries of the optical depth used as constraints are defined as $\tau_{p,up}(0, h) = \tau_{p,b}(0, h) + \Delta\tau_{p,\max}(h)$ and $\tau_{p,low}(0, h) = \tau_{p,b}(0, h) - \Delta\tau_{p,\max}(h)$, where

$$\Delta\tau_{p,\max}(h) = \max[\Delta\tau_p(h_{\min}); \Delta\tau_p(h_{\min} + \Delta h_d); \Delta\tau_p(h_{\min} + 2\Delta h_d); \dots; \Delta\tau_p(h)]. \quad (9)$$

Here Δh_d is the sampling resolution of the data points for the inverted profile of the optical depth, and $\Delta\tau_p(h)$ is the uncertainty boundary found when shaping the optical depth profile [1,7].

For illustration, an example of the synthetic noise-corrupted optical depth and its uncertainty boundaries are shown in Fig. 1. Here the primary optical depth, $\tau_p(0, h)$ versus height is shown as the thick solid curve, and the corresponding profile of the base function, $\tau_{p,b}(0, h)$, is shown by the open diamonds. The profiles of $\tau_{p,up}(0, h)$ and $\tau_{p,low}(0, h)$ are shown as the dashed curves. Note that extreme points of $\tau_p(0, h)$ may be beyond these two marginal profiles.

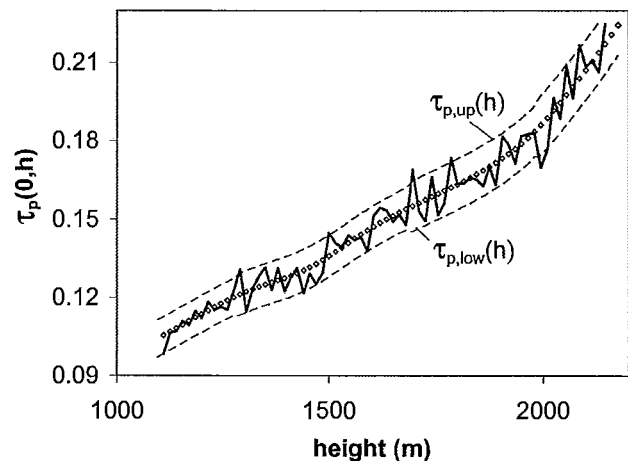


Fig. 1. Synthetic noise-corrupted optical depth, $\tau_p(0, h)$ (thick solid curve), the base function, $\tau_{p,b}(0, h)$ (the open diamonds), and the upper and lower uncertainty boundaries, $\tau_{p,up}(0, h)$ and $\tau_{p,low}(0, h)$ (the dashed curves) versus height.

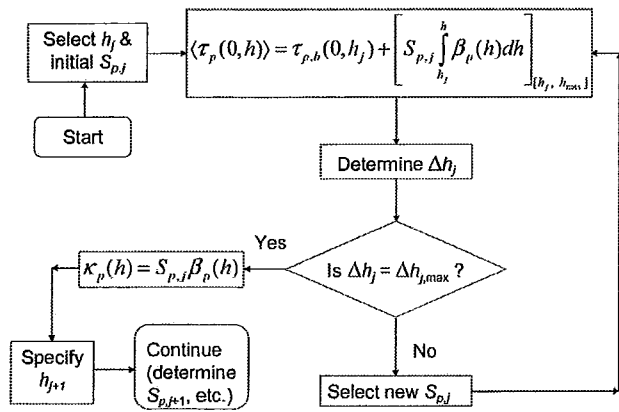


Fig. 2. Simplified flow chart for extracting $\kappa_p(h)$.

C. Determination of the Extinction Coefficient Profile

As mentioned, different methods can be used for the selection of the length of the consecutive intervals $\Delta h_j, \Delta h_{j+1}, \dots, \Delta h_n$ over which the actual column-integrated lidar ratios are replaced by the constants, $S_{p,j}, S_{p,j+1}, \dots, S_{p,n}$. In this study, the constants that maximize the length of the respective intervals are found.

In Fig. 2, the simplified flow chart for extracting $\kappa_p(h)$ over the range interval from h_j to h_{j+1} is shown. The value of $S_{p,j}$ is found that yields the maxima interval $\Delta h_j = h_{j+1} - h_j$. To trigger the retrieval procedure, an initial constant value for $S_{p,j}$ is arbitrarily chosen, for example, equal to the molecular lidar ratio. Using the selected $S_{p,j}$ and the profile of $\beta_p(h)$, the corresponding profile of $\langle \tau_p(h, h) \rangle$ over the whole height interval $h_j - h_{\max}$ is found with Eq. (5). The corresponding profile for $\langle \tau_p(0, h) \rangle$ from ground level is then found as

$$\begin{aligned} \langle \tau_p(0, h) \rangle &= \tau_{p,b}(0, h_j) + \langle \tau_p(h_j, h) \rangle \\ &= \tau_{p,b}(0, h_j) + \left[S_{p,j} \int_{h_j}^h \beta_p(h') dh' \right], \end{aligned} \quad (10)$$

where $\tau_{p,b}(0, h_j)$ is the base optical depth at the point h_j . This calculated function, $\langle \tau_p(0, h) \rangle$ is then compared to the base profile, $\tau_{p,b}(0, h)$ over each height, $h > h_j$. The goal is to determine how well the profiles, $\langle \tau_p(0, h) \rangle$ and $\tau_{p,b}(0, h)$, match each other and at what heights they start to diverge. The interval Δh_j and the corresponding height is determined where the profile $\langle \tau_p(0, h) \rangle$ goes outside the estimated uncertainty boundaries, that is, where it intersects either $\tau_{p,up}(0, h)$, or $\tau_{p,low}(0, h)$. The value of $S_{p,j}$ is varied until the maximal distance, $\Delta h_{j,\max}$, is achieved, where the function $\langle \tau_p(0, h) \rangle$ remains within the zone restricted by the marginal functions, $\tau_{p,up}(0, h)$, or $\tau_{p,low}(0, h)$. Then the extinction coefficient is calculated with Eq. (8).

The procedure of determining the interval within which the profiles, $\langle \tau_p(0, h) \rangle$ and $\tau_{p,b}(0, h)$, match each other is clarified with Fig. 3. Here the open diamonds and dashed curves are the profiles $\tau_{p,b}(0, h)$, $\tau_{p,up}(0, h)$, and $\tau_{p,low}(0, h)$, the same as those in Fig. 1. The initial

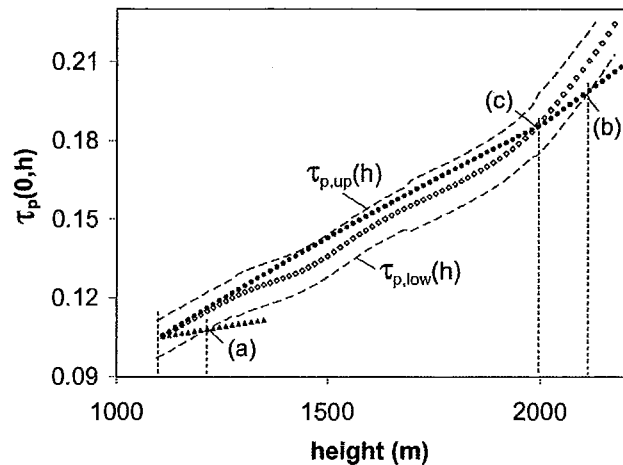


Fig. 3. Base profile, $\tau_{p,b}(0, h)$ (the open diamonds) and the corresponding profiles $\tau_{p,up}(0, h)$ and $\tau_{p,low}(0, h)$ (dashed curves), the same curves as those in Fig. 1. The solid triangles show the profile of $\langle \tau_p(0, h) \rangle$ calculated with $S_{p,j} = 8.38$ sr, and the solid dots show the profile derived with $S_{p,j} \approx 31$ sr.

profile of $\langle \tau_p(0, h) \rangle$ calculated with Eq. (5) using the molecular lidar ratio, $S_{p,j} = 8.38$ sr, is shown with the solid triangles. One can see that when such a small value for $S_{p,j}$ is selected, the profiles $\langle \tau_p(0, h) \rangle$ and $\tau_{p,b}(0, h)$ match each other badly and diverge at a short distance. The intersection point of $\langle \tau_p(0, h) \rangle$ with $\tau_{p,low}(0, h)$, the point (a) at $h \approx 1215$ m, is quite close to the starting point, $h_j = 1100$ m. That is, within the uncertainty boundaries, the profiles $\langle \tau_p(0, h) \rangle$ and $\tau_{p,b}(0, h)$ match each other within the very short height interval, $\Delta h_j \approx 115$ m.

The selection of the larger $S_{p,j}$ shifts the intersection point further from starting point, h_j , and matches $\langle \tau_p(0, h) \rangle$ and $\tau_{p,b}(0, h)$ over larger distances. By incrementally increasing $S_{p,j}$, one should find a value that results in the maximal possible interval, $\Delta h_j = \Delta h_{j,\max}$, over which the calculated optical depth $\langle \tau_p(0, h) \rangle$ remains within the area restricted by the uncertainty boundaries, $\tau_{p,low}(0, h)$ and $\tau_{p,up}(0, h)$. In our case, optimum matching occurs when selecting $S_{p,j} \approx 31$ sr; the corresponding profile of $\langle \tau_p(0, h) \rangle$ is shown in the figure as solid dots. The intersection point of $\langle \tau_p(0, h) \rangle$ with $\tau_{p,low}(0, h)$ has shifted to the maximal height, $h_{j,\max} \approx 2100$ m [point (b)]. Further increasing of $S_{p,j}$ will sharply decrease (rather than increase) Δh_j and degrade the profile match. A slight increase, $S_{p,j} > 31$ sr, will result in reducing the height where $\langle \tau_p(0, h) \rangle$ intersects the uncertainty boundary, this time $\tau_{p,up}(0, h)$, to ~ 1500 m.

Thus, for the established uncertainty boundaries, $\tau_{p,low}(0, h)$ and $\tau_{p,up}(0, h)$, the selection of the constant column-integrated lidar ratio, $S_{p,j} = 31$ sr results in a maximal height interval from $h_j = 1100$ m to $h_{j,\max} \approx 2100$ m, that is, $\Delta h_{j,\max} \approx 1000$ m. The corresponding profile of the extinction coefficient over this interval can be extracted from the backscatter extinction coefficient using Eq. (8). This procedure is performed on each consecutive altitude range until the entire

profile of $\langle \tau_p(0, h) \rangle$ and the corresponding extinction coefficient are found up to the maximal height, $h_{\max}$.

Best inversion results are obtained when the starting point for the next altitude span, h_{j+1} , is chosen such that $h_{j+1} < h_{j,\max}$. It is better to reduce the initial maximal height, $h_{j,\max} = 2100$ m down to the last intersection of the profile, $\langle \tau_p(0, h) \rangle$, with the base function, $\tau_{p,b}(0, h)$. In other words, the starting point for the next interval, Δh_{j+1} , should be selected at $h_{j+1} \approx 2000$ m rather than at 2100 m, that is, at point (c) rather than at point (b).

When determining the column-integrated lidar ratio, $S_{p,k}$, for the uppermost zone, the procedure we described does not provide a unique value of $S_{p,k}$. Here we minimize the difference between $\langle \tau_p(0, h) \rangle$ and $\tau_{p,b}(0, h)$ by selecting $S_{p,k}$ to minimize the function,

$$\xi(h_k, h_{\max}) = \sum_{h_k}^{h_{\max}} [\langle \tau_p(0, h) \rangle - \tau_{p,b}(0, h)]^2. \quad (11)$$

Thus for the last interval, the column-integrated lidar ratio, $S_{p,k}$ and the corresponding extinction coefficient are found by determining the profile of $\langle \tau_p(0, h) \rangle$, which is the closest to $\tau_{p,b}(0, h)$ in a statistical sense. The uncertainties in the primary optical depth, $\tau_p(0, h)$, are involved here indirectly, through the shape of the base function, $\tau_{p,b}(0, h)$. In principle, one can also adjust all previously derived profiles $\langle \tau_p(h_j, h_{j+1}) \rangle$, $\langle \tau_p(h_{j+1}, h_{j+2}) \rangle$, and so on by minimizing $\xi(h_j, h_{j+1})$, $\xi(h_{j+1}, h_{j+2})$, ... for the established intervals, the same as is done when divisions between ranges are uniformly distributed [6]. However, this generally does not significantly change the inversion result when maximal range intervals are chosen.

3. Discussion

The technique described in Section 2 is illustrated with simulated data shown in Figs. 4–7. For the simulations, a synthetic atmosphere is assumed where

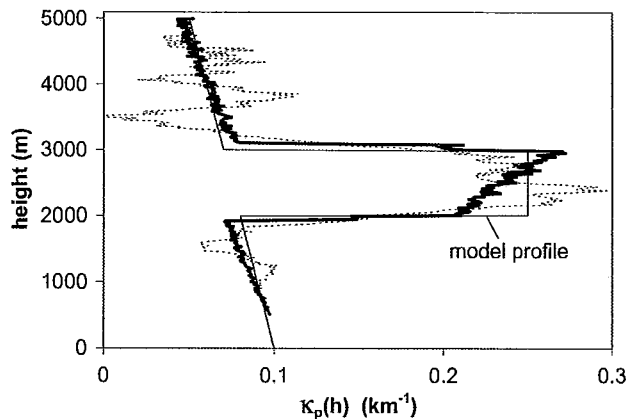


Fig. 4. Thin solid curve is the model profile of the extinction coefficient used for the simulations. The profile of $\kappa_p(h)$ retrieved with the new method is shown as the thick solid curve; the dotted curve shows the extinction coefficient profile calculated using the conventional numerical differentiation with a vertical range resolution of 500 m.

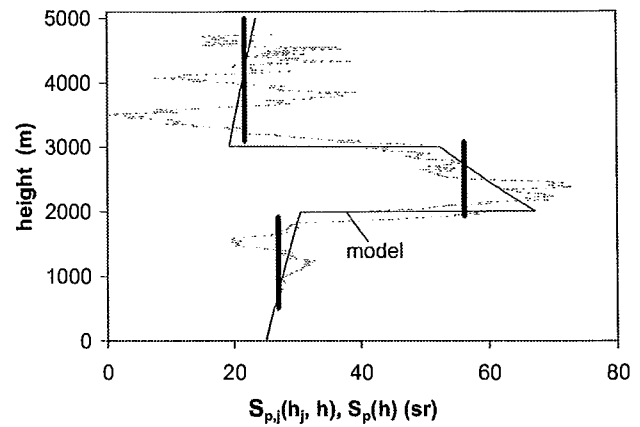


Fig. 5. Model lidar ratio, $S_p(h)$, used for the simulation (the thin solid curve) and the retrieved stepwise column-integrated profile, $S_{p,i}$ (the thick solid lines). The lidar ratio, $S_p(h)$, extracted using the extinction coefficient retrieved with the numerical differentiation is shown as the dotted curve.

the particulate extinction coefficient at the wavelength of 532 nm linearly decreases with height, from $\kappa_p(h) = 0.1 \text{ km}^{-1}$ at ground level down to $\kappa_p(h) = 0.05 \text{ km}^{-1}$ at $h = 5000$ m. Additionally, a turbid layer exists with $\kappa_p(h) = 0.25 \text{ km}^{-1}$ over the heights from 2000 to 3000 m. This model profile of $\kappa_p(h)$ is shown in Fig. 4 as the thin solid curve. It is also assumed that the lidar ratio, $S_p(h)$, linearly varies with height in the range between 20 and 30 sr in the clear air zones and from 50 to 67 sr within the turbid layer; this model profile is shown in Fig. 5 as the thin solid curve.

Now let us suppose that an artificial high spectral resolution lidar is working in such an atmosphere at the wavelength 532 nm. The noise-corrupted profiles of the backscatter coefficient, $\beta_p(h)$, and retrieved optical depth, $\tau_p(0, h)$, obtained from the data of this artificial lidar over the heights from $h_{\min} = 500$ to $h_{\max} = 5000$ m are shown as the dotted curves in Figs. 6 and 7, respectively; the estimated uncertainty boundaries, $\tau_{p,\text{low}}(0, h)$ and $\tau_{p,\text{up}}(0, h)$ are shown in Fig. 7 as the dashed curves. The retrieval procedure described in Subsection 2.C splits the

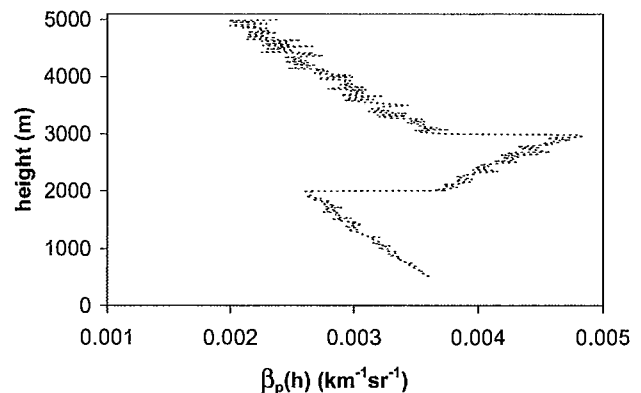


Fig. 6. Backscatter coefficient, $\beta_p(h)$, profile in the simulated atmosphere.

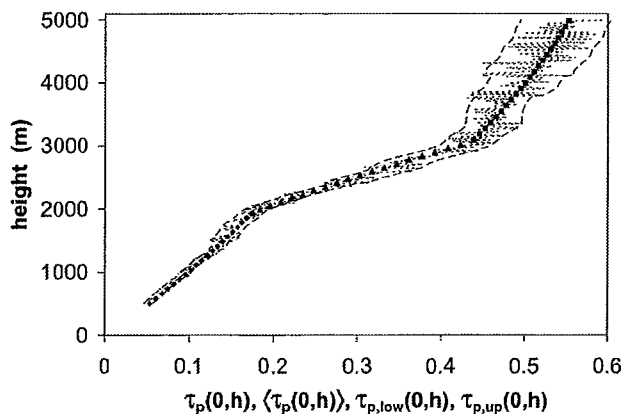


Fig. 7. Profile of the optical depth, $\tau_p(0, h)$, obtained with an artificial lidar (thin dotted curve) and the uncertainty boundaries, $\tau_{p,low}(0, h)$ and $\tau_{p,up}(0, h)$ (dashed curves). The solid diamonds, solid triangles, and solid squares show the sections of the profile $\langle \tau_p(0, h) \rangle$ for the height intervals, 500–1935, 1935–3090, and 3090–5000 m, retrieved with the column-integrated lidar ratios 27, 56.2, and 21.7 sr, respectively.

total measurement range into three adjacent height intervals, 500–1935 m, 1935–3090 m, and 3090–5000 m, in which the column integrated ratios, $S_{p,j}$, are found to be 27, 56.2, and 21.7 sr, respectively; these $S_{p,j}$ are shown in Fig. 5 as the thick vertical lines. Three retrieved pieces of the profiles of $\langle \tau_p(0, h) \rangle$ for these altitude zones are shown in Fig. 7 as the filled diamonds, the filled triangles, and the filled squares, respectively. The profile of the extinction coefficient, $\kappa_p(h)$, over the total measurement range, calculated with Eq. (8) separately for each zone, is shown in Fig. 4 as a thick solid curve. For comparison, the extinction coefficient profile extracted from the primary optical depth, $\tau_p(0, h)$, through conventional numerical differentiation is shown in Fig. 4 as the dotted curve; for the differentiation, the running derivative with a vertical height resolution of 500 m was used. The lidar ratio, $S_p(h)$, extracted from Eq. (1) when using the extinction coefficient retrieved with the numerical differentiation is shown in Fig. 5 as the dotted curve. One can see that these data points are much noisier and are more scattered than the stepwise column-integrated lidar ratio.

Let us consider sources of uncertainty in the retrieved extinction coefficient. As follows from Eq. (1), the fractional uncertainty of the extinction coefficient, $\delta\kappa_p(h)$, obeys the simple formula:

$$\delta\kappa_p(h) = \sqrt{[\delta\beta_p(h)]^2 + [\delta S_p(h)]^2}. \quad (12)$$

Two error components influence the accuracy of the retrieved $\kappa_p(h)$. The first component, the fractional uncertainty, $\delta\beta_p(h)$, is due to the uncertainty in the inverted backscatter coefficient, $\beta_p(h)$. The second fractional uncertainty, $\delta S_p(h)$, is related to the difference between the actual lidar ratio, $S_p(h)$, and the stepwise column-integrated ratio, $S_{p,j}$, used for the retrieval of $\kappa_p(h)$. This uncertainty is due to the use of the approximated formula [Eq. (8)] with the

column-integrated ratio, $S_{p,j}$, instead of the mathematically rigid Eq. (1) with the actual $S_p(h)$. Obviously, for any interval, $\Delta h_j = h - h_j$, the difference between variable $S_p(h)$ and the column integrated lidar ratio generally increases with the increase of Δh_j , so a smaller interval will provide a smaller difference between $S_{p,j}(h_j, h)$ and $S_p(h)$. On the other hand, a constant column-integrated ratio, $S_{p,j}$, is used for the inversion instead of the actual $S_{p,j}(h_j, h)$. As follows from Eqs. (6) and (7), the absolute value of the difference between the selected $S_{p,j}$ and the actual $S_{p,j}(h_j, h)$ can be written as

$$|\Delta S_{p,j}(h)| = |S_{p,j} - S_{p,j}(h_j, h)| \leq \frac{\Delta\tau_{p,max}(h)}{\int_{h_j}^h \beta_p(h') dh'}. \quad (13)$$

Equation (13) determines the maximal possible level of the uncertainty, $\Delta S_{p,j}(h)$. Note that the selection of larger intervals is preferable because of the increase of the denominator in Eq. (13). Thus, the requirements for the selection of the optimal height interval are contradictory.

The determination of the length of the intervals within which $S_{p,j}(h_j, h)$ can be taken as a constant is the key point of the method, and different variants can be used. It is impossible to give unanimous criteria for selecting the length of the interval for all possible measurement conditions, the same as for selecting the range resolution for conventional numerical differentiation of the optical depth. In this study, the maximal possible intervals were selected for determining $S_{p,j}$. Numerous simulations showed that in the case of strong aerosol layering, this variant is more robust than the selection of equal height intervals over the altitude range from h_{min} to h_{max} [6]. Other alternatives are currently being examined.

To show how robust the method is when utilized in a real atmosphere, we present in this section an example of lidar experimental data with an extremely high level of random noise in both the inverted optical depth and the backscatter coefficient. The data were obtained during test measurements in a clear cloudless atmosphere in the spring of 2005 with the scanning lidar that operated at the wavelength 355 nm in a multiangle mode. The modernized Kano–Hamilton inversion method proposed by Adam *et al.* [12] was used to extract the height profiles of the optical depth, $\tau_p(0, h)$, and the relative backscatter coefficient, $C\beta_p(h)$. The constant, C , was determined from the relative backscatter coefficient profile using the assumption of an aerosol-free atmosphere at the altitude ~ 4000 m. The profiles of the backscatter coefficient, $\beta_p(h)$, and the optical depth, $\tau_p(0, h)$, with no smoothing are shown in Figs. 8 and 9 as the dotted curves; thin dashed curves 1 and 2 in Fig. 9 show the estimated uncertainty boundaries, $\tau_{p,low}(0, h)$ and $\tau_{p,up}(0, h)$, respectively, and the thin solid curve 3 is the base profile, $\tau_{p,b}(0, h)$. The column-integrated lidar ratios obtained for two height intervals, from 230 to 740 m and from 740 to 3500 m, are shown as the

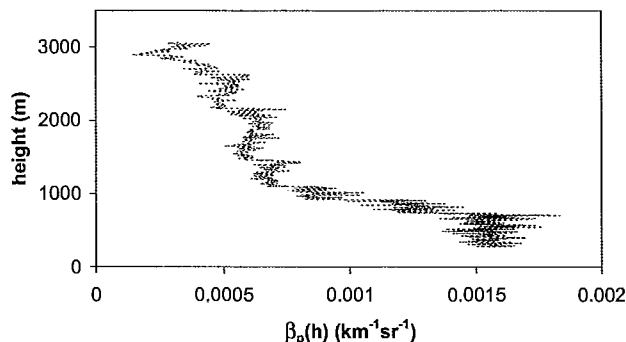


Fig. 8. Noise-corrupted vertical profile of the particulate backscatter coefficient, $\beta_p(0, h)$, measured at the wavelength 355 nm in a clear, cloudless atmosphere.

thick vertical lines in Fig. 10. The values of these are 37.6 and 27.9 sr, respectively. The vertical profile of the particulate extinction coefficient, $\kappa_p(h)$, obtained with the new method is shown as the thick solid curve in Fig. 11. As with the inverted profiles, it is given without any smoothing. Note that the nephelometer readings, which show a good agreement with the lidar data, were not used for the retrieval of the vertical profile of $\beta_p(h)$ from the lidar data. For comparison, the extinction-coefficient profile obtained through conventional numerical differentiation with a range resolution 500 m is shown as the dotted curve; one can see the unphysical negative values over the heights ~ 2200 – 2800 m and a sharp increase over higher altitudes.

There is an important advantage of the new method, which we would like to stress. When processing the shaped profile $\tau_{p,b}(0, h)$ (curve 3 in Fig. 9), the conventional numerical differentiation results in a sharp increase in the derived extinction coefficient at altitudes greater than 2800 m; the calculated $\kappa_p(h)$ here exceeds 2 km^{-1} and is out of the scale in Fig. 10. Obviously, such an increase of $\kappa_p(h)$ in a clear, cloud-

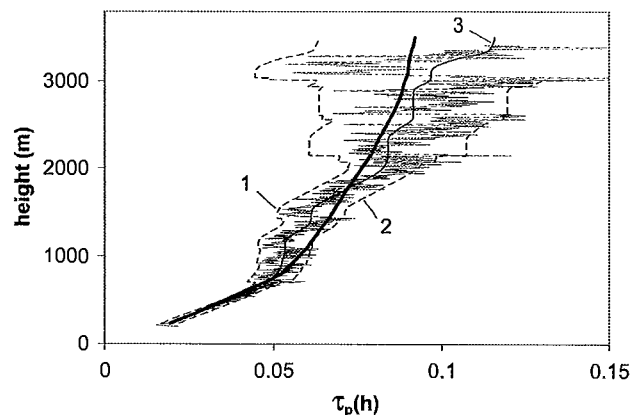


Fig. 9. Dotted curve is the nonsmoothed vertical profile of the particulate optical depth, $\tau_p(0, h)$, measured at 355 nm in a clear, cloudless atmosphere. Dashed curves 1 and 2 show the profiles of $\tau_{p,low}(0, h)$ and $\tau_{p,up}(0, h)$, respectively, and thin solid curve 3 is the base profile, $\tau_{p,b}(0, h)$. The thick solid curve shows the profile $\langle \tau_p(0, h) \rangle$ retrieved assuming a stepwise column-integrated ratio.

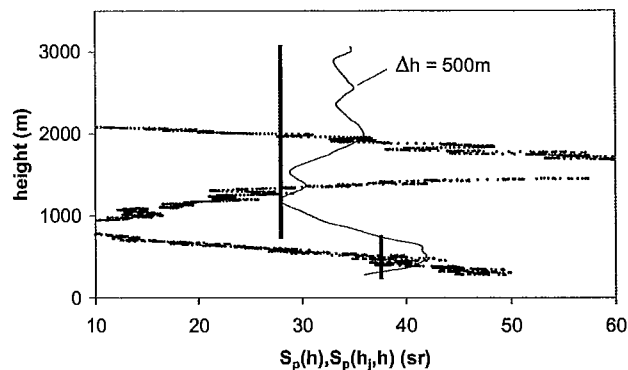


Fig. 10. Column-integrated lidar ratios (the thick vertical lines), the profile of $S_p(h)$ obtained from Eq. (1) (the solid dots), and that obtained with Eq. (14) (the solid curve).

less atmosphere does not appear realistic. Moreover, it disagrees with the profile of $\beta_p(h)$ over these heights (Fig. 8). This disagreement is most likely due to extremely large uncertainty in the derived optical depth and its erroneous increase at the high altitudes.

The profile of $\beta_p(h)$ from which the extinction coefficient is now extracted does not show the increase in aerosol loading over high altitudes. Accordingly, there is no such increase in $\kappa_p(h)$ over high altitudes when using the new method. This effect needs some clarification. As stated above, the profile of the extinction coefficient retrieved with the new method depends on both the optical depth and the backscatter coefficient. If the level of noise in the inverted optical depth, $\tau_p(0, h)$, is small, the profiles $\tau_{p,low}(0, h)$, and $\tau_{p,up}(0, h)$ are close to each other, and the shape of the retrieved extinction coefficient primarily depends on the shape of $\tau_{p,b}(0, h)$. When the uncertainty area between $\tau_{p,low}(0, h)$ and $\tau_{p,up}(0, h)$ increases, the fine structure of the shape $\tau_{p,b}(0, h)$ becomes less influential, whereas the influence of the shape of $\beta_p(h)$ on the profile $\kappa_p(h)$ increases. One can say that the new processing technique “does not trust” the shape of $\tau_{p,b}(0, h)$ when the uncertainty boundaries of the opti-

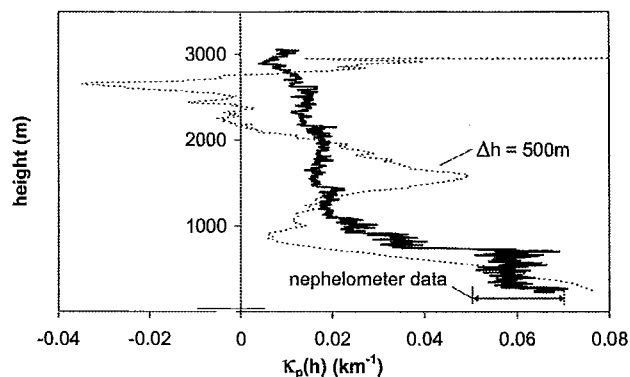


Fig. 11. Profile of $\kappa_p(h)$ retrieved using the new method (the thick solid curve) and that obtained using the conventional numerical differentiation method with a range resolution of 500 m (the dotted curve). In the bottom of the figure, the range of the monotonic change of the ground-based nephelometer data during the lidar measurement is shown.

cal depth are too large; it more relies on the shape of $\beta_p(h)$. In short, when extracting $\kappa_p(h)$, the influence of the profile of $\beta_p(h)$ dominates if the uncertainty in the optical depth is too high.

Using the profile of $\beta_p(h)$ and the extinction coefficient, $\kappa_p(h)$, extracted with the numerical differentiation, one can calculate $S_p(h)$ with the simple dependence between these parameters in Eq. (1). However, as discussed in the study by Shcherbakov [2], the computing of the local values of the lidar ratio, $S_p(h)$, subsequent to $\kappa_p(h)$ can result in extremely noisy profiles for both $\kappa_p(h)$ and $S_p(h)$. The profile of the lidar ratio, $S_p(h)$, calculated with the profile of $\beta_p(h)$ shown in Fig. 8, and the retrieved extinction coefficient, $\kappa_p(h)$, (Fig. 11, dotted curve) is shown in Fig. 10 as solid dots. It is extremely noisy even for lower heights.

With our method, the profile $S_p(h)$ can be extracted more accurately. Now it can be obtained from the column integrated lidar ratio, without using the retrieved extinction coefficient, $\kappa_p(h)$. The differentiation of Eq. (4) and some simple transformations yield the following dependence between the local and column-integrated lidar ratios within the measurement range from $h_{\min}$ to $h_{\max}$,

$$S_p(h) = S_p(h_1, h) + \frac{\int_{h_1}^h \beta_p(h') dh'}{\beta_p(h)} \frac{d}{dh} [S_p(h_1, h)], \quad (14)$$

where h_1 is an arbitrarily selected starting point of the column integrated ratio, $S_p(h_1, h)$. The column-integrated lidar ratio can be found from the original profile of the backscatter coefficient and the base optical depth, $\tau_{p,b}(h_1, h)$, as

$$S_p(h_1, h) = \frac{\tau_{p,b}(h_1, h)}{\int_{h_1}^h \beta_p(h') dh'}. \quad (15)$$

The profile of $S_p(h)$ obtained with Eq. (14) with the numerical differentiation of the profile $S_p(h_1, h)$ using the sliding height resolution 500 m, is shown in Fig. 10 as the solid curve. Up to the heights ~ 1600 m the difference between $S_p(h)$ and the column-integrated lidar ratio does not exceed $\sim 6\%$, and then sharply increases up to $\sim 25\%$, following the sharp increase in the uncertainty boundaries of the optical depth over these heights.

4. Summary

A method is considered that allows determining the extinction coefficient from the backscatter-coefficient profile using the assumption of a stepwise column-integrated lidar ratio. The stepwise lidar-ratio profile is selected that produces the best match between the initial (inverted) profile of the optical depth and that obtained with the inversion of the backscatter coefficient. Unlike conventional methods, the inversion is made without using numerical differentiation.

Numerous simulations and the analysis of experimental data have revealed that the extraction of the extinction coefficient from the backscatter coefficient

using optical depth as a constraint yields much less noisy extinction coefficient profiles as compared to the use of conventional numerical differentiation, especially in a clear atmosphere. When processing data over the interval where the lidar column-integrated ratio is taken as a constant, the profile of the extinction coefficient profile repeats the relative shape of the backscatter coefficient. Accordingly, the relative level of the random noise in the retrieved extinction coefficient is the same as that in the backscatter-coefficient profile from which it is extracted.

The retrieval accuracy may be strongly influenced by the level of uncertainties in the inverted profiles of $\beta_p(h)$ and $\tau_p(0, h)$ and by sharp changes in the local values of the lidar ratio within selected Δh_j . To check whether this effect occurs, the *a posteriori* calculation of the profile of the lidar ratio, $S_p(h)$, with Eq. (14) and the comparison of it with the retrieved column-integrated lidar ratios may be helpful. In this paper, one of the simplest variants of this retrieval technique is analyzed. Alternative variants of the method are currently under examination.

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