Prediction of Height Increment for Models of Forest Growth

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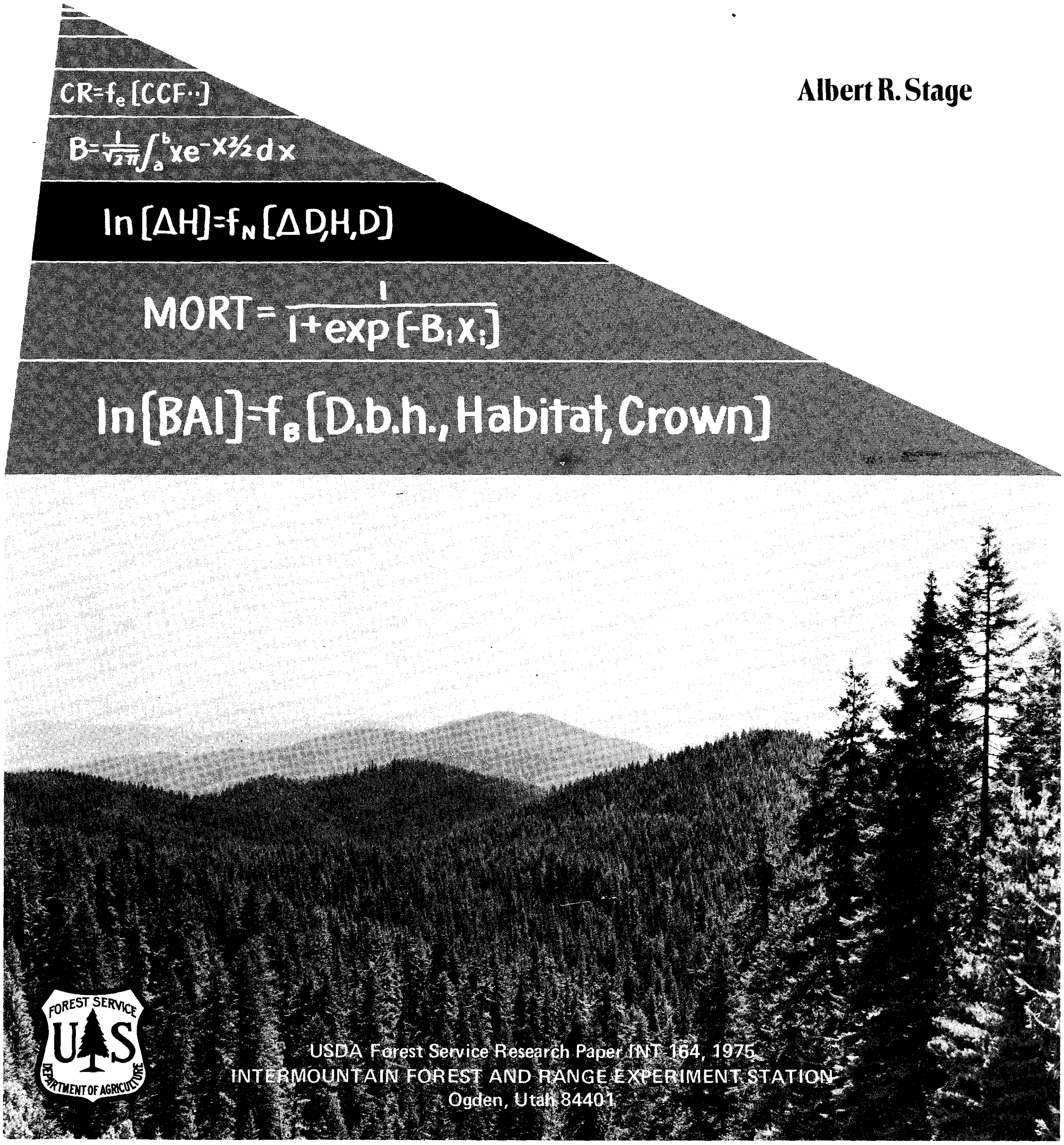

USDA Forest Service
Research Paper INT-164
January 1975

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CONTENTS

ABSTRACT

Functional forms of equations were derived for predicting 10-year periodic height increment of forest trees from height, diameter, diameter increment, and habitat type. Crown ratio was considered as an additional variable for prediction, but its contribution was negligible. Coefficients of the function were estimated for 10 species of trees growing in 10 habitat types of northern Idaho and northwestern Montana. A procedure using these coefficients for calculating the components of current annual volume increment from diameter, height, and past radial increment is described, and its FORTRAN implementation listed.

INTRODUCTION

How fast do trees grow in height? The answer requires a great deal of detail about the tree's environment and about the tree itself, or some specific growth measure on the tree that indicates the combined effects of all environmental factors and the tree's individual characteristics. Diameter increment is such a measure. It is much more easily measured than height increment, and responds to the same growth determinants.

Diameter increment, however, is much more responsive to effects of stand stocking than is height increment. Past effects of stocking on height growth relative to diameter growth are indicated by the tree's form, as measured by the height/d.b.h. ratio, and by its crown ratio (live crown length/total height). In this report, we seek to develop a prediction equation that relates height increment to concurrent diameter increment, tree height, diameter, crown ratio, and habitat type (Daubenmire and Daubenmire 1968).

To use this equation for calculating the height increment component of current volume increment of trees on inventory plots, a computer subroutine is provided in the appendix. For more extended prognoses, the same equation for predicting height increment can be imbedded in a computer model of forest stand development (Stage 1973).

For many years Forest Survey in the Intermountain Forest and Range Experiment Station has calculated current volume increment from changes in height computed from diameter increment and diameter. Wright (1961) recommended a similar procedure. Both approaches were derived from a single curve of total height over d.b.h. by determining the differences in the curve height for current d.b.h. and for d.b.h. plus diameter increment. Hence, these approaches assume that all trees of a particular species follow height/diameter curves that have the same slope for a given d.b.h.

Better prediction equations should be possible where data are available from direct measurements of height increment. Prediction equations presented in this report are based on analysis of 1,165 trees felled as part of management planning inventories of the Kaniksu, Coeur d'Alene, St. Joe, and Lewis and Clark National Forests in northern Idaho and western Montana.

DESCRIPTION OF DATA

Ten species distributed among 10 habitat types are represented in the data (table 1). Distribution of samples with respect to height and d.b.h. are shown by species in figure 1, and by habitat types in figure 2. Periodic diameter growth (inside bark), ΔD, was measured concurrently with periodic height growth, ΔH, for the most recent 10-year period. Values for height (H) and diameter at breast height (D)[1/] were determined as they existed at the start of the increment period because the regression estimate is required to predict the change in height as a function of height, diameter, and projected diameter increment (inside bark). Of necessity, the crown ratio used was observed at the end of the growth period. Fortunately, crown ratio changes very slowly within 10 years.

The trees were measured during the inventories of 1969-1972. Consequently, their most recent 10-year growth is influenced by the climate and other temporal events from about 1959 to 1972.

[1/] Bark increment was not deducted when past d.b.h. was calculated. The slight overestimate of past d.b.h. can cause only a slight bias in the estiamte of height growth.

Table 1.--*Distribution of sample trees by species and habitat type*

Species	Habitat										Total
	Pseudotsuga/ Symphoricarpos	*Pseudotsuga/ Physocarpus*	*Pseudotsuga/ Calamagrostis*	*Abies grandis/ Pachistima*	*Thuja/ Pachistima*	*Tsuga heterophylla/ Pachistima*	*Abies lasiocarpa/ Pachistima*	*Abies lasiocarpa/ Xerophyllum*	*Abies lasiocarpa/ Menziesia*	*Abies lasiocarpa/ Vaccinium*	
Ponderosa pine		12		16	8						36
Douglas-fir	108	53	28	132	13	40	16	6		6	402
Western larch		7		17	9	27	4	14		2	80
Lodgepole pine		6	49	14	3	17	1	8		88	186
Grand fir				49	3	76	1				129
Western white pine				22	2	76		1		1	102
Western redcedar				1	7	39					47
Western hemlock					1	104	8				113
Engelmann spruce			2			3		6	3		14
Subalpine fir						5	13	32	5	1	56
Total	108	78	79	251	46	387	43	67	8	98	1,165

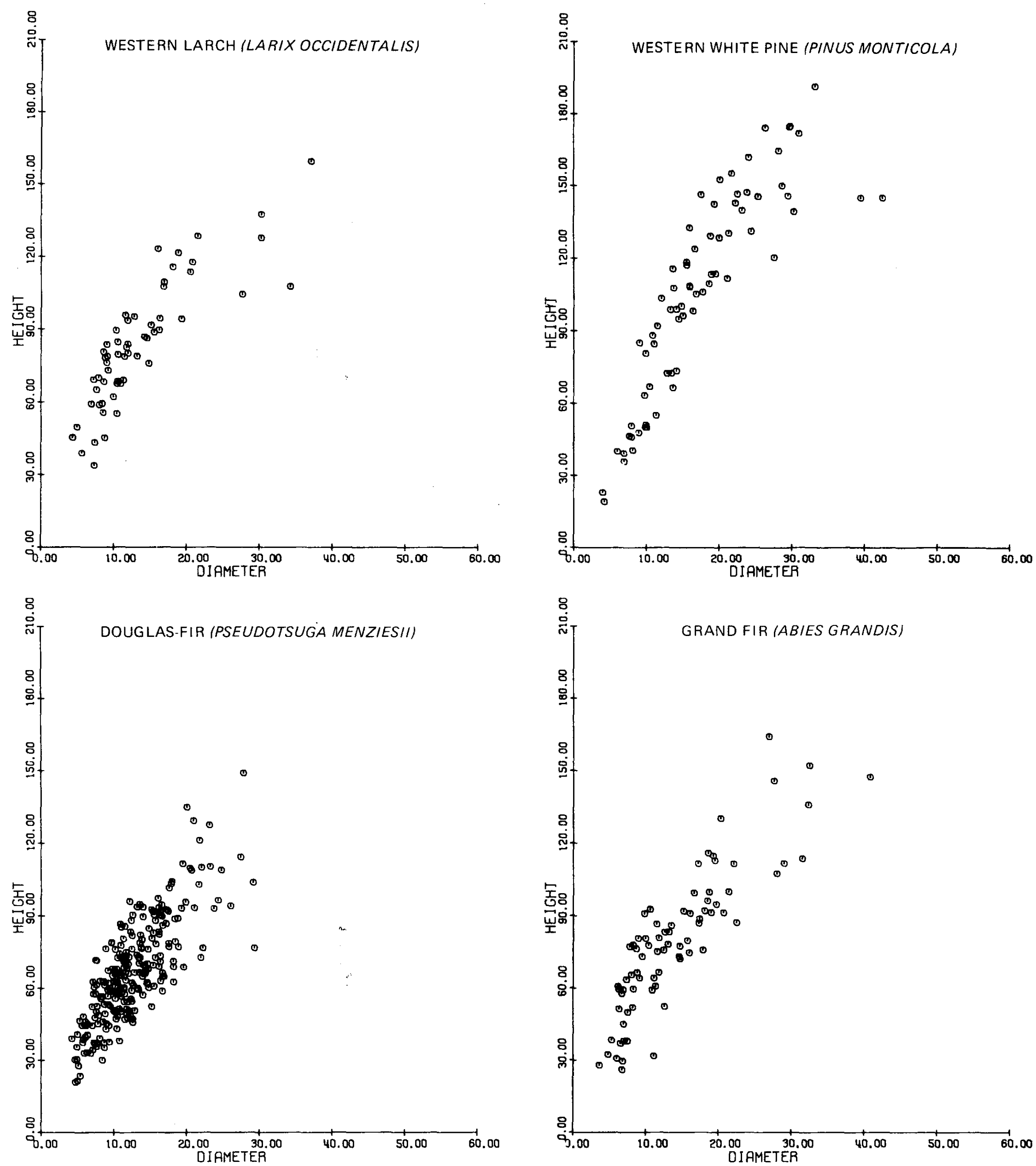
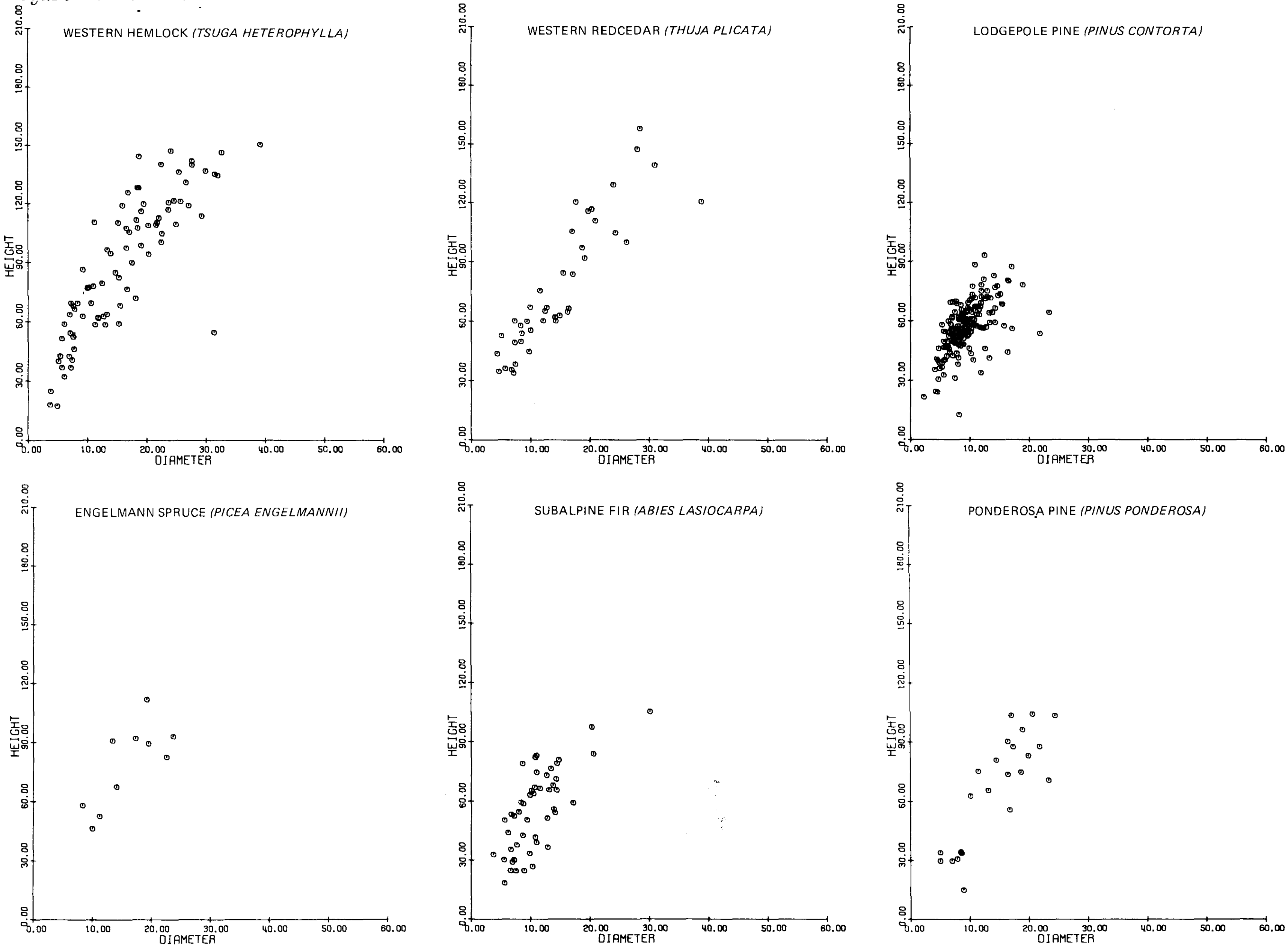


Figure 1. Distribution of sample trees by species, height, and diameter.

Figure 1.--(con.)



WESTERN HEMLOCK *(TSUGA HETEROPHYLLA)*
WESTERN REDCEDAR *(THUJA PLICATA)*
LODGEPOLE PINE *(PINUS CONTORTA)*
ENGELMANN SPRUCE *(PICEA ENGELMANNII)*
SUBALPINE FIR *(ABIES LASIOCARPA)*
PONDEROSA PINE *(PINUS PONDEROSA)*
HEIGHT
DIAMETER
0.00
30.00
60.00
90.00
120.00
150.00
180.00
210.00
10.00
20.00
40.00
50.00

Figure 2. Distribution of sample trees by habitat type, height, and diameter.

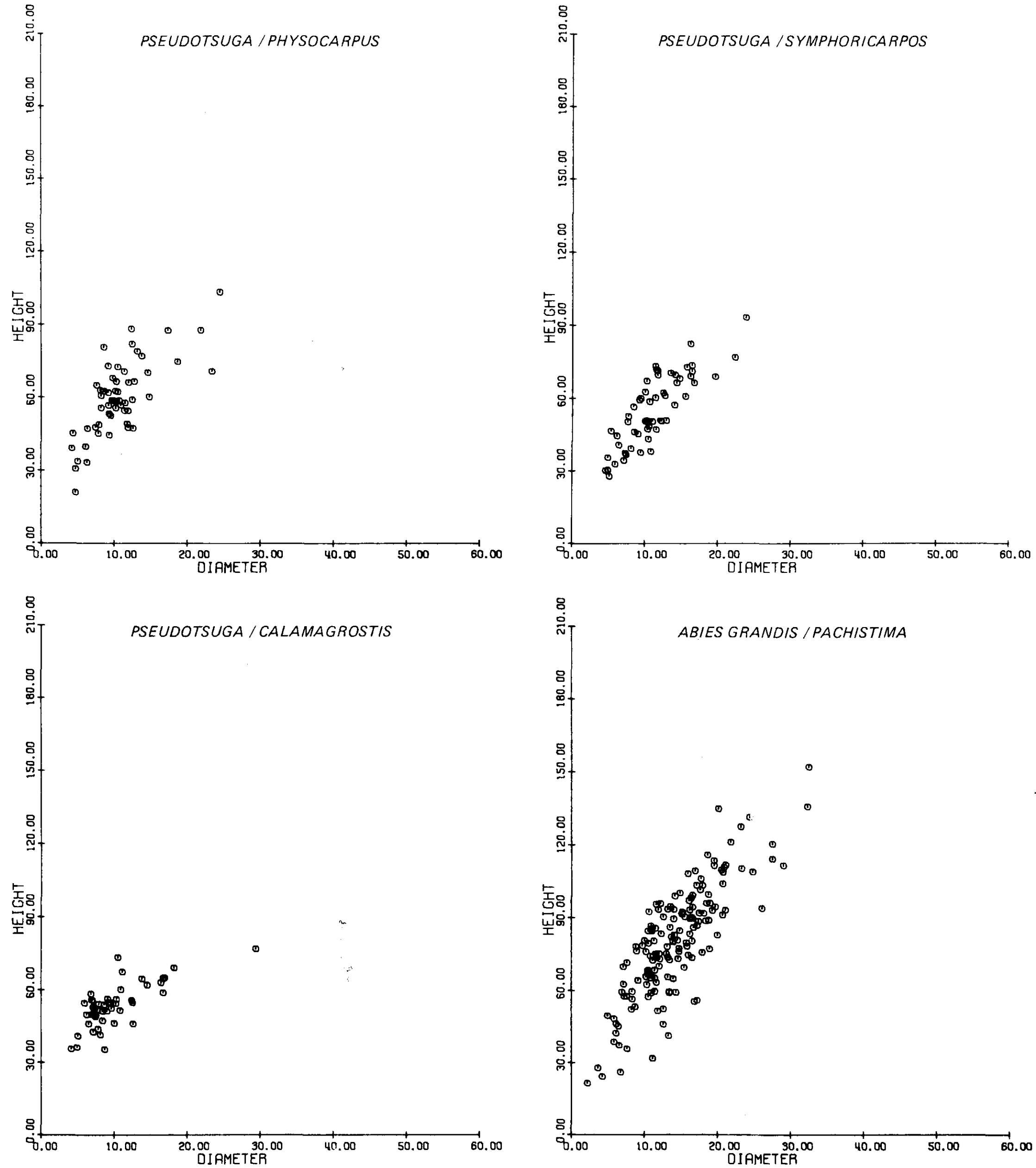
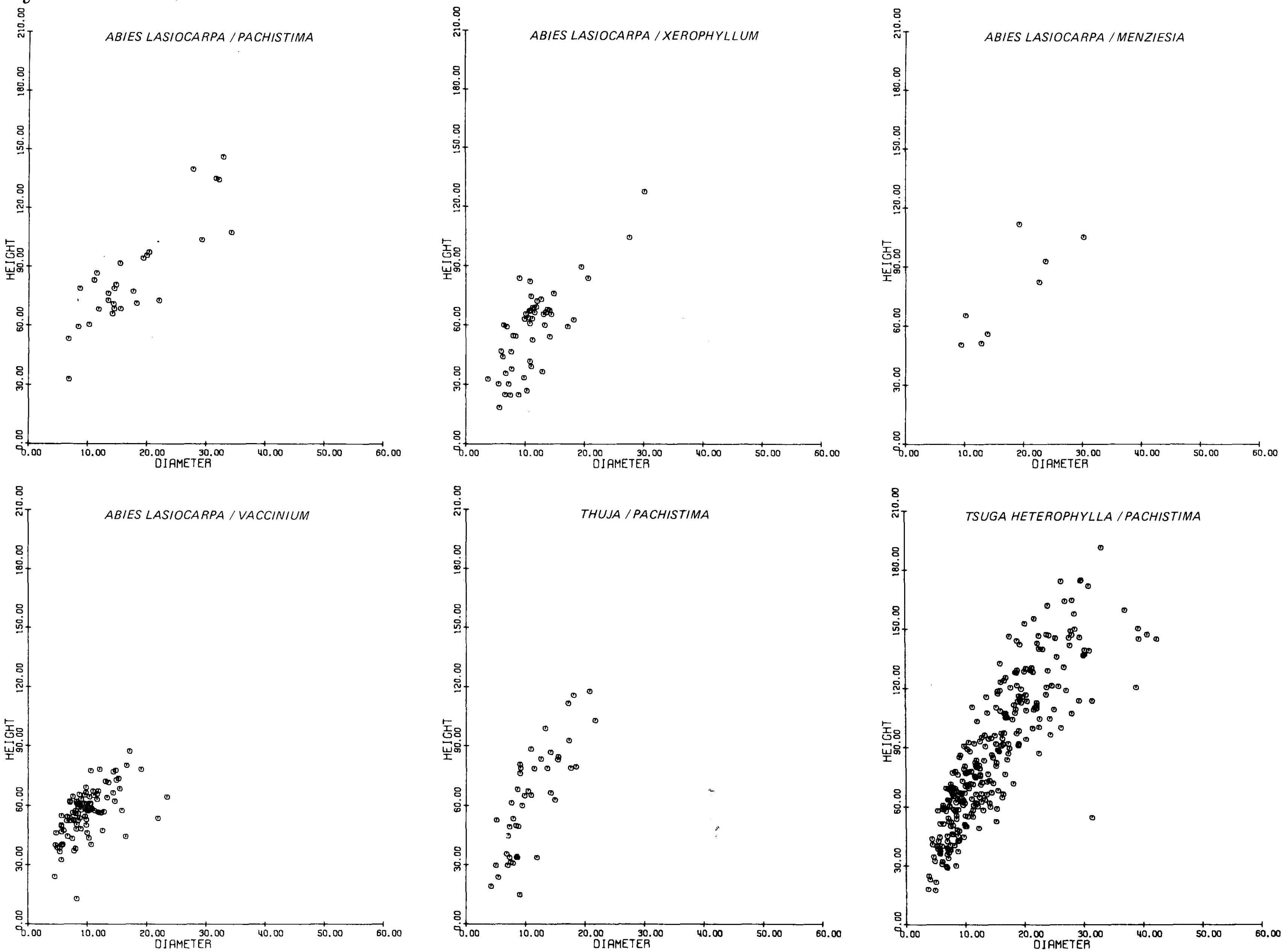


Figure 2.--(con.)


ABIES LASIOCARPA / PACHISTIMA
ABIES LASIOCARPA / XEROPHYLLUM
ABIES LASIOCARPA / MENZIESIA
ABIES LASIOCARPA / VACCINIUM
THUJA / PACHISTIMA
TSUGA HETEROPHYLLA / PACHISTIMA
HEIGHT
DIAMETER
0.00
30.00
60.00
90.00
120.00
150.00
180.00
210.00
10.00
20.00
40.00
50.00

DERIVING THE FUNCTIONAL FORM FOR THE PREDICTION EQUATIONS

A set of height/diameter curves for three even-aged stands of grand fir is shown in figure 3. These curves are plotted solutions of a nonlinear equation for predicting tree height from diameter, site quality, and age of the stand. Analysis of these data confirmed that the allometric relation:

$$\ln(H) = a + b \ln(D) \tag{1}$$

does indeed apply to the development of trees as age increases, but that the coefficients a and b depend upon the competitive status of the trees. Competitive status can be defined by the distance of the (H,D) point for a particular tree from the curve of relation (1) for dominant trees (Perkal and Battek 1955). The dashed line shows the trend followed by the average for the dominant and codominant trees in the stand. Trees in lower crown classes move more steeply upward. Conversely, dominants move along lines of lower slope.

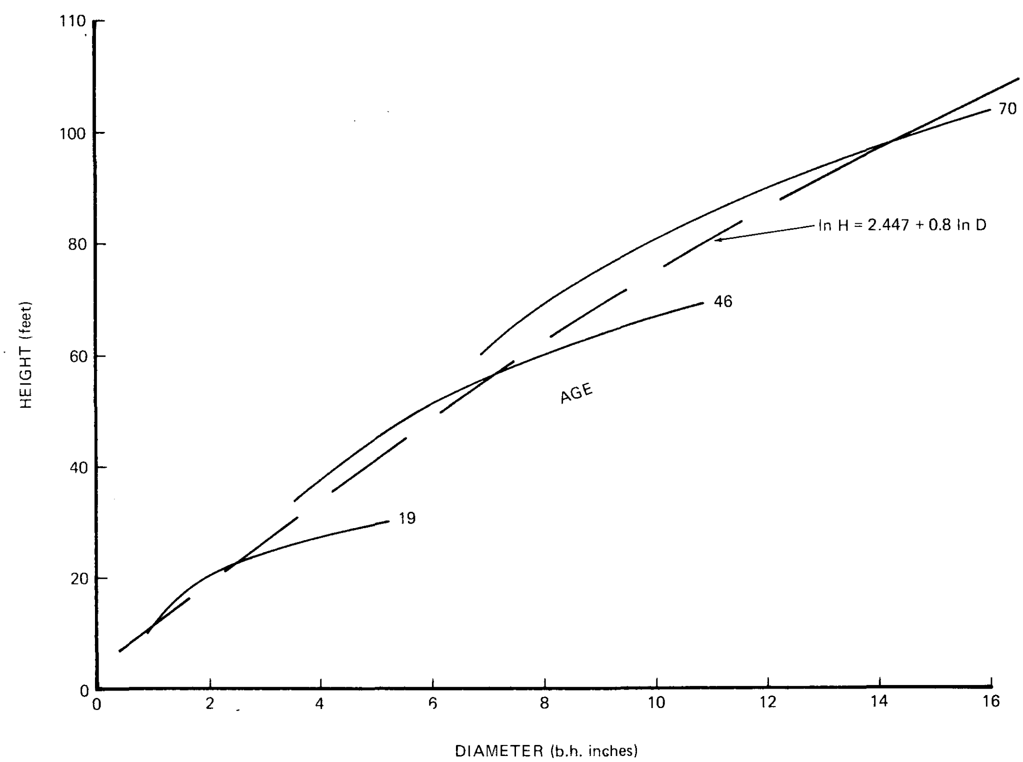


Figure 3. Sequence of height/diameter curves for even-aged stands of grand fir.

Instantaneous growth rates can be obtained by taking the differential of equation (1):

$$\partial H = b \frac{H}{D} \partial D \tag{2}$$

or, in its logarithmic form:

$$\ln(\partial H) = \ln(\partial D) + \ln(H) - \ln(D) + \ln(b) \tag{3}$$

On the other hand, periodic growth rates expressed as a finite difference derived from (1) would be:

$$\ln(H_2) - \ln(H_1) = b(\ln(D_2) - \ln(D_1))$$

which is the same as:

$$\ln(H_2/H_1) = b \ln(D_2/D_1) \tag{4}$$

where the subscripts 1 and 2 designate measured values at the start and end of growth period, respectively.

Models were derived from (2), (3), and (4) by using various transformations of height, the ratio of height to diameter, and crown ratio to estimate the b parameter for each species. The best transformations of these independent variables were selected by combining them in groups of sets for screening overall combinations of the groups by using one set from each group (Grosenbaugh 1967). Coefficients in these alternative models were estimated by least-squares regression. Goodness-of-fit indices (Furnival 1961) for the several transformations of the dependent variable were compared using the best regression for each transformation of the dependent variable.

For the screening of these alternative models, data from 909 trees of 10 species in the northern Idaho forests were used.

Differential Model

The dependent variable in this model is the 10-year periodic height increment, ΔH. Some coefficients (c_j) in the model may be different for each species; other coefficients may be constant for all 10 species. Those that vary with species have an additional subscript i.

$$\Delta H = \frac{H}{D} \Delta D[c_{1_i} + c_{2_i} \ln(CR) + c_3 \frac{\Delta D}{D} + c_4(\frac{\Delta D}{D})^2 + c_5 \ln(H) + c_{6_i} \frac{D}{H\Delta D}]$$

The expression in the brackets represents the estimate of b in equation (2). In computation, c_{6_i} was the set of constant terms in the regression model.

Table 2 is an analysis of variance showing the improvement in the regression sum-of-squares for successively more complex collections of variables. In this table, for example, comparison level 2 indicates that when either the coefficients of $H\Delta D/D$ or the constant terms are allowed to vary by species, then the fit is improved by a significant amount. Of these two alternatives, the former has a slight advantage. At comparison level 3, varying constant terms by species is of little value so long as the coefficients of $H\Delta D/D$ depend on species. However, the variable $\ln(H)$ is shown to be needed to represent the effect of changing height on the b coefficient. At comparison level 4, crown ratio is still of little use, but addition of $\Delta D/D$ and its square results in a considerable improvement.

Table 2.--*Analysis of improvement of regression model attributable to adding variables. Dependent variable* = ΔH

Comparison level	Source	d.f.	Proportion of remaining variance explained[1]	Marginal sum of squares[1]	Mean square	F
1	[1	1		49469.808		
	$H\Delta D/D$]	1		2263.450	2263.450	263.0
2	[$H\Delta D/D_{isp}$]	9	0.065	776.592	86.288	10.0
	1_{isp}	9	.060	728.725	80.969	9.4
3	1_{isp}	9	.0168	260.549	28.950	3.4
	[ln (H)]	1	.151	1491.782	1491.782	173.3
4	ln (CR)	10	.048	479.677	47.967	5.6
	1_{isp}	9	.0158	219.627	23.403	2.7
	[$\Delta D/D, (\Delta D/D)^2$]	2	.108	905.914	452.957	52.6
	Error	878		7556.068	8.606	

[1]Increase in explanatory power for regression due to adding variables to the model composed of those variables bracketed in the levels above.

Finite Difference Model

The dependent variable in this model is $\ln(H_2/H_1)$. When an expression for b analogous to its form in the differential model is inserted in the finite difference model (4), we obtain:

$$\ln(H_2/H_1) = \ln(D_2/D_1)\ [c_{1_i} + c_{2_i} \ln(CR) + c_3 \ln(D_2/D_1) + c_4 (\ln(D_2/D_1))^2$$
$$+ c_{5_i} \ln(H) + c_{6_i} / \ln(D_2/D_1)]$$

To compare the utility of the two alternative forms of the dependent variable, we use the maximum likelihood index of fit (Furnival 1961). The basis for the comparison is the standard error of estimate of the untransformed dependent variable. Standard errors of estimate for other transformations of the dependent variable are converted for comparison by multiplying them by the inverse of the geometric mean of the derivative of the transformation. The derivative of ln (H_2/H_1) with respect to ΔH is:

$$f'(\Delta H) = \frac{\partial \ln(H_2/H_1)}{\partial \Delta H} = \frac{\partial \ln(1. + \Delta H/H_1)}{\partial \Delta H}$$

$$= \frac{H_1}{H_2} \cdot \frac{1}{H_1} = \frac{1}{H_2}$$

Accordingly, the inverse of the geometric mean is given by:

$$\frac{1}{[f'(\Delta H)]} = \exp[\Sigma \ln(H_2)/n]$$

The index-of-fit is 3.54 for the best collection of variables (through level 4) in the finite difference model. This index is compared to the standard error of estimate of the differential model, which is ±2.93 feet. Hence, the differential model is superior.

Table 3 is an analysis of variance for the finite difference model with comparisons similar to those in table 2. The variables in this model that are analogous to those in the differential model are effective as predictors to about the same degree.

Table 3.--*Analysis of improvement of regression model attributable to adding variables. Dependent variable* = $\ln(H_2/H_1)$

Comparison level	Source	d.f.	Proportion of remaining variance explained[1]	Marginal sum of squares[1]	Mean square	F
1	[1	1				
	$\ln(D_2/D_1)$]	1		3.1589	3.1589	1707.5
2	[$\ln(D_2/D_1)_{isp}$]	9	0.091	.3372	.0375	20.3
	1_{isp}	9	.031	.1365	.0152	8.2
3	1_{isp}	9	.015	.0754	.00838	4.5
	[$\ln(H)$]	1	.252	.7702	.7702	416.3
4	[$\Delta D/D, (\Delta D/D)^2$]	2	.258	.5921	.2961	160.0
5	1_{isp}	9	.018	.04633	.00515	2.8
	Error	886			.00185	

[1] Increase in explanatory power for regression due to adding variables to the model composed of those variables bracketed in the levels above.

Differential Model-- Logarithmic Form

When the differences between the observed and predicted values of ΔH are plotted over the predicted values, the variance of the difference increases with larger values of the prediction. As a consequence, faster growing trees are given undue weight in estimating values of the coefficients. The logarithmic form of the model (3) should decrease the variance of the residuals associated with large predictions. The preceding analyses showed that the allometric coefficient (*b*) varied with species and the relation of height, diameter, and diameter increment. These effects could be incorporated quite readily by multiplying each independent variable by a coefficient to be estimated, and by introducing a constant term for each species. In the logarithmic form, the possibility of zero values for ΔD must be considered. Measurements of diameter growth are usually recorded only to the nearest 1/20 inch. By shifting the entire scale up 1/20 inch, zero values can be accommodated without distorting the overall relation.

Thus, the logarithmic form of the differential model is:

$$\ln(\Delta H) = c_{1_i} + c_2 \ln(\Delta D + .05) + c_3 \ln(D) + c_4 \ln(H) \tag{5}$$

For the 909 trees from the northern Idaho forests, the following coefficients for this model gave a better index of fit (±2.73 feet) than the differential model with the loss of fewer degrees of freedom:

$c_{1_i} =$	3.4448	Western white pine
	3.1258	Western larch
	3.1424	Douglas-fir
	3.2939	Grand fir
	3.1508	Western hemlock
	2.9357	Western redcedar
	3.0706	Lodgepole pine
	3.2221	Engelmann spruce
	2.8355	Subalpine fir
	3.4380	Ponderosa pine
$c_2 =$	+0.37401	$\ln(\Delta D + 0.05)$
$c_3 =$	-0.29805	$\ln(D)$
$c_4 =$	-0.13170	$\ln(H)$
$s^2 =$	0.1775	

When the same model was applied to the 265 lodgepole pine and Douglas-fir trees from the Lewis and Clark National Forest, the values of the coefficients were quite different from those above. The estimated residual variance for these data was $s^2 = 0.1630$. The difference between these two populations is apparent in the following averages:

	Northern Idaho Forests	*Lewis and Clark N.F.*
ΔH	6.5	1.4
ΔD	1.8	0.44

Differences in growth rates correspond to marked differences in habitat for those species that are common to the two geographic sources of data. As a consequence of the differing habitats, the Lewis and Clark trees would show a much lower average site index than the northern Idaho trees. Furthermore, the Lewis and Clark trees are generally older. To accommodate these differences, data from the two sources were merged and the differential model modified to permit coefficients to vary with habitat as well as species. If all four coefficients were unique with respect to species and habitats, then there would be 204 coefficients to be estimated by a separate regression solution of equation (5) for each of the 51 entries in table 1. Instead, the four coefficients in the logarithmic form of the differential model were varied according to species and habitat in repeated regression problems (table 4). The final line represents the best model of the series. Coefficients derived from this solution are given in table 5. The ratio of the mean-square deviation from regression to the mean-square deviation about the mean is 0.2722 for this model. That is, the model accounts for about 73 percent of the variance of logarithm of height increment among the 1,165 trees in the sample. Converting the residual error to the scale of feet of height increment (Furnival's index), the mean-square error of estimate would be ±1.97 feet.

To illustrate the implications of this functional form for height increment in the context of stand development, the successive height/diameter curves were plotted for a typical even-aged stand of grand fir carried through four decades of simulated growth (fig. 4). The curves in figure 4 show that the prediction functions for height increment derived in this report can generate successive curves that conform to the general shape and level of typical height/diameter curves in even-aged stands shown in figure 3. Conformation of these curves for simulated stand development also depends on the way diameter increment and mortality change among diameter classes within the stand.

Table 4.--*Effect of varying coefficients by species and habitat on the mean square residual of the logarithmic form of the differential model*

Number of coefficients in model	Variable: 1	Variable: $\ln(\Delta D+.05)$	Variable: $\ln(D)$	Variable: $\ln(H)$	Mean square residual
22	H	*	*	H	0.2045
31	S	*	H	H	.2029
32	S+H	*	*	H	.2008
32	S+H	H	*	*	.1958
40	S	H	H	H	.1875
41	S+H	H	H	*	.1853

Key: * indicates a single coefficient for all species and habitats.
H indicates a unique coefficient of the indicated variable for each habitat.
S indicates a unique coefficient of the indicated variable for each species.

Table 5.--*Coefficients for estimating the logarithm of 10-year periodic height growth*

	Habitat									
	Pseudotsuga/ Symphoricarpos	*Pseudotsuga/ Physocarpus*	*Pseudotsuga/ Calamagrostis*	*Abies grandis/ Pachistima*	*Thuja/ Pachistima*	*Tsuga heterophylla/ Pachistima*	*Abies lasiocarpa/ Pachistima*	*Abies lasiocarpa/ Xerophyllum*	*Abies lasiocarpa/ Menziesia*	*Abies lasiocarpa/ Vaccinium*
					Intercept coefficient					
Ponderosa pine		3.0970		3.8908	3.4091					
Douglas-fir	2.4253	2.7460	2.9966	3.5398	3.0581	3.3143	3.9207	2.4395		2.8073
Western larch		2.7512		3.5450	3.0633	3.3195	3.9259	2.4447		2.8125
Lodgepole pine		2.7973	3.0480	3.5911	3.1094	3.3656	3.9720	2.4908		2.8586
Grand fir				3.6535	3.1718	3.4280	4.0344			
Western white pine				3.8157	3.3340	3.5902		2.7154		3.0832
Western redcedar				3.3143	2.8326	3.0888				
Western hemlock					3.0256	3.2818	3.8881			
Engelmann spruce			2.9080			3.2257		2.3509	1.6339	
Subalpine fir						3.1336	3.7400	2.2588	1.5419	2.6266
					Variable coefficients					
Variable										
$\ln(\Delta D+0.05)$	.96586	.11127	.61883	.44920	.27100	.40908	.61661	.05592	.48219	.75458
$\ln(D)$	-.31462	.00278	-.56538	-.35626	-.23160	-.22856	-.47228	.04938	.21472	-.37079
$\ln(H)$	-.1955	-.1955	-.1955	-.1955	-.1955	-.1955	-.1955	-.1955	-.1955	-.1955

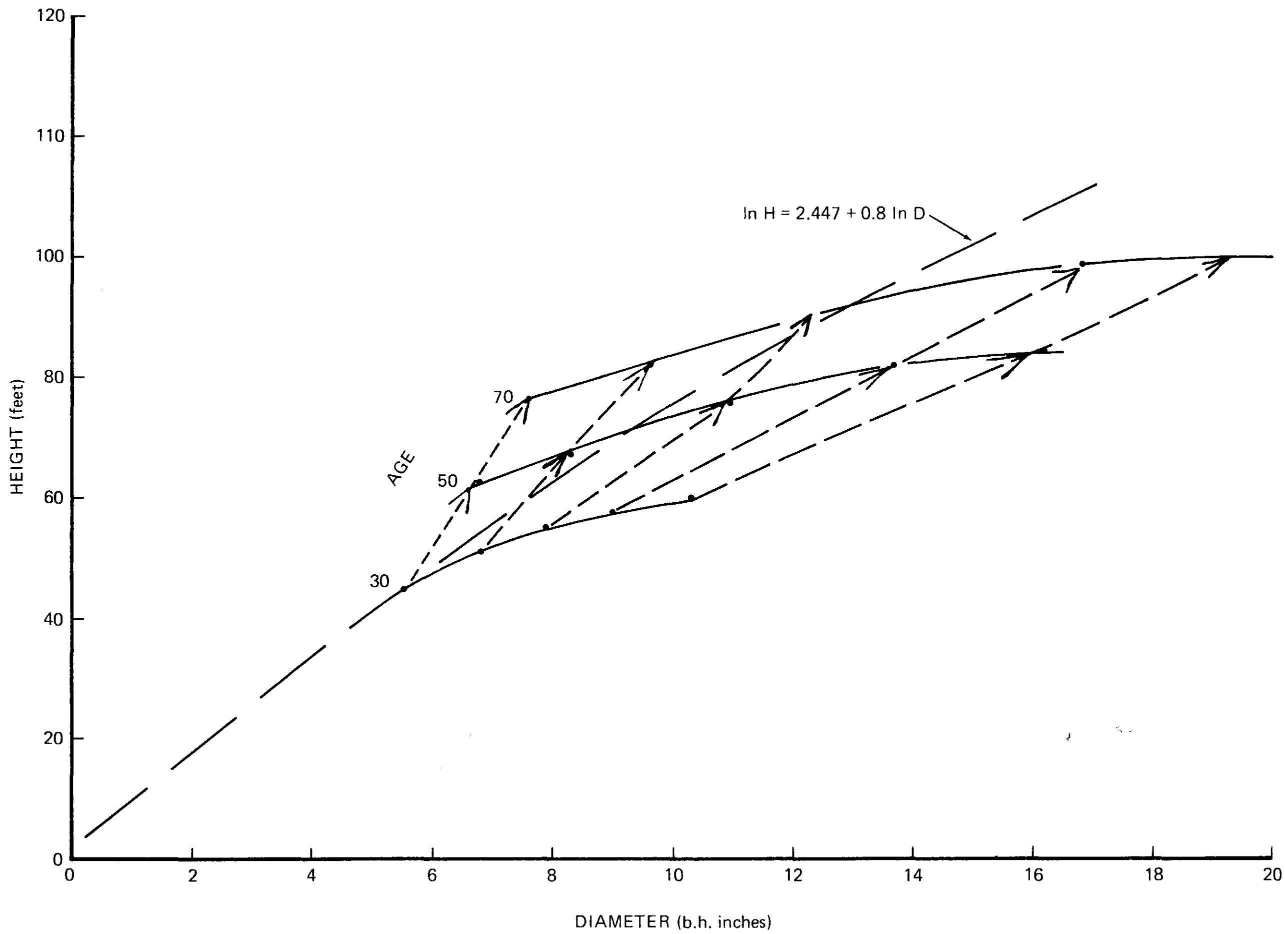


Figure 4.--Projected height/diameter curves for grand fir on Abies grandis/Pachistima habitat.

Removing Bias in Logarithmic Form

When the variable to be predicted is analyzed by making a logarithmic transformation, a bias is introduced when the inverse transformation is used to convert the estimate to the original natural scale. Effect of the bias would be cumulative when these prediction equations are used repetitively to simulate growth of trees and stands through successive periods of time. By removing the bias, repetitive application will produce the same height from the accumulated predictions of increment as would be the result of cumulating observed values of the log-normally distributed height increment.

Size of the bias depends on the residual variance, which is listed as 0.1853 on the last line of table 4, for the logarithmic model.

Bradu and Mundlak (1970) devised a correction for the bias that recognizes the effect of sampling error on the parameters of the regression. Unfortunately, their method requires the values of the inverse covariance matrix which is a 41 X 41 array. As an alternative, a simple correction that is conditional on the sample values of the parameters seems adequate. To determine the corrected estimate of the mean on the original scale, the value of one-half the residual variance is added to the estimate on the logarithmic scale before making the transformation back to the natural scale. In this case, the amount to be added is 0.1853 ÷ 2 = 0.09265. The effect is to multiply the uncorrected estimate of height increment by 1.0971.

APPLICATIONS

Calculating Current Annual Increment

Current annual volume increment is commonly calculated by measuring or estimating the two major dimensions of volume increment: change in tree d.b.h. (inches) and change in tree height (feet). Then, these changes are added to present d.b.h. and total height, and the volume change is determined from the difference between the volumes obtained from a volume table or volume equation that relates tree volume to d.b.h. and total height. The prediction equations developed in this report can be used to calculate the 10-year periodic height increment from the corresponding 10-year periodic diameter increment, initial d.b.h., and initial height. When past periodic diameter increment is used to determine future diameter increment, a problem arises because diameter growth generally declines with age. Consequently, the current annual rate would differ from the periodic rate divided by period length. To remedy this problem, we can invoke the observation that whereas diameter increment declines with age, basal area increment usually remains constant (Spurr 1952, p. 214; Smith 1962, p. 55).

The sequence of calculations is:

1. Determine past increase in diameter squared. If increment was measured outside bark, convert to increment inside bark. If measured for a period of y years, convert to a 10-year basis.

$$DDS = (d_i^2 - d_{i-y}^2) * 10/y$$

2. Use past DDS to estimate future ΔD (10-year periodic increment):

$$\Delta D = \sqrt{d_i^2 + DDS} - d_i \qquad \text{(inches)}$$

3. Solve equation (5) using the set of coefficients from the column of table 5 for the applicable habitat type, and using the first coefficient for the appropriate species:

$$\ln(\Delta H) = c_1 + c_2 \ln(\Delta D + 0.05) + c_3 \ln(D_i) + c_4 \ln(H_i)$$

$$\Delta H = \exp(\ln(\Delta H) + 0.09265) \quad \text{(feet)}$$

4. Convert growth estimates to diameter and height 1 year hence:

$$D_{i+1} = \sqrt{D_i^2 + BKR*DDS/10}$$

$$H_{i+1} = H_i + \Delta H/10$$

where $BKR = (D_i/d_i)^2$ is the square of the ratio of d.b.h. to diameter inside bark at breast height.

A computer subroutine prepared in FORTRAN IV to carry out this sequence of calculations is given in the appendix.

If growth for an arbitrary interval (y) greater than 1 year is desired, then the division by 10 in the last two equations would be replaced by multiplication by the value of $y/10$. That is,

$$D_{i+y} = \sqrt{D_i^2 + BKR*DDS*y/10}$$

$$H_{i+y} = H_i + \Delta H*y/10$$

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APPENDIX

FORTRAN IV Subroutine

```
      SUBROUTINE HTGT(ISP,IHAB,D,H,DD,D2,H2)
C         ISP IS SPECIES SUBSCRIPT
C         IHAB IS HABITAT SUBSCRIPT
C         D IS DIAMETER
C         H IS HEIGHT
C         DD IS 10-YEAR PAST DBH GROWTH
C         D2 IS THE NEW DIAMETER
C         H2 IS THE NEW HEIGHT
      DIMENSION ASPE(10),AHAB(10),BHAB(10),CHAB(10),BKR(10)
C
C     SPECIES SUBSCRIPT                        DEFINITION
C             1              WHITE PINE
C             2              WESTERN LARCH
C             3              DOUGLAS FIR
C             4              GRAND FIR
C             5              WESTERN HEMLOCK
C             6              CEDAR
C             7              LODGEPOLE
C             8              SPRUCE
C             9              SUBALPINE FIR
C             10             PONDEROSA PINE
      DATA ASPE/-0.260831,-0.531522,-0.536742,-0.423059,
     1 -0.569264,-0.762272,-0.485439,-0.625377,-0.717423,-0.185745/
C
C     HABITAT SUBSCRIPT                        DEFINITION
C             1              PSEUDOTSUGA / PHYSOCARPUS
C             2              PSEUDOTSUGA / SYMPHORICARPOS
C             3              PSEUDOTSUGA / CALMAGROSTIS
C             4              ABIES GRANDIS / PACHISTIMA
C             5              ABIES LASIOCARPA / PACHISTIMA
C             6              ABIES LASIOCARPA / XEROPHYLLUM
C             7              ABIES LASIOCARPA / MENZIESIA
C             8              ABIES LASIOCARPA / VACCINIUM
C             9              THUJA / PACHISTIMA
C             10             TSUGA HETEROPHYLLA / PACHISTIMA
      DATA AHAB/3.28270,2.96202,2.53339,4.07652,4.45741,
     1 2.97625,2.25931,3.34406,3.59483,3.85106/
      DATA BHAB/.11127472,.9658621,.61883491,.44919991,.61660659,
     1 .055921353,.48219198,.75457698,.27099925,.40907645/
      DATA CHAB/.0027789974,-0.31462085,-0.56538403,-0.35626483,
     1 -0.47227514,.049378771,.21472478,-0.37078756,-0.2315979,
     1 -0.22855937/
      DATA DSP/-0.19554013/
      DATA BKR /10* 1.09/
      D2 = SQRT(D**2 + DD * (2. * D - DD) * BKR(ISP) / 10.)
      H2 = H + EXP(ASPE(ISP) + AHAB(IHAB) + BHAB(IHAB) * ALOG(DD + 0.05)
     1 + CHAB(IHAB) * ALOG(D) + DSP * ALOG(H)) * 0.109698
      RETURN
      END
```


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USDA Forest Service
Research Paper INT-164
Supplement 1
April 1975

Prediction of Height Increment For Models of Forest Growth

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U.S. Department of Agriculture
Ogden, Utah 84401
Roger R. Bay, Director

During the preparation for publication of Research Paper INT-164 additional trees were obtained from the Colville, Clearwater, and Nezperce National Forests, from research studies of grand fir yield by the author, and from defect distribution studies by Dr. Arthur D. Partridge, College of Forestry, Wildlife and Range Sciences, University of Idaho.

Analysis of these 330 additional trees has provided better coverage of several species and habitats; as shown in the revised table 1. The newly derived coefficients for model (5) (pg. 12) are given in the revised table 5 and the computer subroutine provided in the appendix. The new estimate of the multiplier for removing bias due to the logarithmic transformation equals 1.107.

Table 1.--*Distribution of sample trees by species and habitat type*

Species	Habitat										Total
	Pseudotsuga/ Symphoricarpos	*Pseudotsuga/ Physocarpus*	*Pseudotsuga/ Calamagrostis*	*Abies grandis/ Pachistima*	*Thuja/ Pachistima*	*Tsuga heterophylla/ Pachistima*	*Abies lasiocarpa/ Pachistima*	*Abies lasiocarpa/ Xerophyllum*	*Abies lasiocarpa/ Menziesia*	*Abies lasiocarpa/ Vaccinium*	
Ponderosa pine	1	30		17	8						56
Douglas-fir	108	107	28	143	31	41	27	6		6	497
Western larch		9		17	14	27	4	14		2	87
Lodgepole pine		7	49	14	3	17	3	12	4	90	199
Grand fir				81	48	89	17	3		1	239
Western white pine				22	3	76		1		1	103
Western redcedar				1	52	39					92
Western hemlock					3	109	8				120
Engelmann spruce			2	2		3	3	6	7		23
Subalpine fir						9	16	42	11	1	79
Total	109	153	79	297	162	410	78	84	22	101	1,495

Table 5.--*Coefficients for estimating the logarithm of 10-year periodic height growth*

	Habitat									
	Pseudotsuga/ Symphoricarpos	*Pseudotsuga/ Physocarpus*	*Pseudotsuga/ Calamagrostis*	*Abies grandis/ Pachistima*	*Thuja/ Pachistima*	*Tsuga heterophylla/ Pachistima*	*Abies lasiocarpa/ Pachistima*	*Abies lasiocarpa/ Xerophyllum*	*Abies lasiocarpa/ Menziesia*	*Abies lasiocarpa/ Vaccinium*
					Intercept coefficient					
Ponderosa pine	2.4768	3.9227		3.4004	2.6014					
Douglas-fir	2.1560	3.6019	2.8472	3.0796	2.2806	2.9937	3.0606	1.9315		2.4721
Western larch		3.6605		3.1382	2.3392	3.0523	3.1192	1.9901		2.5307
Lodgepole pine		3.5812	2.8264	3.0589	2.2599	2.9730	3.0399	1.9108	1.7232	2.4513
Grand fir				3.2032	2.4042	3.1172	3.1842	2.0550		2.5956
Western white pine				3.3438	2.5448	3.2579		2.1957		2.7363
Western redcedar				2.7946	1.9956	2.7086				
Western hemlock					2.2455	2.9586	3.0255			
Engelmann spruce			2.7679	3.0003		2.9144	2.9813	1.8522	1.6646	
Subalpine fir						2.9458	3.0128	1.8836	1.6960	2.4242
					Variable coefficients					
Variable										
$\ln(\Delta D+0.5)$	.99173	.56944	.63363	.46651	.49104	.43979	.50854	.33112	.95970	.79813
$\ln(D)$	-.35739	-.62543	-.64966	-.34633	-.09599	-.27165	-.36298	.05146	.02819	-.35941
$\ln(H)$	-.09613	-.09613	-.09613	-.09613	-.09613	-.09613	-.09613	-.09613	-.09613	-.09613

```
      SUBROUTINE HTGT(ISP,IHAB,D,H,DD,D2,H2)
C
C         ISP IS SPECIES SUBSCRIPT
C         IHAB IS HABITAT SUBSCRIPT
C         D IS DIAMETER
C         H IS HEIGHT
C         DD IS 10-YEAR PAST DBH GROWTH
C         D2 IS NEW DIAMETER ONE YEAR HENCE
C         H2 IS THE NEW HEIGHT ONE YEAR HENCE
      DIMENSION ASPE(10),AHAB(10),BHAB(10),CHAB(10),BKR(10)
C
C            SPECIES SUBSCRIPT   DEFINITION
C                    1           WHITE PINE
C                    2           WESTERN LARCH
C                    3           DOUGLAS FIR
C                    4           GRAND FIR
C                    5           WESTERN HEMLOCK
C                    6           CEDAR
C                    7           LODGEPOLE
C                    8           SPRUCE
C                    9           SUBALPINE FIR
C                   10           PONDEROSA PINE
C
      DATA ASPE /-0.731950,-0.937502,-0.996132,-0.872584,-1.03127,
     1          -1.28118, -1.01687, -1.07542, -1.04398, -0.67530/
C
C          HABITAT SUBSCRIPT     DEFINITION
C                  1             PSEUDOTSUGA / PHYSOCARPUS
C                  2             PSEUDOTSUGA / SYMPHORICARPOS
C                  3             PSEUDOTSUGA / CALMAGROSTIS
C                  4             ABIES GRANDIS / PACHISTIMA
C                  5             ABIES LASIOCARPA / PACHISTIMA
C                  6             ABIES LASIOCARPA / XEROPHYLLUM
C                  7             ABIES LASIOCARPA / MENZIESIA
C                  8             ABIES LASIOCARPA / VACCINIUM
C                  9             THUJA / PACHISTIMA
C                 10             TSUGA HETEROPHYLLA / PACHISTIMA
C
      DATA AHAB /4.59802,3.15211,3.84330,4.07574,4.05676,
     1          2.92763,2.74002,3.46821,3.27674,3.98983/
C
      DATA BHAB /.56943679,.99173111,.63362795,.46651012,.50854200,
     1          .33111995,.95970452,.79812711,.49103850,.43978649/
C
      DATA CHAB /-0.62542897,-0.35739297,-0.64966339,-0.34632784,
     1          -0.36297816,.051463176,.028194819,-.35940510,
     2          -.095985949,-0.27164507/
C
      DATA DSP /-0.0961271/
C
      DATA BKR /1.09,1.09,1.09,1.09,1.09,1.09,1.09,1.09,1.09,1.09/
C
      D2 = SQRT(D**2 + DD * (2. * D - DD) * BKR(ISP) / 10.)
C
      H2 = H + EXP(ASPE(ISP) + AHAB(IHAB) + BHAB(IHAB) * ALOG(DD + 0.05)
     1     + CHAB(IHAB) * ALOG(D) + DSP * ALOG(H)) * 0.11066
      RETURN
      END
```

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ALBERT R. STAGE
1975. Prediction of height increment for models of forest growth. USDA For. Serv. Res. Pap. INT-164, 20 p., illus. (Intermountain Forest and Range Experiment Station, Ogden, Utah 84401.)

Functional forms of equations were derived for predicting 10-year periodic height increment of forest trees from height, diameter, diameter increment, and habitat type. Crown ratio was considered as an additional variable for prediction, but its contribution was negligible. Coefficients of the function were estimated for 10 species of trees growing in 10 habitat types of northern Idaho and northwestern Montana.

OXFORD: 561.1; 564. KEYWORDS: height increment, tree-growth models, western white pine, western larch, Douglas-fir, grand fir, western hemlock, western redcedar, lodgepole pine, Engelmann spruce, subalpine fir, ponderosa pine, habitat type.

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Headquarters for the Intermountain Forest and Range Experiment Station are in Ogden, Utah. Field Research Work Units are maintained in:

- Boise, Idaho
- Bozeman, Montana (in cooperation with Montana State University)
- Logan, Utah (in cooperation with Utah State University)
- Missoula, Montana (in cooperation with University of Montana)
- Moscow, Idaho (in cooperation with the University of Idaho)
- Provo, Utah (in cooperation with Brigham Young University)
- Reno, Nevada (in cooperation with the University of Nevada)