ADJUSTMENTS OF INDIVIDUAL-TREE SURVIVAL AND DIAMETER-GROWTH EQUATIONS TO MATCH WHOLE-STAND ATTRIBUTES

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Abstract—Individual-tree models are flexible and can perform well in predicting tree survival and diameter growth for a certain growing period. However, the resulting stand-level outputs often suffer from accumulation of errors and subsequently cannot compete with predictions from whole-stand models, especially when the projection period lengthens. Evaluated in this study were five methods for adjusting the tree-survival equation to match observed number of trees per acre and three methods for adjusting the diameter-growth function to match observed basal area per acre. The evaluation procedures were repeated for adjustments using outputs from a whole-stand model. The different methods for adjusting tree survival probability and diameter growth were found to produce similar results. The methods selected (one for survival probability and another for diameter growth) both involved direct computation of the adjustment factor. Use of observed stand attributes for adjustment resulted in improvement of tree-level predictions of the individual-tree model. On the other hand, when predicted stand densities were used for adjustment, the quality of the tree-level predictions depended on the reliability of the stand predictions.

INTRODUCTION

Forest management decisions are based on information provided by growth and yield models, which ranged from simple whole-stand models, to size-class distribution models, and finally to complicated individual-tree simulation models. Each type of models has advantages and disadvantages; stand-level predictions from individual-tree and size-class models typically suffer from accumulation of errors, whereas whole-stand model outputs are often better behaved but lack information on stand structures (Qin and Cao 2006).

Attempts have been made to link models of different resolutions, such as diameter-distribution and whole-stand models (Baldwin and Feduccia 1987, Matney and Sullivan 1982), individual-tree and diameter-distribution models (Bailey 1980, Cao 1997, Qin and others 2007), stand-table projection and whole-stand models (Cao 2006, Nepal and Somers 1992, Pienaar and Harrison 1988), and individual-tree and whole-stand models (Harrison and Daniels 1988, McTague and Stansfield 1995, Richie and Hann 1997*a*). Ritchie and Hann (1997*b*) reviewed the disaggregative approach that uses information from an individual-tree model to distribute stand growth (obtained from a whole-stand model) among trees in the tree list.

The objective of this study was to evaluate different disaggregation methods for adjusting the individual-tree survival and diameter growth equations to match stand predictions from a whole-stand model.

DATA

Data used in this study were from the Southside Seed Source Study, which included 15 seed sources of loblolly pine (*Pinus taeda* L.) planted at 13 locations across 10 southern states (Wells and Wakeley 1966). Seedlings were planted at a 6 by 6 ft spacing. Each plot of size 0.04-acre consisted of 49 trees. Tree diameters and heights were measured at ages 10, 15, 20, and 25 years. A subset (100 five-year-growth periods) of the original data was randomly selected as the fit data set, to be used for fitting the models. The validation data set, which comprised another 100 growth periods, was randomly selected from the rest of the data. Summary statistics for the fit and validation data sets are shown in table 1.

MODELS

Parameters of the following whole-stand model were simultaneously obtained from the fit data set by use of the seemingly unrelated regression method (SUR) in the SAS MODEL procedure (SAS Institute Inc. 2000):

$$\hat{N}_2 = N_1 / [1 + \exp(16.3284 - 42.4435\, RS_1 - 0.2277\, H_1 - 0.0665\, N_1/A_1 + 50.2900/A_1)] \quad (1)$$
$$s_{y.x} = 99.31;\ R^2 = 0.882$$

$$\hat{B}_2 = B_1\, [1 + \exp(-3.3277 - 0.1790\, B_1/A_1 + 41.0543/A_1)] \quad (2)$$
$$s_{y.x} = 32.00;\ R^2 = 0.647$$

where

$\hat{N}_2$ and $\hat{B}_2$ = number of trees and basal area (square feet) per acre, respectively, at the end of the growth period (age A_2),
N_1 and B_1 = number of trees and basal area per acre, respectively, at the beginning of the growth period (age A_1),
H_1 = average height of the dominants and codominants at time 1, and
$RS_1 = (43{,}560/N_1)^{0.5}/H_1$ = relative spacing at age A_1.

The following individual-tree growth model was also derived from the fit data set:

$$\hat{p}_i = [1 + \exp(1.3586 - 0.0026\, N_1 + 0.0239\, B_1 - 0.7371\, d_{1i})] \quad (3)$$
$$-2 \log L = 2212;\ AIC = 2214$$

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Table 1—Means (and standard deviations) of stand and tree variables in the fit and validation data sets

Age	Dominant Height (ft)	Number of trees per acre	Basal area (sq ft/acre)	Tree diameter (in)
Fit data set				
10	29.4 (4.8)	667 (383)	76.8 (40.1)	11.2 (3.3)
15	28.3 (15.9)	574 (348)	115.2 (50.9)	14.9 (4.1)
20	36.0 (19.9)	437 (206)	128.3 (44.0)	18.1 (4.7)
25	19.5 (2.5)	400 (230)	138.5 (59.3)	19.6 (5.1)
Validation data set				
10	28.0 (5.2)	777 (284)	83.9 (31.2)	10.9 (3.2)
15	27.8 (15.9)	667 (249)	132.3 (33.5)	14.8 (4.0)
20	33.7 (19.9)	499 (167)	146.5 (37.3)	18.1 (4.3)
25	19.7 (2.5)	485 (171)	169.0 (41.8)	19.7 (4.9)

$$\hat{d}_{2i} = d_{1i} + 6.0988\,(A_2/A_1)^{2.0214}\,N_1^{-1.0104}\,B_1^{-0.3168}\,d_{1i}^{1.5122} \qquad (4)$$

$$s_{y.x} = 0.47;\ R^2 = 0.947$$

where

$\hat{p}_i$ = predicted probability that tree *i* will survive the growth period,

d_{1i} = diameter at breast height (d.b.h.) of tree *i* at age A_1, and

$\hat{d}_{2i}$ = diameter at breast height of tree *i* at age A_2.

ADJUSTMENTS

Described below are different methods to adjust the individual-tree survival and diameter growth equation to match estimates of number of trees and basal area per acre from the whole-stand model.

Survival Adjustment

Method 1—The adjusted tree survival probability was expressed as a power function of the unadjusted probability as follows:

$$p_i^* = \hat{p}_i^{\,\alpha}, \qquad (5)$$

where α is the adjustment coefficient.

Method 2—Jin and Cao (2006) assumed proportional ratios of dead and alive probabilities in developing the following adjustment:

$$p_i^* = \frac{\hat{p}_i}{\hat{p}_i + \alpha\,(1 - \hat{p}_i)}\,. \qquad (6)$$

Method 3—The tree survival equation 3 was simplified as:

$$p_i^* = [1 + \exp(\alpha - 0.7371\,d_{1i})]. \qquad (7)$$

Method 4—Eq. 3 can also be rewritten as:

$$p_i^* = [1 + \exp(1.3586 - 0.0026\,N_1 + 0.0239\,B_1 + \alpha\,d_{1i})]. \qquad (8)$$

In methods 1–4 above, SAS procedure MODEL (SAS Institute, Inc. 2000) was used to iteratively solve for the value of α for each plot such that $\Sigma p_i^* = s\hat{N}_2$. Note that eq. (5–8) are properly constrained, so that p_i^* is always between 0 and 1.

Method 5—The adjusted tree survival probability (p_i^*) was obtained from

$$p_i^* = \hat{p}_i + \alpha(1 - \hat{p}_i), \qquad (9)$$

where

$\alpha = \dfrac{s\hat{N}_2 - \Sigma\hat{p}_i}{n_1 - \Sigma\hat{p}_i}$ = adjustment coefficient,

s = plot size in acres,

$\hat{N}_2$ = number of trees per acre at the end of the growth period as predicted from the whole-stand model,

n_1 = number of trees in a plot at the beginning of the period, and the summation signs include values of *i* from 1 to n_1.

Equation 9 has an upper asymptote of 1, but p_i^* could be negative. The latter case occurs when the value of Σp_i is too high compared to $s\hat{N}_2$. It was necessary in this case to employ a two-step procedure as follows:

1. In the first step, the survival probability was adjusted using eq. 9 such that the probability for the smallest diameter (d_{min}) in the plot equaled zero. Let p_{min} be the unadjusted survival probability, computed from eq. 3, of the tree with the smallest diameter. Let α_{min} be the value of α such that the adjusted survival probability from this step equaled zero for the smallest diameter. In other words, α_{min} is the solution of:

$$p_{min} + \alpha_{min}\,(1 - p_{min}) = 0, \text{ or } \alpha_{min} = -p_{min}/(1 - p_{min}). \qquad (10)$$

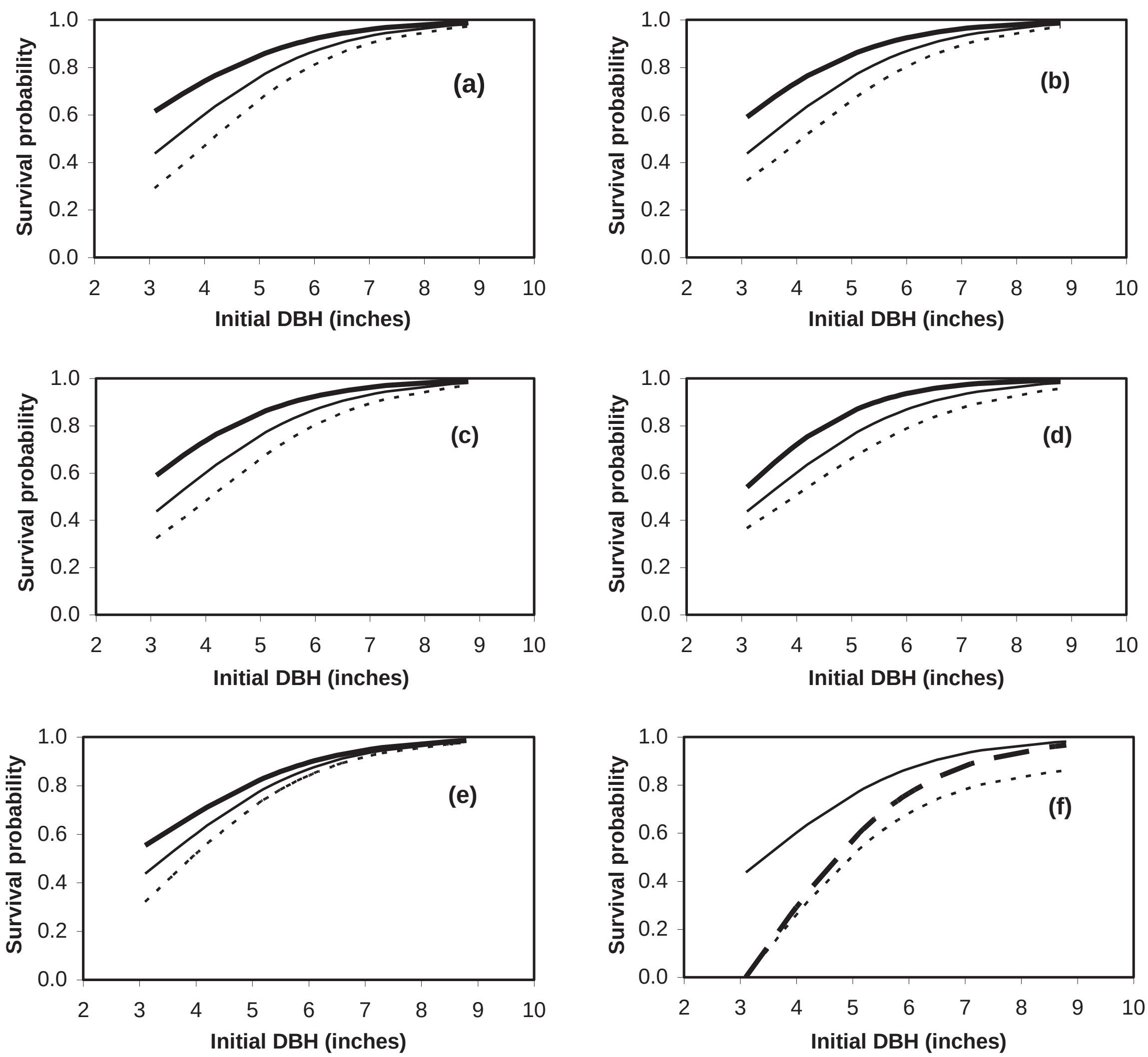

Figure 1—Figures (a) to (e) illustrate, respectively, methods 1 to 5 of adjusting tree survival probability. The unadjusted survival probability (thin solid line) is adjusted upwards (bold solid line) to increase the predicted stand density by 10 percent, or downwards (dotted line) to decrease the predicted stand density by 10 percent. Figure (f) demonstrates the case when the adjustment in method 5 needs to be done in two steps. The survival probability is adjusted downwards (dashed line) using Eq. (9) in the first step to zero probability for the smallest diameter, then downwards further (dotted line) using Eq. (11) in the second step.

2. In the second step, the final probability was proportional to the adjusted survival probability from step 1:

$$p_i^* = \beta\,[\,\hat{p}_i + \alpha_{min}(1 - \hat{p}_i)\,],$$

$$\text{where } \beta = \frac{s\hat{N}_2}{\Sigma\hat{p}_i + \alpha_{min}\left(n_1 - \Sigma\hat{p}_i\right)} \qquad (11)$$

The five methods for adjusting tree survival probability are illustrated in figures (1a–1f).

Diameter Growth Adjustment

Method 1—The tree diameter growth eq. 4 can be simplified by using α to express the effect of stand attributes:

$$d_{2i}^* = d_{1i} + \alpha\, d_{1i}^{1.5122} \qquad (12)$$

Method 2—The power for d_{1i} in Equation 4 was replaced with the adjustment coefficient (α) as follows:

$$d_{2i}^* = d_{1i} + 6.0988\,(A_2/A_1)^{2.0214}\, N_1^{-1.0104}\, B_1^{-0.3168}\, d_{1i}^{\alpha} \qquad (13)$$

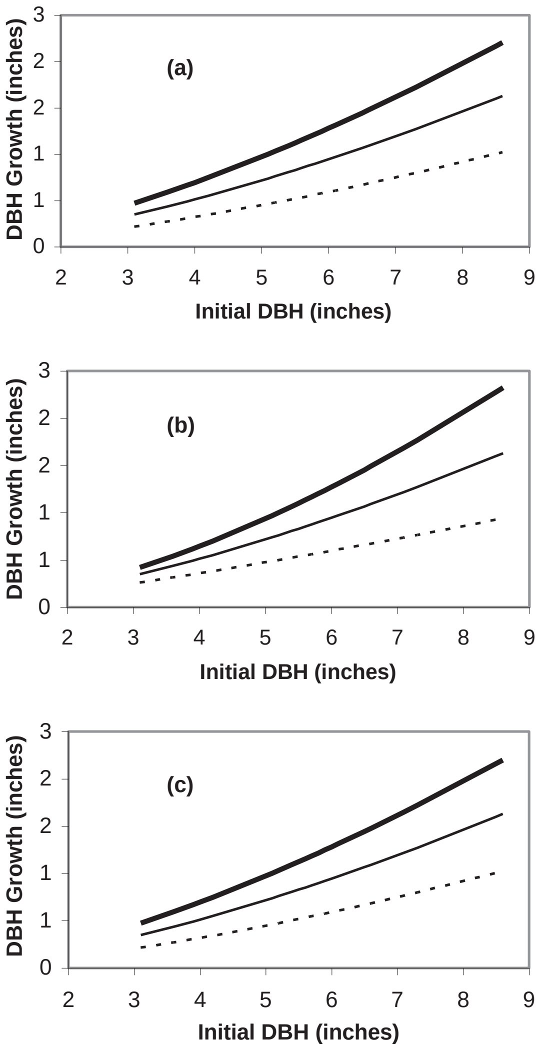

Figure 2—Figures (a) to (c) illustrate, respectively, methods 1 to 3 of adjusting tree diameter growth rate. The unadjusted diameter growth (thin solid line) is adjusted upwards (bold solid line) to increase the predicted stand basal area by 10 percent, or downwards (dotted line) to decrease the predicted stand basal area by 10 percent.

In methods 1 and 2 above, the value of α for each plot was iteratively determined such that $K\Sigma p_i^* d_{2i}^{*2} = s\hat{B}_2$, where K = 0.005454.

Method 3—The adjusted squared diameter growth of each tree was assumed to be proportional to the predicted squared diameter growth, leading to:

$$d_{2i}^{*2} = d_{1i}^2 + \alpha\,(\hat{d}_{2i}^{\,2} - d_{1i}^2),$$

$$\text{where } \alpha = \frac{(s\hat{B}_2 / K - \Sigma(p_i^* d_{1i}^2)}{\Sigma[\,p_i^*(\hat{d}_{2i}^2 - d_{1i}^2)]} \quad (14)$$

Figures (2a–2c) illustrate the three methods for adjusting tree diameter growth.

RESULTS AND DISCUSSION

The validation data set was employed to evaluate the unadjusted and adjusted methods for predicting tree survival probability and diameter growth. The individual-tree equations were adjusted with observed and then predicted number of trees and basal area per acre. Common evaluation statistics for both tree survival probability and diameter growth included mean difference (MD) between observed and predicted values, and mean absolute difference (MAD).

Survival Adjustment

In addition to MD and MAD, –2ln(*L*) was added as the third evaluation statistic, where *L* is the likelihood function. All of the adjustment methods provided better evaluations statistics than did the unadjusted probabilities (table 2), whether the stand attributes used for adjustment were observed or predicted from the whole-stand model. The five adjustment methods were comparable, producing similar evaluation statistics. The first four methods required the adjustment coefficient, α, to be solved in an iterative manner, whereas method 5 allowed α to be directly computed. Therefore method 5 was selected as the appropriate method for adjusting tree survival probability.

Use of observed stand attributes for adjustment greatly reduced the MD value, compared with the unadjusted model, from 0.0250 to almost zero, MAD from 0.2483 to 0.1832, and –2ln(*L*) from 2030 to 1475 for method 5. As expected, predicted stand attributes did not fare as well; the evaluation statistics for method 5 were –0.0168, 0.2146, and 2012 for MD, MAD and –2ln(*L*), respectively. These statistics were, however, still better than those from the unadjusted method. It was clear from these results that the success of the adjustment largely depended on the quality of the stand predictions.

Table 2—Evaluation statistics[†] for the unadjusted and adjusted predictions of tree survival probability from the validation data set

Method	MD	MAD	-2 ln(*L*)
Unadjusted	0.0250	0.2483	2030
Adjusted from observed stand attributes			
Method 1	0.0000	0.1812	1486
Method 2	0.0000	0.1866	1469
Method 3	0.0000	0.1834	1486
Method 4	0.0000	0.1844	1486
Method 5	0.0000	0.1832	1475
Adjusted from predicted stand attributes			
Method 1	–0.0168	0.2148	2015
Method 2	–0.0168	0.2331	1977
Method 3	–0.0168	0.2147	2013
Method 4	–0.0168	0.2142	2098
Method 5	–0.0168	0.2146	2012

[†] MD = mean difference between observed and predicted probabilities, MAD = mean absolute difference, and *L* = likelihood function.

Diameter Growth Adjustment

The survival probability adjusted with method 5 above was used in conjunction with the three methods for adjusting tree diameter growth to match stand basal area. The evaluation statistics employed for this purpose were MD, MAD, and fit index (FI), which is computationally similar to R^2 in linear regression.

The three adjustment methods resulted in similar evaluation statistics (table 3). The adjustment coefficient, α, was directly computed in method 3, but had to be iteratively searched in the first two methods. Therefore method 3 was selected as appropriate for adjusting tree diameter growth. When observed values of number of trees and basal area per acre were used for adjustment, the resulting future diameters from method 3 produced a higher MD value (0.0589 vs. 0.0421) than the unadjusted, but lower MAD (0.3383 vs. 0.3765) and higher R^2 (0.9464 vs. 0.9368). If predicted tree survival probability was further replaced with observed survival in the diameter adjustment process, the results improved to –0.0132 for MD and 0.9537 for R^2. The adjustment for diameter growth thus was partly influenced by how accurate the survival prediction was.

When diameter growth was adjusted using predicted stand densities, MD improved to 0.0332, but MAD and R^2 deteriorated to 0.4258 and 0.9211, respectively, which were even worse than those from the unadjusted method. The reason might be traced to the lack of accuracy and precision in predicting stand basal area, as evidenced in a fairly low R^2 (0.647).

SUMMARY AND CONCLUSIONS

In this study, the different methods for adjusting tree survival probability and diameter growth were found to produce similar results. The methods finally selected (method 5 for survival probability and method 3 for diameter growth) both involved direct rather than iterative computation of the adjustment coefficient (α). Use of *observed* stand attributes for adjustment resulted in improvement of tree-level predictions of the individual-tree model. On the other hand, when *predicted* stand densities were used for adjustment, the quality of the tree-level predictions depended on the reliability of the stand predictions.

Table 3—Evaluation statistics† for the unadjusted and adjusted predictions of tree diameter growth from the validation data set

Method	MD	MAD	FI
Unadjusted	0.0421	0.3765	0.9368
Adjusted from observed stand attributes			
Method 1	–0.0589	0.3384	0.9463
Method 2	–0.0630	0.3385	0.9462
Method 3	–0.0590	0.3383	0.9464
Adjusted from predicted stand attributes			
Method 1	0.0332	0.4258	0.9211
Method 2	0.0328	0.4274	0.9200
Method 3	0.0332	0.4258	0.9211

† MD = mean difference between observed and predicted values, MAD = mean absolute difference, and FI = fit index, computationally similar to R^2 in linear regression.

LITERATURE CITED

Bailey, R.L. 1980. Individual tree growth derived from diameter distribution models. Forest Science. 26: 626-632.

Baldwin, V.C., Jr.; Feduccia, D.P. 1987. Loblolly pine growth and yield prediction for managed West Gulf plantations. Res. Pap. SO-236. U.S. Forest Service, Southern Forest Experiment Station, New Orleans, LA: 27 p.

Cao, Q.V. 1997. A method to distribute mortality in diameter distribution models. Forest Science. 43:435-442.

Cao, Q.V. 2006. Predictions of individual-tree and whole-stand attributes for loblolly pine plantations. Forest Ecology and Management. 236: 342-347.

Harrison, W.C.; Daniels, R.F. 1988. A new biomathematical model for growth and yield of loblolly pine plantations. In: Forest growth modeling and prediction, vol. 1. Gen. Tech. Rep. NC-120. U.S. Forest Service, North Central Forest Experiment Station, St. Paul, MN: 293-304.

Matney, T.G.; Sullivan, A.D. 1982. Compatible stand and stock tables for thinned loblolly pine stands. Forest Science. 28: 161-171.

McTague, J.P.; Stansfield, W.F. 1995. Stand, species, and tree dynamics of an uneven-aged, mixed conifer forest type. Canadian Journal of Forest Research. 25: 803-812.

Nepal, S.K.; Somers, G.L. 1992. A generalized approach to stand table projection. Forest Science. 38: 120-133.

Pienaar, L.V.; Harrison, W.M. 1988. A stand table projection approach to yield prediction in unthinned even-aged stands. Forest Science. 34: 804-808.

Qin, J.; Cao, Q.V. 2006. Using disaggregation to link individual-tree and whole-stand growth models. Canadian Journal of Forest Research. 36: 953-960.

Qin, J.; Cao, Q.V.; Blouin D.C. 2007. Projection of a diameter distribution through time. Canadian Journal of Forest Research. 37: 188-194.

Ritchie, M.W.; Hann D.W. 1997*a*. Evaluation of individual-tree and disaggregative prediction methods for Douglas-fir stands in western Oregon. Canadian Journal of Forest Research. 27: 207-216.

Ritchie, M.W.; Hann D.W. 1997*b*. Implications of disaggregation in forest growth and yield modeling. Forest Science. 43: 223-233.

SAS Institute Inc. 2000. SAS/ETS user's guide, version 8. SAS Institute Inc., Cary, NC: 600 p.

Wells, O.O.; Wakeley P.C. 1966. Geographic variation in survival, growth, and fusiform rust infection of planted loblolly pine. Forest Science Monograph. 11: 40 p.