

Critical length sampling: a method to estimate the volume of downed coarse woody debris

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Abstract In this paper, critical length sampling for estimating the volume of downed coarse woody debris is presented. Using this method, the volume of downed wood in a stand can be estimated by summing the critical lengths of down logs included in a sample obtained using a relascope or wedge prism; typically, the instrument should be tilted 90° from its usual orientation in sampling live trees. The critical length of a downed tree is the length of the log section large enough in diameter to be counted from the selected sample point. A simulation study was conducted to compare critical length sampling with diameter relascope sampling and perpendicular distance sampling. For the population studied, diameter relascope sampling tended to have the smallest variance but exhibited a small design-bias. By combining critical length sampling with critical height sampling—a method previously described for estimating the volume of standing trees—the total volume of dead wood in a stand can be estimated using the same basic principle. One use of the combined method might be in

rapid in-place inventories where a quick estimate of the volume of dead wood is needed for biodiversity management purposes. The combined critical length/height method provides a relatively simple means for obtaining such estimates.

Keywords Critical height sampling · Dead wood · Forest inventory · Relascope sampling · Simulation

Introduction

Coarse woody debris (CWD) has received increasing attention during the last decades due to its importance as habitat for a large number of forest species (e.g. Samuelsson et al. 1994) and its role in carbon balances. Forest management practices are now being developed in order to maintain CWD quantities at adequate levels (e.g. Wikström and Eriksson 2000). As a consequence, efficient inventory methods are needed that can be used to estimate the quantities of dead wood in forest stands and to follow trends over time.

A large number of methods are available for estimating quantities of dead wood. Among traditional forest inventory methods, sample plot inventory and strip surveying (e.g. Schreuder et al. 1993, pages 112–117) can be applied. Line intersect sampling (Warren and Olsen 1964) is a widely used efficient method for sampling downed CWD. Moreover, a number of methods based on the principle of relascope sampling (Bitterlich 1984) have been developed. Transect relascope sampling (Ståhl 1998) and point relascope sampling (Gove et al. 1999) make use of the length of a downed log for determining its inclusion probability and thus for estimating dead wood quantities. Williams and Gove (2003, 2005a) propose perpendicular distance sampling (PDS), a method whereby the volume of downed logs

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in an area can be obtained from a mere count of logs on the sample point; this method has been extended for nearly design-unbiased estimation of surface area as well (Williams et al. 2005b). Bebbler and Thomas (2003) suggest a relascope method for assessing downed CWD; with this method, known as diameter relascope sampling (DRS), the inclusion of a downed log is based on its mid-point diameter and Huber's formula (e.g. Schreuder et al. 1993, page 250) is implicitly used for estimating the volume of logs. In addition, critical point relascope sampling has been proposed for design-unbiased estimation of down wood volume in point relascope sampling inventories (Gove et al. 2005), and a combined line intersect distance sampling strategy has been proposed by Affleck (2008).

The objective of this article is to propose and describe another new method for determining the volume of downed coarse woody debris based on the principle of relascope sampling. The method, critical length sampling (CLS), can be seen as an extension of the critical height sampling method developed by Kitamura (1962) and later expanded by Iles (1979, 1990). We believe that CLS has the potential to become a useful tool in obtaining quick estimates of the volume of dead wood in connection with stand level forest inventories, primarily since it can be easily combined with critical height sampling as thus all types of dead wood can be assessed in one single inventory. Rapid in-place inventories are becoming increasingly important for making biodiversity targets operational in ordinary forest management and management of forest reserves. In this article, we describe the method and compare its performance with similar methods.

Methods

Description of the method

With CLS, relascope sample points are distributed over the area of interest according to some appropriate random sampling principle. For the purpose of describing the theory of CLS, we assume that the points are distributed in accordance with a uniform distribution. At each point, a small angle relascope or wedge prism is used to assess which parts of downed trees are large enough to be counted. To make this determination, the instrument is operated in a tilted manner to normal relascope sampling, such that it is parallel to the down log. For example, assuming that the log lies horizontally on flat terrain, the instrument would be tilted 90° . The length of the section of a tree, within which the diameter is large enough to include the section when aiming at the tree trunk with the relascope, is denoted the *critical length* of the tree. An estimate of the per-hectare volume of downed dead wood in the stand is

obtained by summing the critical lengths of trees and multiplying the sum by the basal area factor of the relascope.

The theory of the method will be described for the case where the relascope is operated in the same horizontal plane as in which the downed trees are located. Moreover, it is assumed that we can aim at a certain point of a stem with the relascope and use the diameter of the stem at that point for determining whether or not the log is included. Thus, the inclusion zone (e.g. Gove et al. 1999) around a certain point of a stem will be a circle, with the same properties as the circular inclusion zones around trees in traditional relascope sampling. This procedure is illustrated in Fig. 1. The critical length of the stem is shown as the darker shaded portion between d_l and d_u . Dashed circles are shown centered on these two diameters and correspond to the borderline distance of the expanded tree diameter circle (inclusion zone) commonly used to illustrate horizontal point sampling. There are only two possible sample points, P_1 and P_2 , which choose d_l and d_u as defining this particular critical length. It may be observed, therefore, that determination of the critical length is facilitated by finding the diameters that are exactly border line when sighted with the relascope.

By definition, the total volume, Y , of dead trees in the area considered is:

$$Y = \int_{l=0}^{\infty} G(l)dl = \int_{l=0}^{\infty} \left(\sum_{i=1}^N g_i(l) \right) dl \quad (1)$$

Here, $G(l)$ is the total cross-sectional area of dead trees at a common length l along the downed trees; $g_i(l)$ is the cross-sectional area of tree i at length l (which is zero if l is greater than the length of the tree; the total cross-sectional area will be 0 for all lengths above the longest down log in

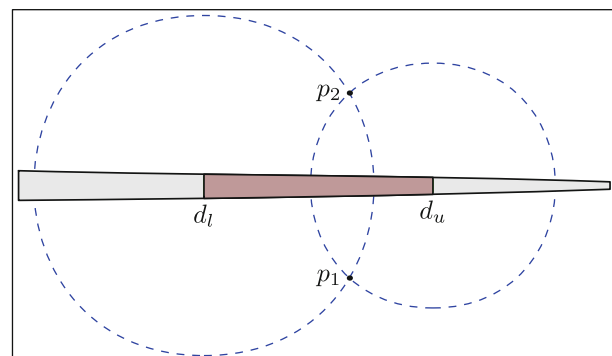


Fig. 1 An illustration of critical length sampling: the critical length is shown as the darker shaded region of the log bounded by diameters d_l and d_u as chosen from sample points P_1 or P_2 . At either of these two points diameters d_l and d_u would appear to be exactly “borderline” through the rotated angle gauge (dashed lines)

the population), and N is the number of downed trees in the area. The inclusion area around a stem at a certain point (at length l) is circular, with size $\frac{g_i(l)}{\sin^2 \theta/2}$, where θ is the relascope angle. This follows from standard relascope theory (e.g. Bitterlich 1984).

Although it would be extremely impractical, we now assume that a relascope be used for estimating the cross-sectional area at all possible lengths along the stems of downed trees, i.e. we estimate the quantity $\sum_{i=1}^N g_i(l)$ for all possible lengths, l , using a relascope at a randomly selected sample point. The per hectare volume estimator would be:

$$\hat{Y} = \int_{l=0}^{\infty} \left(R \sum_{i=1}^N I_i(l) \right) dl \quad (2)$$

$I_i(l)$ is an indicator variable, which is one when the tree is included at length l and zero otherwise; R is the basal area factor of the relascope.

In order to make the estimator operational in practice, the following mathematical rearrangement of (2) is made:

$$\hat{Y} = \int_{l=0}^{\infty} \left(R \sum_{i=1}^N I_i(l) \right) dl = R \sum_{i=1}^N \int_{l=0}^{\infty} I_i(l) dl = R \sum_{i=1}^N c_i \quad (3)$$

Here, c_i is the critical length of a dead tree (which is 0 for all trees in the population that are not included in the sample at all). This length is obtained as the length along the stem where the diameter of the tree is coarse enough for the tree to be included with the relascope, from the specific sample point (see Fig. 1). For simple use in the field, some type of laser range finder would be beneficial or, alternatively, in subjective surveys an ocular estimate of the critical length might be sufficient.

The estimator (3) is found to be identical with the one proposed for standing trees by Kitamura (1962). One difference between the two applications is that the large butt sections of down dead trees sometimes may be excluded in CLS due to their long distance from the sample point, whereas thinner upper parts of a tree that happen to fall closer to the sample point may sometimes be the only parts of a tree that are included. This is due to the orientation of the down tree relative to the sample point.

A more intuitive proof

Below, some intuition is added to the proof given in the former section. In addition, we expand the case to a situation where all dead wood in a stand is estimated, including standing, leaning, and downed dead trees.

As a basis for the intuitive proof, assume all dead wood logs to be chopped up into 1-dm short sections before sampling. Each piece is short enough so that a very precise

volume estimate can be obtained by using Huber's formula (e.g. Schreuder et al. 1993, page 250), i.e. the volume of a piece equals its length times the cross-sectional area at its center point. Although chopped up, we assume that the pieces still are attached to each other to resemble dead trees or logs. Next, we consider our population of interest to be the 1-dm long units of dead wood in the stand; the objective is to estimate their total volume.

For this example, we suppose that a relascope with automatic correction for deviations from the horizontal plane is used, e.g. a Spiegel Relaskop (Bitterlich 1984). Also whenever pieces are not oriented vertically, the relascope is tilted so that the instrument will be oriented parallel with the orientation of the specific piece of wood.

In this case, the inclusion area around each piece of wood will be circular with size $\frac{g_i}{\sin^2 \theta/2}$ where θ is the relascope angle and g_i is the cross-sectional area (m^2) at the mid-point of the piece at which the relascope is aimed (Fig. 2, longer sections are used in this figure to facilitate illustration). Assume a stand of 1 ha size (i.e. the estimate will be obtained as a per-hectare value) and one randomly located point within this area. Ignoring boundary problems, the probability of inclusion of a specific piece will be:

$$\frac{g_i}{10,000 \sin^2 \theta/2} = \frac{g_i}{R} \quad (4)$$

Because a Horvitz-Thompson estimator (e.g. Thompson 1992) always can be set up as the sum of the quantities of interest on sampled units over their corresponding probability of inclusion, an estimator of the total volume of dead wood pieces will be:

$$\hat{Y} = R \sum_{i=1}^n \frac{0.1 g_i}{g_i} = R n \cdot 0.1 = R C = R \sum_{i=1}^N c_i \quad (5)$$

In this formula, the term $0.1 g_i$ is the volume of an individual section and C is the total length of all units

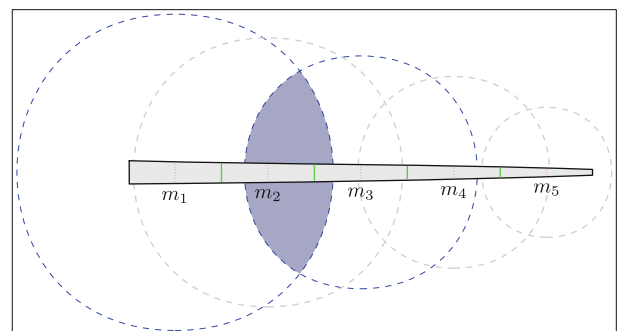


Fig. 2 A log with five segments having midpoints $m_1 \rightarrow m_5$ is depicted with “inclusion” zones (dashed) determined by the rotated angle gauge sighting on diameters at the midpoints; zones for m_1 and m_3 are shown darker with others in gray. The locus of points in the shaded intersection area would select critical length $l_1 + l_2 + l_3$, where l_i is the i th segment length

included ($0.1n$). This is the same result as the estimator (3) presented above.

Geometrical considerations

In practice, CLS requires attention to geometrical factors, at least some of which may require correction in the field. The foremost challenge is that determination of critical length requires accurate identification of the borderline condition when sighting at an angle to the log. This is directly analogous to the situation in critical height sampling. Recall that in ordinary horizontal point sampling of standing trees on level terrain, the relascope would be operated at breast height and the sighting point would be at breast height on the tree. The line of sight would form a right angle with the standing log, such that the relevant cross-section of the log is approximately circular. When one contemplates critical height sampling, or horizontal point sampling on irregular terrain, the ideal relascope would compensate for a line of sight displaced from the horizontal plane and no longer at a right angle to the standing bole, such that the sighting distance is greater than the horizontal distance and the cross-section is approximately an ellipse (cf. Fig. 1 in Großenbaugh 1963). This problem has been well studied; the Spiegel Relaskop is perhaps the most familiar of several relascope designs that meet this objective (Bitterlich 1984).

The problem in CLS is only slightly more complicated than in critical height sampling. In CLS, the log may have an arbitrary orientation; the angle to the log axis, which determines the elongation of the cross-section into an ellipse, is not uniquely defined by the slope of the line of sight relative to the horizontal (see Fig. 3 for an illustration). The most important points on a log for determining critical length—those where the log is approximately borderline—may fall anywhere along the log. However, for the range of

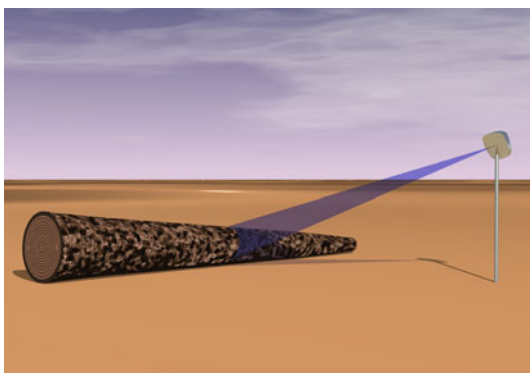


Fig. 3 An illustration of sampling a downed log with an angle gauge in critical length sampling. For geometrical reasons, the gauge generally will sight on a chord of an ellipse rather than a chord of a circle

sighting angles likely to be employed in critical length sampling, the fact that the gauge sights on a chord of an ellipse rather than a chord of a circle almost never causes appreciable error (cf. Großenbaugh 1963). The only exception occurs when the observer must sight very nearly along the axis of the log; in this case, the direct use of the angle gauge will be difficult anyway, and direct measurement of diameters and distances may be required. Thus, in practice the bias caused by log angle will be negligible.

The angle between the line of sight and the horizontal plane is more important, as it alone determines the increase in sighting distance. Practical correction is quite straightforward and follows the same principle as in traditional relascope theory (e.g. Bitterlich 1984) although the relascope is operated in a tilted manner as compared to the standard use. When using a Spiegel Relaskop, the approach is similar to that employed in the measurement of upper-stem diameters on crooked trees to obtain Pressler's point (Bitterlich 1984, pages 110–111). Suppose a point on the log is close to borderline, and a check is required to determine whether that point is within the critical length, or outside it:

- 1) The observer sights on the point, operating the relascope in the ordinary vertical position and locks in the slope of the line of sight.
- 2) The observer rotates the relascope so that the scale is perpendicular to the log.
- 3) The log diameter is compared to the relascope bands to determine if the point falls within the critical length.

When using a prism and performing slope correction following Bruce (1955), the procedure must be modified as follows:

- 1) The observer uses a clinometer or Abney level to determine the slope of the line of site.
- 2) The prism is oriented perpendicular to the log, as if sighting on a leaning tree in ordinary horizontal point sampling.
- 3) The prism is then rotated through an *additional* angle, equal to the slope of the line of sight.
- 4) The edges of the log, as viewed normally and through the prism, are compared to determine if the point falls within the critical length.

The prism is likely to be less accurate than the Spiegel Relaskop because angles of view may be steep (especially for logs close to the observer), and because the “diameter” must be observed on the diagonal. In both cases, a slight bias may occur because the angle gauge subtends a chord through the approximately elliptical cross-section rather than a diameter. In the absence of taper, this bias equals zero. In the presence of taper, the bias is no more severe for downed logs than for tapering portions of standing trees.

As for all methods to estimate downed wood volumes, the potential irregular cross-sectional shapes of decomposed logs may pose problems. That is, bias may occur if log shapes are not circular. Although not further elaborated in this article, we propose that this type of error can be avoided by applying sub-sampling procedures similar to those suggested in Valentine et al. (2008).

Simulation study

The notion of using a standard wedge prism to sample CWD was first introduced by Bebbler and Thomas (2003) in their development of DRS. In addition, both CLS and PDS share the trait that sampling of CWD can be performed with a wedge prism, though the wedge prism is only useful for estimating surface area with PDS. Thus, it seemed reasonable to compare the performance of these three prism-based sampling methods. The comparison was performed with a simulation study.

Since both DRS (Bebbler and Thomas 2003) and PDS (Williams and Gove 2003) have been developed rather recently, and may not be well known, a brief description of the two methods is provided here. With DRS, a relascope is used to sample logs from sample points just like in CLS. In DRS, inclusion is determined based on the mid-point diameter of a log and, by implicitly applying Huber's volume formula, the total log volume within an area is obtained by summing the lengths of logs included (the log lengths need to be measured) and multiplying the sum with the relascope factor. If careful measurements of midpoint diameter and log volume are made more accurate estimates would be obtained, although more time would also be required for the assessments. With PDS, sampling points are allocated as in CLS or DRS, but logs are included based on their diameters at a point perpendicular to the log axis from the sampling point. By selecting an appropriate threshold diameter value that depends on the distance of the log from the sampling point, an unbiased estimator of log volume can be obtained by only counting the logs included in the sample and multiplying by an expansion factor. The ingenious background of the method is that the inclusion area of objects has been constructed to be perfectly proportional to log volume.

The estimators for PDS and DRS can both be written as Horvitz-Thompson estimators with

$$\hat{Y}_{\text{PDS}} = \sum_{i=1}^n \frac{v_i}{\pi_i^{\text{PDS}}} = F_{\text{PDS}} n$$

and

$$\hat{Y}_{\text{DRS}} = \sum_{i=1}^n \frac{\hat{v}_i}{\pi_i^{\text{DRS}}} = R \sum_{i=1}^n L_i,$$

where the π_i in the denominator of each equation is the inclusion probability for a log as defined by the specific sampling design, v_i is the true volume of the log, $\hat{v}_i$ is the estimated volume of the log based on Huber's formula, L_i is the length of log i , and F_{PDS} is the PDS volume factor. The estimators for CLS and PDS are design-unbiased. The estimator for DRS is also design-unbiased, but only under the assumption that the volume, measured by Huber's formula, is equal to the true volume of each log in the data set. Given that this assumption is generally not correct, the actual design-bias for DRS is the weighted sum of the differences between the true volume of each log and the volume estimate derived from Huber's formula.

Constructing a simulator to test the sampling methods is challenging because the diameter at every point on the log and the relationship between the sample point on the ground and the location and orientation of the log must all be known. The simulator used in this study has two components. The first component is a module that derives taper equations to describe the form at every point on the log. The second component distributes the logs over a given area, constructs the response surface, and calculates the mean and variance of the estimator for each of the three sampling techniques. A brief description of each component follows.

Modeling of logs

A set of $N = 183$ ponderosa pine (*Pinus ponderosa* Laws) trees was felled for measurement. For each tree, diameter at breast height (*dbh*) total height (L) and diameter outside bark (d) measurements every 1.22 m (4 ft.) were taken. These data were collected at various timber sale sites on the Black Hills National Forest, U.S.A. Each tree in the data set was treated as a log and the measurements were used to generate individual taper equations for each log. The volume of each log was calculated by integrating each taper equation over the length of the log. In addition, we also calculated the volume that would be obtained by using Huber's formula. Summary statistics for the data set are given in Table 1.

Table 1 Summary statistics for the ponderosa pine data set ($N = 183$)

	Diameter breast height	Total length	Volume	Huber's volume
Minimum	0.18	11.0	0.15	0.13
Mean	0.28	17.5	0.71	0.70
Max	0.56	30.0	3.85	2.79

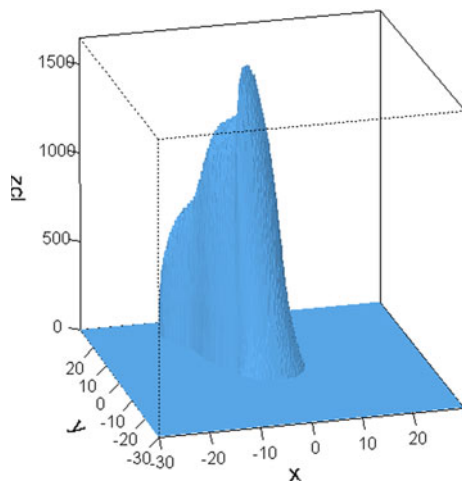


Fig. 4 The response surface of an individual log in critical length sampling

Construction of the response surface

For each log in the data set, the inclusion zone of the log was determined using the sampling methodology for CLS, DRS and PDS. Next, a fine grid of points was established over each inclusion zone. The height of the response surface for log i was Rc_i , v_i/π_i^{DRS} and v_i/π_i^{PDS} and for CLS, DRS and PDS, respectively. The response surface in CLS for a single log is given in Fig. 4. The response surface for each log and sampling technique was generated and located within a square representing an area (i.e., a stand) of approximately $A = 2$ ha. Figure 5 shows an example of the response surface for each of the sampling methods. The final response surface for each sampling technique can be described as a three-dimensional map of every possible estimate within the area A .

The bias and variance of the estimators for CLS, DRS and PDS were determined by calculating the appropriate expected values of each estimator using the results in Williams (2001).¹ The bias (B) and standard errors (SE) of the estimators were expressed as a percentage of the true volume, specifically.

$$B = 100 \frac{\hat{Y} - Y}{Y},$$

$$SE = 100 \frac{\sqrt{\text{var}(\hat{Y})}}{Y}.$$

The simulations were performed under the assumption that an equal number of logs sampled with each of the different methods would imply roughly identical inventory

costs for the methods. Thus, relascope angles were selected so that the average number of logs sampled would be the same for all three methods.

Results of the simulation

The results of the simulation study are given in Table 2. As expected, the DRS estimator exhibits a small bias when compared to the true total volume, with the bias being a roughly 2% underestimate. In contrast, the estimators for both CLS and PDS do not exhibit any bias.

The most surprising result is that the standard error for DRS is smaller than that of the other methods. This seems somewhat illogical, but the reason for the smaller standard error is that the Huber's volume estimates tend to underestimate the volume for large logs and overestimate the volume for small logs (see the summary statistics in Table 1). Note that while the volume for the largest log is underestimated by almost 50%, the distribution of Huber's volumes is significantly less variable than the distribution of true volumes (*cf.* Fraver et al. 2007). Also note that the means are similar, so the bias for the estimator of the overall volume is expected to be reasonably small. What is happening is that the Huber volume estimates are "smoothing" the response surface because the estimated volumes are "more similar" to each other than the true volumes. This trait is apparent in Fig. 5 where the response surface for DRS has fewer peaks than either of the other two surfaces. While the DRS estimator clearly has the best performance for this population, this result probably cannot be generalized to other populations where the distribution of the Huber's estimates of volume could exhibit a significant bias and may not reduce the variance to the same degree.

Discussion

Critical length sampling is one method out of many that can be applied for surveying coarse woody debris. While several other methods (e.g. line intersect sampling, transect relascope sampling, point relascope sampling, perpendicular distance sampling, and diameter relascope sampling) can be effectively applied for estimating the volume of down logs, critical length sampling has the advantage that it can be combined with critical height sampling to become a cost-efficient unified method for the survey of both standing and lying coarse woody debris.

From a practical point of view, one main field of application for the combined critical height/length method might be in rapid forest or biological inventories where there is a need to obtain a reasonable estimate of the

¹ With this approach, instead of applying Monte-Carlo sampling simulation (which is the typical sampling simulation approach), numerical integration was applied to calculate the first and second moments.

Fig. 5 Response surfaces for the methods critical length sampling (CLS), perpendicular distance sampling (PDS), and diameter relascope sampling (DRS)

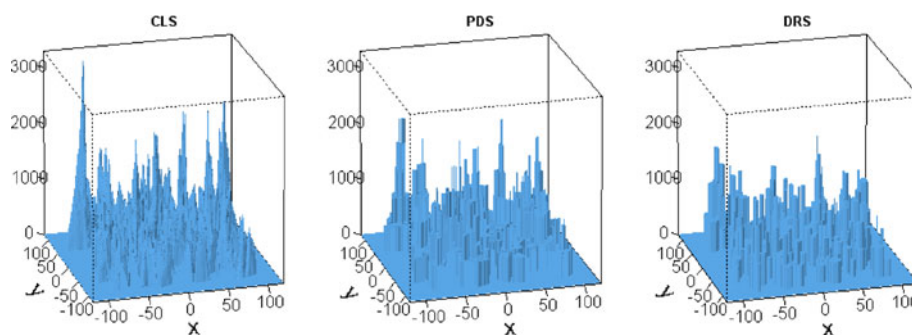


Table 2 Simulation results for the three sampling methods

	CLS	DRS	PDS
Bias	99.7	97.9	99.8
SE	207.3	188.8	191.7

The bias is expressed as a percentage of the true volume, so a result of 100 indicates perfect agreement between the estimated and true volume. The standard error (SE) is expressed for a sample of size 1 and is also expressed as a percentage of the true volume

volume of dead wood in an area at low cost. In this case, “typical locations” within the area of interest can be chosen and the method applied for both standing and downed trees. In Sweden, the method has been tested in connection with stand level inventories with promising results, and it is now applied in biodiversity oriented inventories supplying information to the European Union’s Habitats Directive. However, further tests are needed in order to better assess what accuracy can be achieved under different conditions. Further, to make the method more easily applicable, some technical development of instruments would be needed. The ideal combined relascope device would include facilities for (i) laser distance measurement (ii) horizontal angle measurement, and (iii) vertical angle measurement, in addition to the current functions of a slope-compensating relascope. With an integrated device of this kind, the critical length of a log could be immediately obtained from the sampling point without any need to walk out to the sampled log. The relascope would correct for deviations from the horizontal plane and the length of the log section, i.e. the critical length, would be easily obtained from geometrical relationships involving the distance and the angles (horizontal and vertical) to each of the two critical points. Although no instrument adapted for critical length sampling is available today, instruments with the necessary capabilities are beginning to appear (e.g., Kalliovirta et al. 2005).

The simulations in this study were normalized so that an equal sample size was obtained with each of the three methods. This implied that a larger relascope angle was selected for critical length sampling in comparison with diameter relascope sampling. The motivation for this was

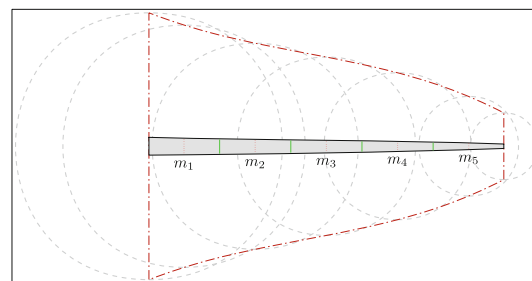


Fig. 6 A log with five segments having midpoint $m_1 \rightarrow m_5$ is depicted with “inclusion” zones at these points plus the butt and tip. Also shown is the inclusion zone for perpendicular distance sampling for surface area with comparable factor. Note that the PDS zone is truncated at the end of the log whereas under critical length sampling, one still could determine a critical length when standing “beyond” the log ends

that equalizing the sample size would imply roughly the same inventory cost for the three methods. However, in practice a large portion of the inventory cost stems from moving between sample points in a forest area and the time spent at individual sample points is a rather small fraction of the total time (i.e. cost) spent on an inventory. Still, the use of a smaller relascope angle for DRS in comparison to CLS most likely favored DRS in this study. Future simulation studies should include explicit cost functions so that proper cost-accuracy relationships can be evaluated for different methods in different types of forests. Before such cost functions can be constructed field studies of time consumption are needed.

It is interesting to note that CLS shares some theoretical similarities with PDS. Figure 6 presents the case where the log has been divided into sections as in Fig. 2; however, the circular “inclusion zones” have also been added for the butt and tip of the log. As noted by Williams et al. (2005b), the inclusion zone for PDS surface area estimation is related to the prism basal area factor, and is shown superimposed in the figure using a comparable PDS factor. Note that the inclusion zone for PDS is truncated at the end of the log, whereas under CLS, one can still determine a critical length when standing “beyond” the ends of the log. In addition, as the length of the log sections determined by

the midpoints, m_i , go to zero, the corresponding inclusion zones will more closely trace the perpendicular part of the corresponding PDS zone.

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