Retransformation bias in a stem profile model

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An unbiased profile model, fit to diameter divided by diameter at breast height, overestimated volume of 5.3-m log sections by 0.5 to 3.5%. Another unbiased profile model, fit to squared diameter divided by squared diameter at breast height, underestimated bole diameters by 0.2 to 2.1%. These biases are caused by retransformation of the predicted dependent variable; the degree of retransformation bias depends upon choice of dependent variable in the regression model, variance of its prediction errors, and the bole position of the desired prediction. Retransformation biases were greatest near the merchantable top of large trees. Equations are given that reduce the magnitude of these biases, but accurate variance models are required. Additional biases are identified for more complex transformations of stem profile models.

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Un modèle non biaisé de profil de tige utilisant le diamètre divisé par le diamètre à hauteur de poitrine surestimait de 0,5 à 3,5% le volume de sections de billes de 5,3 m. Un autre modèle non biaisé utilisant le diamètre au carré divisé par le diamètre au carré à hauteur de poitrine sous-estimait les diamètres de 0,2 à 2,1%. Ces biais sont dus à la retransformation de la variable dépendante calculée; l'importance du biais dû à la retransformation dépend du choix de la variable dépendante dans le modèle de régression, de la variance des erreurs de prédiction du modèle et de la position de la bille pour laquelle la prédiction est désirée. Les biais de retransformation sont maximum près de l'extrémité du diamètre marchand chez les gros arbres. Des équations qui réduisent l'ampleur de ces biais sont présentées, mais des modèles précis de la variance sont nécessaires. D'autres biais sont identifiés dans le cas de transformations plus complexes des modèles de profil de tige.

[Traduit par la revue]

Introduction

Coefficients in profile models are usually fit using unweighted least squares regression methods. To reduce heterogeneous variance, transformations of diameter (d) and diameter at breast height (D) normally serve as the dimensionless dependent variable or response variable; examples include d/D or $(d/D)^2$. However, predictions of these transformed dependent variables must be subsequently retransformed to produce practically important estimates, such as d and d^2, which are needed to predict merchantable log volumes and stem diameters. A regression model produces unbiased predictions of its dependent variable for the entire sample of trees and section measurements that are used to fit the model; however, retransformations of the model predictions are not necessarily unbiased, even across the same sample (Kilkki and Varmola 1979). A retransformation can skew the distribution of prediction errors and can cause retransformation bias by shifting the mean of the distribution. Retransformation bias was first studied in detail by Box and Cox (1964) for the general family of power transformations. Flewelling and Pienaar (1981) discuss various methods to correct for retransformation bias in biological growth models that are fit using a logarithmic transformation, which is a special case of the power transformation. Taylor (1986) provides an approximate method to correct transformation bias in the general family of power transformations.

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Retransformation for volume predictions

First, consider the problem of estimating volume of bole segments. Volume can be computed by integrating a profile model using solids of revolution. Such integration is the summation of volumes for individual cylinders that represent the stem of a tree, where the number of cylinders approaches infinity. Calculation of the volume of a cylinder requires an estimate of the cross-sectional area d^2. Other volume predictors, such as Smalian's formula, also require estimates of cross-sectional area.

If D is a known fixed value (i.e., measured with negligible error), then an unbiased estimate of $(d/D)^2$ can be directly retransformed to an unbiased estimate of cross-sectional area, $\widehat{d^2} = D^2\,(\widehat{d/D})^2$ and therefore, an unbiased estimate of volume. However, the squared retransformation of an unbiased estimate of d/D will produce a biased estimate of $(d/D)^2$ and hence, a biased estimate of volume. Consider the following statistical model for the estimate $\widehat{d/D}$ of the true d/D, where ϵ_1 is the unknown random error:

$$[1] \quad \widehat{d/D} = d/D + \epsilon_1$$

The error propagation model for the squared retransformation of [1] is simply:

$$[2] \quad (\widehat{d/D})^2 = (d/D)^2 + 2(d/D)\epsilon_1 + \epsilon_1^2$$

For an unbiased estimate of d/D, the expected value, denoted by $E[\ ]$, of ϵ_1 is zero (i.e., $E[\epsilon_1] = 0$), and [2] produces the following expectation:

$$[3] \quad E[(\widehat{d/D})^2] = (d/D)^2 + E[\epsilon_1^2]$$

TABLE 1. Estimated coefficients for the Max and Burkhart (1976) regression profile model for grand fir

Regression response variable	b_1	b_2	b_3	b_4	a_1	a_2	MSE[a]
$(d/D)^2$	−2.010	0.957	−0.858	118.5	0.722	0.062	0.01113
d/D	−2.162	0.942	−0.858	121.1	0.699	0.061	0.00415

[a]Residual mean squared error (MSE) has dimensionless units of $(d/D)^4$ for the $(d/D)^2$ model, and $(d/D)^2$ for the d/D mdoel.

(This treats the true d/D as a fixed value, i.e., $E[(d/D)^2] = (d/D)^2$, although the value of d/D is unknown.) A profile model that produces unbiased estimates of d/D will produce biased estimates of $(d/D)^2$ and hence volume, because cross-sectional area tends to be overestimated, as shown in eq. 3. The magnitude of this bias will depend upon $E[\epsilon_1^2]$, i.e., the variance of ϵ_1 assuming $E[\epsilon_1] = 0$.

Retransformation for diameter predictions

Consider estimation of stem diameters, which are needed to apply merchantability standards and board-foot scaling rules. An unbiased estimate of d/D can be directly retransformed to an unbiased estimate of diameter, $\hat{d} = D\widehat{(d/D)}$, if D is a known fixed value. However, the square-root retransformation of an unbiased estimate of $(d/D)^2$ produces a biased estimate of d/D and hence, biased estimates of diameter d. Let the statistical model for $(d/D)^2$ be

$$[4] \quad \widehat{(d/D)^2} = \widehat{(d/D)^2} + \epsilon_2$$

where ϵ_2 is the random error, and $E[\epsilon_2] = 0$. An estimate of the unknown d/D is made using the square-root retransformation of [4]. The Taylor series expansion of [4] near $\epsilon_2 = 0$ produces the following expected value considering error propagation from [4]:

$$[5] \quad E[[\widehat{(d/D)^2}]^{1/2}] = d/D - [E[\epsilon_2^2]\,(d/D)^{-3}]/8 + E[\epsilon_2^3]\,(d/D)^{-5}\,(3/48) - \ldots$$

Equation 5 demonstrates that the square-root retransformation contains bias, the magnitude of which depends upon a complex function of d/D and the higher moments of ϵ_2.

Data

We investigated the effects of retransformation bias using grand fir (*Abies grandis* (Dougl.) Lindl.) from Oregon and Washington, United States (Czaplewski et al. 1989). Diameters were measured at stump and breast heights and at the ends of 5.3-m logs to a 15 cm top diameter. Data were available for 7840 diameter measurements for 4944 logs from 1448 trees at various heights above ground (h). Diameter at breast height (D) averaged 55 cm, and total height (H) averaged 25 m.

Profile models

Two types of stem profile models were employed. Both types were separately fit to two response variables: d/D and $(d/D)^2$. The first type, a "classification" profile model, uses means for 175 classes of D and h/H. The 7840 diameter measurements were partitioned into 7 classes of D and 25 classes of h/H within each D class; each of the resulting 175 classes had at least 40 observations to compute a sample mean. This model is applied in a two-step procedure. First, the appropriate D and h/H classes are determined for the desired prediction; then relative diameter is predicted using the sample mean for d/D or $(d/D)^2$ from the class determined in the first step. Least squares methods are not needed for such a model, as it is important to avoid confounding conditional biases introduced by regression methods (e.g., consistent overestimates for small trees or underestimates for large trees).

The second type of profile equation is the segmented regression model recommended by Max and Burkhart (1976):

$$[6] \quad f[d/D] = b_1[(h/H) - 1] + b_2[(h/H)^2 - 1] + b_3[a_1 - (h/H)]^2 I_1 + b_4[a_2 - (h/H)]^2 I_2$$

where

$$I_i = \begin{cases} 1 \text{ for } (h/H) < a_i \\ 0 \text{ otherwise} \end{cases}$$

a_i = estimated join points (nonlinear with respect to h/H)

b_i = estimated regression parameters

This regression model was developed for two dependent variables: $f[d/D] = (d/D)^2$ and $f[d/D] = d/D$. Model coefficients were estimated by unweighted nonlinear least squares and are given in Table 1.

Heterogeneous variance models

It is usually assumed that variance of d/D or $(d/D)^2$ is homogeneous. However, this assumption is only approximately true. Variance changes with h/H, as shown in Fig. 1. By definition, variance at the top of a tree is zero because diameter is exactly zero. Variance is low near breast height and higher within the upper bole and near stump level. Heterogeneous variance of ϵ_1 and ϵ_2 (i.e., $E[\epsilon_1^2]$ and $E[\epsilon_2^2]$ for $E[\epsilon_1] = E[\epsilon_2] = 0$) affects retransformation bias in eqs. 3 and 5.

Two types of models were used for predicting heterogeneous variance and were fit using the same grand fir data that were used to fit the profile models. Classification variance models use the sample variance for each of the 175 classes of D and h/H that were used for the classification stem profile model. The second type was regression models that predict variance. The following segmented linear regression model was fit to the sample variances $E[\epsilon_1^2]$ for each D and h/H class in Fig. 1A; it had an R^2 value of 0.87:

$$[7] \quad \hat{E}[\epsilon_1^2] = 0.051(1 - h/H)^2 - 0.050(1 - h/H)^3 + 1.081(0.0469 - h/H)I$$

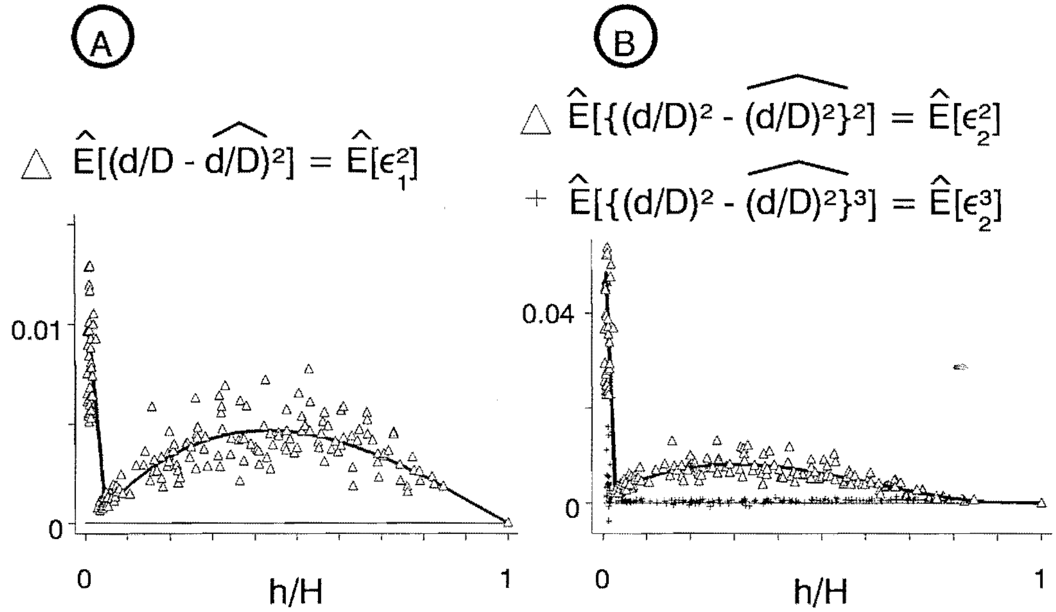

FIG. 1. Estimated variances $\hat{E}[\epsilon_1^2]$ and $\hat{E}[\epsilon_2^2]$ from error models [1] and [4]. Note the heterogeneity of variance. The classification variance models are shown as points, using 175 classes of D and h/H for 7840 measurements. The regression variance models [6] and [7] are shown as solid lines. Higher moments (e.g., ϵ_2^3) are small (B) and have little practical effect on retransformation bias.

$$I = \begin{cases} 1 \text{ for } (h/H) < 0.0469 \\ 0 \text{ otherwise} \end{cases}$$

The following model for the variance of ϵ_2 (Fig. 1B) had an R^2 value of 0.93:

$$[8] \quad \hat{E}[\epsilon_2^2] = -0.0063(1 - h/H) + 0.0674(1 - h/H)^2 - 0.0600(1 - h/H)^3 + 2.3046(0.0469 - h/H)I$$

Both regression models are shown in Fig. 1. The join point ($h/H = 0.0469$) in eqs. 7 and 8 was fixed near the lower extreme of relative height for breast height measurements.

Correction for bias in cross-sectional area predictions

Figure 2A shows the mean of residual errors (i.e., estimated bias) for predictions of $(d/D)^2$ that were retransformed from unbiased estimates of d/D using the classification profile model. Retransformation bias was greatest near stump height and midbole. Equation 3 suggests the following correction for this bias:

$$[9] \quad \widehat{(d/D)^2} = [\widehat{(d/D)}]^2 - \hat{E}[\epsilon_1^2]$$

where the estimated variance $\hat{E}[\epsilon_1^2]$ is used in place of its true, but unknown, value. The approximation $E[\epsilon_1^2] = \hat{E}[\epsilon_1^2]$ was also used by Baskerville (1972) to correct for bias in logarithmic retransformations, and this approximation is consistent and asymptotically unbiased for large sample sizes (Flewelling and Pienaar 1981).

If it is assumed the variance of ϵ_1 is homogeneous (i.e., $E[\epsilon_1^2]$ is estimated by the overall sample variance of residual errors), then [9] corrects the systematic retransformation bias, but a pattern in the bias remains (Fig. 2B). This pattern can be reduced assuming variance is heterogeneous. Regression model [7] for $\hat{E}[\epsilon_1^2]$, when used in correction [9], eliminates most of this pattern (Fig. 2C). Using the classification variance model for $\hat{E}[\epsilon_1^2]$, correction [9] eliminated all of the pattern (Fig. 2D).

Correction for bias in diameter predictions

Figure 3A gives the mean residual errors for estimates of d/D that were retransformed using the square root of unbiased predictions of $(d/D)^2$ from the classification profile model. Estimated bias is largest at stump height and near the merchantable top. Taylor series approximation [5] suggests a correction for this bias:

$$[10] \quad \widehat{d/D} = [\widehat{(d/D)^2}]^{1/2} + \{\hat{E}[\epsilon_2^2]\,[\widehat{(d/D)^2}]^{-3/2}\}/8,$$

where estimates of $(d/D)^2$ and $E[\epsilon_2^2]$ are used in place of their true, but unknown, values. This is a special case of correction given by Taylor (1986) for the more general Box–Cox power transformation. The third-order term of the Taylor series expansion [5] is not used because $\hat{E}[\epsilon_2^3]$ is very small, as shown in Fig. 1B. This term has negligible effect on the correction equation, which was confirmed by adding $\hat{E}[\epsilon_2^3]$ to the correction in eq. 10. Higher order terms in the Taylor series expansion are also omitted because they have less effect than ϵ_2^3.

Correction [10] performed very poorly under the assumption of homogeneous variance, as shown in Fig. 3B (note change in scale of y-axis); it grossly overcorrected for bias near the merchantable top. This problem also occurred when the estimated third moment ($\hat{E}[\epsilon_2^3]$) was included in eq. 10. It is necessary to predict variance accurately near the tree top for correction [10] because $[(d/D)^2]^{-3/2}$ approaches infinity at the tree tip. Assuming heterogeneous variance, correction [10] with the regression variance model [8] for $\hat{E}[\epsilon_2^2]$ greatly reduced retransformation bias (Fig. 3C) and eliminated bias below the midpoint of the bole when using the classification variance model (Fig. 3D).

Practical importance of retransformation bias

The regression and classification profile models were used to estimate volumes of 5.3-m logs. A total of 4944 logs were used from the same data set used to fit the profile and

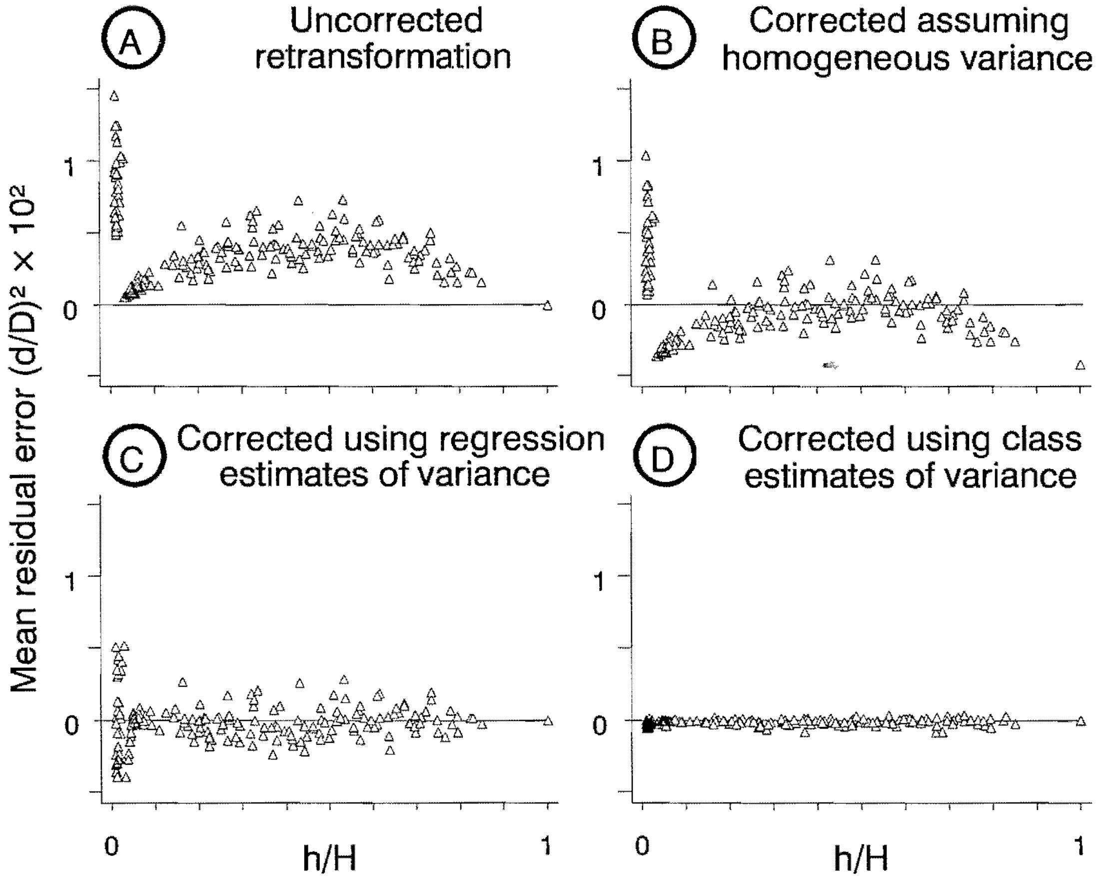

FIG. 2. Estimated bias (i.e., mean residual error) in retransformed predictions of $(d/D)^2$, which are used to predict volume, from the classification profile model fit to d/D. Each point is the mean residual error for one of 175 classes of D and h/H from 7840 measurements. The uncorrected retransformation (A) produces a biased estimate of $(d/D)^2$. The retransformation corrected using [9] assuming homogeneous variance is unbiased, but patterns in the mean residual errors remain (B). The retransformation corrected using regression model [7] to estimate heterogeneous variance, when used with bias correction [9], reduces the pattern in mean residual error (C). Correction [9], using the very accurate class estimates of variance, eliminates retransformation bias (D).

variance models. Estimates from models fit to d/D were retransformed (i.e., squared) to produce estimates of $(d/D)^2$. All estimates of $(d/D)^2$ at log ends were multiplied by D^2, and the resulting cross-sectional area estimates were used in Smalian's formula to predict volume of each log in the data set. Volume estimates retransformed from models that predict d/D were compared with the corresponding volume estimates from models that directly predict $(d/D)^2$. Residual differences between model predictions were partitioned into seven categories of D (the same classes used in the classification models) and three types of logs: the butt log, the top log, and the remaining midbole logs in each tree.

The Max and Burkhart regression profile model that used d/D as the dependent variable produced volume estimates consistently higher than volume estimates from the same model that used $(d/D)^2$ as the dependent variable. Differences in volume estimates between models fit to d/D and $(d/D)^2$ are displayed in Fig. 4A. The mean difference in estimated log volume varied between 0.5 and 1.0% for butt and midbole logs, and ranged from 1.0 to 3.5% for the top logs. (Percent error is computed using the mean residual error divided by the mean butt, midbole, or top log volumes for each D diameter class.) When the retransformation of d/D to $(d/D)^2$ was corrected using [9], with regression model [7] used for $\hat{E}[\epsilon_1^2]$, differences for butt and midbole logs were eliminated, and differences for top logs were reduced by two-thirds (Fig. 4B). The classification profile models produced similar results (Figs. 4C and 4D), although the correction was more successful for top logs.

The regression and classification profile models were also used to estimate upper and diameters (d) for three categories of 5.3-m logs: the butt log, the top log, and the remaining midbole logs in each tree. Such predictions are important for estimating merchantable board-foot volume. The square-root retransformation was applied to models that predict $(d/D)^2$. All estimates of d/D were multiplied by D to produce diameter estimates. Fitting the Max and Burkhart regression profile model to $(d/D)^2$ produced retransformed diameter estimates smaller than those from the same model fit to d/D. The differences in diameter estimates between models fit to d/D and $(d/D)^2$ are given in Fig. 5A. Differences averaged 0.08 to 0.30 cm (0.2 to 0.7%) for butt and midbole log end diameters, and 0.12 to 0.56 cm (0.7 to 2.1%) for the merchantable top diameter. By using correction [10] and regression model [8] for $\hat{E}[\epsilon_2^2]$, these differences were almost eliminated (Fig. 5B) for butt and midbole end diameters, and differences for merchantable top diameters were reduced by half. Improvements from cor-

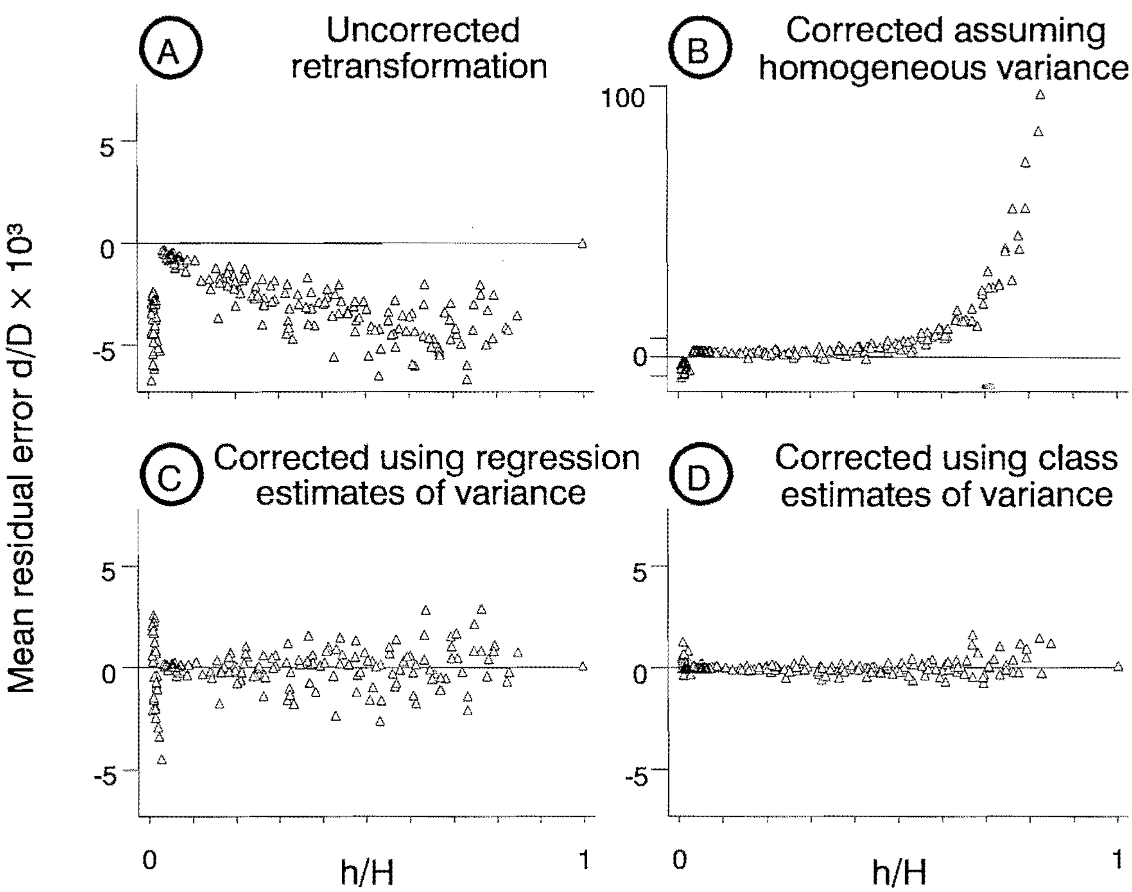

FIG. 3. Estimated bias in predictions of d/D retransformed from unbiased estimates of $(d/D)^2$ from the classification profile model. Such predictions are used to apply merchantable diameter limits and board-foot scaling rules. On average, the direct retransformation produced biased estimates of d/D, especially near stump and upper bole heights (A). Correction eq. 10 performed poorly assuming homogeneous variance (B); note change in scale. However, [10] more reliably corrected retransformation bias using regression model [8] for heterogeneous variance of ϵ_2 (C), and retransformation bias was nearly eliminated using [10] with the highly accurate classification variance model for $E[\epsilon_2^2]$ (D).

rection [10] were greater using the classification profile model with the classification variance model (Figs. 5C and 5D).

Discussion

Retransformation bias affects predictions of either volume or stem diameter in profile models that are fit to transformations of d and D. For profile models fit to d/D, retransformation bias was greatest for estimated volume of top merchantable logs from large trees (Fig. 4A). For profile models using $(d/D)^2$ as the regression response variable, retransformation bias was greatest for estimates of merchantable top estimates from large trees (Fig. 5A). A small error in an estimated top diameter can misclassify the product class of an entire log and can have substantial effect on error in estimated board-foot scale, or in cubic volume of various products from a single stem.

Compared with the classification profile models (Figs. 4D and 5D), differences between the Max and Burkhart regression profile models after bias correction were greater than expected; the remaining differences were most notable near the merchantable top. These differences might be caused by different patterns of heterogeneous variance on the nonlinear regression methods used for these models; more weight is placed on those residuals having higher variance (Fig. 1). Therefore, residuals with low variance might be biased, i.e., $E[\epsilon] \neq 0$, which violates assumptions used in eqs. 3 and 5. Classification profile models, which do not use least squares methods, are unaffected by heterogeneous variance and were used to avoid this confounding source of bias. However, classification profile models are not normally used for practical applications because these models represent stem profile as a stair-step function.

Corrections [9] and [10] are merely approximations because they use estimates of d/D or $(d/D)^2$ and their estimated variances, rather than their true, but unknown, values. Furthermore, correction [10] ignores the higher moments of the residual error distribution. Bias after application of corrections [9] and [10] might be caused in part by these approximations. This source of bias is probably higher for the square-root retransformation of $(d/D)^2$ near the merchantable top in [10] than for the squared retransformation of (d/D) in [9], which is suggested by comparison of bias using the classification models (Figs. 2D and 3D). However, this source of bias was small.

Success of correction eqs. 9 and 10 depends upon reliable predictions of heterogeneous variance, especially near the merchantable top. Some data sets will not have sufficient

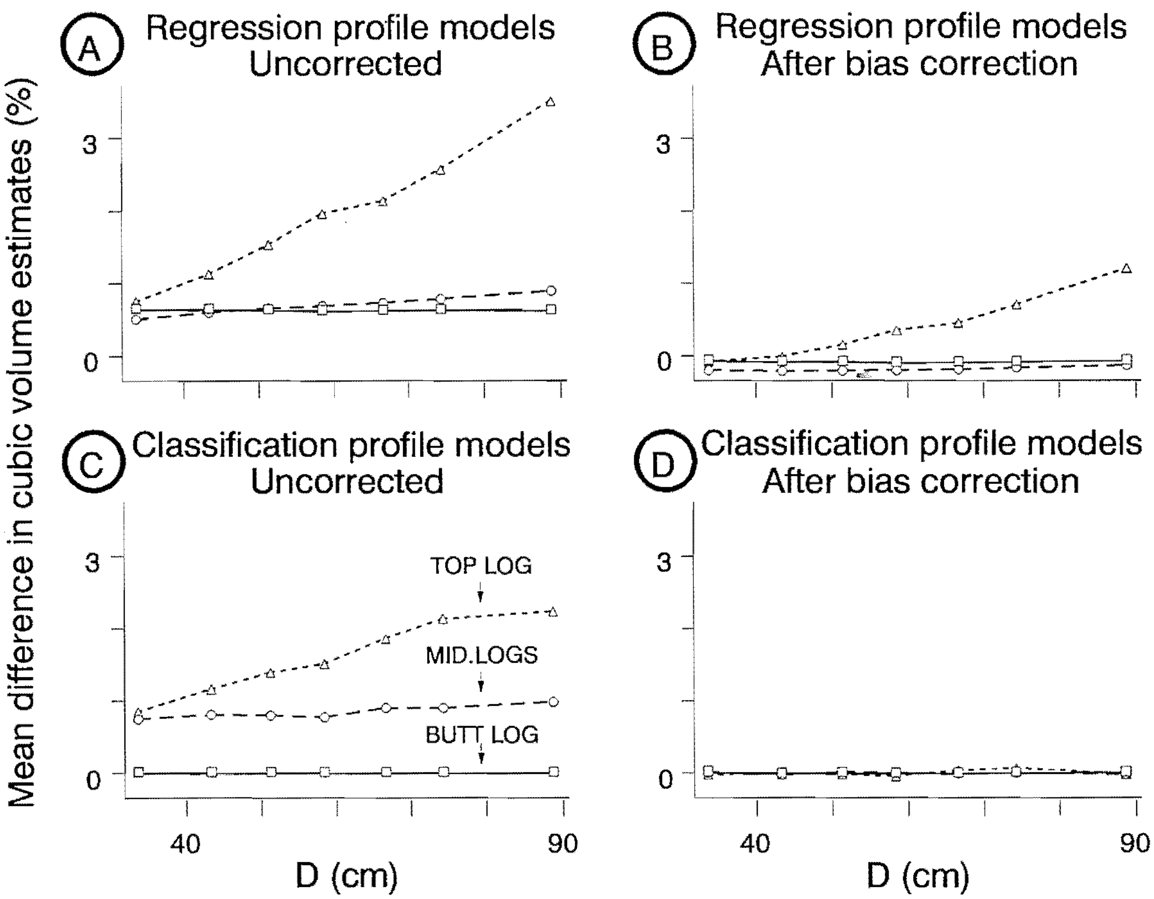

FIG. 4. Mean differences between profile models fit to d/D and $(d/D)^2$ in predicting volume of 5.3-m logs, expressed as a percentage. Predictions from the profile models, which used d/D as the regression dependent variable, were retransformed to estimates of $(d/D)^2$. The uncorrected retransformation of regression profile model [6] produced larger estimates of log volumes, depending on log position and D (A). Bias correction [9], using regression variance model [7], considerably reduced differences (B). Retransformation bias also occurred using the classification profile model (C), which is unaffected by the assumption of homogeneous variance in the regression profile model [6]. Differences between models are effectively eliminated using the classification variance model (D).

sample sizes to independently fit regression models for residual variance, although it is possible to simultaneously fit a regression model and heterogeneous variance model using weighted least squares (Carroll and Ruppert 1988; Gregoire and Dyer 1989). Correction of retransformation bias in models fit to d/D is less sensitive to errors in estimating variance than models fit to $(d/D)^2$. This is apparent when comparing the bias corrections that assume homogeneous variance (Figs. 2B and 3B).

Volume estimates from correction eq. 9 for $(d/D)^2$ require numerical integration; unbiased volume estimates cannot be readily made by integration of the profile model that includes eq. 9. However, this might not be restrictive because actual volume in many management and research applications is defined by numerical techniques, such as Smalian's formula for short sections or log lengths.

Some numerical methods that define log volume use the product of several diameters. Examples include Grosenbaugh's general formula, the frustrum of a cone, and the frustrum of a neiloid (Phillips and Taras 1987). The errors in estimating diameters or cross-sectional areas at different heights for a single tree are often positively correlated; e.g., if a model overestimates diameter at the lower end of a log, it might tend to overestimate the upper end diameter of the same log. Correlated errors will be another source of retransformation bias in these volume estimators.

Retransformation bias will also affect estimates of height to a given top diameter. Bias in estimating merchantable height can affect the accuracy and precision of merchantable volume estimates for different product classes; this effect is most serious when minimum log lengths are part of the merchantability criteria. In an unpublished study, we found that profile models tend to overestimate heights to a 10 cm top diameter merchantability limit, especially for large trees. Correction equations for retransformation bias in height estimates depend upon the form of the profile model, and correction of this bias can be algebraically complex. Correction equations can be formulated using an error propagation model and a Taylor series approximation for nonlinear models, as in eqs. 5 and 10.

It has been assumed that measurement error in D is negligible, which makes D a fixed value. If measurement errors for D, or errors in predicting D from a growth and yield model, are not negligible, then additional retransformation biases are introduced, even if the measurement or prediction errors are unbiased. However, these sources of bias can be predicted in a way similar to [3] or [5], and correction equations can be developed considering errors prop-

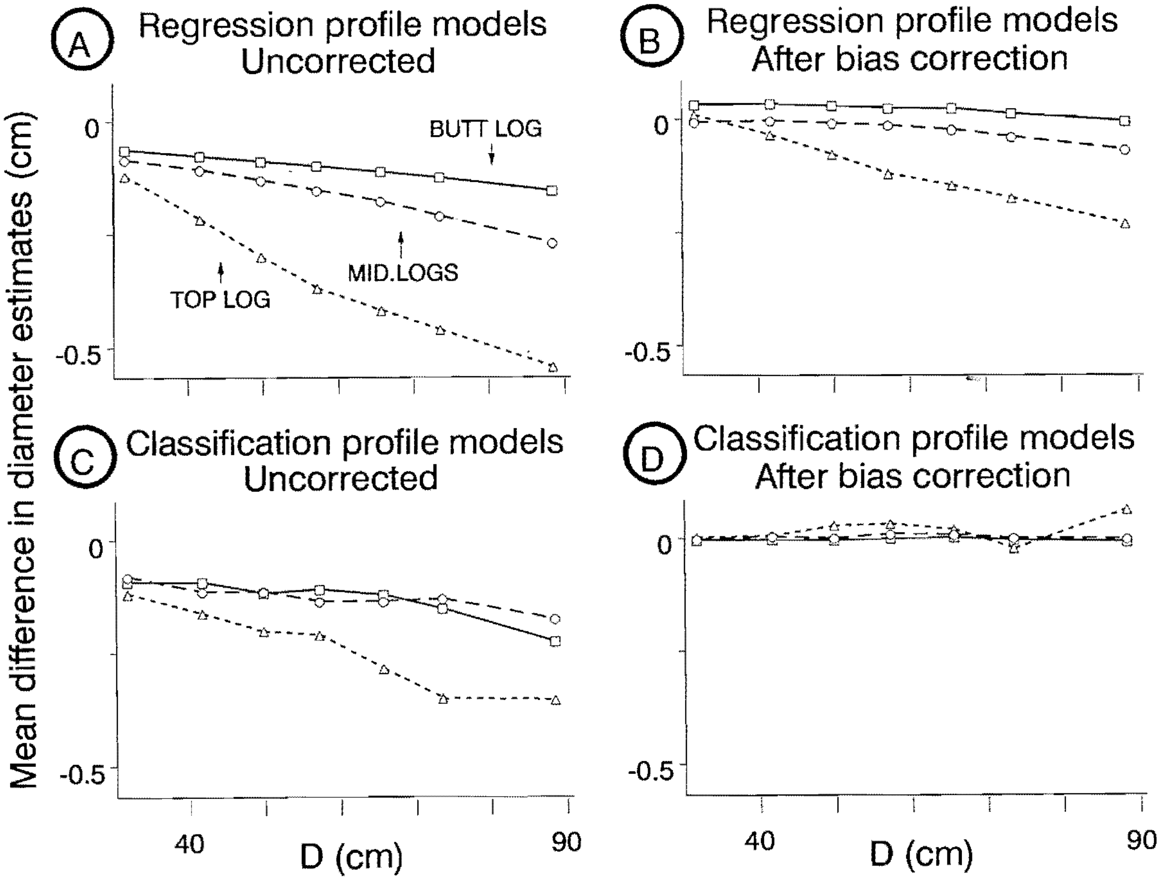

FIG. 5. Mean differences between models fit to d/D and $(d/D)^2$ in predicting upper log end diameters, which are used in merchantability criteria and board-foot scaling rules. The regression profile model [6] fit to $(d/D)^2$ produced smaller diameter estimates, compared with the same model fit to d/D (A and C); bias changed with diameter at breast height and height. The bias correction eq. 10 with a heterogeneous variance estimator reduced these differences (B and D).

agated from the uncertainty in D, similar to [9] or [10]. Czaplewski and McClure (1988) simultaneously fit models for diameter inside bark at breast height and stem profile, which can eliminate correlated error problems for inside bark profile models that are conditioned at breast height.

The effects of retransformation bias were studied using data for lodgepole pine (*Pinus contorta* Dougl.) and Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) from the same study area (Czaplewski et al. 1989). The qualitative results were very similar to those in this paper. However, the magnitude of $\hat{E}[\epsilon_1^2]$ and $\hat{E}[\epsilon_2^2]$ for lodgepole pine and Douglas-fir was less than that of grand fir, and therefore, the magnitude of retransformation bias was less. Also, the results from the classification profile models could depend upon the number of classes chosen. Since each class had at least 40 observations, the sample mean and variance for each of the 175 classes are expected to be accurate estimates of the true class mean and variance. As a test, classification profile models were formulated without partitioning the classes by diameter (D) groupings, and the results were very similar to those reported in this paper. These results suggest that retransformation bias can affect all profile models.

Conclusions

Retransformation bias was greatest near the merchantable top, but can be readily corrected if accurate models for heterogeneous variance of profile prediction errors are available. Corrections for retransformation bias in volume estimates from profile models fit to d/D are less vulnerable to errors in predicting heterogeneous variance than diameter estimates retransformed from models fit to $(d/D)^2$. Therefore, it is recommended that profile models be fit to d/D when volume and diameter estimates are both important. If cubic volume estimates are much more important than diameter or board-foot estimates, then $(d/D)^2$ should be used as the dependent variable.

Even if retransformation bias is eliminated, other sources of bias can occur when applying stem profile models. These include biases produced by heterogeneous errors on ordinary least squares regression; profile models that poorly represent certain types of trees because the models assume constant tree form regardless of tree size, H/D ratio, or crown ratio; biased measurement errors; and lack of true circular cross-sectional areas in tree boles. Each source of bias can either mitigate or aggravate the effect of other biases, and retransformation bias might not be the most important source of bias. Controlling the confounding effect of retransformation bias should clear the way for studying these other sources of bias.

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