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Sampling coarse woody debris along spoked transects

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Summary

Line transects are commonly used for sampling coarse woody debris (CWD). The USDA Forest Service Forest Inventory and Analysis programme uses a variant of this method that involves sampling for CWD along transects that radiate from the centre of a circular plot-like spokes on a wheel. A new approach for analysis of data collected with spoked transects is developed. This approach does not attempt to mimic traditional analysis methods for line transects, which do not readily apply to spoked transects. The method developed here generalizes to any number of spokes. We also demonstrate that there is a close relationship between spoked transects and a particular fixed plot design for sampling CWD.

Introduction

The line intersect method (Warren and Olsen, 1964; Kaiser, 1983) was developed for sampling logging slash and coarse woody debris (CWD). The basic method samples CWD logs proportional to length, which is somewhat related to log volume. Variants of the original method have been proposed that sample CWD with prism sweeps to increase the relationship between selection probability and volume (Gove *et al.*, 1999, 2005; Ducey *et al.*, 2002; Bebber and Thomas, 2003; Jordan *et al.*, 2004; Fraver *et al.*, 2007).

The original line intersect sampling proposal (Warren and Olsen, 1964) called for measuring CWD that intersected straight-line transects randomly located in the forest. Forest Inventory and Analysis (FIA) (Bechtold and Patterson, 2005; Anonymous, 2010) has implemented a variant of this method that is a special case of what we are calling spoked transects. FIA uses three spokes radiating from the centre of a circular plot, equally spaced, at 120° intervals (Woodall and Monleon, 2008). Ell-shaped transects have also been proposed (Gregoire and Valentine, 2003) that consist of two perpendicular spokes.

Traditional line transects would generally be spaced far enough apart to make it rare for the same downed log to intersect multiple transects. However, it is clear that logs will frequently intersect multiple spokes with spoked transects, particularly when the log is near the plot centre. This issue is addressed for Ell-shaped transects (Gregoire and Valentine, 2003) and for the FIA three-spokes design (Affleck *et al.*, 2005) by computing the probability of multiple intersections. With the methods proposed here, a selected CWD piece will only be counted once regardless of the number of spokes it intersects within a single plot.

The analysis methods used for assessing specific spoked designs (Gregoire and Valentine, 2003; Affleck *et al.*, 2005; Woodall and Monleon, 2008) generally follow the seminal methods used in earlier theoretical developments (Warren and Olsen, 1964; de Vries, 1974; Kaiser, 1983). This leads to somewhat complex results that are difficult to generalize to designs with arbitrary numbers of spokes. We follow an alternative analysis approach that leads to general results for spoked transects. This allows us to show how spoked transects relate to sampling CWD on fixed circular plots. In particular, there is a close relationship between a fixed plot method proposed in Gove and Van Deusen (2010), called the sausage method, and spoked transects on circular plots.

Spoked transects converge to the sausage method

A fixed area circular plot method for sampling CWD was developed in Gove and Van Deusen (2010) called the sausage method because the selection area for a downed log looks like a sausage with this method. The sausage method protocol is to measure any piece of CWD that projects into a circular plot. This results in counting an entire log even if just the tip falls into the plot. The selection probability for log i with the sausage method is proportional to

$$p(\mathrm{C})_i \propto \pi r^2 + 2rl_i, \tag{1}$$

where $p(C)$ denotes the probability of being in the circular plot, r is the radius of the plot and l_i is the length of log i. Clearly, selection probability with the sausage method depends on log length.

Consider image 1, row 1 (Figure 1), which depicts a circular plot with one log at the centre. A single spoke is shown with the selection zone lightly shaded. The log will intersect the spoke if its centre point lies within the shaded selection zone. The dash line that encompasses the circular plot traces out the entire sausage method selection zone. The depicted log would be selected with the sausage method if its centre point lies anywhere within the sausage shape because some part of the log would protrude into the circle.

Image 2, row 1 (Figure 1), depicts an Ell-shaped transect and image 3, row 1, depicts the FIA three-spoked transect using the same log for each figure. The shaded spoke-selection zone area increases from left to right. Image 4, row 1, depicts a 50-spoked transect and the selection zone for the log. It is obvious that the entire sausage area will be shaded as the number of spokes increases. Hence, the spoked transect method converges to the sausage method as the number of spokes goes to infinity.

Row 2 (Figure 1) shows the same trend as row 1 but for a longer log with a different angle. It also shows convergence to the sausage method and that the sausage shape is more elongated for long logs and is orientated along the axis of the log.

Spoked transect selection probability

The relationship between spoked transects and the sausage method suggests thinking of the spokes as a second stage to the sausage method. Stage 1 selects all CWD that has any presence within the circular plot. Stage 2 selects a subsample from the stage 1 sample. The joint probability of being selected at both stages is simply

$$p(\mathrm{C},S)_i = p(\mathrm{C})_i\, p(S \mid \mathrm{C})_i, \tag{2}$$

where $p(\mathrm{C})_i$ was defined in equation (1) and $p(S \mid \mathrm{C})_i$ is the stage 2 probability defined below.

Think of $p(S \mid \mathrm{C})_i$ as the probability of log i crossing a spoke, given that log i impinges on the circular plot.

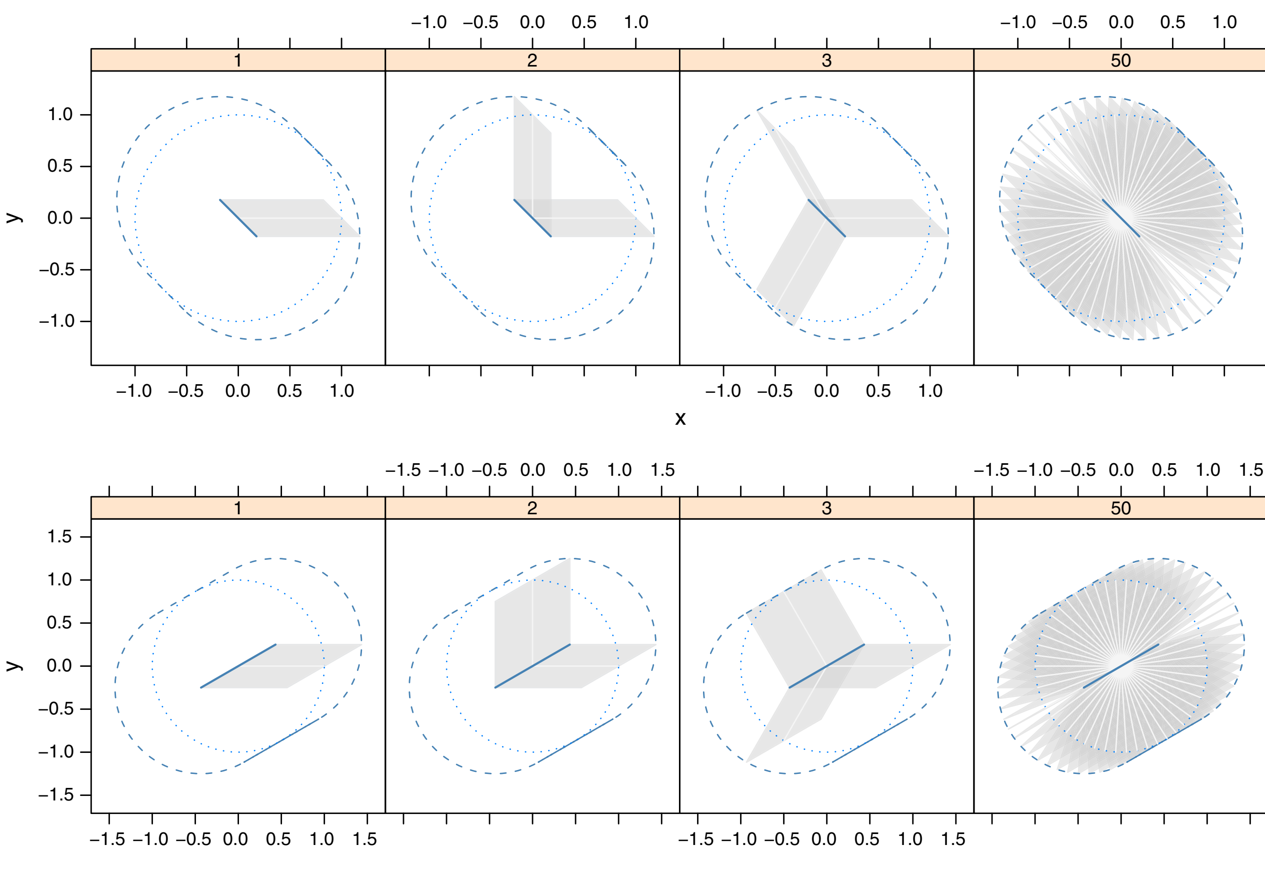

Figure 1. Relationship between spoked transects and the sausage method. Shaded areas depict selection zones along each spoked transect for the log depicted in the centre of the circular plot. As the number of spokes increases, the entire sausage area becomes shaded.

Generally, $0 \leq p(S \mid C)_i \leq 1$ and it approaches 1 as the number of spokes approaches infinity. The spokes can be viewed as being in a fixed configuration consisting of 1, 2, 3, . . ., S spokes where the azimuth of the first spoke is randomly determined. The remaining spokes are placed at intervals of $2\pi/S$ radians or $360/S$ degrees. Think of spinning the pointer for a board game and using the position of the pointer as the random azimuth. With a single spoke, the log would be selected if the transect defined by the pointer intersects the log. The transect radiates from the circular plot centre, so the log's distance from centre, log-length and angle of the log relative to the radii going through the log's midpoint determine its stage 2 selection probability.

Figure 2 shows a log lying within a circular plot, where the dotted lines touching the ends of the log define angle θ. If S spokes were superimposed on this circular plot, the probability of a spoke intersecting the log would be:

$$p(S \mid C)_i = \begin{cases} \theta_i / \theta_S & \text{if } \theta_i \leq \theta_S, \\ 1 & \text{otherwise,} \end{cases} \tag{3}$$

where θ_i is angle θ for log i (Figure 2) and $\theta_S = 2\pi/S$. Angles are in radians and therefore the angle between equally spaced spokes is θ_S. In fact, θ_S adjusts $p(S \mid C)_i$ for the number of spokes. Note that $p(S \mid C)_i$ is constrained to not exceed 1 in cases where θ_i is greater than θ_S.

Application of two-stage sampling with spoked transects

It is not difficult to implement two-stage spoked transect sampling in the field. First, establish the circular plot centre and the spoked transects. The implication is that the transects should be randomly positioned, but they could also be placed at a predetermined orientation within the plot because the plot is randomly located. The sighting angle of each intersected piece of CWD (Figure 2) could be determined with an angle gauge by sighting across the ends of the piece from the circle centre. Note that only the section of the log contained within the circle is used to compute θ_i, although the entire log must be measured for attribute estimation. In other words, imagine the log is truncated at the circle's edge when finding the endpoints for determining θ_i, but use the entire log for computing an attribute such as volume.

An estimate for each circular plot j would be

$$y_j = A \times \sum_{i=1}^{n_j} \frac{y_{ij}}{p(C,S)_i}, \tag{4}$$

where A scales the estimate to either a per hectare or per acre value, n_j is the number of selected CWD pieces on plot j and y_{ij} is the value of interest (perhaps volume) for each selected piece of CWD. For example, if plot radius and log lengths are measured in metres, then A = 10 000 will scale $p(C,S)_i$ to represent a per hectare selection probability. If the measurements are in feet, then A = 43 560 scales $p(C,S)_i$ to acres.

Perhaps it is not practical to measure angles by sighting the within-circle ends of each selected CWD piece from plot centre. Alternatively, these angles can be computed by measuring the distance from plot centre to the centre of the within-circle section of each selected CWD piece. Then determine the angle of the selected piece relative to the radii that goes through the within-circle-piece centre point. The computations required to determine θ_i from these measurements are derived in the Appendix.

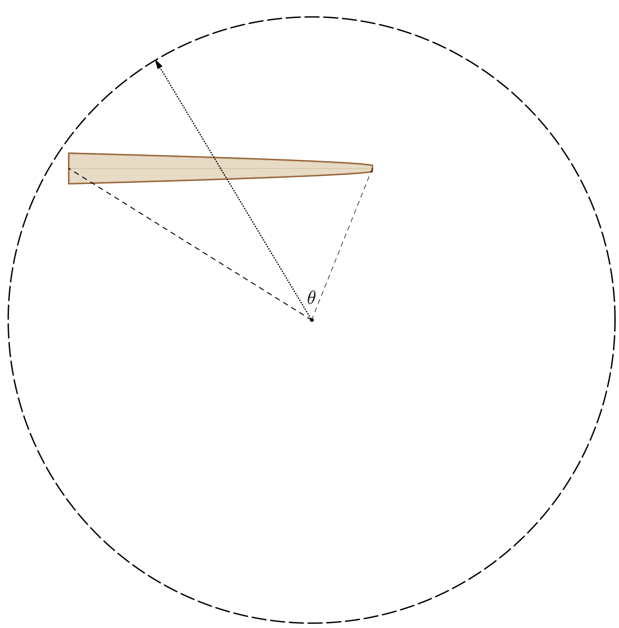

Figure 2. Angle θ determines the probability that a log will intersect a random spoke emanating from the centre of the circle.

Alternative second-stage selection probabilities

Second stage spoked-transect selection probabilities involve the CWD piece length, angle and distance from plot centre. For traditional line transect sampling, an expected probability is used that is computed by integrating over all possible angles (Warren and Olsen, 1964) to effectively remove the angle from the equation.

In fact, it seems undesirable to have a sample protocol that samples proportional to a random angle that has no relationship to the variable of interest. This is akin to sampling proportional to random noise, which might result in unbiased estimates, but will add additional variance. The approximate expected value for the stage 2 selection probability is developed here, which is necessary for this sampling design to have practical appeal.

The computational method for the actual stage 2 selection probabilities outlined in the Appendix requires measurements of distance from the centre of the circle, an angle relative to a midpoint radii and a log length. However, log length is the only thing that is typically measured in the field for line intersect sampling. Hence, we seek an expected selection probability that utilizes log length but does not require measurements of angles and distances. The

Appendix outlines a highly non-linear algorithm for computing selection probabilities. Rather than attempting to integrate over distance and angles, we propose substituting expected distance from the circle centre and the expected angle relative to a radius into the algorithm to arrive at the desired expected stage 2 selection probability.

First, consider the expected angle relative to a radius passing through the midpoint of the CWD piece. Since the angle can range from 0 to $\pi/2$ radians, with each possible value being equally likely, the expected angle is just the midpoint of this range,

$$\hat{\Psi} = \frac{\pi}{4} \tag{5}$$

We are defining 0 radians to mean the log is parallel to the radii and $\pi/2$ to mean it is perpendicular. The log is treated like a symmetric needle for the purposes of determining this angle, so we only need to consider log angles from 0 to $\pi/2$ radians.

The distance from the centre of the circle to the log-midpoint for a log that protrudes into the circle can range from 0 to D, where $D = r + l/2$. A log at the maximum distance (D) can just reach the edge of the circular plot.

Computation of expected distance requires a probability density function (pdf) for distance. The cumulative density function (cdf) for distance is $p(d < d') = d'^2 / D^2$. The pdf is the first derivative of the cdf, hence $p(d) = 2d/D^2$. Now the expected distance is

$$\hat{d} = \frac{2}{D^2}\int_0^D d^2 \mathrm{d}d = \frac{2}{3}D. \tag{6}$$

Our proposal is to simply plug $\hat{\Psi}$ and $\hat{d}$ into the algorithm outlined in the Appendix, which results in an expected selection probability that requires only measuring piece length (l) in the field. Specifically, plug $\hat{\Psi}$ and $\hat{d}$ into equations (A-3) through (A-7) to determine $\hat{\theta}_i$. Then use $\hat{\theta}_i$ to compute expected stage 2 probability with equation (3).

Simulated application

The simulation is designed to compare the sausage method with the FIA three-spoke transect method. The three-spoke method is applied with actual and expected stage 2 selection probabilities. The simulation is based on a large circular region with a radius of 600 m. The large area contains 5000 CWD pieces. The piece length varies from 3 to 50 m in length. Large end diameter ranges from 10 to 100 cm, and piece volume ranges from 0.0014 to 16.4 m^3.

Each sample replication involves measuring the CWD pieces that fall into a 1-ha circular plot located at the centre of the large circle. The positions of the 5000 CWD pieces are randomized between each of the 4000 replications. Piece distances from the centre of the 600-m sampling circle are generated using the inverse transform method based on the cdf used to derive equation (6), and piece angles are randomly distributed between 0 and $\pi/2$. The simulation results (Table 1) give the minimum, maximum, median, mean and standard error values computed over the 4000 replications for the volume estimate from each method. The mean estimate from the sausage method is closest to the true total-volume estimate, as might be expected.

The mean estimate from the three-spoke method with expected stage 2 probabilities is a little high. However, the median is much closer to the correct estimate with the expected probabilities than with the actual probabilities. Likewise, the maximum is closer with expected probabilities. The standard error with the expected probabilities is about twice as large as the sausage method standard error. However, the standard error for the spoke method using actual stage 2 probabilities is almost 10 times larger than the sausage method.

Discussion

We have shown that spoked transects can be viewed as part of a two-stage sampling process. Stage 1 is just the sausage method as discussed in Gove and Van Deusen (2010), where all CWD that protrudes into a circular plot is part of the initial sample. Stage 2 involves selecting a subsample from the stage 1 sample. The overall selection probability can be described using standard conditional probability formulae (equation 2).

The stage 2 selection probability (equation 3) requires field measurements that might be impractical, e.g. the angle of each selected CWD piece relative to a plot radii. An approximate method to compute expected stage 2 probabilities was proposed that requires only a measurement of CWD piece length, which will generally be available in existing field data. In fact, our simulation of the FIA three-spoke design suggested (Table 1) that the expected

Table 1: Simulation results for sausage method (V-sausage), three-spoked transects with correct stage 2 selection probability (V-spoke) and expected stage 2 probability (V-expected)

	V-sausage	V-spokes	V-expected	n-sausage	n-spokes
Minimum	3782.15	0.00	0.00	27.00	0.00
Maximum	22 142.33	11 90 670.01	35 880.50	81.00	16.00
Median	11 500.65	8392.38	11 068.72	51.00	6.00
Mean	11 637.28	11 567.75	11 716.30	51.23	6.00
SE	2495.69	23 424.11	5505.87	7.20	2.46

Sample size statistics are shown for the sausage and spoke methods. The true volume is 11 629.42.

probabilities result in estimates with better overall statistical properties than the exact stage 2 selection probabilities.

The simulated example application showed that the sausage method performs better than the three-spoke design. This is primarily because the sausage method does not involve the second stage subsampling that reduced the mean sample size from 51 to 6 (Table 1). Although the sausage method has half the standard error of the three-spoke method using expected probabilities, this comes at the expense of measuring more than eight times as many downed logs. This suggests that the FIA system for subsampling CWD within circular plots is a rather efficient design if properly analysed.

The concept of spoked transects implies that spokes emanate from a central point and are equally spaced around the circular plot. The FIA three-spoke design follows this paradigm, but the Ell-shaped design (Gregoire and Valentine, 2003) does not. However, the Ell-shaped design with two transects at right angles can be viewed as half of a four-spoked design that would have four transects spaced at 90° intervals around the circle. Thus, the stage 2 selection probability for the Ell-shaped design is half the selection probability for the four-spoked design, which can be easily computed with equation (3).

It is also useful to consider if transects could be replaced with another mechanism for selecting samples at stage 2. The focus here is on transects, so we do not want to get into alternative stage 2 options in depth. However, we mention two possibilities in passing. First, one could simply flip a coin to decide if a CWD piece selected by the stage 1 sausage method should be included in the stage 2 sample. This would result in a stage 2 selection probability of 0.5 for each CWD piece. Second, one could use sector sampling (Iles and Smith, 2006), where the piece is selected if its within-circle midpoint falls into the selected sector. This stage 2 probability would be proportional to the sector size. The stage 1 probability will remain the same (equation 1) regardless of the stage 2 method employed.

Conclusions

Line intersect sampling is widely used for sampling CWD. Spoked transects are a variant that has not been specifically categorized prior to this. In particular, spoked transects are designs where spokes emanate from a central point and would typically be spaced at equal intervals around a circular plot. FIA employs a three-spoked design and Ell-shaped transects (Gregoire and Valentine, 2003) can be viewed as one half of a four-spoked design.

The theory for Ell-shaped and three-spoke designs has been developed elsewhere (Gregoire and Valentine, 2003; Affleck *et al.*, 2005; Woodall and Monleon, 2008). However, the two-stage approach taken here simplifies the development and provides a unifying theory for any number of spokes. In particular, we showed that a spoked design converges to the sausage method (Gove and Van Deusen, 2010) as the number of spokes increases. The sausage method is also newly identified and is itself a viable method for sampling of downed wood.

We showed that the sausage method can be viewed as stage 1 for a spoked design. The spoked transect, in stage 2, selects a subsample from the first stage sample. This view leads to a simple conditional probability formula to describe the joint probability of selecting a sample over the two stages. The two-stage view also makes it obvious that methods other than transect sampling could be substituted for stage 2 or stage 2 could be omitted to result in regular sausage sampling.

One could ask if this two-stage method has relevance for traditional line intersect sampling. We suggest that traditional line intersect theory is already well developed and might not derive much benefit from this approach. The main benefit for spoked designs is that they are already making implicit use of a circular plot, whereas traditional straight line transects are not. Also, the general theory for spoked transects seemed somewhat intractable without taking the two-stage approach.

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Appendix: Computing CWD selection angles relative to plot centre

The selection angle for two-stage spoked transect sampling can be computed from three measurements, the distance (d) from plot centre to the midpoint of the within-circle section of the selected CWD piece, the within-circle length (l) of the piece and the angle (Ψ) of the piece relative to the radii drawn through the within-circle centre of the piece. Note that the definitions for d, l, Ψ apply to the within-circle part of the log rather than the entire log for the purposes of this Appendix. The depiction (Figure A1) shows the circular plot with the radii (shown as an arrow) going through the centre of the CWD piece.

There are two triangles (Figure A1) where enough information is available to define the selection angle (θ) for each CWD piece, which are shown as ΔBCD and $\Delta B'C'D'$. Note that $\theta = \angle C + \angle C'$ in Figure A1. Therefore, our goal is to compute $\angle C$ and $\angle C'$.

Following standard practice, angles are denoted by uppercase letters and the opposite side by the corresponding lowercase letter. The cosine rule and the sine rule are used to find the desired angles,

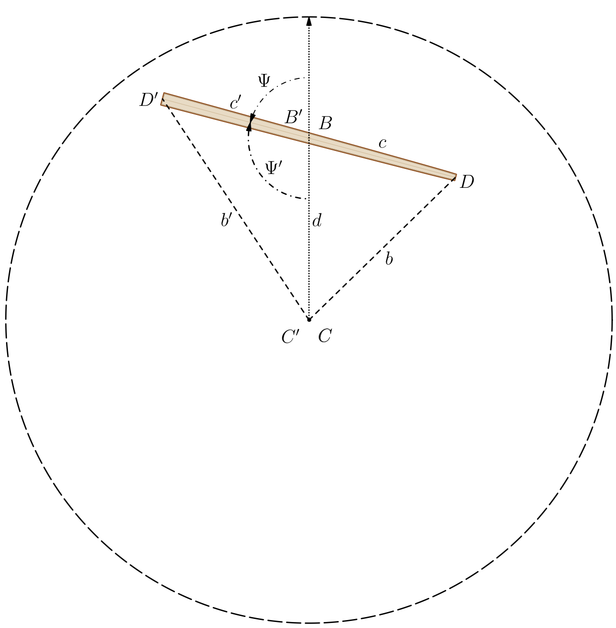

Figure A1. Using basic trigonometric properties to compute the selection angle for two-stage spoked transect sampling.

$$\text{cosine rule}: b^2 = c^2 + d^2 - 2cd \times \cos(B), \quad \text{(A-1)}$$

$$\text{sine rule}: \frac{\sin(B)}{b} = \frac{\sin(C)}{c} = \frac{\sin(D)}{d}. \quad \text{(A-2)}$$

We know the length of side d by measuring it in the field. Likewise, we measure the length (l) of the within-circle section of the log and therefore $c = c' = l/2$. Finally, we measure $\angle B = \Psi$, where $0 \le \Psi \le \pi/2$ in radians. The angle of the log relative to the central radii is 0 if it is parallel to the radii and it is $\pi/2$ if it is perpendicular. Likewise, $\angle B' = \Psi' = \pi - \Psi$. This is enough information to compute the required angles.

Use the cosine rule to find the lengths of sides b and b',

$$b^2 = \frac{l^2}{4} + d^2 - ld \times \cos(\Psi), \quad \text{(A-3)}$$

$$b'^2 = \frac{l^2}{4} + d^2 + ld \times \cos(\Psi), \quad \text{(A-4)}$$

since $\cos(\pi - \Psi) = -\cos(\Psi)$.

Now use the sine rule to find the sine of $\angle C$ and $\angle C'$,

$$\sin(C) = l\frac{\sin(\Psi)}{2b}, \quad \text{(A-5)}$$

$$\sin(C') = l\frac{\sin(\Psi)}{2b'}, \quad \text{(A-6)}$$

since $\sin(\pi - \Psi) = \sin(\Psi)$. Now we find the selection angle for this CWD piece as,

$$\theta = \arcsin(\sin(C)) + \arcsin(\sin(C')), \quad \text{(A-7)}$$

where arcsin is the inverse sine function.

The mathematical approach outlined here can be extended to apply to the situation where d is the distance to the centre of the log, l is total log length and Ψ is the angle relative to the radii that passes through the centre of the log. However, four different cases have to be considered that each use the sine and cosine rules somewhat differently. Case 1 is the case where the entire log is in the circle, which is what this Appendix applies to. Case 2 is where the midpoint and all of one half of the log are in the circle, but part of the other half lies outside the circle. Case 3 is where the midpoint is outside the circle and only part of 1 half falls in the circle. Case 4 is the unusual situation where the midpoint is in the circle, but both ends of the log fall outside the circle.