Chapter 6
Estimating the Carbon in Coarse Woody Debris with Perpendicular Distance Sampling

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Abstract Perpendicular distance sampling (PDS) is a design for sampling the population of pieces of coarse woody debris (logs) in a forested tract. In application, logs are selected at sample points with probability proportional to volume. Consequently, aggregate log volume per unit land area can be estimated from tallies of logs at sample points. In this chapter we provide protocols and formulae for estimating the carbon in coarse woody debris with PDS. We also provide formulae for estimating components of change in the log population between two points in time.

Keywords Components of change, limiting distance, logs, two-phase sampling, volume factor

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6.1 Introduction

Strategies for the estimation of attributes of fallen coarse woody debris on forested tracts have employed a variety of sampling designs, The most popular designs are line intersect sampling (e.g., de Vries 1986, Affleck et al. 2005) and plot sampling (e.g., Gregoire and Valentine 2008, Chap. 7), though several recently conceived designs – transect relascope sampling (Ståhl 1998), point relascope sampling (Gove et al. 1999), a prism sweep method (Bebber and Thomas 2003), and perpendicular distance sampling (Williams and Gove 2003, Williams et al. 2005) – also are applicable.

Coarse woody debris comprises the woody remains of fallen trees and large branches. Customarily, individual pieces of coarse woody debris are called 'logs.' In line intersect sampling, the probability that a particular log is included in the sample is proportional to log length. In biological organisms, plant or animal, length tends to scale with the 1/4 power of volume (e.g., West et al. 1999). Volume, in turn, tends to scale isometrically with carbon stock, our parameter of interest. In fixed-radius plot sampling or quadrat sampling, all logs in a sample are included with equal probability regardless of volume, so plot sampling is, in effect, sampling with probability proportional to the zeroth power of volume, which most likely is less efficient than line intersect sampling for estimating carbon stock. By contrast, in point relascope sampling, a log is included in a sample with probability proportional to the log's length-square, which tends to scale with the 1/2 power (or square root) of volume, so on theoretical grounds we should expect point relascope sampling to be more efficient than line intersect sampling, and this expectation is supported by results from field tests (Brissette et al. 2003). Ideally, the inclusion probability of a log should scale directly with its volume, and this is achieved with perpendicular distance sampling (PDS). Thus, in this chapter, we describe how the carbon in coarse woody debris can be estimated with PDS.

6.1.1 Coarse Woody Debris

Our objective is the estimation of areal carbon density for the population of pieces of coarse woody debris on the floor of a forested tract. By areal carbon density, we mean the carbon stock per unit land area (kg C ha^{-1}). The question at the outset is: what constitutes a piece of coarse woody debris?

We define any fallen stem and all its connected branches and sub-branches to be a single piece of woody debris. The debris is 'coarse' if its diameter meets or exceeds the minimal diameter (0.1 m = 10 cm) at some point on the main stem. Thus, under this protocol, every piece of wood on the ground, whether branched or not, is either coarse woody debris or fine woody debris. This definition is applicable to all sampling designs. However, since fallen wood gradually decays, we also need a way of distinguishing decomposed coarse woody debris from organic soil. We shall adopt a

Table 6.1 Decay classes of logs

Class	Integrity	Texture
1	Sound, freshly fallen	Intact, no rot
2	Sound	Intact, sapwood partly soft
3	Heartwood sound, log supports its own weight	Sapwood can be pulled apart by hand or sapwood is absent
4	Heartwood rotten, log does not support its own weight, but maintains its shape	Soft, small blocky pieces; a metal pin can be pushed into heartwood
5	None; log spreads on the ground	Soft, powdery when dry

decay classification used by the U.S. Forest Service (Table 6.1). Decay classes 1–4 connote wood; decay class 5 is organic soil.

For ease of presentation, we shall henceforth follow custom and call all pieces of coarse woody debris 'logs.' Moreover, unless otherwise specified, all log lengths and diameters are measured in meters; cross-sectional areas of logs are measured in square meters; volume is measured in cubic meters, and carbon mass is measured in kilograms. With one exception, land area is measured in ha. A symbol table is provided in the appendix (Table 6.4).

6.2 Perpendicular Distance Sampling

We presume that an investigator selects the locations of m sample points, P_s, $s = 1, 2, ..., m$ within the tract of interest. These points may be selected independently at random, but ordinarily the sample points are arranged either singly or in clusters on a systematic grid, where the anchor point of the grid is selected independently at random. The acceptance-rejection method for selection of a point at random is described in the appendix (p. 84).

A log in the vicinity of P_s is a potential member of the sth sample if there exists a line from the sample point that intersects the central axis of the log perpendicularly. If so, the log is selected into the sample if the perpendicular distance, D, from the central axis of the log to the sample point does not exceed $\kappa_v a$, where a is the cross-sectional area of the log at the point of intersection by the perpendicular line, and κ_v ($\text{m}^2\ \text{m}^{-3}$) is a constant, which is chosen by the investigator (Fig. 6.1).

If the log is curved or branched or both, a straight and horizontal central axis is defined by range poles erected at the farthest end points of the log. Cross-sectional area is measured perpendicular to the central axis and the horizon. If the log is branched, then the cross-sectional area for the log is the sum of the areas for the branches where they are intersected by the perpendicular line. Note, however, that the perpendicular line is most likely perpendicular only to the central axis of the log, and so branch cross-sectional areas are measured on the slant (see the appendix for methods), but perpendicular to the horizon (Fig. 6.2).

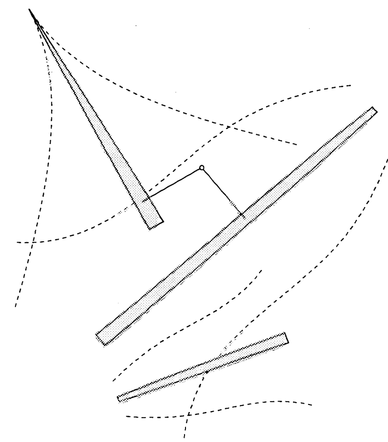

Fig. 6.1 The dashed lines on either side of a log depict the borders of the log's inclusion zone. The area of an inclusion zone is proportional to the log's volume. A log is selected by a sample point (○) if the sample point occurs in the log's inclusion zone, and this is determined by measuring the perpendicular distance, D, from the log to the sample point. If $D \leq \kappa_v a$, the log is selected. In this example, two logs are selected by the sample point

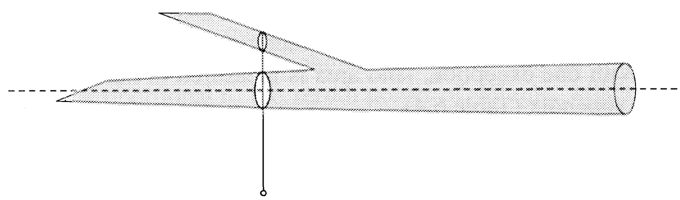

Fig. 6.2 The cross-sectional areas on a multi-stemmed log are measured perpendicular to the central axis of the main stem and coincident with the perpendicular line, and then summed

The probability that a log is included in the sth sample is proportional to the log's volume because cross-sectional area integrated over the length of a log's central axis equals the log's volume, v. Consequently, the probability of inclusion is proportional to $2\kappa_v v$, the factor of 2 arising because the log may be selected by a sample point on either side of the central axis.

The inclusion probability may be diminished if the log is close to a boundary of any region where sample points could not occur, for example, across a property boundary, on a road, in a lake or river, or over the edge of a cliff. Hence, if a selected log, say log k, appears as though it may be closer to a boundary than to the sample point, we suggest implementing the walkthrough method (Ducey et al. 2004, Williams and Gove 2003) to determine whether log k should be tallied twice to correct for the truncation effect by the boundary on the inclusion probability. To wit, measure D_k from the sample point to the perpendicular point on the central axis, then proceed beyond the perpendicular point an additional distance D_k. If a boundary is intercepted before traversing the additional distance, log k is tallied twice, i.e., $t_k = 2$; otherwise, the log is tallied once, i.e., $t_k = 1$.

Let y_k denote an attribute of log k. The aggregate amount of log attribute per unit land area (λ_y) is unbiasedly estimated from the sth sample by

$$\tilde{\lambda}_{y,s} = \frac{1}{2\kappa_v} \sum_{k=1}^{n_s} \frac{y_k t_k}{v_k}, \tag{6.1}$$

where v_k is the volume of the kth log in the sample, and n_s is the number of logs selected into the sample at the sth sample point.

If log volume is the attribute of interest, then $y_k = v_k$, so aggregate log volume per unit land area (λ_v) is unbiasedly estimated from the tallies of the logs in the sth sample, i.e.,

$$\tilde{\lambda}_{v,s} = \frac{1}{2\kappa_v} \sum_{k=1}^{n_s} t_k, \tag{6.2}$$

where $\tilde{\lambda}_{v,s}$ has dimensions cubic meters per square meter. Note that cross-sectional area is not used in the estimator, even though it is measured to determine whether a log is selected into the sample. In the absence of boundary effects, $t_k = 1$ for all k, so

$$\tilde{\lambda}_{vs} = \frac{n_s}{2\kappa_v}. \tag{6.3}$$

Williams and Gove (2003) defined a volume factor $F_v = 10^4/(2\kappa_v)$ ($\mathrm{m^3\,ha^{-1}}$), which is used to estimate log volume on a per hectare basis. For example, for $\kappa_v = 250\,\mathrm{m^2\,m^{-3}}$, we obtain

$$F_v = \frac{10{,}000\,\mathrm{m^2\,ha^{-1}}}{2 \times 250\,\mathrm{m^2\,m^{-3}}} = 20\,\mathrm{m^3\,ha^{-1}}.$$

Whence,

$$\hat{\lambda}_{v,s} = 10^4\,\tilde{\lambda}_{v,s} = F_v \sum_{k=1}^{n_s} t_k \tag{6.4}$$

unbiasedly estimates log volume per hectare of land. In the absence of boundary effects,

$$\hat{\lambda}_{v,s} = F_v n_s. \tag{6.5}$$

A combined or 'replicate' estimate obtains from m sample points, i.e.,

$$\hat{\lambda}_{v,\mathrm{rep}} = \frac{1}{m} \sum_{s=1}^{m} \hat{\lambda}_{v,s}. \tag{6.6}$$

The variance of $\hat{\lambda}_{v,\mathrm{rep}}$ is estimated by

$$\hat{v}\left[\hat{\lambda}_{v,\mathrm{rep}}\right] = \frac{\sum_{s=1}^{m}\left(\hat{\lambda}_{v,s} - \hat{\lambda}_{v,\mathrm{rep}}\right)^2}{m\,(m-1)}. \tag{6.7}$$

This estimator is unbiased if the sample points are selected independently at random. The standard error of $\hat{\lambda}_{v,\mathrm{rep}}$ is estimated by

$$\widehat{\mathrm{se}}\left[\hat{\lambda}_{v,\mathrm{rep}}\right] = \sqrt{\hat{v}\left[\hat{\lambda}_{v,\mathrm{rep}}\right]}. \tag{6.8}$$

6.2.1 Example

For illustrative purposes, PDS was applied on a systematic square grid comprising 18 sample points within a 0.8 ha riparian forest fragment in Durham, New Hampshire. The forest succeeded over the past 120 years from a former pasture, and has become dominated by eastern white pine (*Pinus strobus* L.) along with northern red oak (*Quercus rubra* L.), red maple (*Acer rubrum* L.), and eastern hemlock (*Tsuga canadensis* L.).

Logs were tallied at each sample point with a volume factor of $F_v = 14\,\text{m}^3\,\text{ha}^{-1}$ (or, equivalently, 200 $\text{ft}^3\,\text{ac}^{-1}$). This volume factor corresponds to $\kappa_v = 357.1\,\text{m}^2$ m^{-3} (or, equivalently, $\kappa_v = 108.9\,\text{ft}^2\,\text{ft}^{-3}$). A reference table provided the limiting perpendicular distance, D_{limit}, for a log of a given log diameter (d), i.e, $D_{\text{limit}} = \kappa_v \times \pi d^2/4$, where $\pi d^2/4$ approximates log cross-sectional area. Some limiting distances by volume factor and log diameter are provided in Table 6.5 (p. 86).

Since both distance and log diameter are easily ocularly estimated, measurements were taken at a sample point only if a log was borderline, or if the log was forked, because cross-sectional area must be accumulated over the branches. Boundary effects were corrected with the walkthrough method. Logs were categorized by decay class, by diameter class (A, $<$ 14.9 cm; B, 15 – 29.9 cm; C, $\geq$ 30 cm), and type (H, hardwood; S, softwood). Field work for the entire project required approximately 1 h, including walking between points. Tally time per point was typically 2 min or less. Data from the PDS are provided in Table 6.2. Estimates of total log volume per hectare and estimates of hardwood and softwood log volume per hectare, $\hat{\lambda}_{vs}(\text{H})$ and $\hat{\lambda}_{vs}(\text{S})$, respectively, were calcualted with (6.4). The results are provided in Table 6.3. The reader may verify that replicate estimates ($\text{m}^3\,\text{ha}^{-1}$) for the forest fragment are:

$$\hat{\lambda}_{v,\text{rep}} = 30.33, \quad \widehat{\text{se}}\left[\hat{\lambda}_{v,\text{rep}}\right] = 4.56,$$

$$\hat{\lambda}_{v,\text{rep}}(\text{H}) = 3.89, \quad \widehat{\text{se}}\left[\hat{\lambda}_{v,\text{rep}}(\text{H})\right] = 1.52,$$

$$\hat{\lambda}_{v,\text{rep}}(\text{S}) = 26.44, \quad \widehat{\text{se}}\left[\hat{\lambda}_{v,\text{rep}}(\text{S})\right] = 4.92.$$

6.3 Carbon Stock

Of course the real parameter of interest is not volume, but rather the carbon stock per unit land area. Woody dry matter is approximately 50% carbon by weight so, unless bioassay is undertaken, woody biomass is converted to carbon stock with the constant factor $\rho_{\text{C}} = 0.5$ kg C (kg wood)$^{-1}$. The bulk density of wood, i.e., the dry weight of wood per unit volume (kg m^{-3}), may vary among logs because it

Table 6.2 Log data from 18 PDS points in Durham, NH ($F_v = 14$ m^3 ha^{-1})

Point	Species	Decay class	Diameter class	Hollow	Branches	Walk-through	t_k
1	H	3	B	-	-	-	1
2	S	3	C	Yes	-	-	1
3	S	3	B	-	-	-	1
3	S	3	C	Yes	-	-	1
3	S	3	B	-	-	-	1
4	H	4	B	-	Yes	-	1
4	S	4	A	-	-	-	1
5	S	3	B	-	-	-	1
5	S	3	B	-	Yes	-	1
6	S	4	C	-	-	Yes	2
6	H	3	A	-	-	-	1
7	-	-	-	-	-	-	-
8	S	1	C	-	-	Yes	1
8	S	3	B	-	-	-	1
9	S	3	C	-	-	-	1
9	S	4	B	-	-	-	1
10	S	4	B	-	-	-	1
10	S	3	C	-	-	-	1
10	S	4	C	-	-	-	1
11	S	3	B	-	-	-	1
11	S	3	B	-	-	-	1
11	S	3	C	-	-	Yes	2
12	S	4	C	-	-	-	1
12	S	4	A	-	-	-	1
12	S	4	C	-	-	-	1
12	S	3	B	-	-	-	1
13	S	4	B	-	-	-	1
13	S	3	C	-	-	Yes	2
13	S	3	C	-	-	Yes	2
14	S	3	B	-	-	-	1
14	S	3	A	-	-	-	1
15	H	3	B	-	-	Yes	1
16	H	4	B	-	-	-	1
16	S	3	C	-	-	Yes	1
16	S	3	B	-	-	-	1
17	-	-	-	-	-	-	-
18	S	3	B	-	-	-	1

depends on species and degree of decay, among other factors. A core of wood may be extracted from the kth log for the measurement of bulk density ($\rho_{w,k}$), where the volume of the core obtains from the known diameter of the coring device and the 'trimmed length' of the core. Hence the kth log's carbon mass per unit volume (kg C (m^3 wood)$^{-1}$) is estimated by

$$\hat{C}_k = \rho_C \rho_{w,k}. \tag{6.9}$$

Table 6.3 Estimates of log volume, by sample point, calculated from the data in Table 6.2

Point	$\hat{\lambda}_{v,s}$	$\hat{\lambda}_{v,s}$(H)	$\hat{\lambda}_{v,s}$(S)	Point	$\hat{\lambda}_{v,s}$	$\hat{\lambda}_{v,s}$(H)	$\hat{\lambda}_{v,s}$(S)
1	14	14	0	10	42	0	42
2	14	0	14	11	56	0	56
3	42	0	42	12	56	0	56
4	28	14	14	13	70	0	70
5	28	0	28	14	28	0	28
6	42	14	28	15	14	14	0
7	0	0	0	16	42	14	28
8	28	0	28	17	0	0	0
9	28	0	28	18	14	0	14

Our target parameter, carbon stock per unit land area (kg C ha^{-1}), is estimated by

$$\hat{\lambda}_{C,s} = F_v \sum_{k=1}^{n_s} t_k \hat{C}_k = \rho_C F_v \sum_{k=1}^{n_s} t_k \rho_{w,k}. \tag{6.10}$$

With m sample points,

$$\hat{\lambda}_{C,rep} = \frac{1}{m} \sum_{s=1}^{m} \hat{\lambda}_{C,s}. \tag{6.11}$$

The variance of $\hat{\lambda}_{C,rep}$ is estimated by

$$\hat{v}\left[\hat{\lambda}_{C,rep}\right] = \frac{\sum_{s=1}^{m}\left(\hat{\lambda}_{C,s} - \hat{\lambda}_{C,rep}\right)^2}{m(m-1)}. \tag{6.12}$$

6.4 Two-phase Sampling

An assumption of homogeneous bulk density among logs may be indistinguishable from wishful thinking in the majority of most, if not all, sampling situations. On the other hand, coring every log may be too much work, or too expensive. And, since coarse woody debris usually is a small carbon pool, compared to the standing trees and soil, it does not deserve a sampling effort or budget that is incommensurate with its importance.

As a compromise, we may estimate the average bulk density, $\bar{\rho}_w$, at a subset of sampling locations or for a subset of logs (perhaps the closest log to the sample point) at all locations, and apply this average to all logs at all locations. Hence, for the sth perpendicular distance sample,

$$\hat{\lambda}_{C,two} = \rho_C \bar{\rho}_w \hat{\lambda}_{v,rep} \tag{6.13}$$

estimates the carbon stock in logs per unit land area. The variance of $\hat{\lambda}_{\mathrm{C,two}}$ may be estimated with Goodman's (1960) formula

$$\hat{v}\left[\hat{\lambda}_{\mathrm{C,two}}\right] = \rho_{\mathrm{C}}^2 \left(\bar{\rho}_w \hat{v}\left[\hat{\lambda}_{v,\mathrm{rep}}\right] + \hat{\lambda}_{v,\mathrm{rep}} \hat{v}\left[\bar{\rho}_w\right] - \hat{v}\left[\hat{\lambda}_{v,\mathrm{rep}}\right] \hat{v}\left[\bar{\rho}_w\right]\right). \tag{6.14}$$

The variance of $\bar{\rho}_w$ is problematic, since the inclusion probabilities are proportional to the log volumes, which are unknown. We suggest assuming that the sample cores constitute a random sample from an infinite population, in which case the variance is consistently estimated by

$$\hat{v}\left[\bar{\rho}_w\right] = \frac{\sum_{i=1}^{n} \left(\rho_{w,i} - \bar{\rho}_w\right)^2}{n(n-1)}, \tag{6.15}$$

where $\rho_{w,i}$ is the bulk density of the ith of n cores in the sample. And, of course, the variance of $\hat{\lambda}_{v,\mathrm{rep}}$ is estimated by (6.7).

6.5 Components of Change

The total carbon in coarse woody debris changes over time owing to recruitment, decomposition, and mobility. Recruitment ordinarily results from trees and branches falling to the ground, but in flooded riparian sites, logs may float from off-site to on-site, and vice versa, or change their locations within a tract. The change in the classification of a tree or branch from standing to fallen wood does not, in and of itself, affect the total carbon stock of a tract or the net ecosystem exchange (kg C ha^{-1} year^{-1}). However, the migration of logs to and from the tract and the heterotrophic respiration associated with decomposition do affect both the total carbon stock and the net ecosystem exchange of carbon.

Because a piece of coarse woody debris, by definition, has a minimum size, a log may shrink from coarse to fine woody debris, effecting a loss from the population of logs and an addition to the population of pieces of fine woody debris. Even in the absence of log mobility, transitions from branches to fallen logs to fine woody debris or organic soil complicates the estimation of the contribution of the population of logs to the overall net ecosystem exchange between two points in time.

In the absence of log mobility, the change from time $\mathcal{t}_1$ to $\mathcal{t}_2$ in log carbon per unit land area (Δ_{C}) is attributable to: (i) the carbon added by recruited logs, which we denote by $\Delta_{\mathrm{C,\,gain}}$; (ii) the carbon lost from residual logs present in the population at $\mathcal{t}_1$ and $\mathcal{t}_2$, which we denote $\Delta_{\mathrm{C,\,res}}$; and (iii) the carbon in logs that transition to fine woody debris or organic soil between $\mathcal{t}_1$ and $\mathcal{t}_2$, which we denote by $\Delta_{\mathrm{C,\,loss}}$. Whence,

$$\begin{aligned}\Delta_{\mathrm{C}} &= \lambda_{\mathrm{C}}(\mathcal{t}_2) - \lambda_{\mathrm{C}}(\mathcal{t}_1) \\ &= \Delta_{\mathrm{C,\,gain}} - \Delta_{\mathrm{C,\,res}} - \Delta_{\mathrm{C,\,loss}}.\end{aligned}$$

If, during PDS, we measure the distance and azimuth of sample logs at each sample point, P_s, at time $\mathcal{t}_1$ and reuse the same sample point at time $\mathcal{t}_2$, then we can distinguish new logs not present at t_1 from residual logs present in the samples at both t_1 and t_2. Moreover, we can locate and examine each log, which was in the sample at t_1 but absent at t_2, for the purpose of determining whether the log at t_1 still qualifies as a log at t_2, albeit a smaller one, or whether the log at t_1 transitioned to fine woody debris or organic soil by time t_2.

Let $\mathcal{R}_s$ denote the set of residual logs in the samples at P_s at both t_1 and t_2, and let $\mathcal{G}_s$ denote the set of new logs in the sample at t_2. Further, let $\mathcal{L}_s$ denote the set of logs in the sample at t_1, which transitioned to fine woody debris by time t_2, and let $\mathcal{S}_s$ denote the set of logs that were in the sample at t_1 and still classify as logs at t_2, but, owing to shrinkage from decomposition, are not in the sample at t_2. The union of sets $\mathcal{R}_s$ and $\mathcal{G}_s$, i.e., $\mathcal{R}_s \cup \mathcal{G}_s$, comprises all the logs in the sample at t_2, and $\mathcal{R}_s \cup \mathcal{S}_s \cup \mathcal{L}_s$ comprises all the logs in the sample at t_1.

The three components of change are estimated from the samples at P_s with

$$\hat{\Delta}_{\mathrm{C,\,gain},\,s} = \rho_{\mathrm{C}} F_v \textstyle\sum_{k \in \mathcal{G}_s} t_k \rho_{w,k}(t_2) \tag{6.16}$$

$$\hat{\Delta}_{\mathrm{C,\,loss},\,s} = \rho_{\mathrm{C}} F_v \textstyle\sum_{k \in \mathcal{L}_s} t_k \rho_{w,k}(t_1) \tag{6.17}$$

$$\hat{\Delta}_{\mathrm{C,\,res},\,s} = \rho_{\mathrm{C}} F_v \left\{ \textstyle\sum_{k \in \mathcal{S}_s} t_k \rho_{w,k}(t_1) + \sum_{k \in \mathcal{R}_s} t_k \left[\rho_{w,k}(t_1) - \rho_{w,k}(t_2) \right] \right\}. \tag{6.18}$$

The investigator may choose to substitute $\bar{\rho}_w(t_1)$ and $\bar{\rho}_w(t_2)$, respectively, for $\rho_{w,k}(t_1)$ and $\rho_{w,k}(t_2)$. And, of course, the total change in the carbon stock per unit land area is estimated by

$$\begin{aligned} \hat{\Delta}_{\mathrm{C},s} &= \hat{\lambda}_{\mathrm{C},s}(t_2) - \hat{\lambda}_{\mathrm{C},s}(t_1) \\ &= \hat{\Delta}_{\mathrm{C,\,gain},\,s} - \hat{\Delta}_{\mathrm{C,\,res},\,s} - \hat{\Delta}_{\mathrm{C,\,loss},\,s}. \end{aligned} \tag{6.19}$$

A combined or replicate estimate of any component of change obtains by averaging the separate estimates from the m sample points, e.g.,

$$\hat{\Delta}_{\mathrm{C,\,loss,\,rep}} = \frac{1}{m} \sum_{s=1}^{m} \hat{\Delta}_{\mathrm{C,\,loss},\,s}. \tag{6.20}$$

The rate of change in the carbon stock in logs per unit land area between t_1 and t_2 is $\Delta_{\mathrm{C}}/(t_2 - t_1)$, which is estimated by

$$\begin{aligned} \frac{\hat{\Delta}_{\mathrm{C,\,rep}}}{t_2 - t_1} &= \frac{\hat{\lambda}_{\mathrm{C,\,rep}}(t_2) - \hat{\lambda}_{\mathrm{C,\,rep}}(t_1)}{t_2 - t_1} \\ &= \frac{\hat{\Delta}_{\mathrm{C,\,gain,\,rep}} - \hat{\Delta}_{\mathrm{C,\,res,\,rep}} - \hat{\Delta}_{\mathrm{C,\,loss,\,rep}}}{t_2 - t_1}. \end{aligned} \tag{6.21}$$

By definition, net ecosystem exchange of carbon (NEE) is the rate of change in the total carbon stock per unit land area (kg C ha^{-1} year^{-1}). Equation (6.21) is an appropriate estimator of the log component of NEE only if this component is defined as the rate of change in the carbon stock in logs per unit land area.

Alternatively, it seems natural to define the log component of NEE as the rate at which carbon from logs is lost to respiration between t_1 and t_2, but owing to the population dynamics of logs, this definition would be problematic. Under the respiration definition, (6.21) is not an appropriate estimator of the log component of NEE. Carbon is added to the log population the instant a large branch or tree falls, but the carbon of interest is not this addition, but rather that which is lost to respiration between the instant of fall and t_2. There is the possibility that a branch may fall and become a log after t_1, but transition to fine woody debris before t_2, in which case the carbon lost between fall and transition is of interest. Residual logs lose carbon between t_1 and t_2, and $\hat{\Delta}_{\mathrm{C,res,rep}}$ does unbiasedly estimate the change in the carbon stock of residual logs per unit land area. By contrast, $\hat{\Delta}_{\mathrm{C,loss,rep}}$ subtracts all the carbon in logs that transition to fine woody debris between t_1 and t_2. Of interest, however, is the carbon lost to respiration from each of these logs between t_1 and the time of transition.

6.6 Other Considerations

Carbon stock per unit land area may be estimated by decay class, species, or both. The separate estimates add to provide a total estimate for the tract of interest. However, the use of published estimates of bulk densities by decay class, species, or both, generally would not result in unbiased estimation.

We have suggested PDS as an efficient means to estimate carbon stock per unit land area in coarse woody debris in forested tracts. The superior efficiency of PDS compared to alternative methods, including line intersect sampling, has been demonstrated in coniferous forests in the Western United States, and in northern hardwood forests in New Hampshire. However, these results are not yet published. PDS requires measurements of bulk density from samples of wood, but so do the other methods. However, in its favor, PDS does not require the measurement or estimation of the volumes of the logs in a sample. A measurement of cross-sectional area is needed for selection of a log, but a 'precise measurement' is needed only to determine whether a borderline log is in or out of the sample; the cross-sectional area measurement is not used in the estimator. Log volume per unit area is estimated from the tallies of the logs in the samples.

Since selection of a log into a sample is with probability proportional to volume, samples will comprise, on average, the larger logs in the neighborhood of a sample point. Moreover, since the limiting distance for selection is proportional to log cross-sectional area, the larger sections of these logs are more apt to be intersected by perpendicular lines, and this facilitates coring.

In order to implement PDS, one must decide on a volume factor. A small factor such as 14 $\mathrm{m^3\ ha^{-1}}$ (or 200 $\mathrm{ft^3\ ac^{-1}}$) is feasible only in locations where the logs are small. Doubling the volume factor halves the limiting distance of a given log diameter, so a large volume factor is suggested where large logs are expected (see Table 6.5, p. 86).

Two advances in PDS may be considered as alternatives to the conventional PDS described in this chapter. 'Distance limited PDS' (Ducey et al. 2008a) affords the use of a small volume factor even where large logs occur. In essence, this strategy (a) incorporates a maximum search distance for logs larger than a fixed diameter, (b) requires a diameter measurement on these large logs, and (c) uses a slightly more complicated estimator. Another strategy, called 'omnibus PDS' (Ducey et al. 2008b, Gregoire and Valentine 2008, Chap. 10), provides for efficient estimation of log attributes other than volume, mass, and carbon. Finally, the line intersect distance strategy (LIDS) of Affleck (2008) combines elements of omnibus PDS and line intersect sampling (LIS). Investigators who are already using LIS should find it easy to switch to LIDS.

6.7 Appendix

6.7.1 Acceptance-rejection Method

Surround the region of interest with an imaginary rectangle with dimensions $X \times Z$. Draw two uniform random numbers, u_1 and u_2, with values between 0 and 1. Determine whether coordinates (u_1X, u_2Z) occur within the region of interest. If so, accept the location as a sample point or the anchor point of a systematic grid. Otherwise, start over with two new random numbers.

6.7.2 Estimating Cross-Sectional Area on the Slant

Practical methods for estimating cross-sectional area of logs in the field include (cf. Valentine et al. 2001):

(i) Under the assumption that the cross-section of a log is round when viewed perpendicular to its axis of length, the cross-section sliced by an oblique vertical plane will be elliptical unless the plane is parallel to the axis of length or nearly so. Let Ω be the horizontal width of the log sliced (on the slant) by the vertical plane. Let Ω' denote depth of wood calipered vertically in the same plane. Then the cross-sectional area, a, of the log is

$$a \approx \frac{\pi}{4}\Omega\Omega' \tag{6.22}$$

(ii) If Ω' can not be measured, substitute d, which is the diameter calipered horizontally and perpendicular to the axis of length. If the piece of debris is round and not tilted, then d should equal Ω'.

Table 6.4 Symbol table

Symbol	Definition	Units
a, a_k	Cross-sectional area of log or branch.	m^2
$\hat{C}_k$	Estimated carbon per unit wood volume for the kth log in a sample.	kg C (m^3 wood)$^{-1}$
d, d_k	Log diameter.	m
t_k	Tally for the kth log in a sample.	count
y_k	Any attribute of the kth log.	
v_k	Volume of the kth log.	m^3
n_s	Number of logs in the sth sample, i.e., the sample at P_s.	count
P_s	The sth of m sample points.	
F_v	Volume factor, which is equivalent to $10^4/(2\kappa_v)$.	$m^3\,ha^{-1}$
D, D_k	Perpendicular distance from a sample point to the central axis of a log.	m
D_{limit}	Maximum perpendicular distance from a sample point for the inclusion of a log in the sample.	m
Δ_C	Change in log carbon stock per unit area.	kg C ha^{-1}
$\hat{\Delta}_{C,s}$	Estimate of Δ_C from repeated samples at P_s.	kg C ha^{-1}
$\hat{\Delta}_{C,\text{rep}}$	Estimate of Δ_C from m repeated samples.	kg C ha^{-1}
$\Delta_{C,\text{ gain}}$	Gain in log carbon from recruitment.	kg C ha^{-1}
$\hat{\Delta}_{C,\text{ gain},\, s}$	Estimate of $\Delta_{C,\text{ gain}}$ from repeated samples at P_s.	kg C ha^{-1}
$\Delta_{C,\text{ loss}}$	Loss of log carbon by transition to fine wood.	kg C ha^{-1}
$\hat{\Delta}_{C,\text{ loss},\, s}$	Estimate of $\Delta_{C,\text{ loss}}$ from repeated samples at P_s.	kg C ha^{-1}
$\Delta_{C,\text{ res}}$	Loss in log carbon from decomposition.	kg C ha^{-1}
$\hat{\Delta}_{C,\text{ res},\, s}$	Estimate of $\Delta_{C,\text{ res}}$ from repeated samples at P_s.	kg C ha^{-1}
κ_v	Constant design parameter.	$m^2\,m^{-3}$
λ_C	Carbon stock in logs per unit land area.	kg C ha^{-1}
$\hat{\lambda}_{C,s}$	Estimate of λ_C from the sample at P_s.	kg C ha^{-1}
$\hat{\lambda}_{C,\text{rep}}$	Average estimate of λ_C from m samples.	kg C ha^{-1}
$\hat{\lambda}_{C,\text{two}}$	Two-phase estimate of λ_C	kg C ha^{-1}
λ_v	Aggregate log volume per unit land area.	$m^3\,ha^{-1}$
$\tilde{\lambda}_{v,s}$	Estimate of λ_v from the sample at P_s.	$m^3\,m^{-2}$
$\hat{\lambda}_{v,s}$	Estimate of λ_v from the sample at P_s.	$m^3\,ha^{-1}$
$\hat{\lambda}_{v,\text{rep}}$	Average estimate of λ_v from m samples.	$m^3\,ha^{-1}$
ρ_C	Carbon concentration of woody debris.	kg C (kg wood)$^{-1}$
$\rho_{w,k}$	Bulk density of a core from the kth log in a sample.	kg m^{-3}
$\bar{\rho}_w$	Average bulk density in a second-phase sample of cores.	kg m^{-3}
Ω	Horizontal width of a log, coincident with the perpendicular distance line.	m
Ω'	Horizontal width of a log, perpendicular to the log's central axis.	m

Table 6.5 Limiting distance (D_{limit}) by volume factor and log diameter

	F_v ($m^3\ ha^{-1}$)				
Diameter	14	28	56	112	224
(cm)	D_{limit} (m)				
2	0.11	0.06	0.03	0.01	0.01
4	0.45	0.22	0.11	0.06	0.03
6	1.01	0.50	0.25	0.13	0.06
8	1.80	0.90	0.45	0.22	0.11
10	2.80	1.40	0.70	0.35	0.18
12	4.04	2.02	1.01	0.50	0.25
14	5.50	2.75	1.37	0.69	0.34
16	7.18	3.59	1.80	0.90	0.45
18	9.09	4.54	2.27	1.14	0.57
20	11.22	5.61	2.80	1.40	0.70
22	13.58	6.79	3.39	1.70	0.85
24	16.16	8.08	4.04	2.02	1.01
26	18.96	9.48	4.74	2.37	1.19
28	21.99	11.00	5.50	2.75	1.37
30	25.24	12.62	6.31	3.16	1.58
32	28.72	14.36	7.18	3.59	1.80
34	32.43	16.21	8.11	4.05	2.03
36	36.35	18.18	9.09	4.54	2.27
38	40.50	20.25	10.13	5.06	2.53
40	44.88	22.44	11.22	5.61	2.80
42	49.48	24.74	12.37	6.19	3.09
44	54.30	27.15	13.58	6.79	3.39
46	59.35	29.68	14.84	7.42	3.71
48	64.63	32.31	16.16	8.08	4.04
50	70.12	35.06	17.53	8.77	4.38
52	75.85	37.92	18.96	9.48	4.74
54	81.79	40.90	20.45	10.22	5.11
56	87.96	43.98	21.99	11.00	5.50
58	94.36	47.18	23.59	11.79	5.90
60	100.98	50.49	25.24	12.62	6.31
62	107.82	53.91	26.96	13.48	6.74
64	114.89	57.45	28.72	14.36	7.18
66	122.19	61.09	30.55	15.27	7.64
68	129.70	64.85	32.43	16.21	8.11
70	137.44	68.72	34.36	17.18	8.59
72	145.41	72.71	36.35	18.18	9.09
74	153.60	76.80	38.40	19.20	9.60
76	162.02	81.01	40.50	20.25	10.13

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