Local constitutive behavior of paper determined by an inverse method

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The macroscopic behavior of paper is governed by small-scale behavior. Intuitively, we know that a small-scale defect with a paper sheet effectively determines the global behavior of the sheet. In this work, we describe a method to evaluate the local constitutive behavior of paper by using an inverse method.

Many different techniques have used an inverse method to determine specific, hard-to-measure properties. An inverse method is generally described as a procedure that varies constitutive behavior in a numerical model to agree with experimental measurements made on a material.

FPL has successfully used digital image correlation (DIC) to measure the local deformation of a paper tensile specimen. The full-field deformation has been measured on handsheet specimens at two grammages with specifically-introduced formation features. Using ANSYS® FEM (finite element method) model, the tensile behavior was characterized with a 4-node, plane stress, nonlinear, quadrilateral element.

Figure 1 shows the local caliper of the center portion of a handsheet specimen that had a low-grammage region. The nominal grammage of the specimen was 60 gm/m^2. The local caliper was measured via light transmission. The caliper in the low-grammage region was approximately 30% less than the surrounding area. The segments indicate the elements used in the FEM model. The actual specimen was 100 mm. long and 15 mm. wide. To improve resolution, the camera viewed only the center portion.

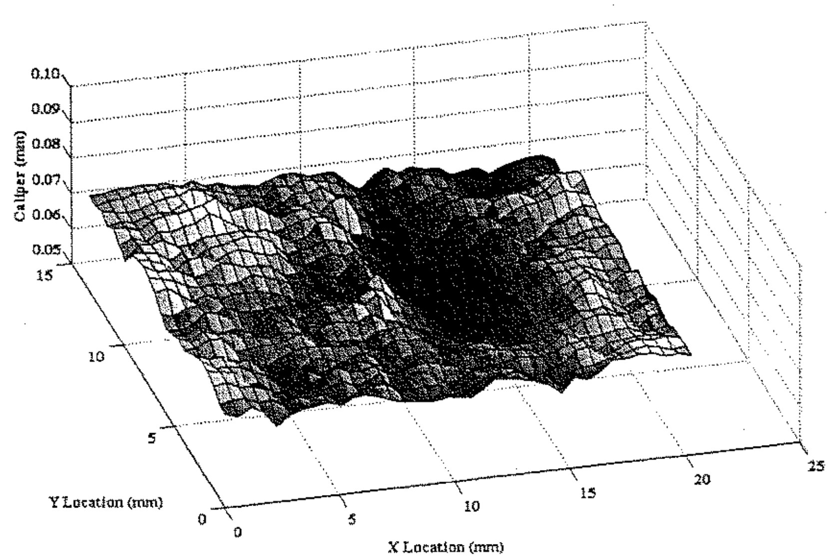

Figure 1: Caliper of Handsheet Specimen with Low-Grammage Region

The global constitutive behavior was measured during tensile testing. The σ-ε behavior for a specimen is shown in Figure 2. ANSYS® contains a non-linear, isotropic constitutive law called NLISO and its functional form is:

$$\sigma = k + R_0\varepsilon + R_\infty\left(1 - e^{-b\varepsilon}\right) \qquad (1)$$

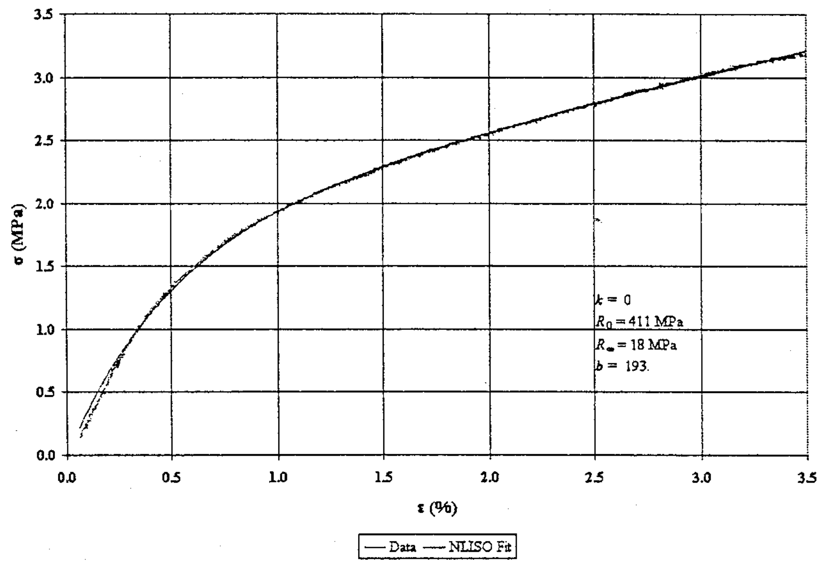

Figure 2: NLISO Fit for ANSYS Input

Initially, each element in the FEM model was given the identical σ-ε relation as determined from a typical tensile test. The only difference in properties for each element was the caliper.

For the same specimen shown in Figure 1, the strains as determined by the FEM model and as measured by DIC are shown in Figure 3 (FEM) and Figure 4 (DIC). The global stress in the FEM model was 21 MPa. The global mean strain at this stress was 0.0186 (m/m). The image with the nearest global mean strain was selected for comparison with the FEM model. The autocorrelation for the strain contours in Figure 3 and Figure 4 was r^2 0.94.

Clearly, the strain surface shown in Figure 4 has more features than given by the FEM strain in Figure 3. One aspect not included in the FEM model is the viscoelasticity, or time-dependent nature, of paper. Comparison of the sequential strain contours as determined by DIC showed strain 'waves' traversing the length of the specimen. These waves occur because the deformation of one end of a viscoelastic tensile specimen will take a finite length of time to equalize strain throughout the specimen. Some of these waves are visible in Figure 4. One reason to believe that these waves are not artifacts of the measurement system was the global mean strain as measured by DIC and as measured by grip deformation agreed very well.

Another aspect of the strain variation shown in Figure 4 could be the non-uniform σ - ε behavior of the material. An inverse method can be used to better model this aspect. The strain measured by DIC had a very high coefficient of variation, often greater than 50%. By comparing regions of greatest difference between the FEM model and DIC, an element's constitutive behavior can be changed to better correlate with the DIC measurements.

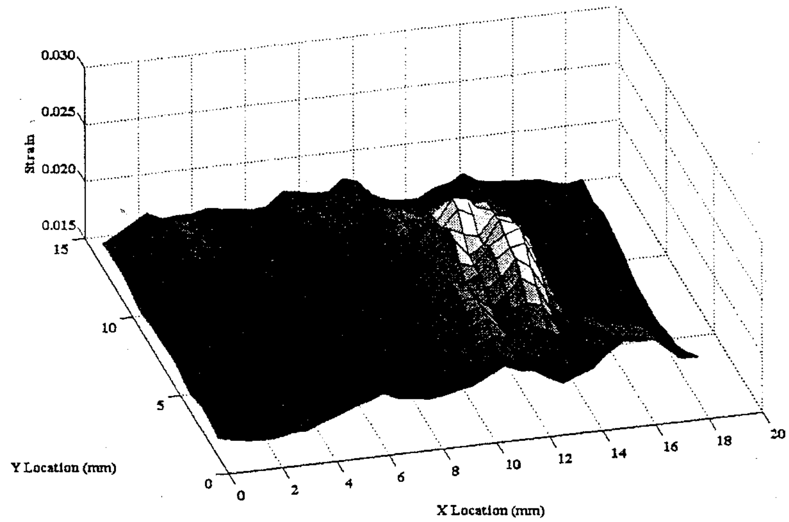

Figure 3: FEM Model Strain at 21 MPa

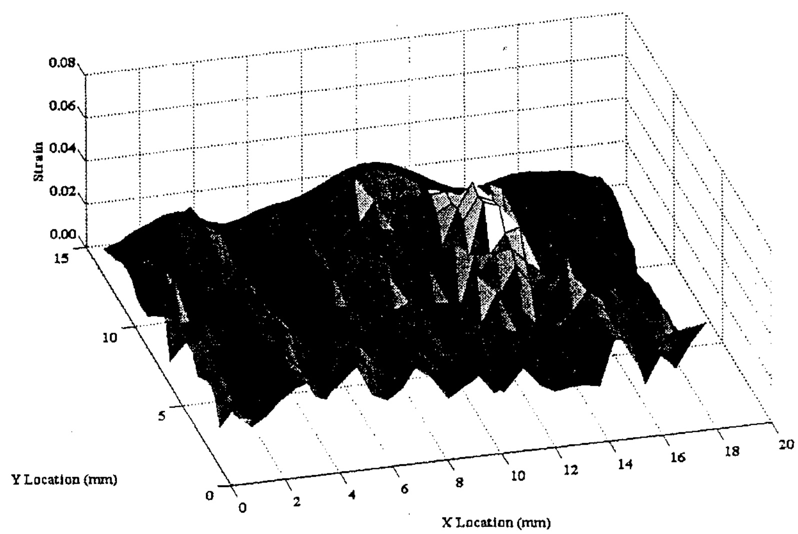

Figure 4: DIC strain results for particular image with strain similar to Figure 3.

The differences between the FEM calculated strain and the DIC measured strain were compared at several stress levels. The parameters in Equation 1 were changed for elements with the greatest differences across the stress levels. Some elements had the constitutive behavior changed to produce more strain (made 'softer') and other elements were changed to produce less strain (made 'stiffer').

Figure 5 shows the local σ_x at a global stress of 21 MPa after 4 iterations. The correlation of FEM strain and the DIC strain was improved. The result of these iterations was that the elements within the low-grammage region needed to be stiffened and elements on the left and right of the low-grammage region needed to be softened.

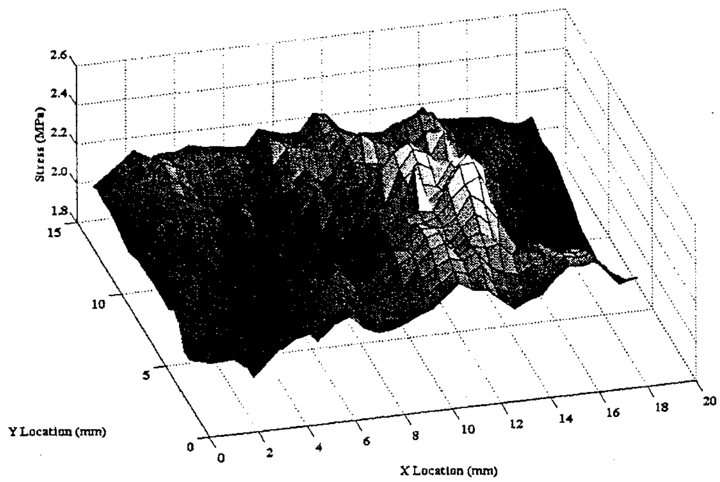

Figure 5: Local σ_x at σ_{global} = 21 MPa after 4 iterations.

This work showed that local constitutive behavior varies significantly throughout a paper handsheet specimen. In the example given, the low grammage region was shielded by the surrounded area. An inverse method was used to determine local constitutive behavior. While the results of an inverse method do not yield a unique solution for local behavior, the method can provide insight as to the nature and severity of flaws within paper materials.

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