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ABSTRACT: Multivariate regression equations were developed for predicting board-foot(Int. 1/4-inch log rule) and cubic-foot volumes in each 8.15-foot section of trees of four Hawaii hardwood species. The species are koa (*Acacia koa*), ohia (*Metrosideros polymorpha*), robusta eucalyptus (*Eucalyptus robusta*), and saligna (*E. saligna*). The four independent variables used are d.b.h., merchantable length, form class, and the diameter at the top of the merchantable length.

Predicting Volumes in Four Hawaii Hardwoods

. . . first multivariate equations developed

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In making volume predictions, foresters often need estimates for portions of trees as well as whole trees. Such volume equations are available for the first time for four Hawaii hardwoods. The four species are: koa (*Acacia koa*), ohia (*Metrosideros polymorpha*), robusta eucalyptus (*Eucalyptus robusta*), and saligna eucalyptus (*E. saligna*). The equations were developed for predicting the board-foot (Int. 1/4-inch rule) and cubic foot volume in each 8.15-foot section of a tree.

To develop these equations, it was necessary to find a predictor for a set of interdependent variables (section volumes) based on another set of variables (tree measurements). The usual method of solving this problem is to construct taper curves. This approach is useful if one aim of the equations is to determine the distribution of log volumes by small end diameter class. But nothing is to be gained by making two predictions when one will be enough.

Instead of predicting log diameters and using these data to predict volumes, we predict volumes directly. To do so, we would find a separate univariate multiple regression for predicting volume in each section of a tree. When section volumes are added up to give total tree volume, however, no estimate of the variance of each prediction could be made unless the interdependence of section volumes in a single tree is ignored. This interdependence can be handled in the multivariate regression framework used in this study.

The form of the multivariate model for each of the four species is as follows:

$$\begin{aligned}
Y_{1k} &= \beta_{10} + \beta_{11}X_{1k} + \beta_{12}X_{2k} + \cdots \\
&\quad + \beta_{1j}X_{jk} + \cdots + \beta_{1q}X_{qk} + e_{1k} \\
Y_{2k} &= \beta_{20} + \beta_{21}X_{1k} + \beta_{22}X_{2k} + \cdots \\
&\quad + \beta_{2j}X_{jk} + \cdots + \beta_{2q}X_{qk} + e_{2k} \\
&\ \ \vdots \\
Y_{ik} &= \beta_{i0} + \beta_{i1}X_{1k} + \beta_{i2}X_{2k} + \cdots \\
&\quad + \beta_{ij}X_{jk} + \cdots + \beta_{iq}X_{qk} + e_{ik} \\
&\ \ \vdots \\
Y_{pk} &= \beta_{p0} + \beta_{p1}X_{1k} + \beta_{p2}X_{2k} + \cdots \\
&\quad + \beta_{pj}X_{jk} + \cdots + \beta_{pq}X_{qk} + e_{pk}
\end{aligned}$$

in which Y_i = dependent variable of the ith equation.

= volume of the ith 8.15-foot section; board-foot volume is used in models with suffix a; cubic-foot volume is used in models with suffix b.

X_j = jth independent variable.

= jth tree measurement; the same X's are used in each equation.

β_{ij} = p·(q+1) regression coefficients to be estimated.

p = number of dependent variables.

= total number of sections in tallest tree in the species.

q = number of independent variables.

e = random term.

N = the number of observations.

k = 1,2,...,N.

The Y_i for top sections of trees which are shorter than the tallest tree of the species are set equal to zero.

Data Collection and Processing

Two sets of data were collected. The trees in each set were selected in the same way, but were measured with different instruments.

The tree selection procedure was designed to give a good geographical and size distribution without introducing personal bias in the selection of individual trees to measure. Details of the selection procedure as well as details of tree measurement have been described by Nelson et al.[1] Briefly, stands were selected from all the stands that had been visited for previous work. A point on a road or trail in or next to a stand was selected at random. A distance of 200 feet was paced into the stand at right angles to the road or trail. This point was considered the plot center. The 15 trees closest to the plot center which met the diameter class requirements were selected as sample trees (table 1).

The trees in the first set of data had diameters measured at stump, breast height, and top of first 16.3-foot log by steel tape. The rest of the logs were measured with a Relaskop with the distance to the tree measured with a steel tape. The trees in the second set were measured with a Barr & Stroud optical dendrometer.[2]

Data from both sets were processed by Grosenbaugh's STX computer program.[3] A special subroutine of STX, ST4S, interpolated between measurement points to find the diameter at the top of each 8.15-foot log section. The subroutine also found the top of the merchantable bole when this was not determined in the field. The board-foot and cubic-foot volume were then calculated from these diameter and length measurements.

Previous univariate regressions developed by the Hawaii Forest Survey staff using the Relaskop data and preliminary calculations with the dendrometer data led to the following two models:

Model 1 (a & b):

$$Y_i = \beta_{i0} + \beta_{i1} \cdot \mathrm{DBH} + \beta_{i2} \cdot \mathrm{TDIB} + \beta_{i3} \cdot \mathrm{DBH}^2 \cdot \mathrm{H} + \beta_{i4} \cdot \mathrm{TDIB}^2$$

[1]Nelson, R. E., Tagawa, T. K., Honda, Nobuo, and Hornibrook, E. M. *Manual of instructions for initial survey of the timber resource in the State of Hawaii.* 81 pp. U.S. Forest Serv., Pacific SW. Forest & Range Exp. Sta., Berkeley, Calif., and Div. Forestry, Hawaii Dep. Land & Natur. Resources. 1958. Rev. 1964.

[2]Grosenbaugh, L.R. *Optical dendrometers for out-of-reach diameters: a conspectus and some new theory.* 47 pp. For. Sci. Monogr. 4. 1963.

[3]Grosenbaugh, L.R. *STX--Fortran 4 program--for estimates of tree populations from 3P sample-tree-measurements.* 49 pp. U.S. Forest Serv. Pacific SW. Forest & Range Exp. Sta., Berkeley, Calif. Res. Pap. PSW-13. 1964.

$\cdot\ H + \beta_{i5} \cdot NS \quad [i = 1,2, \ldots, p]$

Model 2 (a & b):

$$Y_i = \beta_{i0} + \beta_{i1} \cdot DBH + \beta_{i2} \cdot TDIB + \beta_{i3} \cdot DBH^2 \cdot H \cdot FC + \beta_{i4} \cdot TDIB^2 \cdot H \cdot FC + \beta_{i5} \cdot FC \quad [i = 1,2, \ldots, p]$$

DBH = d.b.h. to nearest .1 inch.

TDIB = diameter inside bark at the top of the merchantable length to nearest .1 inch.

Merchantable length is the distance between the stump and either 9-inch d.o.b. or the point where the bole breaks up such that none of the branches contains a 12-foot log with a small end d.o.b. of at least 9 inches.

H = total merchantable length in feet.

FC = Form Class = (d.i.b. at top of first 16.3 ft. log)/DBH.

NS = number of 8.15-ft. sections in merchantable length = H/8.15 ft.

Model 3 was formulated after coefficients were estimated for Models 1 and 2. Adding the variable H to Model 2 we get

Model 3 (a & b):

$$Y_i = \beta_{i0} + \beta_{i1} \cdot DBH + \beta_{i2} \cdot TDIB + \beta_{i3} \cdot DBH^2 \cdot H \cdot FC + \beta_{i4} \cdot TDIB^2 \cdot H \cdot FC + \beta_{i5} \cdot FC + \beta_{i6} \cdot H \quad [i = 1,2, \ldots, p].$$

Table 1.--*Distribution of sample, by species and d.b.h. class*

D.b.h. class (inches)	Species			
	Koa	Ohia	Robusta	Saligna
	Number of trees			
11 - 16.9	18	21	35	17
17 - 22.9	19	20	28	17
23 - 28.9	13	17	27	15
29 - 34.9	16	14	25	15
35 - 40.9	13	12	2	14
41 - 46.9	10	7	--	8
47 - 52.9	8	5	--	6
53 +	3	2	--	3
Total	100	98	117	95

Model Selection and Final Results

The multivariate regression calculations were made by two computer programs written for this study. The programs calculated the estimates of β_{ij}'s, predicted values, residuals, sums of products of residuals matrix, and Anderson's U statistic.[4]

[4] Anderson, T.W. *An introduction to multivariate statistical analysis.* 374 pp. New York: John Wiley. 1958.

Anderson's U statistic, used to choose between the models, is defined as:

$$U = \frac{\left| N \hat{\Sigma}_{\Omega} \right|}{\left| N \hat{\Sigma}_{\omega} \right|}$$

in which $\left| N \hat{\Sigma}_{\Omega} \right|$ is the determinant of the matrix of sum of products of residuals about the regression surface using the β_{ij} as estimated from the data; $\left| N \hat{\Sigma}_{\omega} \right|$ is the determinant of the matrix of sum of products of residuals about the regression surface using the β_{ij} of the null hypothesis.

If we set $\beta_{i1} = \beta_{i2} = \cdot \cdot \cdot = \beta_{iq} = 0$ for each i as the null hypothesis, then the estimate of β_{i0} is the mean of the Y_{ik}. The U statistic is then somewhat analogous to the univariate ratio of residual sum of squares to the total sum of squares. In fact, if p = 1, $U = 1 - R^2$. The smaller the value of U becomes, the larger the percentage reduction of variance about the regression line as compared to the variance about the mean. No hypotheses were tested. The model with the smaller value of U is considered the better model.

Weighted regression was used because the generalized variance of the residuals within subgroups increased as $DBH^2 \cdot H$ increased. The first set of weights used was from Gedney and Johnson.[5] Because these weights eliminated the trend of the variance in all species, no further weights were sought.

The dendrometer data were used to calculate regression coefficients for all species in models 1 and 2 (table 2). The number of observations and the number of dependent variables (p) used for each species are also shown in the table. Form class appears to be an important variable. The U values for model 2 are smaller by almost a factor of 10 than those for model 1.

To further compare the models, the residuals of the Relaskop data about the surface estimated by the dendrometer data were found for three model-species combinations. Their U values were .692, .749, and 3.024. The last of these--ohia (model 1a)--indicates that the mean of Y_i of the Relaskop data would make a better predictor for section volume in these trees than the regression surface estimated from the dendrometer data. All three values indicate that the two samples probably differ. Both samples were from the population for which predictions were wanted. Either the samples are from different subpopulations or this difference is due to sampling error. In either case I think the best predictive ability can be obtained by pooling the data to estimate the coefficients for the whole population.

Since the computations took a large amount of computer time, the pooled data were first used for only four model-species combinations (table 3). The pattern of U values is similar to that found in table 2. From the above results model 3 was formulated. Again only a few model-species regressions were calculated. Because model 3 appeared better than either models 1 or 2, only the remaining model 3 regressions were run (table 4). Tables 6 to 13 give the estimated regression coefficients ($\hat{\beta}_{ij}$) for predicting board foot and cubic foot volumes for 8.15-foot sections of trees of the four species. The final row of each table gives the coefficients for predicting total tree volume. These coefficients equal $\sum_{i=1}^{p} \hat{\beta}_{ij}$ for each j. They are also the estimated coefficients of model 3*. Model 3* has the same independent variables as model 3 but only one dependent variable, $Y = \sum_{i=1}^{p} Y_i$. These univariate regressions were calculated for board foot and cubic foot volume for each species. Table 5 gives the R^2's for these regressions and the weighted standard error in percent. That is, percent S.E. = 100x (weighted standard deviation of residuals/weighted mean of Y).

To predict section or tree volumes, measure DBH, TDIB, H, and FC. Then, the appropriate row of coefficients is substituted for the β's in

[5] Gedney, Donald R. and Johnson, Floyd A. *Weighting factors for computing the relation between tree volume and DBH in the Pacific Northwest.* 5 pp. U.S. Forest Serv. Pacific NW Forest & Range Exp. Sta. Res. Note 174. 1959.

Table 2.--*Values of Anderson's U for models 1 and 2; q=5, dendrometer data*

Species	Model number				p	Observations
	1a	2a	1b	2b		
						Number
Koa	$.143 \times 10^{-3}$	$.215 \times 10^{-4}$	$.899 \times 10^{-4}$	$.157 \times 10^{-4}$	9	30
Ohia	$.499 \times 10^{-4}$	$.534 \times 10^{-5}$	$.138 \times 10^{-4}$	$.145 \times 10^{-5}$	11	28
Robusta	$.408 \times 10^{-2}$	$.612 \times 10^{-3}$	$.322 \times 10^{-2}$	$.268 \times 10^{-3}$	15	55
Saligna	$.146 \times 10^{0}$	$.341 \times 10^{-5}$	$.365 \times 10^{-1}$	$.513 \times 10^{-5}$	18	40

Table 3.--*Values of Anderson's U for models 1 and 2; q=5, pooled data*

Species	Model number				p	Observations
	1a	2a	1b	2b		
						Number
Koa	--	--	$.174 \times 10^{-2}$	--	10	100
Ohia	$.239 \times 10^{-2}$	--	--	--	11	98
Robusta	--	$.191 \times 10^{-2}$	--	--	15	117
Saligna	--	--	--	$.179 \times 10^{-3}$	18	95

Table 4.--*Values of Anderson' U for model 3; q=6, pooled data*

Species	Model		p	Observations
	3a	3b		
				Number
Koa	$.837 \times 10^{-3}$	$.285 \times 10^{-3}$	10	100
Ohia	$.398 \times 10^{-3}$	$.217 \times 10^{-3}$	11	98
Robusta	$.101 \times 10^{-2}$	$.546 \times 10^{-3}$	15	117
Saligna	$.999 \times 10^{-4}$	$.979 \times 10^{-4}$	18	95

Table 5.-*Values of R^2 for univariate model 3*; q=6, pooled data*

Species	Model 3a*		Model 3b*		p	Observations
	R^2	Percent S.E.	R^2	Percent S.E.		
						Number
Koa	.986	12.45	.989	9.51	1	100
Ohia	.985	13.53	.988	10.50	1	98
Robusta	.973	18.21	.979	14.02	1	117
Saligna	.984	16.72	.988	12.69	1	95

model 3 along with the measurements of the independent variables. For example, to predict the board foot volume in the first 1/2 log of a 24-inch d.b.h. 57 foot koa with a form class of .75 and a 10-inch TDIB, we use the first row of coefficients in table 6 to get the following equation:

$$V_1 = 95.82708 + 10.18193 \times 24 + .17171 \times 10.0 + .00285 \times 24^2 \times 57.0 \times .75 + .00270 \times 10^2 \times 57.0 \times .75 + 31.1948 \times .75 - 1.216054 \times 57.0 = 186.1 \text{ bd. ft.}$$

To predict the total volume of the same tree, the last row of coefficients in table 6 is used as follows:

$$V_T = - 98.0469 + 14.5390 \times 24 - 10.9328 \times 10 + .0175 \times 24^2 \times 57.0 \times .75 + .0285 \times 10^2 \times 57.0 \times .75 + 85.4659 \times .75 - 2.4364 \times 57.0 = 619.54 \text{ bd. ft.}$$

If the first half log is cull, the net board foot volume of the tree is

$$V_T - V_1 = 619.5 - 186.1 = 433.4 \text{ bd. ft.}$$

This koa, without any cull, has 7 sections. Therefore an unbiased estimate of total tree volume could be obtained by summing the predicted section volumes from the first seven equations. The model gives zero volume to sections not present, however, because the tree is shorter than the tallest tree in the species. For this reason the total volume equation for all koa trees is the sum of the 10 section equations. The average error will be smaller using the total volume equation than by using the sum of j equations for trees with j sections.

In general, to predict volume for a large portion of a tree, the excluded portion should be predicted and then subtracted from the prediction of total tree volume, except when the top section is cull. Excluding portions at the upper end of the stem should be done by shortening H, changing TDIB, and then predicting total tree volume. For example, if the top 8.15-foot section of the above koa were cull, the net tree volume would be estimated by calculating H as 57.0 - 8.15 = 48.85 feet. If the top section had a 1-inch taper, the TDIB at 48.85 feet would be 11 inches. The volume would then be predicted by the equation:

$$V_{T'} = - 98.0369 + 14.5390 \times 24 - 10.9328 \times 11 + .0175 \times 24^2 \times 48.85 \times .75 + .0285 \times 11^2 \times 48.85 \times .75 + 85.4659 \times .75 - 2.4364 \times 48.85 = 569.25 \text{ bd. ft.}$$

This procedure is opposite to the usual one in which full height to a specified top is used to get the proper amount of taper. H can be changed in the present case because TDIB is one of the variables in the prediction equation. The change is necessary because the error in predicting section volumes becomes larger as section number increases. There is less error in predicting the short tree volume than in predicting the difference in volume between the tall tree and its top section.

To find the total volume of a group of trees or sections which use the same set of regression coefficients, the following form of the equation simplifies calculation:

$$\sum_{k=1}^{n} Y_k = n \cdot \beta'_0 + \beta'_1 \cdot \sum_{k=1}^{n} DBH_k + \beta'_2 \cdot \sum_{k=1}^{n} TDIB_k + \beta'_3 \cdot \sum_{k=1}^{n} DBH_k^2 \cdot H_k \cdot FC_k + \beta'_4 \cdot \sum_{k=1}^{n} TDIB_k^2 \cdot H_k \cdot FC_k + \beta'_5 \cdot \sum_{k=1}^{n} FC_k + \beta'_6 \cdot \sum_{k=1}^{n} H_k$$

in which the subscript k indicates the measurement in the kth tree in a group of n trees. The β_i' are the $\hat{\beta}_{ij}$ that apply to the particular group of trees.

To illustrate how the volumes vary with changes in variables, a small portion of a volume table is given in table 14. Because of the number and range of the independent variables, the construction of a complete volume table is impractical; a complete table for all four species would take nearly 5,000 pages. The possibility of making a condensed table by using average values per DBH class of one variable was considered, but was discarded because none of the measured variables was suitable for elimination in this way. H is obviously too important a variable to use average H per DBH class. Form class was shown earlier to reduce residual variance by a factor of almost 10. The only measured variable this leaves is TDIB. TDIB was used to explain a source of variation which is poorly correlated with DBH. The size of TDIB, along with H, reflects the breakup of the main bole into small branches. In fact, the correlation between DBH and TDIB is less than 0.5. Therefore, using average TDIB per DBH is not reasonable. It should also be noted that the residual variance is quite large. Although the regressions explain a large proportion of the original variance, the original variance is very large. The variables used reduced the residual variance by a significant amount.

A procedure for estimating the variances of the predicted volumes of parts of trees, trees, and groups of trees is now being developed. This procedure, along with the necessary numbers, will be presented in a subsequent report.

Appendix

Table 6. *Regression coefficients for predicting board-foot volume of koa (Int. 1/4-in. rule)*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	-95.82708	10.18193	0.17171	0.00285	0.00270	31.19480	-1.216054
2	-98.9874	9.3395	6.9892	.0016	.0102	110.0857	-1.500098
3	-68.87314	5.28700	-7.41769	.00200	.01008	99.45291	-.741615
4	7.695041	-.480727	-2.175746	.002980	.004667	4.461617	.137687
5	48.52265	2.85065	.95769	.00296	.00188	-48.85356	.331848
6	45.65307	-2.84193	1.81631	.00236	-.00044	-48.79502	.303930
7	41.76758	-2.31423	1.27148	.00160	.00022	-39.71543	.117924
8	15.86447	-1.28068	1.04170	.00083	-.00057	-16.60251	.105860
9	5.825181	-.442723	.334543	.000280	-.000149	-5.615163	.025345
10	.3226912	-.0585186	.0563598	.0000520	-.0000766	-.1474525	-.001241
Totals	-98.0369	14.5390	-10.9328	.0175	.0285	85.4659	-2.4364

Table 7.--*Regression coefficients for predicting cubic-foot volume of koa*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	- 9.686990	1.214020	0.023089	0.000394	0.000362	3.576347	-0.158056
2	-11.91520	1.13800	- .86295	.00022	.00120	14.77558	.175340
3	-11.10862	.79387	-1.04888	.00015	.00135	14.72008	.050964
4	- 1.440857	.004906	.288692	.000334	.000574	1.995580	.063705
5	5.209516	- .377899	.191257	.000387	.000170	- 6.460742	.079629
6	5.789749	- .400744	.312052	.000331	- .000154	- 7.159193	.066171
7	5.915681	- .352759	.231965	.000237	- .000029	- 6.129322	.030363
8	2.427013	- .200374	.169549	.000127	- .000095	- 2.670028	.019788
9	.8881901	- .0669086	.0512612	.0000412	- .0000215	- .8806413	.004412
10	.04958669	- .00913714	.00884829	.00000812	- .00001206	- .02262095	- .000186
Totals	-13.8719	1.7332	-1.2125	.0022	.0033	11.7450	- .1205

Table 8.--*Regression coefficients for predicting board-foot volume of ohia (Int. 1/4-in. rule)*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	-170.3914	8.2664	8.6611	0.0039	-0.0049	62.3521	-0.843890
2	-120.5771	5.6555	2.1029	.0030	.0019	88.9183	- .899460
3	- 44.18240	5.46496	- 3.78737	.00098	.01464	88.82999	-1.009962
4	52.81054	2.30632	-10.04607	.00109	.01558	1.28497	- .584072
5	41.31205	-1.51536	- .69208	.00201	.00396	-41.87429	.282015
6	42.13990	-3.03324	1.94653	.00227	.0022	-48.86516	.440775
7	19.19443	-2.68394	3.15507	.00235	- .00444	-30.33555	.380373
8	11.08655	-1.92976	2.11924	.00187	- .00350	-14.02272	.181155
9	12.47474	-1.24408	.86864	.00124	- .00140	- 8.82283	.012869
10	9.690916	- .466431	- .161346	.000458	.000283	- 3.608517	- .062947
11	2.818997	- .042842	- .179804	.000044	.000298	- .794831	- .026993
Totals	-143.6228	10.7775	-1.0132	.0192	.0222	93.0615	-2.1301

Table 9.--*Regression coefficients for predicting cubic-foot volume of ohia*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	-19.41844	1.01349	1.03132	0.00050	-0.00066	8.00497	-0.104521
2	-14.47685	.76226	.19734	.00033	.00024	11.53981	- .099139
3	- 8.82702	.84825	-1.21373	.00002	.00187	14.47147	- .099642
4	5.926268	.415849	1.487051	.000056	.002205	1.052171	- .048797
5	5.301151	- .198231	- .083528	.000231	.000582	- 6.326220	.071528
6	5.977388	- .465099	.340442	.000311	- .000055	- 7.753169	.088066
7	2.533931	- .422026	.542706	.000358	- .000767	- 4.849127	.071285
8	1.419499	- .308002	.360319	.000297	- .000604	- 2.098870	.033883
9	1.649707	- .207882	.177219	.000206	- .000293	- 1.308878	.006680
10	1.322752	- .081551	.001187	.000080	- .000001	- .509818	- .007166
11	.4058990	- .0073059	- .0247008	.0000074	.0000407	- .1117655	- .003833
Totals	-18.1857	1.3498	- .1585	.0240	.0026	12.1106	- .0917

Table 10.--*Regression coefficients for predicting board-foot volume of Robusta eucalyptus (Int. 1/4-in. rule)*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	-147.4436	6.9941	1.3161	0.0023	0.0023	135.0112	-0.590282
2	- 87.9867	5.3265	-2.1968	.0016	.0021	100.2585	- .529313
3	- 71.08105	4.38309	-1.86725	.00113	.00204	62.60464	- .215083
4	- 10.02939	2.68275	-3.92930	.00108	.00363	18.36333	- .153004
5	- 25.13546	.32897	.76797	.00140	.00057	13.65443	.082405
6	6.878988	- .368779	- .313218	.001442	.001460	- 6.488287	- .012552
7	29.20325	-1.45658	- .44369	.00163	.00170	- 17.50768	- .081734
8	30.21361	-1.61878	- .27416	.00151	.00160	- 16.36067	- .118055
9	30.43162	-1.59213	- .05596	.00141	.00061	- 17.37143	- .124815
10	23.61735	-1.44648	.26886	.00111	.00040	- 14.23213	- .080985
11	13.70695	-1.07266	.39112	.00081	- .00051	- 7.27223	- .019399
12	10.27594	- .73192	.37068	.00052	- .00026	- 7.77690	- .011313
13	1.846884	- .441627	.595504	.000311	- .000636	- 3.182155	.019253
14	.096042	- .246899	.400580	.000165	- .000475	- 1.085088	.019980
15	- .4799474	- .0378753	.1016402	.0000274	- .0001236	.0163414	.004886
Totals	-196.0776	10.7017	4.8679	.0164	.0098	238.6319	-1.8100

Table 11.--*Regression coefficients for predicting cubic-foot volume of Robusta eucalyptus*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	-17.82186	0.90209	0.14525	0.00026	-0.00028	17.48416	-0.068875
2	-10.98505	.70596	- .27203	.00018	.00024	13.34293	- .059877
3	-10.93912	.68559	- .24740	.00008	.00024	8.95076	- .005218
4	- 1.851224	.498383	- .723060	.000062	.000610	3.034923	- .000017
5	- 5.584770	.091009	.138066	.000139	.000050	3.111704	.041055
6	.0086459	- .0178570	- .0535476	.0001619	.0002073	- .6397084	.022332
7	3.753958	- .218191	- .032923	.000216	.000229	- 2.725338	.008064
8	4.093590	- .254223	- .007515	.000214	.000224	- 2.473237	.004053
9	4.430871	- .254877	.019384	.000216	.000066	- 2.692627	- .012634
10	3.668604	- .240210	.055913	.000177	.000051	- 2.266640	- .008779
11	2.441994	- .185444	.050881	.000136	- .000075	- 1.207162	- .002140
12	1.749710	- .131611	.072097	.000092	- .000055	- 1.362329	- .000462
13	.3331091	- .0818189	.1102678	.0000568	- .0001179	- .5863276	.003916
14	.0366694	- .0473497	.0715013	.0000312	- .0000847	- .2227333	.003627
15	- .09175286	- .00728397	.01950573	.00000526	- .00002374	.00257417	.000946
Totals	-26.7566	1.4442	- .6536	.0020	.0013	31.7509	.0821

Table 12.--*Regression coefficients for predicting board-foot volume of Saligna eucalyptus (Int. 1/4-in. rule)*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	139.4222	13.4385	-34.3848	0.0009	0.0150	77.1701	-1.656012
2	- 27.16889	7.92880	-11.19857	.00093	.00798	80.57326	- .799945
3	-120.2769	6.0060	- 5.6035	.0015	.0012	159.8142	- .600258
4	-163.0663	2.6343	6.9076	.0017	- .0042	98.9372	.103804
5	-101.3343	.9542	7.6232	.0016	- .0035	23.6212	.317791

Table 12.--*Regression coefficients for predicting board-foot volume of Saligna eucalyptus (Int. 1/4-in. rule) continued*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
6	53.70727	-.49243	6.19461	.00162	.00212	-.49348	.291428
7	72.34748	-1.57365	-3.36042	.00163	.00266	-44.21681	.022817
8	74.52870	2.42129	2.18962	.00157	.00247	-47.86710	.065601
9	72.81351	3.23532	-.53876	.00150	.00230	-52.75278	.112728
10	85.20497	-2.92933	-1.87182	.00130	.00293	-56.59252	.044585
11	97.67553	-2.76203	-2.42418	.00120	.00318	-66.32821	.054258
12	62.83060	-2.37526	1.09586	.00103	.00160	-66.32523	.000638
13	48.14582	-1.89902	1.08436	.00078	.00203	-51.76450	-.065606
14	87.60065	-1.18257	-4.77535	.00050	.00395	-42.41969	-.242557
15	41.87644	-.79009	-2.09000	.00033	.00153	-18.49814	-.111464
16	23.01003	-.57185	-1.19807	.00021	.00025	-7.53108	-.020979
17	10.02657	-.18220	-.63880	.00007	.00022	-2.79567	-.017864
18	3.543246	-.061057	-.218041	.000020	.000119	1.096220	-.009133
Totals	353.4721	10.4857	-47.5863	.0184	.0376	22.5655	2.6187

Table 13.--*Regression coefficients for predicting cubic-foot volume of Saligna eucalyptus*

Section number	B_0	B_1	B_2	B_3	B_4	B_5	B_6
1	25.55281	1.72446	5.79898	0.00021	0.00236	20.35287	0.314010
2	10.75487	1.09651	3.63853	.00020	.00163	20.43980	.216379
3	11.48821	.76538	.95195	.00018	.00033	18.59479	-.071356
4	-33.17864	.44743	1.77712	.00018	.00097	16.57859	.057898
5	-19.78817	.23582	1.37509	.00016	-.00065	5.31403	.071517
6	10.82043	.01000	.97277	.00018	-.00036	1.00509	.060427
7	12.42869	-.15716	.85970	.00019	.00055	-5.96517	.006107
8	12.72470	.30661	.65015	.00019	.00050	-6.56025	.014429
9	12.46305	.45758	.36738	.00019	.00045	-7.27538	.023199
10	14.69305	-.41632	.59230	.00017	.00055	-7.96605	.011469
11	16.81795	-.40787	.61724	.00017	.00056	-10.15834	.004836
12	10.27860	-.35576	.10614	.00015	.00023	-10.63589	.007449
13	7.441342	-.293802	.177972	.000117	.000275	-8.282357	-.003108
14	14.04546	.18383	-.78781	.00008	.00060	-6.68389	.034834
15	6.530406	-.122940	-.331550	.000052	.000221	-2.834447	-.016085
16	3.960105	-.093894	.216559	.000035	.000043	-1.266801	-.003338
17	1.620516	-.029141	.104540	.000011	.000034	.443422	-.002728
18	.5015038	-.0093961	-.0297491	.0000031	.0000159	-.1562058	.001259
Totals	74.5376	1.4463	10.5373	.0025	.0064	14.0570	-.4154

Table 14.--*Board-foot volume of koa (Int. 1/4-inch rule); form class: 0.85*

		9-INCH TDIB									
		SECTION NUMBER									
DBH	HT.	1	2	3	4	5	6	7	8	9	10
14.0											
	2	65.7	53.8								
	3	61.2	49.4	29.9							
	4	56.7	45.1	32.2	16.3						
	5	52.2	40.8	34.6	24.1	14.6					
	6	47.6	36.5	36.9	31.9	22.4	13.4				
16.0											
	3	85.1	70.1	43.0							
	4	81.8	66.5	46.1	20.3						
	5	78.4	62.8	49.3	29.3	15.1					
	6	75.1	59.1	52.4	38.4	24.1	13.7				
	7	71.8	55.5	55.6	47.4	33.1	20.1	9.9			
18.0											
	3	109.5	91.1	56.4							
	4	107.5	88.1	60.5	25.0						
	5	105.5	85.2	64.6	35.4	16.3					
	6	103.5	82.3	68.7	45.8	26.7	14.7				
	7	101.5	79.4	72.8	56.3	37.1	22.2	10.5			
20.0											
	3	134.4	112.3	70.1							
	4	133.9	110.2	75.3	30.3						
	5	133.4	108.1	80.4	42.3	18.4					
	6	132.9	106.1	85.6	54.3	30.4	16.5				
	7	132.4	104.0	90.7	66.3	42.3	25.3	11.8			
	8	131.9	101.9	95.9	78.3	54.3	34.1	17.3	8.3		
22.0											
	3	159.7	133.7	84.2							
	4	160.9	132.6	90.5	36.3						
	5	162.0	131.5	96.8	50.0	21.3					
	6	163.2	130.3	103.1	63.7	35.0	19.1				
	7	164.4	129.2	109.4	77.5	48.7	29.2	13.7			
	8	165.5	128.1	115.8	91.2	62.4	39.4	20.1	9.6		
24.0											
	4	188.5	155.4	106.2	42.9						
	5	191.5	155.2	113.8	58.5	25.1					
	6	194.5	155.1	121.3	74.2	40.6	22.4				
	7	197.4	155.0	128.9	89.8	56.2	34.1	16.2			
	8	200.4	154.9	136.5	105.4	71.8	45.8	23.7	11.2		
26.0											
	4	216.8	178.5	122.3	50.2						
	5	221.7	179.5	131.3	67.9	29.6					
	6	226.7	180.5	140.2	85.6	47.2	26.6				
	7	231.6	181.4	149.2	103.3	64.8	39.9	19.3			
	8	236.6	182.4	158.2	121.0	82.5	53.3	27.9	13.3		
28.0											
	4	245.7	201.9	138.8	58.1						
	5	252.7	204.1	149.3	78.1	35.0					
	6	259.8	206.3	159.8	98.0	54.8	31.6				
	7	266.9	208.5	170.3	117.9	74.6	46.7	23.1			
	8	274.0	210.7	180.7	137.9	94.5	61.8	32.8	15.7		
30.0											
	4	275.2	225.8	155.8	66.8						
	5	284.6	229.2	167.9	89.1	41.2					
	6	293.9	232.7	180.0	111.4	63.4	37.3				
	7	303.3	236.2	192.1	133.7	85.6	54.3	27.4			
	8	312.7	239.7	204.2	156.0	107.8	71.3	38.5	18.4		

Table 14.--*Board-foot volume for koa (Int. 1/4-inch rule); form class: 0.85; continued*

DBH	HT.	SECTION NUMBER									
		1	2	3	4	5	6	7	8	9	10
32.0											
	4	305.3	249.9	173.3	76.0						
	5	317.2	254.8	187.1	100.9	48.2					
	6	329.0	259.6	200.9	125.8	72.9	43.8				
	7	340.8	264.5	214.7	150.7	97.7	62.9	32.4			
	8	352.6	269.3	228.5	175.6	122.5	81.9	44.9	21.6		
34.0											
	4	336.1	274.5	191.2	86.0						
	5	350.6	280.8	206.8	113.6	56.0					
	6	365.0	287.1	222.4	141.2	83.5	51.2				
	7	379.4	293.4	238.1	168.8	110.9	72.4	38.1			
	8	393.8	299.7	253.7	196.4	138.4	93.6	52.0	25.1		
36.0											
	4	367.5	299.4	209.5	96.6						
	5	384.7	307.2	227.1	127.1	64.7					
	6	401.9	315.1	244.6	157.6	95.0	59.3				
	7	419.1	322.9	262.2	188.1	125.3	82.8	44.3			
	8	436.3	330.8	279.8	218.6	155.7	106.3	59.7	29.0		
38.0											
	5	419.7	334.1	247.9	141.4	74.1					
	6	439.8	343.6	267.5	174.9	107.5	68.2				
	7	459.9	353.1	287.1	208.5	140.9	94.1	51.1			
	8	480.1	362.6	306.8	242.0	174.2	120.0	68.2	33.2		
	9	500.2	372.1	326.4	275.6	207.6	146.0	85.3	42.1	13.6	
40.0											
	5	455.5	361.4	269.3	156.5	84.4					
	6	478.7	372.7	291.1	193.3	121.0	77.8				
	7	501.9	383.9	312.8	230.1	157.6	106.3	58.6			
	8	525.1	395.1	334.6	266.8	194.1	134.8	77.4	37.8		
	9	548.3	406.4	356.4	303.6	230.7	163.3	96.3	47.6	15.4	
42.0											
	5	492.0	389.2	291.2	172.5	95.5					
	6	518.5	402.2	315.3	212.7	135.5	88.3				
	7	544.9	415.3	339.3	252.8	175.4	119.5	66.7			
	8	571.3	428.3	363.4	293.0	215.3	150.7	87.3	42.8		
	9	597.8	441.4	387.4	333.1	255.3	181.9	108.0	53.5	17.4	
44.0											
	5	529.4	417.4	313.7	189.3	107.5					
	6	559.2	432.4	340.1	233.0	150.9	99.6				
	7	589.0	447.3	366.6	276.7	194.4	133.6	75.4			
	8	618.9	462.3	393.0	320.4	237.9	167.6	98.0	48.2		
	9	648.7	477.2	419.4	364.1	281.3	201.6	120.5	59.8	19.5	
46.0											
	6	600.9	463.0	365.7	254.3	167.4	111.6				
	7	634.3	480.0	394.6	301.8	214.5	148.6	84.8			
	8	667.7	496.9	423.5	349.2	261.7	185.6	109.3	53.9		
	9	701.0	513.9	452.5	396.6	308.8	222.5	133.8	66.6	21.7	
	10	734.4	530.8	481.4	444.0	356.0	259.5	158.4	79.3	26.0	5.1
48.0											
	6	643.5	494.2	391.9	276.7	184.8	124.5				
	7	680.6	513.2	423.4	328.0	235.8	164.5	94.7			
	8	717.7	532.3	455.0	379.3	286.8	204.6	121.3	60.0		
	9	754.8	551.3	486.5	430.6	337.8	244.6	148.0	73.8	24.1	
	10	791.9	570.3	518.0	481.9	388.8	284.7	174.6	87.6	28.7	5.7
50.0											
	6	687.1	525.9	418.7	300.0	203.2	138.1				
	7	728.1	547.1	453.0	355.3	258.2	181.4	105.3			
	8	769.0	568.3	487.2	410.7	313.3	224.6	134.1	66.4		
	9	810.0	589.5	521.5	466.0	368.3	267.9	162.9	81.4	26.7	
	10	851.0	610.7	555.7	521.4	423.3	311.2	191.7	96.3	31.6	6.3

Table 14.--*Board-foot volume for koa (Int. 1/4-inch rule); form class: 0.85, continued*

		SECTION NUMBER									
DBH	HT.	1	2	3	4	5	6	7	8	9	10
52.0											
	6	731.6	558.1	446.3	324.3	222.6	152.5				
	7	776.6	581.6	483.4	383.8	281.8	199.1	116.5			
	8	821.6	605.1	520.4	443.4	341.0	245.7	147.6	73.3		
	9	866.6	628.6	557.5	503.0	400.2	292.4	178.6	89.3	29.4	
	10	911.6	652.0	594.6	562.5	459.4	339.0	209.7	105.4	34.7	6.9
54.0											
	6	777.1	590.9	474.5	349.6	243.0	167.7				
	7	826.3	616.7	514.5	413.5	306.6	217.8	128.3			
	8	875.5	642.6	554.5	477.5	370.1	267.9	161.7	80.4		
	9	924.6	668.4	594.5	541.4	433.7	318.0	195.1	97.8	32.2	
	10	973.8	694.2	634.5	605.3	497.2	368.1	228.5	115.1	37.9	7.5
56.0											
	6	823.5	624.2	503.3	375.9	264.4	183.7				
	7	877.1	652.5	546.4	444.3	332.4	237.4	140.8			
	8	930.6	680.8	589.5	512.8	400.5	291.1	176.6	88.0		
	9	984.1	709.0	632.5	581.3	468.6	344.8	212.5	106.6	35.1	
	10	1037.6	737,3	675.6	649.8	536.6	398.6	248.3	125.2	41.3	8.2
58.0											
	6	870.9	658.1	532.9	403.1	286.7	200.5				
	7	928.9	688.9	579.1	476.3	359.5	257.9	153.8			
	8	987.0	719.6	625.3	549.5	432.2	315.4	192.2	95.9		
	9	1045.0	750.4	671.5	622.7	504.9	372.8	230.6	115.8	38.2	
	10	1103.0	781.2	717.8	695.9	577.7	430.3	268.9	135.7	44.9	8.9
		18-INCH TDIB									
20.0											
	1	141.5									
	2	145.5	85.8								
	3	149.6	100.9	54.3							
	4	153.6	116.0	76.4	42.1						
	5	157.7	131.1	98.5	62.0	42.9					
	6	161.7	146.2	120.6	81.8	58.0	28.4				
22.0											
	1	163.5									
	2	169.2	106.3								
	3	174.9	122.3	68.3							
	4	180.6	138.4	91.6	48.1						
	5	186.3	154.4	114.9	69.7	45.8					
	6	192.0	170.4	138.2	91.3	62.6	31.0				
24.0											
	1	185.7									
	2	193.2	127.0								
	3	200.7	144.1	82.7							
	4	208.2	161.1	107.3	54.7						
	5	215.8	178.2	131.8	78.2	49.5					
	6	223.3	195.2	156.4	101.7	68.2	34.3				
26.0											
	1	208.0									
	2	217.5	147.9								
	3	227.0	166.1	97.4							
	4	236.5	184.3	123.4	62.0						
	5	246.0	202.4	149.3	87.6	54.0					
	6	255.5	220.6	175.3	113.1	74.8	38.5				

Table 14.--*Board-foot volume for koa (Int. 1/4-inch rule); form class: 0.85, continued*

		SECTION NUMBER									
DBH	HT.	1	2	3	4	5	6	7	8	9	10
28.0											
	2	242.1	169.0								
	3	253.8	188.4	112.5							
	4	265.4	207.7	140.0	70.0						
	5	277.0	227.1	167.4	97.8	59.4					
	6	288.6	246.4	194.8	125.6	82.4	43.5				
	7	300.3	265.8	222.3	153.3	105.4	57.8	37.1			
30.0											
	2	267.1	190.3								
	3	281.0	210.9	127.9							
	4	294.9	231.5	157.0	78.6						
	5	308.8	252.2	186.0	108.8	65.6					
	6	322.7	272.8	215.1	139.0	91.0	49.2				
	7	336.7	293.5	244.1	169.1	116.4	65.5	41.5			
32.0											
	2	292.3	211.7								
	3	308.7	233.7	143.6							
	4	325.1	255.7	174.4	87.9						
	5	341.4	277.7	205.2	120.6	72.6					
	6	357.8	299.8	235.9	153.4	100.5	55.7				
	7	374.2	321.8	266.7	186.1	128.5	74.0	46.5			
34.0											
	3	336.9	256.8	159.7							
	4	355.9	280.2	192.3	97.8						
	5	374.8	303.7	224.9	133.3	80.5					
	6	393.8	327.2	257.5	168.7	111.1	63.1				
	7	412.8	350.7	290.1	204.2	141.7	83.5	52.1			
36.0											
	3	365.5	280.1	176.1							
	4	387.3	305.1	210.6	108.4						
	5	409.0	330.2	245.2	146.8	89.1					
	6	430.7	355.2	279.7	185.1	122.6	71.2				
	7	452.5	380.2	314.2	223.5	156.1	93.9	58.3			
38.0											
	4	419.3	330.4	229.4	119.7						
	5	444.0	357.0	266.0	161.1	98.6					
	6	468.6	383.7	302.6	202.5	135.1	80.1				
	7	493.3	410.4	339.2	243.9	171.6	105.3	65.2			
	8	518.0	437.1	375.7	285.3	208.2	130.5	82.6	34.9		
40.0											
	4	452.0	356.0	248.6	131.6						
	5	479.8	384.4	287.4	176.2	108.9					
	6	507.5	412.8	326.1	220.9	148.6	89.7				
	7	535.2	441.2	364.9	265.5	188.3	117.5	72.7			
	8	563.0	469.6	403.6	310.1	228.1	145.3	91.8	39.5		
42.0											
	4	485.3	381.9	268.3	144.2						
	5	516.3	412.1	309.3	192.2	120.0					
	6	547.3	442.4	350.3	240.2	163.1	100.2				
	7	578.3	472.6	391.3	288.2	206.2	130.7	80.7			
	8	609.2	502.8	432.4	336.2	249.3	161.1	101.8	44.5		
44.0											
	4	519.3	408.2	288.4	157.4						
	5	553.6	440.4	331.8	209.0	131.9					
	6	588.0	472.5	375.2	260.6	178.5	111.5				
	7	622.4	504.6	418.6	312.1	225.2	144.8	89.5			
	8	656.8	536.7	462.0	363.7	271.8	178.0	112.4	49.9		

Table 14.--*Board-foot volume of koa (Int. 1/4-inch rule); form class: 0.85, continued*

		SECTION NUMBER									
DBH	HT.	1	2	3	4	5	6	7	8	9	10
46.0											
	4	553.9	434.9	308.9	171.3						
	5	591.8	469.0	354.8	226.6	144.7					
	6	629.7	503.1	400.7	281.9	195.0	123.5				
	7	667.6	537.3	446.6	337.2	245.3	159.8	98.8			
	8	705.6	571.4	492.5	392.4	295.6	196.0	123.7	55.6		
48.0											
	4	589.1	461.9	329.9	185.9						
	5	630.7	498.1	378.4	245.1	158.2					
	6	672.3	534.3	426.9	304.2	212.4	136.4				
	7	714.0	570.5	475.4	363.4	266.6	175.7	108.8			
	8	755.6	606.7	523.9	422.5	320.8	215.0	135.8	61.7		
50.0											
	4	624.9	489.3	351.4	201.1						
	5	670.4	527.6	402.6	264.3	172.6					
	6	715.9	566.0	453.8	327.5	230.8	150.0				
	7	761.4	604.4	505.0	390.7	289.0	192.5	119.3			
	8	806.9	642.8	556.2	453.9	347.2	235.1	148.5	68.1		
52.0											
	4	661.4	517.0	373.2	217.0						
	5	710.9	557.6	427.3	284.4	187.9					
	6	760.5	598.3	481.3	351.8	250.2	164.4				
	7	810.0	638.9	535.4	419.3	312.6	210.3	130.5			
	8	859.5	679.5	589.4	486.7	375.0	256.2	162.0	75.0		
54.0											
	4	698.5	545.1	395.6	233.5						
	5	752.2	588.1	452.5	305.3	203.9					
	6	805.9	631.0	509.5	377.1	270.6	179.6				
	7	859.7	674.0	566.5	448.9	337.3	229.0	142.4			
	8	913.4	717.0	623.5	520.7	404.0	278.3	176.1	82.1		
56.0											
	4	736.2	573.5	418.3	250.8						
	5	794.3	618.9	478.4	327.1	220.7					
	6	852.4	664.4	538.4	403.4	292.0	195.6				
	7	910.4	709.8	598.4	479.8	363.2	248.6	154.8			
	8	968.5	755.2	658.5	556.1	434.4	301.5	191.0	89.7		
58.0											
	4	774.6	602.3	441.5	268.6						
	5	837.2	650.2	504.7	349.7	238.4					
	6	899.7	698.2	567.9	430.7	314.3	212.4				
	7	962.3	746.2	631.1	511.7	390.2	269.1	167.9			
	8	1024.9	794.1	694.3	592.8	466.1	325.8	206.6	97.6		

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