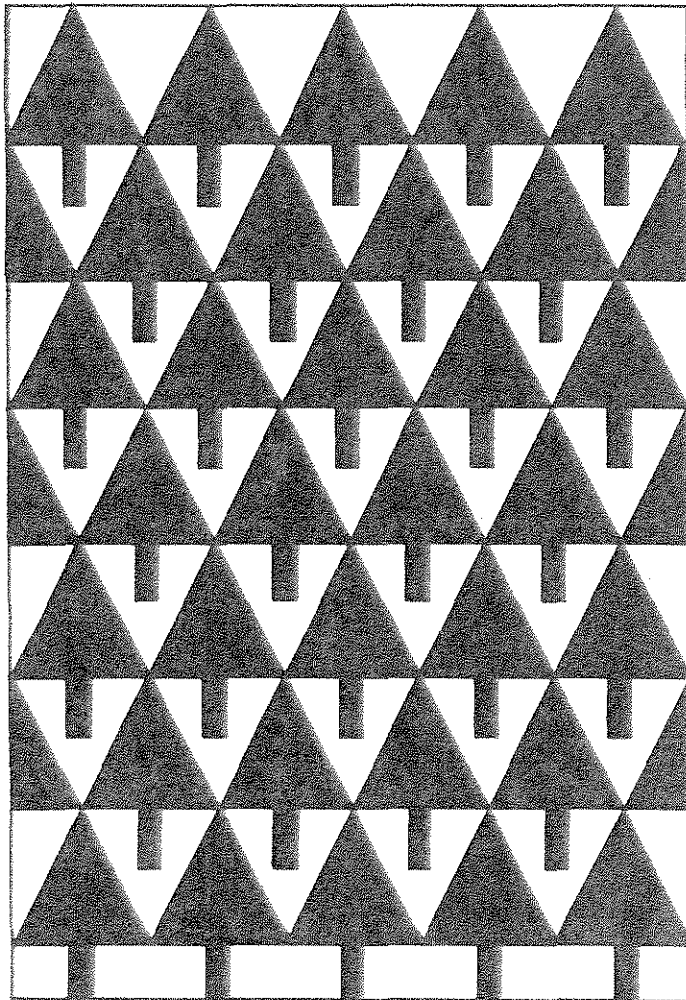
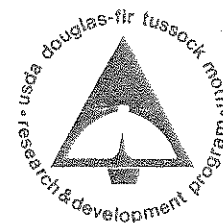


SITE INDEX AND HEIGHT GROWTH CURVES

FOR MANAGED,
EVEN-AGED STANDS OF WHITE OR GRAND FIR
EAST OF THE CASCADES
IN OREGON AND WASHINGTON



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Contents

	Page
INTRODUCTION	1
Data Collection	1
Curve Construction.	3
RESULTS AND APPLICATION.	4
LITERATURE CITED	10
APPENDIX	10

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Site Index and Height Growth Curves for Managed, Even-Aged Stands of White or Grand Fir East of the Cascades in Oregon and Washington

Reference Abstract

Cochran, P. H.

1979. Site index and height growth curves for managed, even-aged stands of white or grand fir east of the Cascades in Oregon and Washington. USDA For. Serv. Res. Pap. PNW-252, 13 p., illus. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.

Height growth and site index curves and equations for white or grand fir growing in managed, even-aged pure stands or mixed with other conifers east of the Cascade Range in Oregon and Washington are presented. Data were collected in stands where height growth apparently has not been suppressed by high density or top damage.

KEYWORDS: Site index, increment (height), stem analysis, measurement systems, even-aged stands, white fir, *Abies concolor*, grand fir, *Abies grandis*, Oregon (eastern), Washington (eastern).

RESEARCH SUMMARY

Research Paper PNW-252

1979

Height growth and site index curves and equations for white or grand fir (*Abies grandis*-*A. concolor* species complex) east of the Cascades were constructed from stem analysis data collected from 8 plots in Washington and 26 plots in Oregon.

Height growth curves give estimates of expected heights at different ages for stands of known site index. Site index curves give estimates of site index of managed stands where present breast-high age and present total height can be determined.

The appropriate curves provide valid estimates of site index and potential height growth for stands where height growth has not been retarded by high density or related factors. They *do not* represent the average of existing stands. The height growth curves are most appropriate for use in constructing yield tables for managed, even-aged stands of white or grand fir or even-aged, mixed conifer stands that have white or grand fir as major components.

Curves are based on measurements of the tallest tree for its breast-high age in a 1/5-acre plot.

Introduction

Over 9 million of the 20 million acres of commercial forest land in eastern Oregon and eastern Washington is in mixed conifer forests.¹ White fir (*Abies concolor* (Gord. & Glend.) Lindl.) or grand fir (*Abies grandis* (Dougl.) Lindl.)² is a component of many of these forests and often occurs in pure stands or mixed with Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco), Engelmann spruce (*Picea engelmannii* Parry), western larch (*Larix occidentalis* Nutt.), or ponderosa pine (*Pinus ponderosa* Laws.). The site index and height growth curves presented here represent growth potential for grand or white fir in pure or mixed, even-aged, managed stands where relatively low density, lack of top damage, and absence of vegetative competition early in the life of the stand permit full height development. A managed stand is being manipulated toward some goal, usually a "target" average diameter and height within a set length of time. Some combination of precommercial and commercial thinnings and perhaps early suppression of competing vegetation will, at times, be employed. No stands exist that have been under this kind of management through a rotation. Therefore, I chose stands for sampling that approximated densities believed desirable in managed stands, not merely the average of existing stands. This work was conducted as part of the Douglas-fir Tussock Moth Research and Development Program. The results came from a study undertaken to ascertain potential production of stands susceptible to attack by tussock moth.

The site index curves and height growth curves in this publication are two separate sets of curves constructed for two different purposes. Site index curves are used to determine an index to potential production from current height and age. Height growth curves are used for describing the development of height growth as a function of age and site index in construction of yield tables (Curtis et al. 1974).

DATA COLLECTION

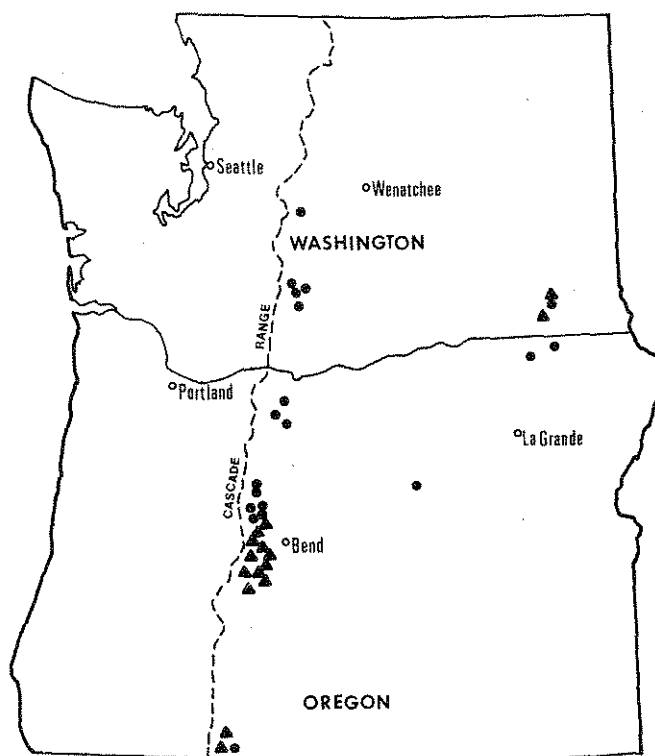
The data came from thirty-four 1/5-acre circular plots in pure and mixed stands (fig. 1) with these characteristics:

1. The average breast-high age was greater than 50 years. The stands were single storied, and at ground line the ages of the youngest trees were at least 80 percent of the ages of the oldest trees regardless of species.

¹ 1 acre = 0.4047 hectare.

² An *Abies grandis*-*A. concolor* species complex is recognized in the central Oregon Cascade Range. Farther south populations resemble *A. concolor*, whereas populations to the north become more like *A. grandis* (Zobel 1973). In this study no attempt was made to separate white and grand fir trees from each other or their hybrids. Data were handled as if they were one species.

Figure 1.--Distribution of plots used in construction of curves. Triangles indicate plots where more than 80 percent of the basal area was white or grand fir. Solid circles represent mixed plots where 20 percent or more of the basal area consisted of conifer species other than white or grand fir.



2. The crown canopy was closed or nearly closed at the time of sampling. Mortality, if present, was due to suppression; and its volume was less than 5 percent of the plot volume. Stumps were absent. Volume growth patterns for the plot determined from stem analysis of at least 12 sample trees across the range of diameter classes indicated that the highest periodic annual volume increment had occurred within the last 15 years. These factors indicate that there was no severe competition between trees in the past.

3. The dominant trees on the plot did not contain a group of narrow annual rings, which would indicate stress in the past.

4. Trees were not visibly infected with disease or insects.

5. Dominant trees did not exhibit crook in the bole, and internodal lengths did not indicate past top damage. Several plots, including six in the area of Lakeview, Oregon, were rejected after sampling because the shape of the height growth curves suggested substantial top damage 30 or more years earlier, even though this damage was not apparent at the time of selection.

6. Clumps of trees were not sampled. The plots were in homogeneous stands, and each plot had a buffer strip equivalent in width to tree height.

Diameter at breast height (d.b.h.) for each tree in each plot was measured, and 15 trees of each species on each plot were felled to determine past periodic annual increments for the plot. Included in this group were the three largest diameter trees of each species. If taller trees existed on the plot, they were

felled and their stem analysis data used for construction of the site index and height growth curves but not in volume growth determinations. The two to five tallest trees of each species at the time of sampling were sectioned at a 1-foot stump, 4.5 feet (bh), 10 feet, and then at 10-foot intervals up the stem after total height was measured.³ Sections at ground line were also taken from at least two of the largest diameter trees, two of the smallest diameter trees, and one tree with close to the mean diameter. Rings were counted for all sections and recorded for the appropriate height.

CURVE CONSTRUCTION

An age of 50 years at breast height (4.5 feet) was chosen as the index age. For each plot, total heights of each of the three to five tallest trees were plotted as a function of bh age for each tree on a single sheet of graph paper. The bh age for each tree was used as the independent variable in the initial plotting rather than average bh age for two reasons: (1) Height growth to 4.5 feet seems to be greatly influenced by competition immediately adjacent to a seedling and perhaps by early animal damage. Thus, it is possible for dominant trees of the same species to be the same age at ground line but to differ as much as 10 years at 4.5 feet. In this case, use of an average bh age for plotting heights results in an underestimate of the height growth potential of the site. (2) Use of an average bh age for mixed species when heights are plotted as a function of age does not give a realistic picture of height growth for each species. For mixtures of seedlings germinating the same year, larch reaches bh height first, followed by the pines, then Douglas-fir, and finally white or grand fir. There will usually be more than a 5-year bh age spread between the larch and the white or grand fir.

In plotting, shifts in relative position of individual trees occurred for the white or grand fir on 50 percent of the plots. These shifts in the tree of maximum height for its bh age indicate that the tree that was tallest for its bh age at the time of sampling was not always tallest for its bh age in the past.

Freehand curves were drawn for each tree on the single sheet of graph paper for each plot, and the highest points at each decadal age were used in subsequent construction of curves. Site index for white or grand fir on each plot was defined as the tallest height for that species complex at bh age 50.

From this point the methodology outlined by Barrett (1978) is used; it includes the recent improvements in curve construction methods suggested by Curtis et al. (1974) and Dahms (1975). Tabulations and figures used in construction of the curves are presented in the appendix. For aid in understanding this appendix, these tabulations and figures can be compared with similar data and explanations for Douglas-fir (Cochran 1979).

³ 1 foot = 0.3048 meter.

Some understanding of how the curves were constructed is helpful for proper use of the curves. Therefore, it is suggested that even occasional users of these curves read the appendix.

The 34 plots used in construction of curves for white fir or grand fir had site indexes ranging from 63.5 to 130 feet although only 1 sample plot had a site index above 110 feet. Average site index was 89.43 feet:

Number of plots of white fir or grand fir	Site index (feet)
0	50-59
2	60-69
7	70-79
5	80-89
14	90-99
5	100-109
1	>110

Results and Application

Site index curves (fig. 2) can be used for rough field estimates of site index. For a more precise estimate, the appropriate a and b values in table 1 can be used to solve the equation,

$$\text{Site index} - 4.5 \text{ feet} = a + b (\text{height} - 4.5 \text{ feet}).$$

The equation,

$$\begin{aligned} \text{Site index} = & (\text{height} - 4.5 \text{ feet})(e^{x_1}) - e^{x_1} (e^{x_2} \text{ feet}) \\ & + 89.43 \text{ feet}, \end{aligned}$$

can be used with a calculator; $x_1 = 3.8886 - 1.8017 (\log_e \text{ age}) + 0.2105 (\log_e \text{ age})^2 - 0.0000002885 (\log_e \text{ age})^9 + 0.0000000000000000001187 (\log_e \text{ age})^{24}$, and $x_2 = -0.30935 + 1.2383 (\log_e \text{ age}) + 0.001762 (\log_e \text{ age})^4 - 0.0000054 (\log_e \text{ age})^9 + 0.0000002046 (\log_e \text{ age})^{11} - 0.0000000000000404 (\log_e \text{ age})^{18}$.

Age in all equations in this paper is at breast height.

The following steps should be used in applying any of the procedures for estimating site index:

1. Select suitable plots with the following characteristics:
 - (a) Even-aged at the ground line; no older remnants from earlier stands; present stand one-storied.
 - (b) No visible symptoms of disease or insect attack to reduce height growth.
 - (c) No narrow ring groups to indicate suppression.
 - (d) No sign of damage on internodal lengths on taller trees.

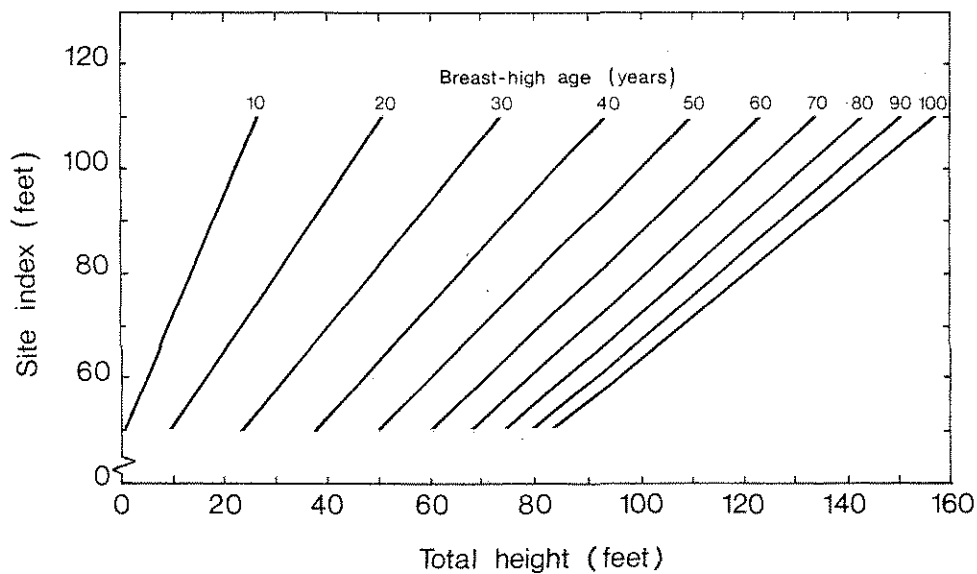


Figure 2.--Site index curves for managed, even-aged stands of white or grand fir east of the Cascades in the Pacific Northwest.

Table 1--Values for *a* and *b* by years for the family of regressions¹ for estimating site index for white or grand fir east of the Cascades in the Pacific Northwest) derivation of these *a* and *b* values is described in the appendix)

Breast-high age	Years between decades																			
	0		1		2		3		4		5		6		7		8		9	
	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>
Years																				
10	53.778	2.353	52.315	2.177	50.808	2.035	49.266	1.917	47.699	1.819	46.112	1.735	44.511	1.663	42.903	1.600	41.290	1.546	39.678	1.497
20	38.069	1.455	36.467	1.416	34.876	1.382	33.296	1.350	31.732	1.322	30.185	1.296	28.656	1.273	27.149	1.251	25.663	1.231	24.201	1.212
30	22.764	1.195	21.353	1.179	19.968	1.164	18.612	1.150	17.283	1.137	15.983	1.125	14.711	1.113	13.470	1.102	12.257	1.092	11.075	1.082
40	9.922	1.073	8.799	1.064	7.706	1.056	6.641	1.048	5.607	1.040	4.601	1.033	3.624	1.026	2.675	1.019	1.754	1.012	.860	1.006
50	0	1.000	-.846	.994	-1.659	.989	-2.447	.983	-3.210	.978	-3.948	.973	-4.662	.968	-5.353	.963	-6.020	.958	-6.665	.954
60	-7.288	.949	-7.889	.945	-8.469	.941	-9.029	.936	-9.569	.932	-10.090	.928	-10.591	.924	-11.074	.920	-11.539	.917	-11.987	.913
70	-12.417	.909	-12.831	.906	-13.228	.902	-13.609	.899	-13.975	.895	-14.326	.892	-14.661	.888	-14.983	.885	-15.290	.882	-15.584	.878
80	-15.863	.875	-16.130	.872	-16.384	.869	-16.625	.866	-16.853	.863	-17.069	.859	-17.273	.856	-17.466	.853	-17.646	.850	-17.815	.847
90	-17.973	.844	-18.119	.842	-18.255	.839	-18.379	.836	-18.493	.833	-18.596	.830	-18.688	.827	-18.770	.824	-18.841	.823	-18.901	.819
100	-18.951	.816																		

¹/To estimate site index, measure total height of up to 5 tallest trees per 1/5-acre plot; determine breast-high age for each. Select appropriate *a* and *b* values above. Substitute values in the equation, Site index - 4.5 feet = *a* + *b* (height - 4.5 feet). For example, for a tree 53 years old at breast height and 80 feet in total height, solve the equation, Site index - 4.5 feet = -2.447 feet + 0.983 (80 feet - 4.5 feet), for a site index of 78.7 feet. Determine the site index for each sample tree. The highest site index determined is the site index for white or grand fir on the 1/5-acre plot.

- (e) Little remnant understory vegetation or suppression mortality to indicate competition early in the life of the stand and possible reduction of height growth.
2. Establish boundaries of 1/5-acre plot with a prespecified shape.
3. Measure the height of up to five tallest dominant or codominant white or grand fir and Douglas-fir, if present, on the plot.
4. Extract increment cores from these trees to determine their breast-high age.
5. Using the breast-high age and total height for each tree, determine a site index value for each tree. Curves, tables, and equations for Douglas-fir are given by Cochran (1979).
6. Record for each species on each plot the highest value obtained for site index.
7. The site index for the area of concern would be the average of the highest site indexes determined on the 1/5-acre sample plots. As will be pointed out, there may be no practical differences between the site indexes of white or grand fir and Douglas-fir. Therefore, in stands that are mixtures of Douglas-fir and white or grand fir, I recommend averaging the highest site indexes of each plot regardless of species. It is very important to use accurate age and height measurements; bias from height measurement error could easily occur.

Height growth curves (fig. 3) define the average pattern of height development for the tallest trees in stands of a given site quality. They are appropriately used for constructing yield tables but do not provide optimum estimates of site index from measured height and age in an existing stand (Curtis et al. 1974). Comparisons of height growth curves and site index curves are given in figure 4.

The following are alternatives for estimating the anticipated height of the tallest trees of a managed, even-aged stand on land of known site index:

1. Use figure 2 for rough field estimates.
2. For more precise estimates, use table 2 to solve the equation:

$$\text{Total height} - 4.5 \text{ feet} = a_1 + b_1 (\text{site index} - 4.5 \text{ feet}).$$

3. The estimating procedure can be programed on a calculator using the equation,

$$\begin{aligned} \text{Total height} - 4.5 \text{ feet} = & e^{x^2} \text{ feet} - (84.93 \text{ feet}) e^{x^3} \\ & + (\text{site index} - 4.5 \text{ feet}) e^{x^3}. \end{aligned}$$

Here, x^2 is as previously defined, and

$$\begin{aligned} x^3 = & -6.2056 + 2.097 \log_e \text{ age} - 0.09411 (\log_e \text{ age})^2 \\ & - 0.00004382 (\log_e \text{ age})^7 + 0.00000000002007 (\log_e \text{ age})^{16} \\ & - 0.0000000000000002054 (\log_e \text{ age})^{24}. \end{aligned}$$

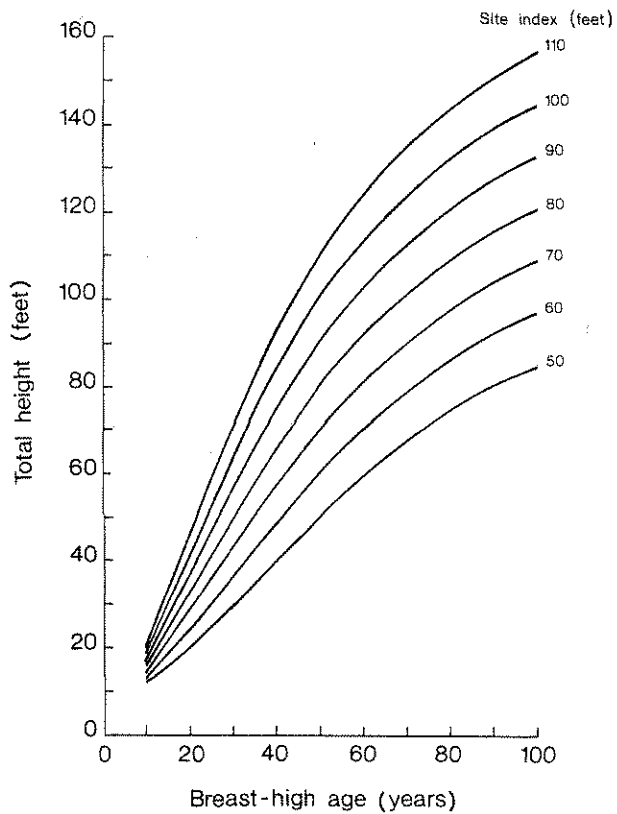


Figure 3.--Height growth curves for grand or white fir east of the Cascades in managed, even-aged stands.

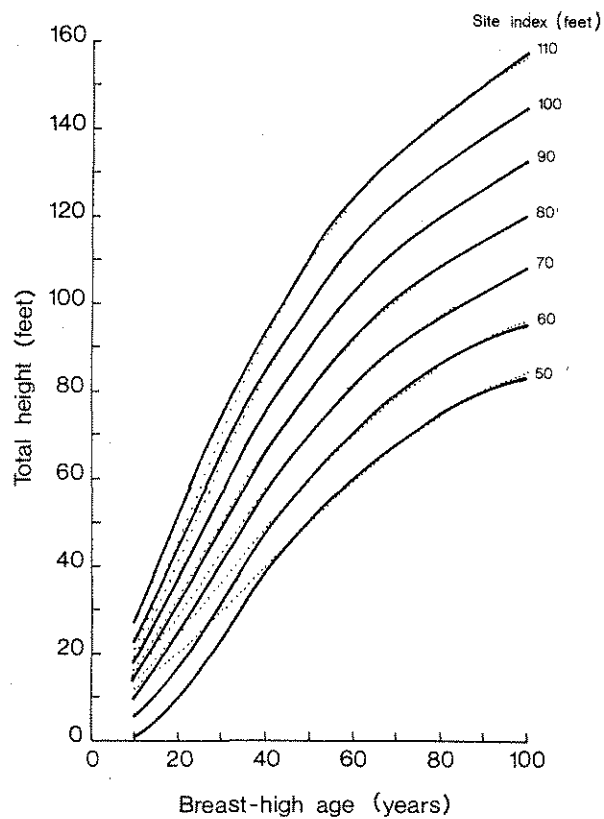


Figure 4.--Site index (solid lines) and height growth curves (dashed lines) for white or grand fir in managed, even-aged stands east of the Cascades in Oregon and Washington.

Table 2--Values for a_1 and b_1 by years for the family of regressions¹ for estimating height of the tallest trees in a newly established stand of white or grand fir where site index and age are known (derivation of a_1 and b_1 values is described in the appendix)

Breast-high age	Years between decades																			
	0		1		2		3		4		5		6		7		8		9	
	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
Years																				
10	0.423	0.151	0.048	0.176	-0.353	0.202	-0.771	0.228	-1.200	0.255	-1.631	0.283	-2.060	0.310	-2.481	0.338	-2.888	0.366	-3.278	0.394
20	-3.648	.422	-3.993	.450	-4.312	.477	-4.602	.504	-4.861	.531	-5.088	.557	-5.280	.583	-5.438	.608	-5.561	.632	-5.647	.656
30	-5.698	.680	-5.712	.702	-5.690	.724	-5.633	.745	-5.540	.766	-5.412	.785	-5.251	.805	-5.057	.823	-4.830	.840	-4.572	.857
40	-4.285	.874	-3.968	.889	-3.624	.904	-3.253	.918	-2.857	.932	-2.437	.945	-1.995	.957	-1.532	.969	-1.048	.980	-.546	.990
50	0	1.000	.508	1.010	1.058	1.019	1.621	1.027	2.196	1.035	2.782	1.043	3.378	1.050	3.981	1.057	4.592	1.063	5.209	1.070
60	5.831	1.075	6.457	1.081	7.085	1.086	7.715	1.091	8.346	1.095	8.976	1.100	9.605	1.104	10.233	1.108	10.857	1.111	11.477	1.115
70	12.093	1.118	12.704	1.121	13.308	1.125	13.905	1.127	14.495	1.130	15.077	1.133	15.649	1.136	16.212	1.138	16.765	1.141	17.308	1.144
80	17.839	1.146	18.358	1.148	18.865	1.151	19.359	1.153	19.840	1.156	20.307	1.158	20.760	1.161	21.198	1.163	21.621	1.166	22.029	1.168
90	22.421	1.171	22.798	1.173	23.157	1.176	23.500	1.179	23.825	1.182	24.133	1.184	24.424	1.187	24.696	1.190	24.950	1.193	25.186	1.197
100	25.403	1.200																		

¹/Height at a future date of the tallest portion of a young stand may be estimated on land of known site index by selecting a_1 and b_1 values for the appropriate breast-high age. Substitute a_1 and b_1 values in the equation, Height - 4.5 feet = $a_1 + b_1$ (Site index - 4.5 feet). For example, to determine the height of the tallest trees in the stand at breast-high age 75 on land with a known site index of 100 feet, solve the equation, Height - 4.5 feet = 15.077 feet + 1.133 (100 feet - 4.5 feet), for a total height of 126.3 feet.

Data for these curves were collected from natural, even-aged stands where densities prior to sampling resembled those assumed desirable in the managed stands of the future. Therefore, use of these curves should be restricted to managed, even-aged stands where height growth has not been reduced by stand density. The curves should not be used for:

1. Precommercially thinned stands showing a tight core of rings.
2. Commercially thinned stands with numerous stumps indicating a high initial density.
3. Plantations with large numbers of trees thinned long after competition between trees occurred.
4. Stands where height growth was severely restricted by brush competition early in the life of the stand. This usually results in a tight core of rings at the center of the b.h. increment core and should be watched for in widely spaced older stands that appear to have never experienced severe competition.

These curves may have limited application in the field for some time because only a small amount of land under management fits the constraints of the curves. Greatest use is in forecasting future stand performance in stand growth simulators.

Reliability of the curves can be partially determined by the r^2 values and standard errors associated with the slope and intercept values of the basic equations. This information is presented in the appendix. Also, one may judge the equation fit from figures 5 and 6.

There were 13 plots that had both dominant Douglas-fir and dominant white or grand fir trees. Site index values for Douglas-fir were determined as outlined by Cochran (1979) and compared with the site index values obtained for the white or grand fir. For these plots, the relationship between the site indexes of the two species is ($r^2 = 0.7942$; standard error = 4.2 feet): Douglas-fir SI = $-2.7894 + 1.0367$ (white or grand fir SI).

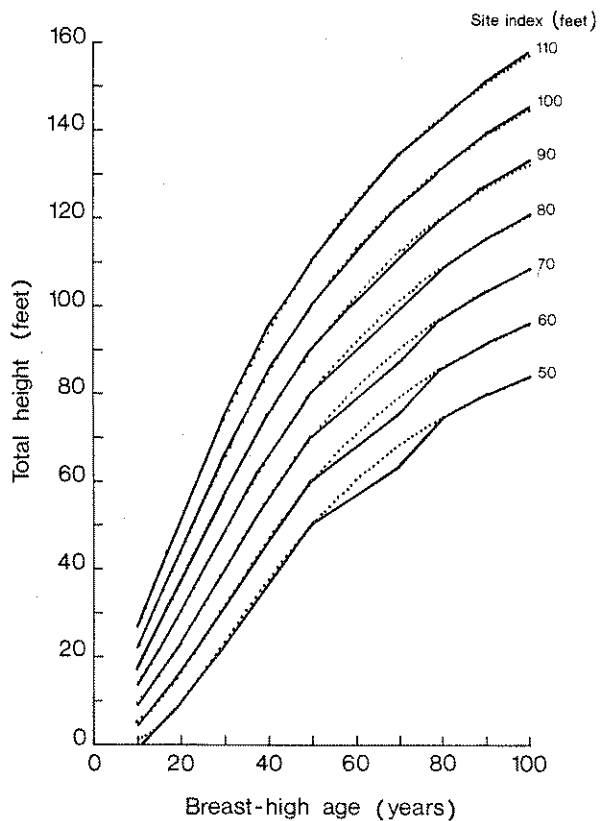


Figure 5.--Site index curves for managed, even-aged stands of white or grand fir east of the Cascades in the Pacific Northwest. Solid lines connect decadal points derived from the unsmoothed basic data regressions of $SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet})$. Dashed lines represent smooth curves from a rearrangement of the estimating equation.

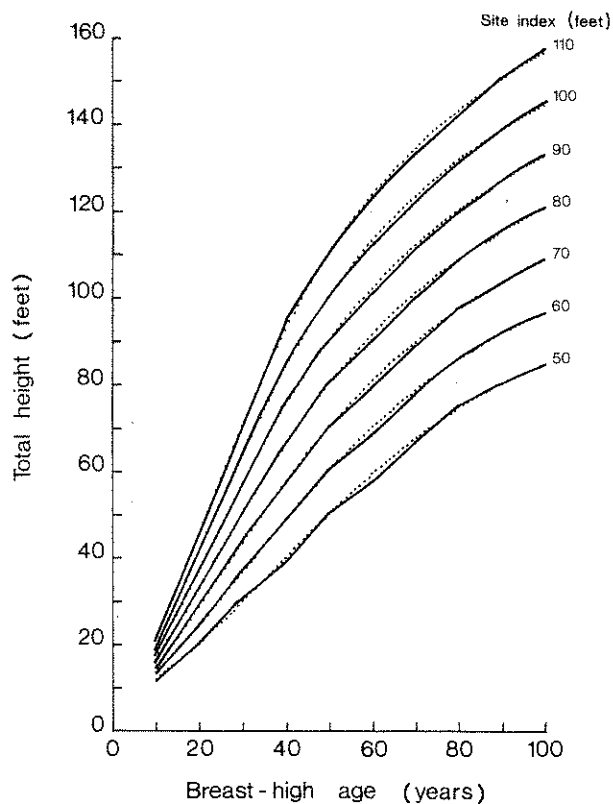


Figure 6.--Height growth curves for managed, even-aged stands of white or grand fir in the Pacific Northwest. Solid lines connect decadal points derived from the unsmoothed basic data regressions of $HT - 4.5 \text{ feet} = a_1 + b_1 (SI - 4.5 \text{ feet})$. Dashed lines represent smooth curves from the estimating equation.

Although the relationship of site indexes between species needs more investigation, it may indicate that within the ranges of sites where these species occur as mixtures there is no practical difference in the 50-year-bh-age-base site index between the two species even though the shapes of the curves are different.

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Appendix

In construction of both site index and height growth curves, a curve of average height for the samples as a function of age at 4.5 feet is determined. This height curve is then adjusted upward or downward to the desired site index using the linear relationship existing between height and site index at any age with appropriate estimates of slope and intercept. The curves are different because slope and intercept values of the basic equations,

$$SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet}) \text{ and}$$

$$HT - 4.5 \text{ feet} = a_1 + b_1 (SI - 4.5 \text{ feet}),$$

are different for all ages except the index age (50 years at bh for these curves).

Values for a , b , a_1 , and b_1 are determined for each 10-year-bh age interval using heights for each 10-year interval and site indexes tabulated from the freehand curves for the sample plots. The b and b_1 values are then smoothed (figs. 7 and 8) by fitting equations which describe b and b_1 as a function of breast-high age. Each of these equations are forced through 1 at age 50.

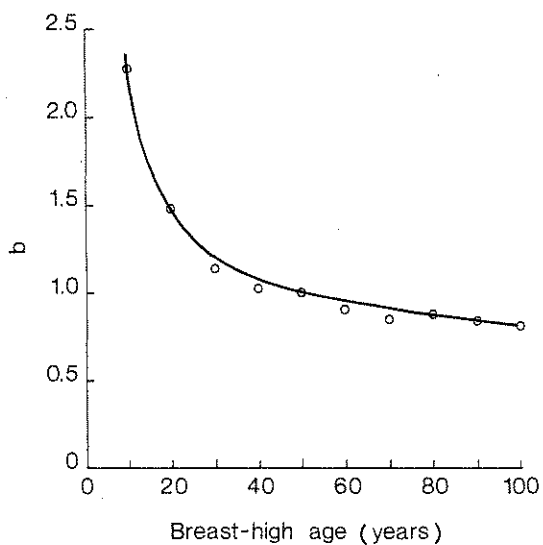


Figure 7.-- b values in the equation $SI - 4.5 \text{ feet} = a + b$ ($HT - 4.5 \text{ feet}$) as a function of age. Plotted points are actual b values. Solid lines are curves expressed by the equation $\hat{b} = e^{x1}$. Coefficient $x1$ is defined in the text. Standard error is 0.063, and R^2 equals 0.9891.

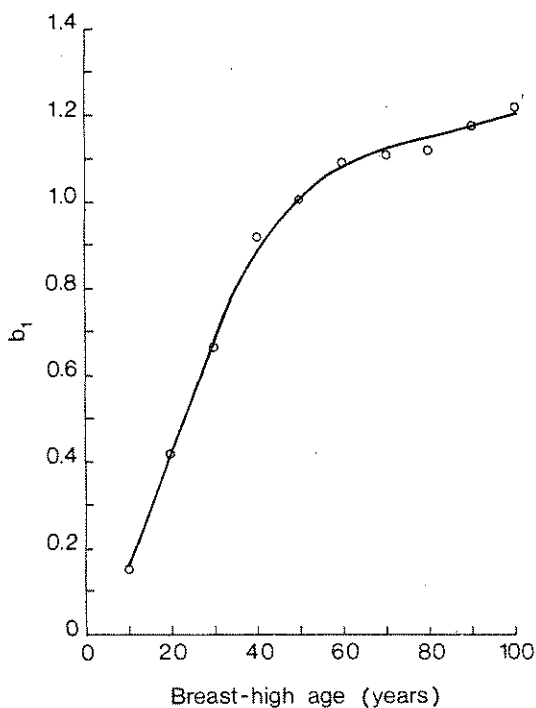


Figure 8.-- b_1 values in the equation $HT - 4.5 \text{ feet} = a_1 + b_1$ ($SI - 4.5 \text{ feet}$) as a function of age. Plotted points are actual b_1 values. Solid lines are curves expressed by the equation $\hat{b}_1 = e^{x3}$. Coefficient $x3$ is defined in the text. Standard error and R^2 values are 0.031 and 0.9966, respectively.

The R^2 and standard error values associated with the b and b_1 values in figures 7 and 8 are given merely to show how well the fitted equations describe the slope values for each decadal age. These measures of variation have no direct association with the original population. Next, an average height curve is defined (fig. 9). For ages where the sample size decreases, points for fitting the height curve are determined by using actual a_1 and b_1 values from the regression calculated for that age and the average site index for all the plots in:

$$HT - 4.5 \text{ feet} = a_1 + b_1 (\overline{SI} - 4.5 \text{ feet}).$$

In fitting the average height curve, the equation describing $HT - 4.5$ feet as a function of bh age is forced through the average site index - 4.5 feet at bh age 50 years.

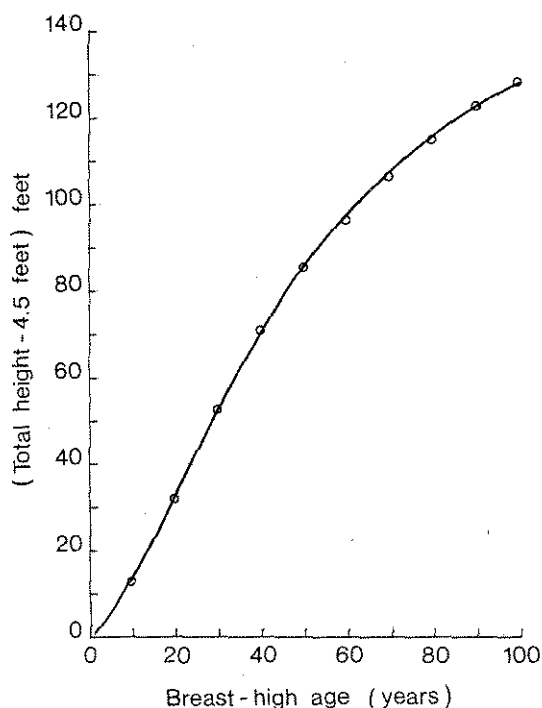


Figure 9.--Average height of sectioned trees as a function of breast-high age. Plotted points are actual average heights minus 4.5 feet. Solid lines are curves expressed by the equation $\hat{HT} - 4.5 \text{ feet} = e^{x^2}$. Coefficient x^2 is defined in the text. The standard error is 1.25 feet, and R^2 equals 0.995.

Once equations for the smoothed $\hat{b}$ and $\hat{b}_1$ values and the average height curve are obtained, equations for the smoothed $\hat{a}$ and $\hat{a}_1$ values in tables 1 and 2 and the equations for the site index and height growth curves are obtained by substitution of appropriate expressions and rearrangement of the basic equations. All b and b_1 values are without units; all a and a_1 values have units of feet.

Following are some estimates for the equation $SI - 4.5 \text{ feet} = a + b (HT - 4.5 \text{ feet})$, where HT = total height and SI = site index:

White or grand fir, bh age	a	b	r^2	Standard error of the estimate	Number of observations
(Years)	(Feet)	(Feet)		(Feet)	
10	56.3805	2.2724	0.3423	10.85	34
20	38.3353	1.4729	.6086	8.37	34
30	25.7738	1.1341	.7492	6.70	34
40	13.1296	1.0211	.9373	3.35	34
50	0	1	1	0	34
60	-1.1024	.9001	.9773	2.01	33
70	-4.1854	.8438	.9316	3.73	23
80	-15.5229	.8768	.9782	2.01	13
90	-17.2781	.8363	.9820	1.83	8
100	-18.5481	.8091	.9839	1.74	8

The eight plots with bh ages of 90 years and above have site indexes of 63.5, 74.8, 75, 75.5, 86.5, 90, 94.5, and 102 feet.

Some estimates follow for the equation $HT - 4.5 \text{ feet} = a_1 + b_1 (SI - 4.5 \text{ feet})$:

White or grand fir, bh age	a_1	b_1	r^2	Standard error of the estimate	Number of observations
(Years)	(Feet)	(Feet)		(Feet)	
10	-0.2285	0.1506	0.3423	2.79	34
20	-3.4601	.4132	.6086	4.32	34
30	-3.9426	.6606	.7492	5.11	34
40	-7.6391	.9179	.9373	3.18	34
50	0	1	1	0	34
60	3.3556	1.0858	.9773	2.21	33
70	11.8894	1.1041	.9316	4.27	23
80	19.7133	1.1157	.9782	2.27	13
90	22.3436	1.1743	.9820	2.17	8
100	24.4805	1.2161	.9839	2.13	8

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