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Transverse Vibrations of Wood-Based Products: Equations and Considerations

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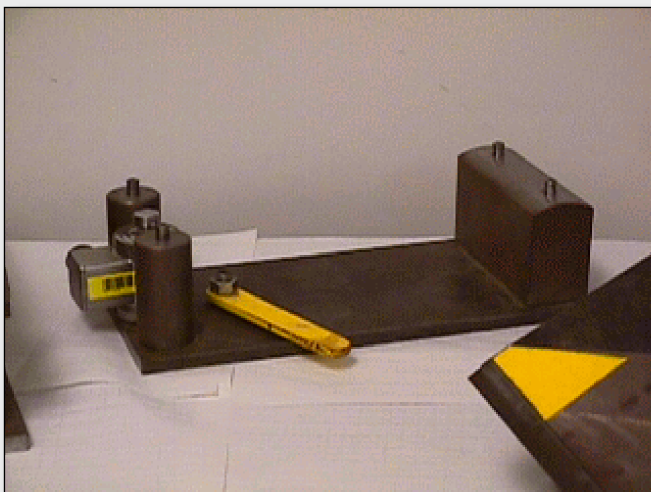

Abstract

Four equations are presented to determine bending stiffness using transverse vibration. These equations are used for constant cross-section products, panels, rectangular cross-section products, and logs with and without taper. Practical considerations for their use are discussed and concluding remarks are included.

Keywords: transverse vibration, bending stiffness, constant cross section, panels, rectangular cross section, logs, taper

Content

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Transverse Vibrations of Wood-Based Products: Equations and Considerations

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Introduction

For the past 15 plus years, the Forest Products Laboratory, specifically the Engineering Mechanics and Remote Sensing Laboratory, has been using transverse vibration of wood-based products to nondestructively measure bending stiffness, which then can be correlated with strength (Green and others 2006). We use the basic vibration equation to measure bending stiffness that was determined by Timoshenko and others 1974) and can be found in general engineering books, such as that authored by Young (1989). Whereas the basic equation was determined for slender isotropic materials with constant cross section, we have successfully applied it to wood, wood-based composites, and even tapered logs. This paper presents the equations to measure bending stiffness, analytical solutions for specific classes of products, and practical considerations.

Analytical Solutions

Equations

Four equations are used to determine bending stiffness using transverse vibration for various types of wood products. The most important equation is for products with a constant cross section. The bending stiffness EI is calculated from two terms. The first term uses just the fundamental frequency and the length of the span on which the specimen is simply supported. The second term is the specimen mass per unit length. No information about the cross section is required, as it is buried in the moment of inertia. Equation (1) accounts for overhang as noted in Murphy (1997) and is an excellent approximation for a span/specimen length (S/L) ratio ≥ 0.80. The other three equations use the same two right-hand terms of Equation (1) but multiply these terms by geometric modifiers.

Constant cross-section products

$$EI = \frac{f^2 S^4}{\pi^2/4}\,\frac{W}{gL} \tag{1}$$

Panels

$$EI/b = \frac{f^2 S^4}{\pi^2/4}\,\frac{W}{gL}\,\frac{1}{b} \tag{2}$$

Rectangular cross-section products

$$E = \frac{f^2 S^4}{\pi^2/4}\,\frac{W}{gL}\,\frac{12}{bh^3} \tag{3}$$

Logs

$$E_o = \frac{f^2 S^4}{\pi^2/4}\,\frac{W}{gL}\,\frac{64\pi^3}{c_b^4}\,\frac{1}{\theta} \tag{4a}$$

where

$$\begin{aligned} \theta &= \frac{(m^2 + 6mn + n^2)}{32}\,\frac{(1 + a + a^2)}{3} \\ a &= c_t/c_b \\ m &= 2 + (a-1)(1 + S/L) \\ n &= 2 + (a-1)(1 - S/L) \end{aligned} \tag{4b}$$

b is	horizontal width of specimen,
c_b	butt circumference of log,
c_t	tip circumference of log,
E	modulus of elasticity,
EI	bending stiffness,
E_o	modulus of elasticity of log,
f	fundamental frequency of vibration,
g	gravitational (acceleration) constant (980.665 cm/s^2, 386.089 in/s^2),
h	vertical height of specimen,
I	moment of inertia,
L	length of specimen,
S	support span ($< L$),
W	total weight of specimen, and
π	3.1415926...

Constant Cross-Section Products

Equation (1) is valid for all wood and wood–plastic members with constant cross section. We use it for “I” and “T” beams as well as glulam members. The plane of vibration is vertical with the member on edge. The moment of inertia does not need to be known because displacement is important for engineering applications, and bending stiffness (EI) is used directly without separating out modulus of elasticity.

Panels

Equation (2) is used for panel products. The panel must be vibrated in one dimension so that it vibrates like a very wide beam. Care must be taken so that the panel does not vibrate in two directions simultaneously. Here again bending stiffness (EI) per unit width is important to determine displacement, and specific knowledge of moment of inertia is not needed for plywood or oriented strandboard (OSB).

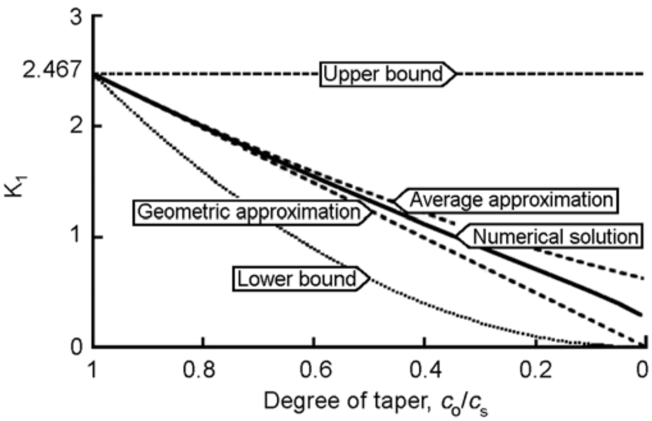

Figure 1. Transformed fundamental root of the frequency equation as a function of the degree of taper. c_S, circumference of log at the support near the butt; c_o, circumference of log at the support near the tip.

Rectangular Cross-Section Products

Equation (3) is used for dimension lumber as well as any solid sawn (or wood–plastic) member that has a rectangular cross section. For wood–plastic or even glulam beams, modulus of elasticity is an effective modulus because uniform material properties across the gross cross section are assumed. In some wood–plastic composites, the modulus of elasticity for the skin and core differs, but an effective modulus of elasticity can be determined for a particular cross section.

Logs

Equation (4) is used for logs, either doweled ($\theta = 1$) or tapered. When the tip circumference is equal to the butt circumference, $\theta = 1$. The second term on the right-hand side of the $\theta = \ldots$ equation, Equation (4b), accounts for the volume of a tapered cylinder compared with a straight cylinder. The first term on the left accounts for measuring the tip and butt circumferences at the ends of the log and not at the supports and the effect of taper on the fundamental root of the frequency equation. While this term is technically most accurate, if the ratio of tip circumference to butt circumference is greater than or equal to 0.80, then, as an excellent approximation, the log can be treated as a straight cylinder ($\theta = 1$) with a constant circumference equal to the average of the tip and butt circumferences (Murphy 2000b, fig. 1).

Practical Considerations

Transverse Vibrations of Logs

Practical considerations for the transverse vibration of logs require the use of two load cells because one end of a log can weigh up to twice as much as the other end. Figure 2 shows a log end support. When swung to one side, the disengaging lever raises the support off the load cell. The 120° angled ridges provide a corner edge for centering the log. With the log centered, the load cell measures half the weight at the end of the log. The total weight of the log is calculated by multiplying each load cell reading by 2 and adding. If the log is bowed, it is placed bowed side down for safety and to ensure only one plane of vibration, i.e., the vertical plane. A sewing tape measure is used to measure circumference, assuming the log is round in cross section.

If care is not taken while debarking, debarked logs may have processing taper at both ends. In this case, measuring the circumference at each end underestimates the volume, resulting in unrealistically high modulus of elasticity values (E_o). To avoid this problem, we measure the circumference "z" distance from the ends, with c_1 near the tip end and c_2 near the butt end. We then use the following equations to estimate the tip and butt circumferences.

$$c_t = c_2\,[2 + (\beta - 1)(1 + 1/X)]/2$$
$$c_b = c_2\,[2 + (\beta - 1)(1 - 1/X)]/2$$

where

$$X = 1 - (2z/L)$$
$$c_1 \le c_2$$
$$\beta = c_1/c_2$$

Consider now the transverse vibration of a log with the bark still attached. The bark can act like springs or nonrigid supports, which affect the fundamental frequency of the system (Murphy 2000a.) Moreover, taking circumference measurements around the bark without accounting for the bark thickness (i.e., without subtracting $2\pi \times \text{thickness}_{\text{bark}}$ from the circumference) skews the resulting log modulus of elasticity E_o. Each variable—log volume, weight, density, the moment of inertia, and weight/unit length—includes both log and bark. Therefore, the resulting E_o is off by the value of I, while the fundamental frequency compensates for the (log + bark weight)/unit length.

Dynamic and Static Modulus of Elasticity

We have found that using transverse vibration gives essentially the same modulus of elasticity in bending as a dead load static bending test with the specimen in the same orientation. Modulus of elasticity values are different when the orientation is changed. For example, a piece of dimension lumber (nominal 2- by 4-in., standard 38- by 89-mm; hereafter called 2 by 4) is transversely vibrated flatwise. The flatwise modulus of elasticity is the same as that from a flatwise dead load static test. When the specimen is subjected to an edgewise dead load static test, the edgewise modulus of elasticity can be 15% less than the flatwise modulus of elasticity. The difference is not dynamic as opposed to static testing, but flatwise as opposed to edgewise orientation. This misconception is apparently the result of dynamically testing dimension lumber flatwise to estimate a static edgewise modulus of elasticity. Dynamic and static modulus of elasticity values are the same in the same orientation. Following are two examples of dynamic and static testing with the specimens in the same orientation.

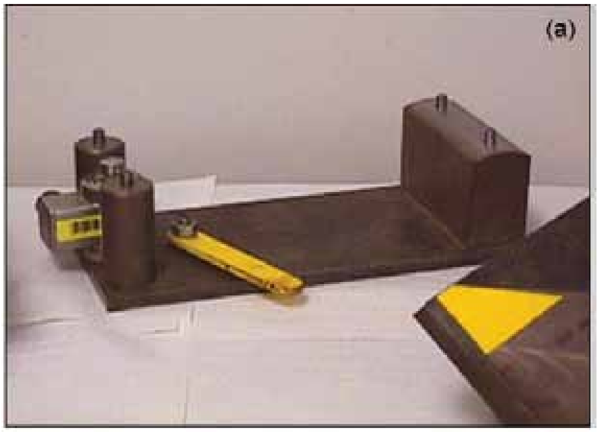

Figure 2. Log end support: (a) unassembled; (b) assembled. Load cell and disengaging lever shown.

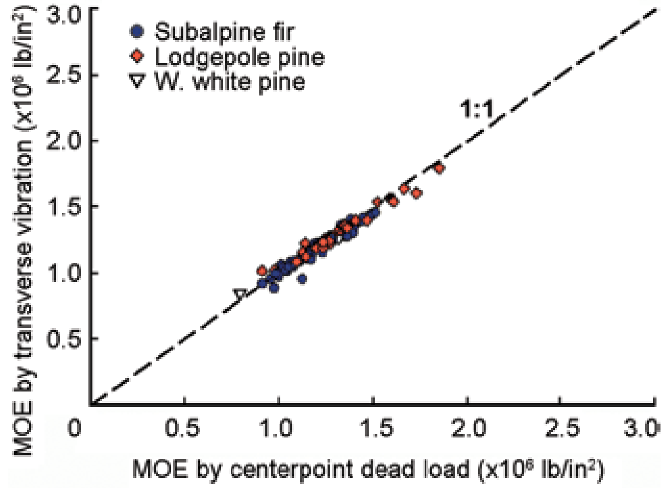

Figure 3. Dynamic versus static modulus of elasticity for 119, dry, 9-in. (229-mm) uniform-diameter logs.

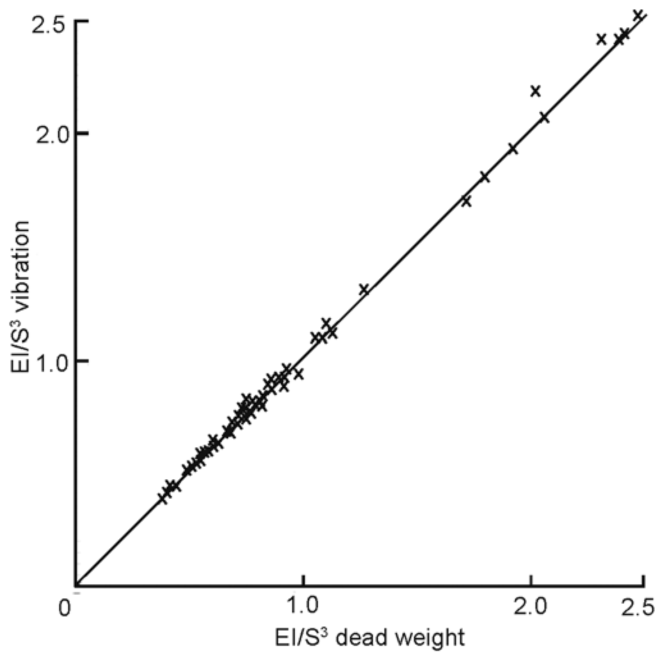

Figure 4. Dynamic stiffness versus static stiffness.

Log Tests

In the first example, 119 doweled logs were transversely vibrated (Green and others 2006). Then, without moving the logs on the supports, we added a 50-lb (22.7-kg) dead weight at midspan and measured midspan deflection. We then plotted dynamic versus static modulus of elasticity (Fig. 3).

Lumber Tests

In the second example, we tested 59 pieces of 2 by 4 lumber, varying in length from 96 to 102 in. (244 to 259 cm). For 50 pieces, the span was 96 in. (244 cm); for the remaining 9 pieces, the span was 92 in. (234 cm). Each piece was weighed. After the transverse vibration test (and without moving the lumber from the support stands) we added a 5-lb (2.3 kg) dead weight, P, at midspan and measured midspan deflection, δ. Cross section was not measured. We plotted calculated EI/S^3 from the vibration tests using Equation (1), against calculated EI/S^3 from the static dead weight tests using $\delta = PS^3/(48EI)$ (Fig. 4). A best-fit line yielded a slope of 1.00977 and an offset of 0.009947 with a correlation coefficient r^2 of 0.997. The offset corresponds to a difference of only 0.0036 in. (0.0914 mm) in the static deflection measurement. With twisted boards, we found that adding the 5-lb (2.3-kg) weight could increase the twist and skew the deflection measurement if care was not taken.

With the spans used and the moment of inertia of a flatwise 2 by 4, $EI/S^3 = 1.0$ corresponds approximately to $E = 1.0 \times 10^6$ lb/in^2 (6.9 GPa).

Effect of Moisture Content

If the wood is not frozen, then the bending stiffness EI as measured is at the moisture content at the time of measurement. The added weight of the water affects the frequency of vibration appropriately such that the product frequency and weight (f^2W) remains a constant. This can be proven by measuring the *EI* of a member and then attaching a chain along the length of the member and measuring *EI* again. The added weight of the chain increases the weight per unit length, W/L, but the frequency of vibration, f, is slowed such that f^2W does not change.

Concluding Remarks

The Engineering Mechanics and Remote Sensing Laboratory of the Forest Products Laboratory uses four equations in the nondestructive assessment of bending stiffness properties of structural wood and wood–plastic members. The first two terms of all the equations account for overhang of the test specimen over the supports and are very accurate for a ratio of support span to specimen length (S/L) equal to or greater than 0.80. The result of Equation (1) is used in the other equations. The general Equation (1) is merely modified by geometric specifics to apply transverse vibration to panels (Eq. (2)), rectangular cross sections (Eq. (3)), and logs (Eq. (4)). If the log taper is not severe ($c_t/c_b \geq 0.80$), then the log can be treated accurately as a straight cylinder ($\theta = 1$) with a constant circumference equal to the average of the tip and butt circumferences.

Dynamic bending stiffness *EI* as measured by transverse vibration is the same as static bending stiffness *EI* as measured by dead load *if* the support setup and specimen orientation are not changed.

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