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Enhancing Paper Strength by Optimizing Defect Configuration

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Abstract
Poor formation in paper, as denoted by large local variation of mass, tends to reduce maximum tensile strength but has not been well characterized. The effect of grammage variation on tensile strength was studied by introducing carefully placed holes in tensile specimens made of three different paper materials. Previous researchers demonstrated that the point-stress criterion (PSC) accurately predicts failure for paper specimens containing a single hole. Results of comparing the PSC for multiple holes in paper indicated the PSC provided a conservative strength estimate for specimens with multiple holes.

Introduction
The expense of cellulose fibers is one important factor in the cost of cellulosic materials. As nations with high fiber demands increase their dependence on fiber-rich regions, the price of fiber products will become less stable. Cellulose-fiber materials having equivalent performance, but produced with less fiber, offer competitive advantages over traditionally formed products.

The goal of this research was to examine the applicability of the PSC (point-stress criterion) to predict failure in specimens with multiple artificially introduced defects. It is believed that multiple defects in paper products could be located in a beneficial manner to reduce stress and strain in critical regions and therefore increase tensile strength. This research provides additional understanding of the effect of formation and related stress-induced behavior on paper and paperboard.

The implications of formation on local stress and strain behavior have not been characterized in cellulose fiber materials. Variables that contribute to a well-made sheet, such as pulp constituents, grammage, density, drying restraint, and fiber orientation, all have microscopic gradients that contribute to local behavior. The ability to analyze pre-failure stress/strain fields will offer opportunities to improve paper and paperboard behavior.

Researchers have tried to relate tensile strength, a macroscopic behavior, to formation, but no clear picture has emerged. For most newsprint, the tensile failure zone generally passes through regions of low grammage [1]. Norman [2] found a direct relationship between tensile strength loss and large variation of local mass. Nazhad and co-workers [3] observed that tensile strengths of handsheets made from softwood mechanical pulps decrease with decreasing mass uniformity. Mohlin [4] found no general correlation between formation and tensile strength of machine-made papers prepared with hardwood or softwood pulps; in fact, Wathén and Niskanen [5] reported only a weak correlation between formation and strength for machine-made papers. On the other hand, Nordström [6] found that the relationship between formation and tensile strength depends on grammage. Koskinen et al. [7], who introduced artificial “formation” defects to examine their effect on web breaks, noted that defect size, orientation, type (hole or crack), and location were important variables.

Natural defects in poorly formed sheets increase strain disorder [8] and cause regions that exhibit elastic and plastic response to occur simultaneously under uniform global stress. A continuum model developed by Korteoja et al. [9] indicates that large strain variations within a sheet reduce tensile strength. Researchers have mapped full-field displacements of tensile-loaded paper and paperboard using DIC (digital image correlation) to characterize variations in strain [10-13]. These mappings showed large strain distributions even in uniform papers. DIC was able to measure displacements on a sub-millimeter scale and demonstrate large strains near low-grammage regions or holes. In particular, Wong et al. [12] found that local grammage and local tensile strain were inversely proportional. Considine et al. [13] observed compressive strain near low-grammage regions of tensile specimens.

The scale of measurement is an important aspect of evaluating behavior of cellulose fiber materials. Hristopulos and Uesaka [14] examined the strength distribution in paper and suggested that the important representative element was less than 1 mm^2. Other researchers have suggested a larger element [15] based on floc size. Flocs are small regions of uniform grammage within a large region of different grammage. Floc size, grammage variation, and local fiber orientation each contribute to strength behavior [16].

The present research follows the approach by Rhee et al. [17] of introducing multiple holes in a tensile specimen in order to examine changes in local stress and strength caused by stress interaction. This extends the work of Um and Perkins [11, 18], who measured strain in the vicinity of a single hole in a tensile paper strip and compared their measurements with a FEA (finite element analysis) simulation. Using the point-stress criterion, one can predict tensile strengths of specimens with a single hole [18, 19].

Materials

Three types of cellulosic materials were examined in this work. Their physical and mechanical properties are given in Table 1. Sheet thickness was measured with a Mitutoyo® (Kawasaki, Japan) 543-396B Digital Indicator equipped with a 4-mm-diameter ball tip. Elastic moduli and Poisson's ratio were obtained ultrasonically with a Nomura Shoji Corporation (Tokyo, Japan) Sonic Sheet Tester (SST). The SST is equipped with one sensor pair, which operates at 25 kHz; measurements were taken at 5° intervals by rotating the sample on a turntable

Material C is a commercial copy paper, containing about 8% ash. Material L is an unbleached, kraft single-ply linerboard, a material commonly used in structural paperboard products such as corrugated containers. Material F is a commercial filter paper, manufactured by Whatman® International (Maidstone, Kent, UK) and identified as Chromotography Paper, Model 3MM CHR. The latter was chosen as it is 100% cellulose.

Table 1: Properties of materials evaluated

	Material designation		
Property	C	L	F
Grammage (g/m^2)	76	209	187
Thickness (mm)	0.11	0.30	0.31
Density (kg/m^3)	721	688	603
E_{11} (GPa)	7.82	7.75	4.52
E_{22} (GPa)	2.56	3.73	2.12
G_{12} (GPa)	1.63	2.15	1.27
v_{12}	0.17	0.23	0.18
E_{11}/E_{22}	3.05	2.08	2.13

Tensile Testing

Tensile tests were performed on an Instron® (Norwood, Massachusetts) Model 5865 test machine equipped with line-clamp pneumatic grips. Gage length was 125 mm. The test sequence started with a pre-load to 1 N at 12 N/min followed by displacement at a constant speed of 1.5 mm/min that continued to specimen failure. Load and grip displacement data were collected at 10 Hz. All tests were performed in a controlled environment at 50% RH and 23°C.

Some combinations of material type and specimen geometry, described in the next section, were susceptible to buckling along the width of the specimen when loaded in axial tension. To ensure test similarity, all specimens were restrained transversely with glass plates. The two 100-mm-long glass plates that were used as restraints were separated by a gap of twice the specimen thickness and placed at the vertical center of the tensile specimen. These plates were held independent of the test machine and were stationary during tensile testing.

Specimen Geometry

The specimen geometry is shown in Figure 1. The center and auxiliary hole diameters ($2R$) are identical. The auxiliary holes are placed equidistant, c, from the center hole. Hole diameters and locations for the nine specimen geometries investigated are given in Table 2.

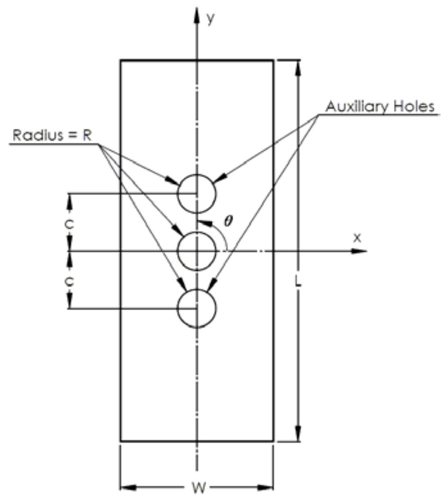


Figure 1: Specimen geometry;
***L* = 125 mm, *W* = 25 mm.**

Table 2: Diameters and locations of holes in tensile specimens

Hole diameter, $2R$ (mm)	Distance between hole centers, *c* (mm)
2.50	3.50, 5.50, 11.50
3.75	4.75, 5.75, 7.75
5.00	6.00, 8.00, 14.00

Specimen Preparation

Holes were created with specially designed tool-steel machined punches that have an inner cutting taper to prevent densification of the material near the hole boundary. Holes were located with alignment fixtures that consisted of a different specific fixture for each specimen geometry. The alignment fixtures ensured hole location along the longitudinal centerline and held the specimen firmly to a backing plate during the cutting process.

Failure Criterion Development

For an infinite uniaxially loaded orthotropic sheet containing a central circular hole of radius R, the tensile stress at the edge of the hole, $\sigma_\theta(R,0)$, (θ = 0, Fig. 1) is given by [20]

$$\sigma_\theta(R,0) = K_T^\infty \sigma_\infty = \sigma_\infty(1+n) \tag{1}$$

where

K_T^∞ is tensile stress concentration factor for an infinite plate,

σ_∞ is far-field tensile stress applied normal to sheet edge parallel to *y*-direction,

$n = \sqrt{2\left(\sqrt{\frac{E_{11}}{E_{22}}} - \nu_{12}\right) + \frac{E_{11}}{G_{12}}}$, and

$E_{11}, E_{22}, \nu_{12}, G_{12}$ are elastic properties, with the 1-direction aligned with the *y*-axis.

The Whitney and Nuismer [21] failure criterion predicts that a sheet containing a central circular hole of radius R fails when the longitudinal stress, σ_y at a characteristic distance, d_0, from the hole edge achieves the unnotched tensile strength of the material in the *y*-direction, σ_U (i.e., failure occurs when $\sigma_y(R+d_0,0)=\sigma_U$). This theory is called the point-stress criterion (PSC) and intuitively suggests that failure occurs when the longitudinal stress throughout the distance adjacent to the edge of the hole, d_0, exceeds the unnotched tensile strength, σ_U. They used the following equation, which is strictly valid for an infinite plate with a hole in it, to describe the stress distribution along the centerline through the hole that is perpendicular to the far-field applied stress, σ_∞:

$$\sigma_y(x,0)=\frac{\sigma_\infty}{2}\left\{2+\left(\frac{R}{x}\right)^2+3\left(\frac{R}{x}\right)^4-(K_T^\infty-3)\left[5\left(\frac{R}{x}\right)^6-7\left(\frac{R}{x}\right)^8\right]\right\} \quad (2)$$

where $x \geq R$.

The PSC can be formally written as

$$\sigma_U=\frac{\sigma_H^\infty}{2}\left\{2+\left(\frac{R}{R+d_0}\right)^2+3\left(\frac{R}{R+d_0}\right)^4-(K_T^\infty-3)\left[5\left(\frac{R}{R+d_0}\right)^6-7\left(\frac{R}{R+d_0}\right)^8\right]\right\} \quad (3)$$

where σ_H^∞ is the strength of the material containing the central circular hole.

Using the same technique as Kortschot and Trakas [19], the stress concentration factor for an infinite plate, K_T^∞, is replaced in Equation (3) by that for our finite-width specimens, K_T, according to the following equation developed by Tan [22]:

$$\frac{K_T^\infty}{K_T}=\left[2-(2R/W)^2-(2R/W)^4\right]/2+(2R/W)^6(K_T^\infty-3)\left[1-(2R/W)^2\right]/2 \quad (4)$$

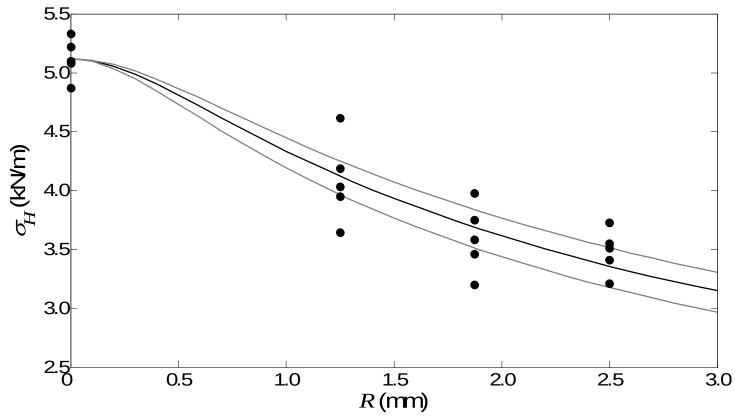
Application of PSC: Specimens without Auxiliary Holes (Single Hole Geometry)

Figure 2 shows the results of applying the PSC of Equations (3) and (4) to the specimen geometries examined in the present work. Each point represents the result from a single tensile test; five replications were performed for each specimen geometry. The solid black line represents a nonlinear regression fit of Equation (3) to the test data, where σ_U is the measured unnotched strength (results shown at R = 0) and d_0 is the sole fit parameter. The failure criterion characterizes the data well, even for specimens with 2.5-mm radius holes, which result in a 20% reduction in effective specimen width. The solid gray lines represent the span of the 95% confidence interval of d_0.

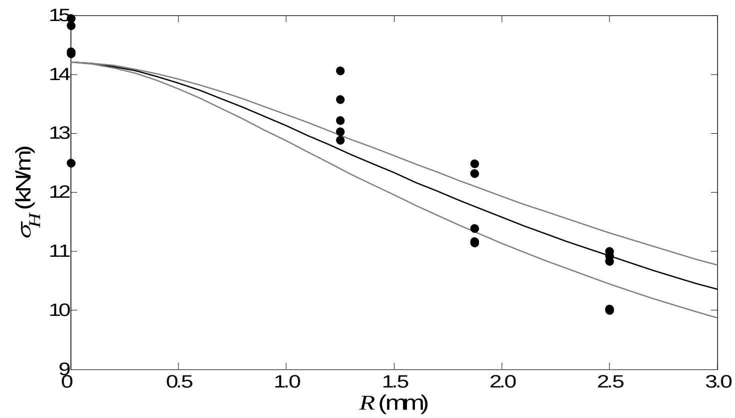
Table 3 gives d_0 and its 95% confidence interval for the present materials as determined from regression analyses of Figure 2. Although pulp fiber lengths were not measured for these materials, we offer the observation that these calculated d_0 values approximately correspond to expected fiber lengths for these materials. Copy papers, such as Material C, are made from hardwood fibers that have an average fiber length of 1 mm. Linerboards, such as Material L, are made from recycled softwood fibers and generally have an average fiber length less than 2.0 mm. Cotton linters, the fibers in Material F, can range from 2.0 to 8.0 mm in length. Inter-fiber bonding also likely contributes to d_0. Density is one measure of inter-fiber bonding, and the values for d_0 in Table 3, which are obtained from the PSC of Figure 2, are ranked inversely proportional to the densities of Table 1.

Application of PSC: Specimens with Auxiliary Holes

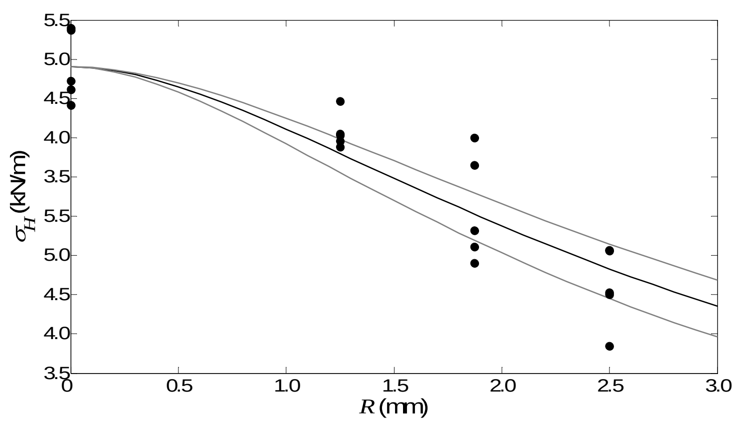
Table 4 lists mean tensile strengths of specimens with auxiliary holes. In many cases, mean tensile strength decreases with greater distance, c, between the central and auxiliary holes. Most specimens with auxiliary holes failed at either the upper or lower auxiliary hole suggesting the central hole was shielded by the adjacent holes. Shaded cells in Table 4 indicate that the multi-hole specimen had a mean tensile strength that was greater than a specimen with a single hole of same radius. More statistical analysis is required, but the presence of auxiliary holes tended to improve tensile strength.



(a) Material C



(b) Material L



(c) Material F

Figure 2: PSC calculations for single hole specimens; data at *R* = 0 represent tensile strength of specimens without holes, σ_U . Gray lines represent the 95% confidence interval.

Table 3: Distance from hole edge, d_0, for PSC

Material	d_0 (mm)	95% confidence interval (mm)
C	1.07	±0.15
L	1.86	±0.26
F	2.07	±0.27

Table 4: Mean tensile strengths of specimens with auxiliary holes[a]

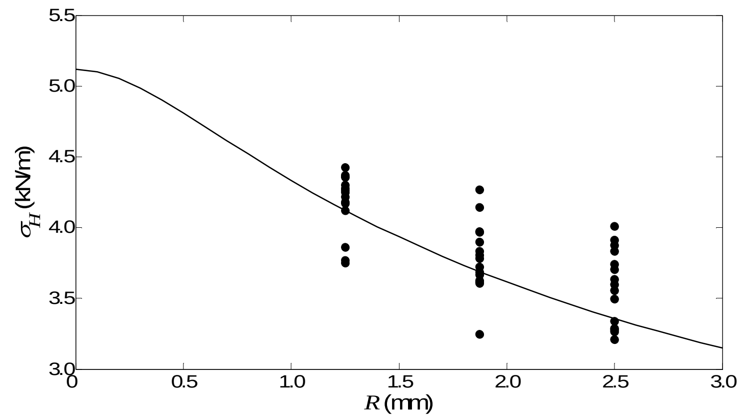
		Mean tensile strength (kN/m)								
		$2R$ = 2.50 mm			$2R$ = 3.75 mm			$2R$ = 5.00 mm		
Material	c (mm):	3.50	5.50	11.50	4.75	5.75	7.75	6.00	8.00	14.00
C		4.34	4.24	3.96	3.89	3.81	3.67	3.80	3.57	3.38
L		13.82	13.38	12.77	12.14	11.60	12.24	11.53	11.03	11.05
F		4.73	4.50	4.59	4.26	4.31	4.39	3.78	3.98	3.82

[a] Shaded cells indicate greater mean strength than for specimen with single hole of same diameter.

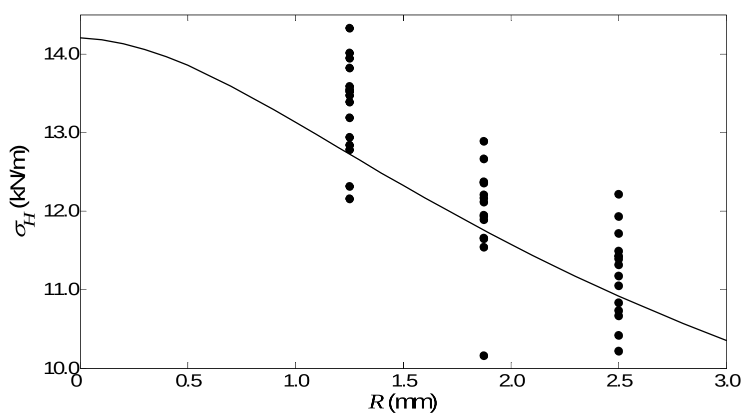
A material property, the parameter d_0, which is the horizontal distance beyond the edge of a hole in Figure 1 over which σ_y exceeds the ultimate stress of the specimen without holes, σ_U , should be applicable to specimens with multiple defects. Figure 3 shows the same nonlinear regression lines as in Figure 2 but with strength data from specimens with auxiliary holes (i.e., no new regression was performed). All strength data for a specimen with auxiliary holes, regardless of hole spacing, are plotted at the same radii (recognizing that for any particular case, the size of the central and auxiliary holes are equal).

In general, application of PSC to specimens with auxiliary holes produced a conservative strength estimate, particularly for Materials C and L, which suggests lower-bound strength can be estimated with knowledge of the largest defect in the sample. Unmodified papers always contain low-mass regions, called voids, analogous to holes in the present study. The results offer a tool for estimating the effect of voids on paper's inherent strength. A common void size in copy papers is R = 1.5 mm. At this radius, Figure 3a suggests that reduction of strength from the peak possible at R = 0 mm is about 25%. Near R = 1 mm, the slope of the PSC curve is greatest, indicating greatest sensitivity of strength to process changes that influence R.

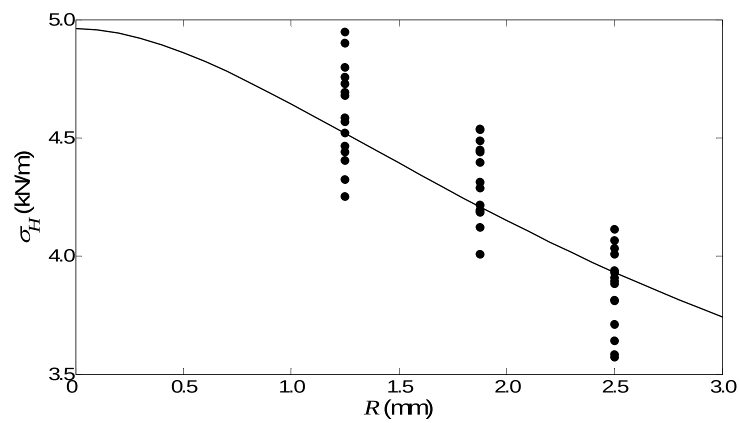
Light-weight papers are valued for the appearance of visual uniformity when viewed in transmitted light. Small void diameter is preferred. The same conditions favorable for sheet appearance are therefore favorable for strength. Conditions that compromise appearance also compromise strength. However, Figure 3a suggests that the sensitivity of strength to process changes is reduced if void size is reduced below R = 0.5 mm, where the PSC curve flattens. This is desirable from both a strength and appearance point of view. It helps to explain the papermaker's constant attention to formation.



(a) Material C



(b) Material L



(c) Material F

Figure 3: PSC applied to specimens with multiple holes.

Conclusion

Large variations of mass distribution in paper are known to compromise strength. To model the effect, holes have been added to three paper materials subjected to tensile testing. A single, central hole reduces strength according to its radius in a manner well described by the point-stress failure criterion (PSC). The PSC has an adjustable parameter used to fit single-hole data. It corresponds to the distance d_o from the hole perimeter over which local stresses exceed the failure stress of the unmodified sheet. This distance was found to range from 1–2 mm, which is comparable to mean lengths of many papermaking fibers. Among the three materials studied here, copy paper alone incorporated shorter hardwood fibers with longer softwood fibers, and it was found to be at the lower end of this range. For specimens having adjacent auxiliary holes of the same radius, PSC provided a conservative strength estimate, with 66% of the individual test results having a higher strength than the value predicted for a single hole. This suggests that the present approach may ultimately add insight to the relation between the structure of paper and its strength.

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