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HEIGHT GROWTH and SITE INDEX CURVES for MANAGED, EVEN-AGED STANDS of PONDEROSA PINE in the PACIFIC NORTHWEST

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Height Growth and Site Index Curves for Managed, Even-Aged Stands of Ponderosa Pine in the Pacific Northwest

Reference Abstract

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This paper presents height growth and site index curves and equations for even-aged, managed stands of ponderosa pine east of the Cascade Range in Oregon and Washington where height growth has not been suppressed by high density or related factors.

KEYWORDS: Increment (height), site index -)stand height/age, measurement systems, stem analysis, ponderosa pine, *Pinus ponderosa*, Oregon (eastern), Washington (eastern).

RESEARCH SUMMARY

Research Paper PNW-232

1978

Height growth and site index curves and equations for even-aged managed stands of ponderosa pine (*Pinus ponderosa* Laws.) east of the Cascade Range in Oregon and Washington were derived from stem analysis data from 27 plots in Oregon and 3 plots in Washington.

Height growth curves estimate expected heights at different ages for stands of known site index. Site curves estimate site index of managed stands where breast height age and total height can be determined.

The curves provide valid estimates of site index and potential height growth for stands where height growth has not been retarded by high density or related factors. They do not represent the average of existing stands. The curves are most appropriate for use in constructing yield tables for managed, even-aged stands of ponderosa pine.

Curves are based on measurements of the tallest tree in a 1/5-acre plot.

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Introduction

This paper presents curves for estimating height growth and site index for even-aged, managed stands of ponderosa pine (*Pinus ponderosa* Laws.) east of the Cascade Range crest in Oregon and Washington. The curves represent growth potential under density conditions approximating those anticipated in managed stands of the future. The site curves are applicable to stands where relatively low density has permitted full height development so that the site estimate represents the full potential of the site. The data plots were deliberately chosen where stand development approximated densities believed desirable in managed stands, not merely the average of existing stands.^{1/} Therefore, the curves provide valid estimates of site index and potential height growth for stands where height growth has not been retarded by high density or related factors.

Impressive acreages of second-growth ponderosa pine east of the Cascade Range are being precommercially thinned to between 150 and 300 trees per acre. Additional acres are being planted at these densities. The thinning is being done to stimulate height and diameter growth so that we may grow usable trees in a much shorter time than if we allowed nature to follow its usual course of suppression, stagnation, insect attack, and then indiscriminate mortality. After thinning, the stand takes on attributes of

what is commonly referred to as a managed stand or a stand that is being manipulated toward some predetermined goal. This goal is usually a "target" average stand diameter and height within some time frame.

The most "desirable density" is the density at which trees continue growing at the predetermined goal rate. As the stand grows older, relatively low density will be maintained by periodic thinnings to some basal area level or tree number--average diameter combination.

Height of the dominant or tallest portion of the stand is the one measure of growth found most independent of stand factors and, consequently, the most reliable for site evaluation (Lynch 1958). Generally speaking, we can conclude from the literature that height is not materially affected in stands that are not extremely overstocked or understocked (Spurr 1952). In the Pacific Northwest, however, Weaver (1947), Baker (1953), Mowat (1953), Lynch (1958) and Barrett (1973) have shown that dense stocking of ponderosa pine does reduce height growth.

Reducing the density below levels found in natural stands usually affects the height growth patterns shown by Meyer (1961) for "normal" stands. Fortunately, there seems to be a rather broad density range in ponderosa pine stands where density does not affect height growth of the larger trees in the stand. This range appears to fall within the density range now used by most ponderosa pine managers.

In this study I chose plots in stands that were 100 years or older and contained at least 60

^{1/} Summerfield, Edward R. Site index, height growth and productivity of ponderosa pine in eastern Washington. In preparation for publication by the State of Washington Department of Natural Resources.

trees per acre on the lower sites and no more than 200 trees per acre on the higher sites. Plots were selected in extensive, homogeneous, even-aged stands, not in clumps of even-aged trees. I did this to avoid the selection of exceptionally fast-growing trees and to incorporate the total stand effect into the site index and height growth system.

As time progresses, it will be necessary for managers to evaluate the productive capacity of these newly created stands. They will need a method of rating site and a method of predicting the course of stand height with age so that they may develop yield tables for second-growth ponderosa pine in the Pacific Northwest. This paper is a first step toward this goal. Therefore, these curves are most appropriately used in constructing yield tables for managed stands of ponderosa pine. The site curves will classify the growth potential of sample plots, and the height growth curves will express height development of even-aged stands attaining specified heights at index age.

When Meyer's (1961) site index curves are used in intensively managed, even-aged stands, it is difficult to select what Meyer refers to as "representative dominant and co-dominant trees." After trees are of merchantable size, all trees often appear to be dominants or codominants and selection can be subjective. In this study, I chose to eliminate this difficulty by dealing with only the tallest trees in the stand and densities likely to be found in *managed stands*.

Data Collection

Thirty 1/5-acre circular plots were located throughout Oregon and parts of Washington (fig. 1). The plots had the following tree and stand characteristics:

1. The stand was pure ponderosa pine 95 to 180 years old at breast height and even-aged within 10 years.
2. Selective cutting or mortality had not removed the tallest tree within the age span of the existing stand.
3. The six tallest trees on the plot did not contain a group of more than four fine annual rings, which would indicate stress during a portion of the stand's life.
4. Trees were not infected with mistletoe.

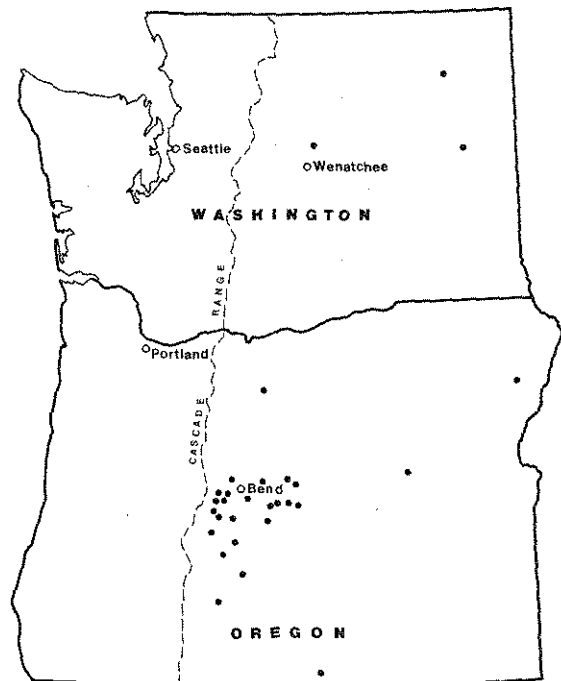


Figure 1.--Distribution of ponderosa pine stem analysis study plots in Oregon and Washington.

5. The six tallest trees on the plot did not exhibit crook in the bole which would indicate terminal damage or reduced internodal length from recurring insect attack.
6. The stand or plot did not contain large, old dead, or old living brush that suggested heavy competition from understory vegetation for the trees during the early part of stand development. The usual scattered understory of living brush and/or grass was permitted.

Using a clinometer and tape, I selected the six tallest trees per plot. An increment core was extracted at breast height from each of the six tallest trees and also from some lower crown-class trees to be sure the stand was even aged within 10 years. The six tallest trees per plot were felled, and a section was removed at ground line, at breast height, and at 10-foot intervals above ground until half the total height was reached. The remainder of the tree was sectioned at 5-foot intervals. Tree sections were frozen to prevent molding. Rings were counted on all sections and recorded for the appropriate height.

The data base consists of 3,500 sections from 177 trees at 30 plot locations in Oregon and Washington (fig. 1). Site indices ranged from a high of 145 to a low of 72 at breast-high age of 100 years:

<u>Number of plots</u>	<u>Site index</u>
6	72-89
12	90-109
9	110-129
3	130-145

The number of trees on sample stands ranged from a low of 52 per acre to a maximum of 152 on some of the better sites.

Results

The methodology used in developing these curves includes the recent improvements in curve construction suggested by Curtis et al. (1974) and Dahms (1975). This methodology is described in the appendix for the reader who may wish to construct his own curves. Poor estimates of site, however, often result from a lack of adherence to the principles on which the index system was founded. Some understanding of how curves were constructed leads to an appreciation of how they should be used. Therefore, the appendix section on curve construction is recommended reading for even the occasional user of a curve or equation.

Height-over-age relationships for all plots were consistent and reasonably smooth between the ages of 20 and 130 years. Not many plots were found that qualified under the constraints of plot selection beyond 130 years. Under age 20, some curves were erratic; but smoothness improved markedly after age 20 and was consistent thereafter.

ESTIMATING THE AVERAGE COURSE OF HEIGHT GROWTH FOR STANDS OF A GIVEN SITE QUALITY

Height growth curves define the average pattern of height development on stands of a given site quality. They are appropriately used for constructing yield tables but do not provide optimum estimates of site index from measured height and age in an existing stand (Curtis et al. 1974).

The following are alternatives for estimating the anticipated height of the tallest trees of a stand on land of known site index:

1. Use figure 2 for rough field estimates.
2. For a more precise estimate, table 1 can be used to solve the equation,

$$\text{Total height} - 4.5 \text{ feet} = a + b (\text{site index} - 4.5 \text{ feet}).$$

3. The equation shown in the appendix will be useful for those wishing to program the estimating procedure in a computer.

ESTIMATING SITE INDEX

Site index curves relate to the productivity potential of land. They give the estimated height of the tallest tree at 100 years in relation to its height and breast-high age. Many of the same plot qualifications used in the study are applicable in selecting plots for estimating site index. The following steps and precautions are recommended for estimating the site index of a managed stand.

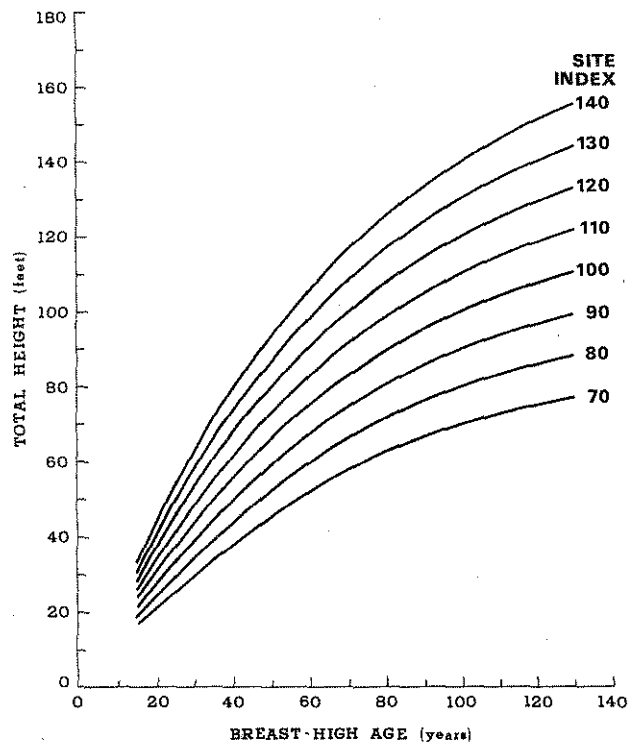


Figure 2.--Height growth curves for managed, even-aged stands of ponderosa pine in the Pacific Northwest. Heights represent the tallest trees in the stand.

Table 1--Values for *a* and *b* by years for the family of regressions^{1/} for estimating height of the tallest trees in a newly established stand of ponderosa pine where site index and age are known

Breast-high age	Years between decades																	
	0		1		2		3		4		5		6		7		8	
	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
20	-4.834	0.325	-5.132	0.342	-5.381	0.359	-5.587	0.375	-5.755	0.390	-5.887	0.406	-5.988	0.420	-6.061	0.434	-6.108	0.448
30	-6.136	.475	-6.121	.488	-6.089	.500	-6.043	.512	-5.982	.524	-5.910	.536	-5.828	.547	-5.735	.559	-5.635	.569
40	-5.412	.591	-5.292	.601	-5.167	.611	-5.037	.621	-4.905	.631	-4.769	.640	-4.631	.649	-4.491	.659	-4.350	.668
50	-4.066	.685	-3.924	.694	-3.781	.702	-3.639	.711	-3.499	.719	-3.359	.727	-3.220	.735	-3.083	.743	-2.948	.750
60	-2.684	.766	-2.555	.773	-2.429	.780	-2.305	.787	-2.184	.794	-2.066	.801	-1.951	.808	-1.838	.815	-1.729	.822
70	-1.520	.835	-1.420	.841	-1.324	.848	-1.230	.854	-1.141	.860	-1.054	.867	-.971	.873	-.892	.879	-.816	.885
80	-.674	.896	-.608	.902	-.546	.908	-.487	.913	-.431	.919	-.379	.924	-.331	.930	-.285	.935	-.244	.940
90	-.170	.951	-.138	.956	-.109	.961	-.084	.966	-.062	.971	-.043	.976	-.027	.981	-.014	.986	-.005	.991
100	.005	1.000	.006	1.005	.004	1.009	-.002	1.014	-.010	1.018	-.021	1.023	-.034	1.027	-.051	1.032	-.070	1.036
110	-.116	1.045	-.143	1.049	-.173	1.053	-.205	1.057	-.239	1.061	-.276	1.065	-.316	1.069	-.357	1.073	-.401	1.077
120	-.496	1.085	-.547	1.089	-.599	1.093	-.654	1.097	-.711	1.100	-.770	1.104	-.831	1.108	-.894	1.112	-.958	1.115
130	-1.093	1.122																

^{1/} Height at a future date of the tallest portion of a young stand may be estimated on land of known site index by selecting *a* and *b* values for the appropriate breast-high age from the table. Substitute *a* and *b* values in the equation $(\text{HT} - 4.5 \text{ feet}) = a + b(\text{SI} - 4.5)$ for the particular breast-high age wanted. For example, for the height of the tallest trees in the stand at breast-high age 80 on land with a known site index of 100, solve the equation, $\text{HT} - 4.5 = -0.674 + 0.896(100 - 4.5)$, for a total height of 89.4 feet.

1. Select a suitable plot with the following characteristics:
 - (a) even-aged within 10 years,
 - (b) no disease symptoms that would affect height,
 - (c) no fine ring groups to indicate growth suppression,
 - (d) internodal lengths consistent on taller trees,
 - (e) no remnant understory vegetation that would indicate competition during early part of the stand's life.
2. Establish boundaries of a circular 1/5-acre plot.
3. Measure height of the tallest tree on the plot.
4. Extract increment cores from the three tallest trees on the plot to obtain an average breast-high stand age.
5. From breast-high age and total height, estimate the site index by one or more of the following alternatives.
 - (a) Use figure 3 for rough field estimates.
 - (b) For a more precise estimate, table 2 can be used to solve the equation,

$$\text{Site index} - 4.5 \text{ feet} = \frac{a + b (\text{height} - 4.5 \text{ feet})}{\dots}$$
 - (c) The equation shown in the appendix will be useful for those wishing to program the estimating procedure in a computer.

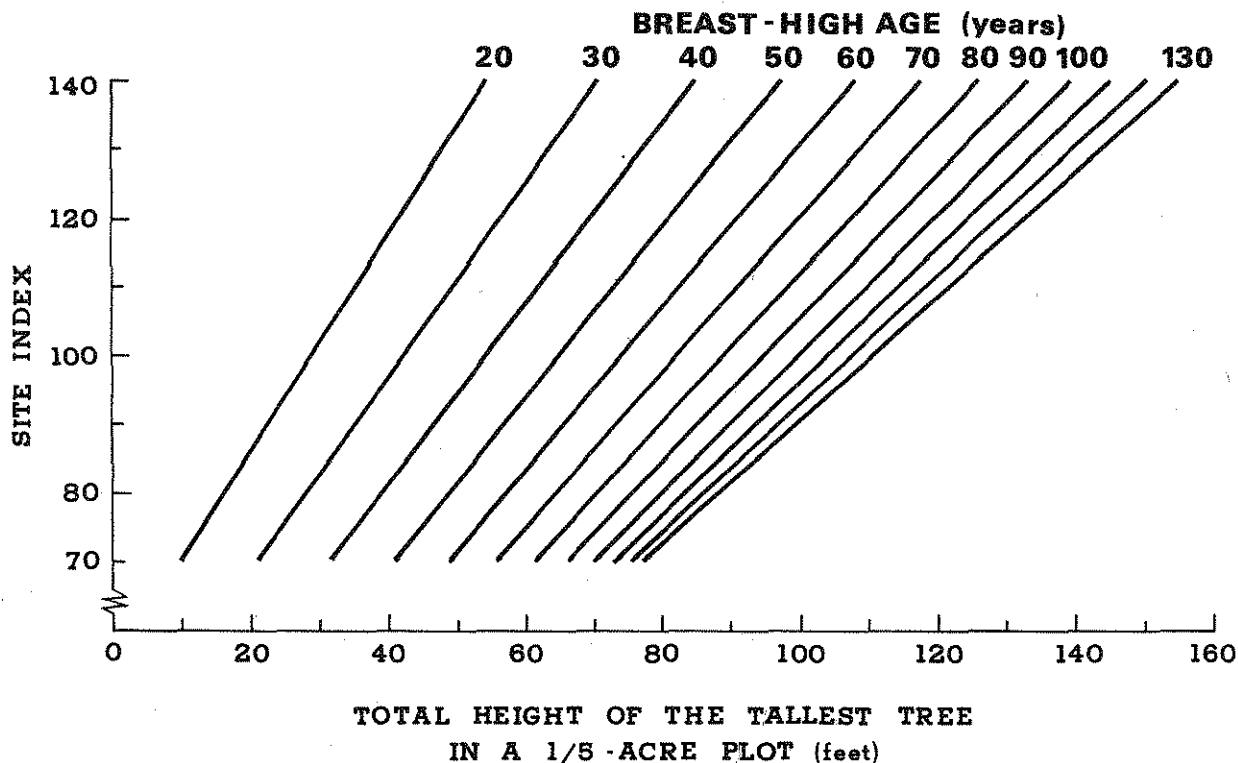


Figure 3.--Site index curves for even-aged, managed stands of ponderosa pine in the Pacific Northwest.

Table 2--Values for *a* and *b* by years for the family of regressions^{1/} for estimating site index for ponderosa pine in the Pacific Northwest

Breast-high age	Years between decades																	
	0		1		2		3		4		5		6		7		8	
	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
Years																		
20	56.947	1.564	55.378	1.541	53.835	1.520	52.318	1.501	50.828	1.483	49.365	1.466	47.929	1.450	46.519	1.435	45.135	1.421
30	42.447	1.395	41.143	1.383	39.864	1.372	38.611	1.361	37.384	1.351	36.182	1.341	35.005	1.331	33.854	1.322	32.727	1.313
40	30.546	1.297	29.492	1.289	28.461	1.281	27.454	1.273	26.470	1.266	25.510	1.259	24.571	1.252	23.655	1.245	22.762	1.239
50	21.039	1.226	20.210	1.220	19.402	1.214	18.615	1.208	17.847	1.202	17.100	1.196	16.373	1.191	15.665	1.185	14.977	1.180
60	13.656	1.170	13.024	1.164	12.410	1.159	11.813	1.154	11.234	1.149	10.673	1.145	10.128	1.140	9.600	1.135	9.089	1.130
70	8.114	1.121	7.651	1.117	7.203	1.112	6.770	1.108	6.353	1.103	5.949	1.099	5.561	1.095	5.186	1.090	4.826	1.086
80	4.146	1.078	3.827	1.074	3.520	1.069	3.226	1.065	2.945	1.061	2.676	1.057	2.419	1.053	2.175	1.049	1.942	1.045
90	1.510	1.038	1.312	1.034	1.124	1.030	.947	1.026	.781	1.022	.625	1.019	.479	1.015	.343	1.011	.217	1.007
100	-.005	1.000	-.103	.996	-.191	.993	-.270	.989	-.340	.985	-.402	.982	-.455	.978	-.500	.975	-.536	.971
110	-.585	.964	-.598	.960	-.603	.957	-.601	.953	-.591	.950	-.574	.947	-.550	.943	-.519	.940	-.481	.936
120	-.386	.929	-.328	.926	-.264	.922	-.194	.919	-.118	.916	-.036	.912	.052	.909	.146	.906	.245	.902
130	.460	.896															.350	.899

^{1/}To estimate site index, measure total height of the tallest tree per 1/5-acre plot. Determine average breast-high stand age from at least three of the tallest trees per plot. Select appropriate *a* and *b* value above. Substitute values in the equation $(SI - 4.5 \text{ feet}) = a + b(HT - 4.5)$. For example, for a tree 53 years old at breast height and 60 feet in total height, solve the equation, $SI - 4.5 = 18.615 + 1.208(60 - 4.5)$, for a site index of 90.

Application

In this study, site index is a number representing the height of the tallest tree on a 1/5-acre plot at a breast-high age of 100 years. Since site has been found to be closely correlated with volume, site (as discussed here) will later be used in a yield study to categorize volume productivity potentials of managed stands of ponderosa pine. Past experience has shown that height objectively reflects site where stands are not overstocked or understocked. Managed stands, in contrast to natural stands, probably will not be overstocked to the point of substantially affecting height growth. Therefore, use of these curves should be restricted to managed stands where height growth competition between trees has been held to a minimum.

Typical examples of when the curves should not be used are:

1. precommercially thinned stands showing a tight core of rings;
2. commercially thinned stands with numerous stumps indi-

cating a high initial density; and

3. plantations with large numbers of trees and thinned long after competition between trees occurred.

These curves may have limited field application for some time because there is only a limited amount of land under intensive management that fits the constraints of the curves. Greatest use is in forecasting future stand performance in a stand growth simulator. One should note, however, that these curves represent the tallest trees. Stand projections should assign appropriate lesser heights to the other trees.

Reliability of the curves can be partially judged by the r^2 values and the standard errors of estimate shown in the appendix. Also, one may judge equation fit from figures 8 and 9 in the appendix. A portion of the data from 20 plots was destroyed in the Bend Silviculture Laboratory fire, of January 15, 1974. If this data had been available, an annual comparison of actual and predicted values could have been made, similar to that made by Johnson and Worthington (1963).

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Appendix

CONSTRUCTION OF SITE INDEX AND HEIGHT GROWTH CURVES^{2/}

The following steps were used in constructing the site index curves:

1. Average breast-high stand age for each plot was calculated from the six felled trees. Total height was plotted over breast-high age. The six trees were plotted on the same graph. Height of the tree that was tallest at a specific age to 130 years was read from the graph. Shifts^{3/} in relative height similar to those reported by Dahms (1963) were found in 90 percent of the plots. Site index was read from the graph of the tallest tree at breast high age of 100 years.
2. The basic equation form is

$$SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet});$$

where, HT = total height and
SI = site index.

Linear regressions of this form were calculated for each decade starting with 10 years and ending with 130 years. Following is a list of these regressions:

Breast-high age (years)	Site index (SI)	a	b	Total height (HT)	r ²	Standard error of estimate	Number of observations
10	-4.5	= 69.8003 +	1.9825	-4.5	0.23	15.8	30
20	-4.5	= 52.8113 +	1.6807	-4.5	.50	12.7	30
30	-4.5	= 43.6947 +	1.3845	-4.5	.64	10.8	30
40	-4.5	= 35.2987 +	1.2269	-4.5	.74	9.2	30
50	-4.5	= 23.5422 +	1.1966	-4.5	.83	7.3	30
60	-4.5	= 16.1510 +	1.1322	-4.5	.90	5.7	30
70	-4.5	= 10.0630 +	1.0986	-4.5	.94	4.3	30
80	-4.5	= 5.1357 +	1.0717	-4.5	.97	3.1	30
90	-4.5	= 1.3892 +	1.0456	-4.5	.99	2.0	30
100	-4.5	= 0.0000 +	1.0000	-4.5	1.00	0.0	30
110	-4.5	= 3.4683 +	0.9164	-4.5	.99	1.2	28
120	-4.5	= 5.5218 +	0.8598	-4.5	.99	1.8	25
130	-4.5	= 1.0876 +	0.8848	-4.5	.98	2.0	11

^{2/} The methodology developed by Dahms (1975) was used to derive the site index and height growth equations. The methodology is repeated here so the reader may judge the quality of various relationships. In addition, equation derivation is explained for those who wish to use this method to construct their own curves.

^{3/} For example, the tallest tree in the stand at age 50 may not be the tallest at age 80.

3. The above decadal estimates of b were smoothed over age (fig. 4) by the equation,

$$\hat{b} = 1.198632 - 0.0028307A + 8.4441 (1/A);$$

where, A = breast high age

The resulting $\hat{b}$ values are those appearing in table 2.

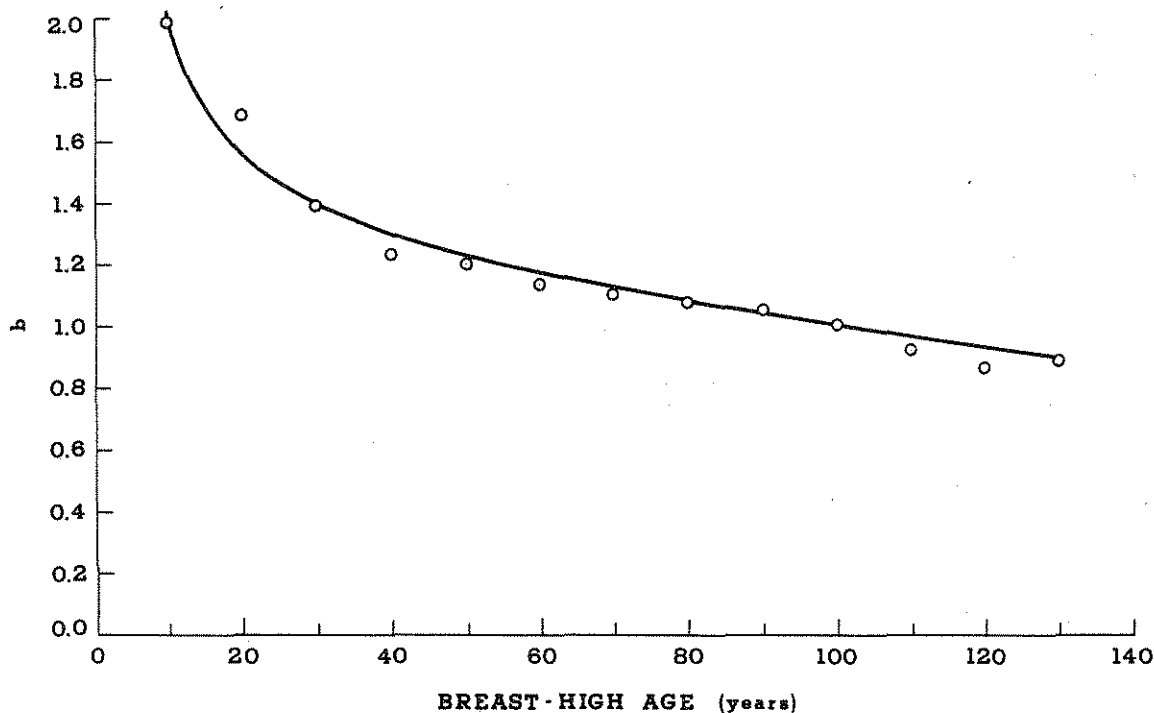


Figure 4.-- b values in the equation $SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet})$ as a function of age. X = actual b values. Solid line is curve expressed by the equation, $b = 1.198632 - 0.0028307 \text{ age} + 8.4441 (1/A)$.

4. The following equation, expressing decadal mean heights as a function of age, was conditioned to pass through mean site index ($\bar{SI}$) 100.43 (+4.5) at 100 years (fig. 5).

$$\hat{HT} - 4.5 = 128.895 \left(1 - e^{-0.016959A} \right) 1.23114$$

At ages beyond 100 years the sample became progressively smaller and mean site index was slightly different. Heights were adjusted to the mean overall site index before fitting the average height curve.

5. Since average height, $\hat{HT}$, from years 10 through 130 was known as well as slope $\hat{b}$ of regressions for each year, the corre-

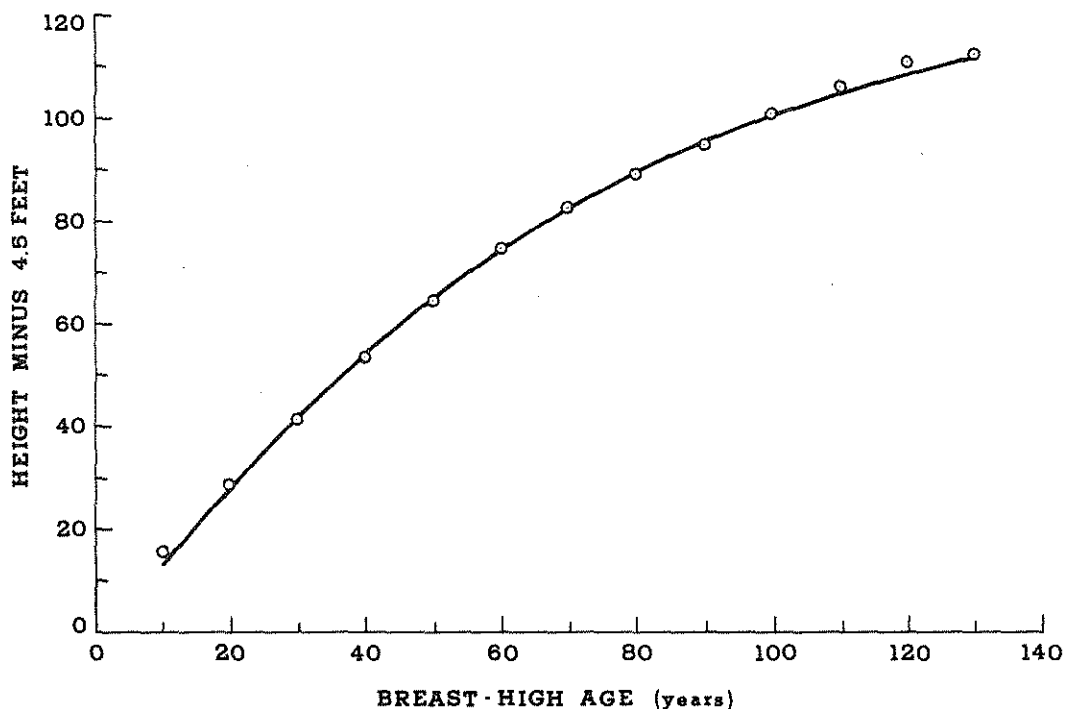


Figure 5.--Average height of sectioned trees as a function of breast-high age. $\bar{X}$ = average height. Solid line is curve expressed by equation

$$\hat{HT} - 4.5 \text{ feet} = 128.895 \left[1 - e^{-0.016959 \text{ age}} \right]^{1.23114}$$

sponding intercept $\hat{a}$ could be calculated. By substituting the following in the basic equation of step 2:

$$\bar{SI} - 4.5 = \hat{a} + \hat{b} (\hat{HT} - 4.5),$$

$$\text{therefore, } \hat{a} = \bar{SI} - 4.5 - \hat{b} (\hat{HT} - 4.5).$$

The resulting $\hat{a}$ values are those shown in table 2.

6. Substituting $\hat{a}$, $\hat{b}$, and $\hat{HT}$ in the basic equation of step 2 gives the final equation used to estimate site index as a function of age and height (fig. 3).

$$\begin{aligned} SI - 4.5 = & 100.43 - [1.198632 - 0.00283073 \text{ age} + 8.44441 / \text{age}] \\ & \cdot \left[128.8952205 \left[1 - e^{-0.016959 \text{ age}} \right]^{1.23114} \right] + \left[(1.198632 - \right. \\ & \left. 0.00283073 \text{ age} + 8.44441 / \text{age}) \cdot (\hat{HT} - 4.5) \right]. \end{aligned}$$

Construction of the height growth curves was similar. Since we are now interested in attainable height when age and site index are known, the basic equation is reversed from the previous example where site index was estimated and we now have

$$HT - 4.5 = a + b (SI - 4.5).$$

1. The data accumulated in step 1 of site index curve construction is used for constructing height growth curves.
2. Linear regressions of this form were calculated for each decade starting with 10 and ending with 130 years. Following is a list of these regressions:

Breast-high age (years)	Total height (HT)	a	b	Site index (SI)	r ²	Standard error of estimate	Number of observations
10	-4.5	= 3.8003	+ 0.1162	-4.5	.23	3.8	30
20	-4.5	= -1.5177	+ 0.2974	-4.5	.50	5.3	30
30	-4.5	= -5.5801	+ 0.4637	-4.5	.64	6.2	30
40	-4.5	= -7.2401	+ 0.6008	-4.5	.74	6.5	30
50	-4.5	= -5.7960	+ 0.6976	-4.5	.83	5.6	30
60	-4.5	= -5.5043	+ 0.7960	-4.5	.90	4.7	30
70	-4.5	= -3.9442	+ 0.8583	-4.5	.94	3.8	30
80	-4.5	= -1.9355	+ 0.9046	-4.5	.97	2.9	30
90	-4.5	= -0.1844	+ 0.9450	-4.5	.99	1.9	30
100	-4.5	= 0.0000	+ 1.0000	-4.5	1.00	0.0	30
110	-4.5	= -3.2803	+ 1.0861	-4.5	.99	1.3	28
120	-4.5	= -5.1488	+ 1.1500	-4.5	.99	2.1	25
130	-4.5	= 0.4011	+ 1.1111	-4.5	.98	2.2	11

3. The above decadal estimates of b were smoothed over age (fig. 6) by the equation,

$$\hat{b} = -0.7864 + 2.49717 \left(1 - e^{-0.0045042A} \right)^{0.33022}.$$

The resulting $\hat{b}$ values are those appearing in table 1.

4. The same expression for decadal mean height used in determining site index was used again.

$$\hat{HT} - 4.5 = 128.895 \left(1 - e^{-0.016959A} \right)^{1.23114}.$$

5. Substituting mean site index $\overline{SI} = 100.43$ in the basic equation:

$$\hat{HT} - 4.5 = a + \hat{b} (\overline{SI} - 4.5);$$

$$\text{therefore, } \hat{a} = \hat{HT} - 4.5 - \hat{b} (\overline{SI} - 4.5).$$

The resulting a values are those shown in table 1.

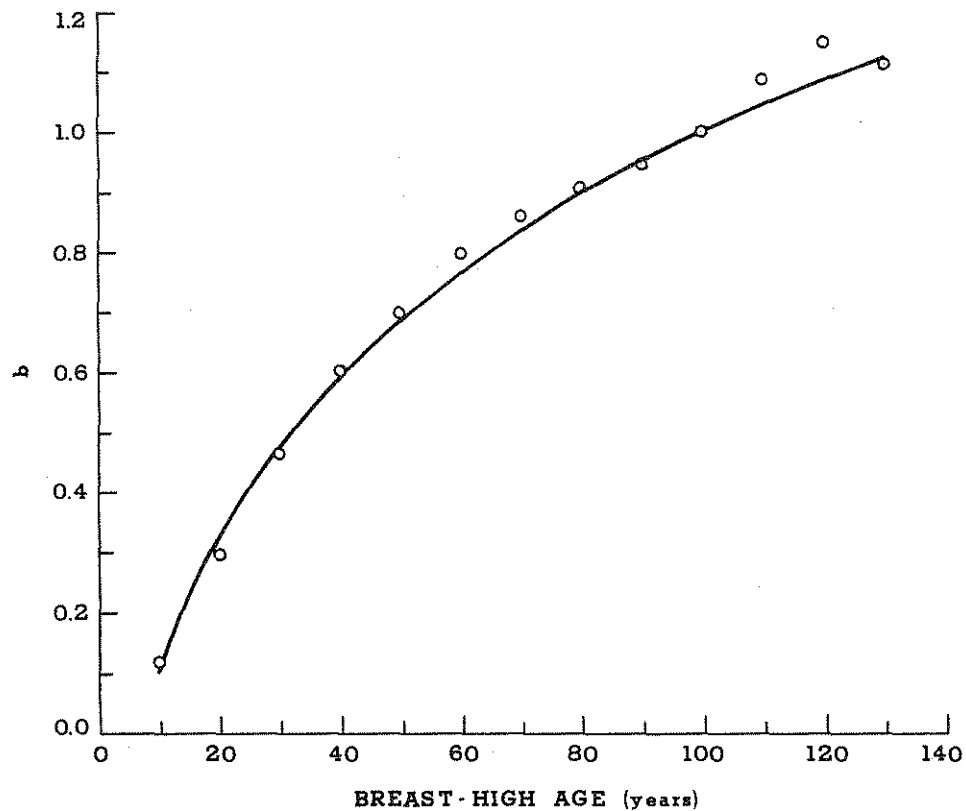


Figure 6.--b values in the equation $HT - 4.5 \text{ feet} = a + b$
 $(SI - 4.5 \text{ feet})$ as a function of age. X = actual b value.
 Solid line is curve expressed by the equation

$$b = -0.7864 + 2.49717 \left(1 - e^{-0.0045042 \text{ age}} \right)^{0.33022}$$

6. Substituting $\hat{a}$, $\hat{b}$, and $\hat{HT}$ in the basic equation $HT - 4.5 = a + b$ ($SI - 4.5$) gives the final equation used to estimate height as a function of age and site index as shown in figure 2.

$$HT - 4.5 = \left[128.8952205 \left(1 - e^{-0.016959 \text{ age}} \right)^{1.23114} \right] -$$

$$\left[\left(-0.7864 + 2.49717 \left(1 - e^{-0.0045042 \text{ age}} \right)^{0.33022} \right) \cdot (100.43) \right] +$$

$$\left[\left(-0.7864 + 2.49717 \left(1 - e^{-0.0045042 \text{ age}} \right)^{0.33022} \right) \cdot (SI - 4.5) \right].$$

A graphic comparison between site index and the height growth curves is shown in figure 7.

The final estimating equations for both site and height fit the basic data regression points well. Some deviations occurred at sites and ages above 100 (figs. 8 and 9).

Figure 7.--Site index (solid lines) and height growth curves (dashed lines) for even-aged managed stands of ponderosa pine in the Pacific Northwest.

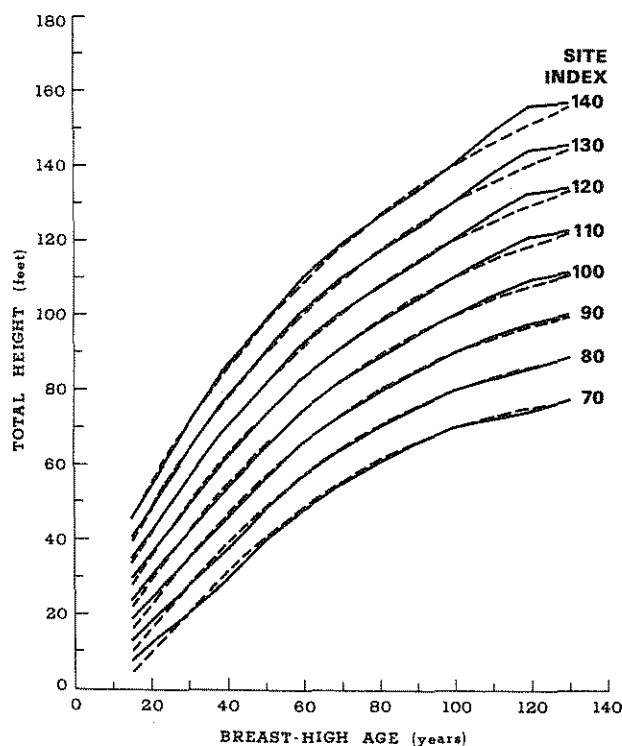
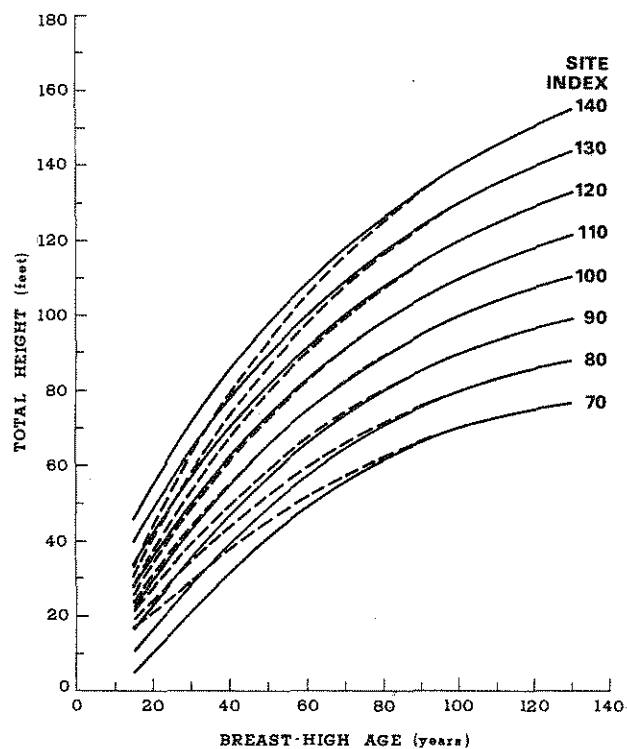


Figure 8.--Site index curves for managed even-aged stands of ponderosa pine in the Pacific Northwest. Solid lines connect decadal points derived from the unsmoothed basic data regressions of $SI - 4.5 = a + b(HT - 4.5)$. Dashed lines represent smooth curves from the estimating equations.

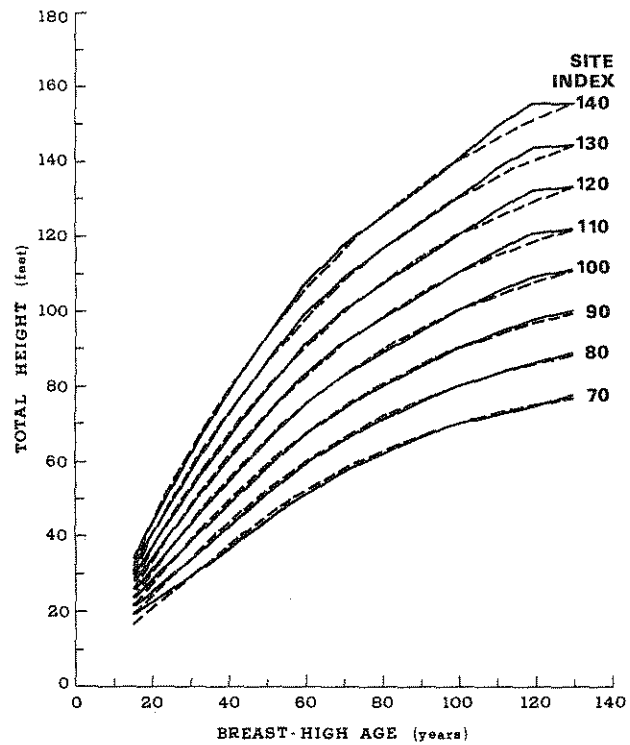


Figure 9.--Height growth curves for managed even-aged stands of ponderosa pine in the Pacific Northwest. Solid lines connect decadal points derived from the unsmoothed basic data regressions of $HT - 4.5 = a + b(SI - 4.5)$. Dashed lines represent smooth curves from the estimating equations.