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Effect of Diameter Limits and Stand Structure on Relative Density Indices: A Case Study

Robert O. Curtis

ABSTRACT

An understory of shade-tolerant species often develops in stands in the Douglas-fir region of western Washington and Oregon and can have a disproportionate effect on relative density indices, such as Reineke stand density index and Curtis relative density. The effects of such understories and of other departures from the even-aged condition are illustrated with selected stand data. In general, the summation methods are less influenced by departures from the even-aged condition than are the conventional calculatons based on quadratic mean diameter. Recommendations are made for consistent definition of the lower diameter limit of trees to be included in such computations

Keywords: density control, relative density, stand density index, stocking control, Reineke stand density index, Curtis relative density

Stand density measures are numerical values or indices that express the average degree of crowding (competition) existing within a forest stand. They are important tools in silviculture, most commonly as guides to thinning and stand density control. They are also related to suppression mortality, crown development, tree growth, understory development, and risk of windfall and snow breakage.

Although basal area or number of trees per unit area are often referred to as density measures and are easily determined, they do not-considered alone-express the degree of crowding (competition) existing within a stand. A given basal area or number of trees represents very different conditions in a young stand than in an old stand, or on a good site compared with a poor one. Any useful expression must combine basal area or number of trees with some measure of stage of stand development, such as average diameter, height, or site and age.

A considerable number of such combinations, or relative density measures, have been used or proposed. Most of these are basically quite similar, although they are derived by different routes and expressed in somewhat different forms (Curtis 1970, 1971, West 1982). Most are interpretable as explicit or implicit comparisons of average area available per tree in the subject stand compared with that in some reference stand condition of the same average diameter, height, or site and age. Most were derived from data from unimodal single-cohort stands with one principal species, and their application to other types of stand is somewhat problematic.

Those measures calculated from the diameter distribution are particularly useful because they do not require the frequently difficult and inaccurate determination of stand height, stand age, or site index. Two such measures that have had considerable use in the Pacific Northwest are Reineke stand density index (SDI) (Reineke 1933) and Curtis relative density (RD) (Curtis 1982). These are independent of site and age, or nearly so. (They are *not* expressions of productivity.) Although superficially different, these measures are actually very similar, and there is no clear basis for saying that one *is* better than the other.

This report considers some problems that arise in application of RD and SDI and makes some empirical comparisons of alternatives to conventional procedures that may alleviate these problems in applications to stand structures other than the simple single-cohort, single-species condition. The problems are likely to become considerably more important in the future as a result of the current popularity of various types of partial cuts in lieu of the older practice of clearcutting, with or without intermediate thinning.

Alternative Forms of RD and SDI

These are usually defined as follows (with all values on a unit area basis and coefficients assuming English units):

$$RD_{qmd} = BA/(QMD)^{0.5} \quad (1)$$

$$= 0.00545415T * TPA * (QMD)^{1.5},$$

$$SDI_{qmd} = TPA * (QMD/10)^{1.6} \quad (2)$$

$$= 0.02512 * TPA * (QMD)^{1.6},$$

where BA is basal area in square feet per acre, TPA is number of trees per acre, QMD is quadratic mean diameter in inches, and the subscript qrnd indicates the conventional calculation using QMD (as opposed to alternatives to be discussed).

Equations 1 and 2, the usual forms based on QMD, are nearly equivalent. They differ only by a scale factor and a small difference in the exponent. Given a value of either index and QMD and solving for TPA, 1/TPA is then an estimate of average area per tree m acres.

Both measures were derived from even-aged stands with one principal species (Douglas-fir *[Pseudotsuga menziesii* (Mirb) Franco var. menziesii] in the case of RD). Long and Daniel (1990) and

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Long (1995) pointed out that an alternative method of computation gives a related measure, which they designated by SDI * and which is here designated by the more descriptive SDI_{sum}'

$$SDI_{sum} = \sum(d_i/10)^{1.6}/\text{area} \quad (3)$$

$$= 0.02512 * \sum(d_i^{1.6})/\text{area},$$

where d_i is the diameter of an individual tree and summation is over all trees in the plot or stand larger than some specified small minimum diameter. This gives nearly the same value as the conventional computation when applied to a symmetrical diameter distribution. Values calculated by Equation 2 versus Equation 3 differ when diameter distributions are skewed or irregular, that for Equation 3 being smaller. The authors considered Equation 3 to be the more general form and one that is applicable to uneven-aged as well as even-aged stands.

Analogous to the Long and Daniel (1990) procedure, one can also calculate a value RD_{sum} as

$$RD_{sum} = 0.00545415 * \sum(d_i^{1.5})/\text{area}. \quad (4)$$

Curtis (1971) applied a variation of Chisman and Schumacher's (1940) tree area ratio (TAR) procedure to even-aged Douglas-fir, deriving the equation

$$TAR = 7.5432 * \sum(d_i^{1.55}) * (10^{-5})/\text{area} \quad (5)$$

and concluded that the simple expression $E(d_i^{1.55})$ per unit area could be used as a measure of relative density.

Equations 3, 4, and 5, which are calculated by summation of diameters raised to a power, are nearly equivalent, although they differ in their derivation. They are essentially the same as Chisman and Schumacher's (1940) TAR, with a power function substituted for the second-degree polynomial used in the original form. For the Douglas-fir data used in Curtis (1971), the two curve forms were practically identical. Equations 3, 4, and 5 differ only by scale factors, and small and probably inconsequential differences in the exponent.

Zeide (1983) argued that SDI should not be calculated from QMD but that a more appropriate method would use an average diameter D_R, calculated as

$$D_R = (\sum d_i^{1.6}/TPA)^{1/1.6} = (\sum d_i^{1.6}/TPA)^{0.625}, \quad (6)$$

and thence,

$$SDI_{DR} = 0.02512 * TPA * D_R^{1.6}. \quad (7)$$

Equation 7 gives values identical with those calculated with Equation 3 above, shown by Shaw (2000). Zeide's. D_R (Equation 6) can be interpreted as the diameter of the tree of mean area occupancy.

Shaw (2000), Ducey and Larson (2003), and Ducey (2009) discussed the behavior of conventional SDI_{qmd} (Equation 2) compared with that of SDI_{sum}(Equations 3) when applied to a number of theoretical diameter distributions. They showed that differences increase as the diameter distribution departs from the unimodal single-cohort condition, and that differences between the methods are strongly affected by truncation of the distribution. The definition of the lower limit of measurement can have a considerable effect on calculated values. These conclusions are equally applicable to RD_{qmd} versus RD_{sum}.

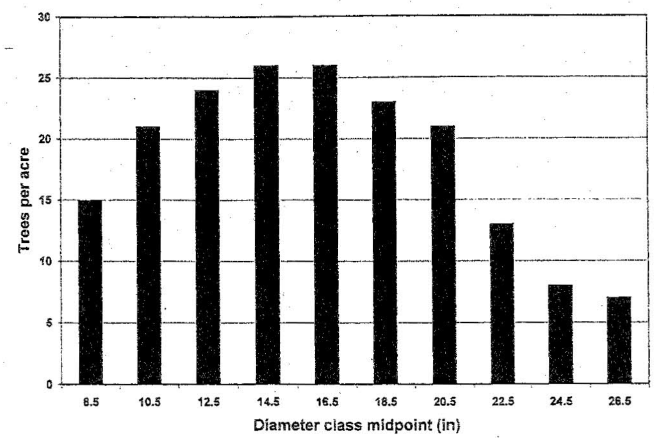

Figure 1. Diameter distribution in "normal" stand, site II, age 100, from McArdle et al. (1961).

In the following sections, parallel computations of RD and SDI are made using the conventional and summation procedures, with different minimum diameters, for selected examples of actual stand data illustrating the effect of contrasting stand structures. Similar computations are also done for a large set of rather heterogeneous older stand data, with comparisons made in terms of overall averages.

Empirical Comparisons

Case 1: Even-Aged, Well-Stocked, Unmanaged Stand

This used the stand table given in Table 11 of McArdle et al. (1%1) for site II, age 100. The diameter distribution is only slightly skewed, as shown in Figure 1. The corresponding calculated values were: QMD = 16.92 in.; RD_{qmd} = 69.8; SDI_{qmd} = 427; RD_{sum} = 67.7; and SDI_{sum} = 416, Results of the computations based on QMD are only slightly different from those using the summation method, although the latter values are slightly smaller, as expected.

Note, however, that understory trees were excluded (as stated specifically by McArdle and others [1961]). Their inclusion might have produced somewhat different results, as discussed in following sections.

Case 2: Heavily Thinned Douglas-Fir with an Understory of Much Smaller Stems of Shade-Tolerant Species

Example: Iron Creek LOGS, Treatment 1

This example uses data from the 1994 measurement of treatment 1 in the Iron Creek Levels-of-Growing-Stock (LOGS) study (Curtis and Marshall 2009) . This is a uniform site II Douglas-fir plantation, age 47 in 1994. Repeated heavy thinning has resulted in establishment of a younger cohort of the shade tolerant species hemlock *(Tsuga heterophylla* [Raf.] Sarg.) and redcedar *(Thuja plicata* Donn ex D. Don) (Figure 2), under a rather open overstory of Douglas-fir (Figure 2).

Table 1 compares values calculated for (1) the Douglas-fir main stand only, understory excluded, (2) all trees 2:1.6 in., and (3) trees larger than D40/4, where D40 is mean diameter of the largest 40 TPA, 18.97 in. in this case. Unlike QMD, D40 is not influenced by understory trees. Curtis and Clendenen (1994, p. 11) suggested D40/3 as a basis for excluding understory trees; D40/4 is used here, as a more conservative value. Exclusion of understory has a relatively small effect on basal area butlarge effects on TPA and QMD, which

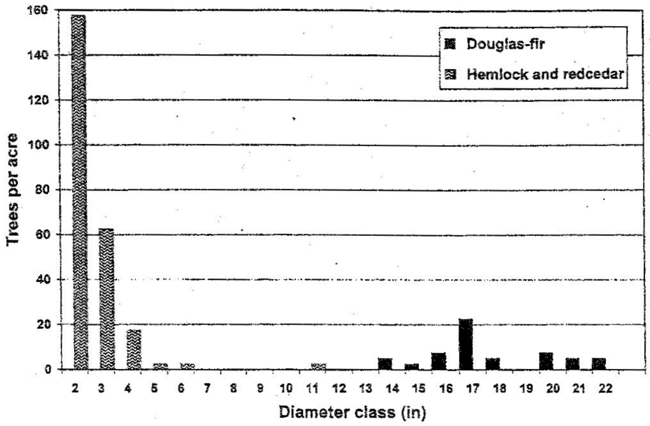

Figure 2. Diameter distribution in Iron Creek LOGS, treatment 1, age 47.

Table 1. Comparison of values for Iron Creek LOGS study, treatment 1, age 47.

Measure	Trees ≥1.6 in.	Understory[a] excluded	Trees ≥D40/4 = 4.75 in.
Basal area (ft^2/ac)	115.3	106.4	107.2
TPA	305.5	62.5	67.0
QMD (in)	8.3	17.8	17.1
RD_{qmd}	40.0	25.2	25.9
RD_{sum}	30.4	25.1	25.4
SDI_{qmd}	228	155	158
SDI_{sum}	182	154	156

[a] Ingrowth, not present at study establishment.
TPA, trees per acre; QMD, quadratic mean diameter; RD_{qmd}, Curtis relative density; RD_{sum}, summation form of Curtis relative density; SDI_{qmd}, Reineke stand density index; SDI_{sum}, summation form of Reineke stand density index; D40, mean diameter of the largest 40 TPA.

also affect RD_{qmd} and SDI_{qmd}, increasing them considerably compared with RD_{sum} and SDI_{sum}. Differences disappear when understory stems are excluded.

Example: Mount Walker Thinning, Plot 7

Plot 7 from the old Mount Walker thinning study provides an example of a two-cohort stand developing after heavy thinning. This site IV stand has developed very slowly. The stand was thinned very heavily in 1934 at age 60. It was originally nearly pure Douglas-fir. A few windfalls were removed about 1950, otherwise there has been no further treatment.

Subsequent to thinning, there was abundant natural regeneration of hemlock and redcedar, plus a few Douglas-fir. When remeasured in 1991, a few moribund small Douglas-fir were still present. The hemlock and redcedar have continued development in the understory, with a few reaching merchantable size. Hence, the distinction between main stand and understory is becoming somewhat indistinct. The stand diameter distribution in 1991 consisted of an even-aged original cohort with a peak around 22 in., plus a tail of much younger trees in the 2-10-in. diameter range (Figure 3). One cannot argue that the younger cohort has no influence on the older cohort, but effects are probably not large. The general impression in 1991 was that the stand was in excellent condition, closed but vigorous and not excessively dense.

Basal area, TPA, QMD, RD_{qmd}, RD_{sum}, SDI_{qmd}, and SDI_{sum} were calculated for several stand components (Table 2). Inclusion of the "tail" of small trees gives very different values of QMD, RD_{qmd}, and SDI_{qmd} compared with those obtained when trees smaller than

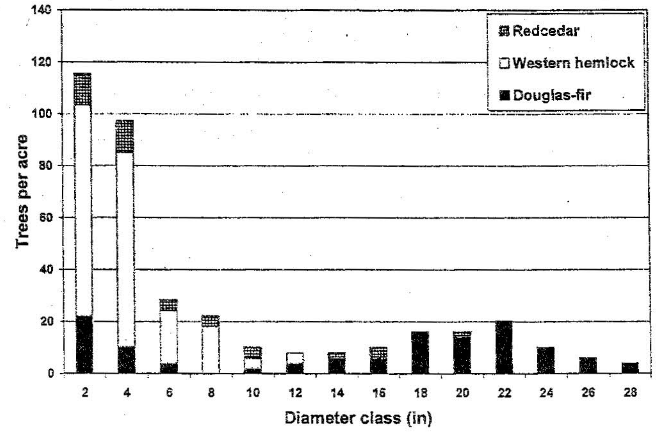

Figure 3. Diameter distribution in Mount Walker plot 7, 1991.

Table 2. Comparison of values for Mt. Walker thinning, age 117.

Measure	Trees ≥1.6 in.	Trees ≥D40/4 = 5.88 in.
Basal area (ft^2/ac)	250.8	238.0
TPA	373.2	146.0
QMD (in.)	11.1	17.3
RD_{qmd}	75.3	57.2
RD_{sum}	61.1	54.2
SDI_{qmd}	441	351
SDI_{sum}	371	336

Abbreviations are as defined in Table 1.

D40/4 are excluded. For this stand, reasonable values are obtained by the summation method or by the conventional calculation using QMD after omitting the 2-, 4-, and 6-in. classes. Estimates of RD -55-60 and SDI -350 seem consistent with visual impressions of the stand.

Example: Black Rock Plot 31

This 1.0-ac plot was established in 1957 by the late Alan Berg of Oregon State University, in a Douglas-fir stand on an excellent site. It was thinned very heavily at age 48, to a residual stand of 53 TPA, and underplanted with hemlock in 1959 in an effort to create a two-storied stand (Tappeiner et al. 2007, p. 356-359). The most recent available complete measurement that includes the hemlock understory was made at the end of the 1990 growing season, age 81.

This stand has a very pronounced bimodal distribution (Figure 4), in which nearly all the volume and most of the growth is in the overstory Douglas-fir. D40 was 33.9 in.; only one hemlock tree was greater than D40/4 = 8.48. Hemlock made up 21% of the basal area, 8% of the standing cubic volume, 39% of the $Ed_i^{1.5}$, and 35% of the $Ed_i^{1.6}$ in 1990 but only 11% of the cubic volume increment in the most recent growth period (1985-1990). Statistics similar to those given earlier were calculated for (1) the Douglas-fir component, (2) the hemlock component, and (3) all trees combined (Table 3).

This is an extreme example of the effect of numerous understory trees on QMD and on density measure computations based on QMD. The additive forms SD1surnand RDsum clearly give values that are more consistent with the visual impression and vigorous condition of the stand than do the forms based on QMD.

One can plausibly hypothesize that volume increment should be proportional to growing space occupancy. However, volume increment of the hemlock component is a much smaller proportion of the

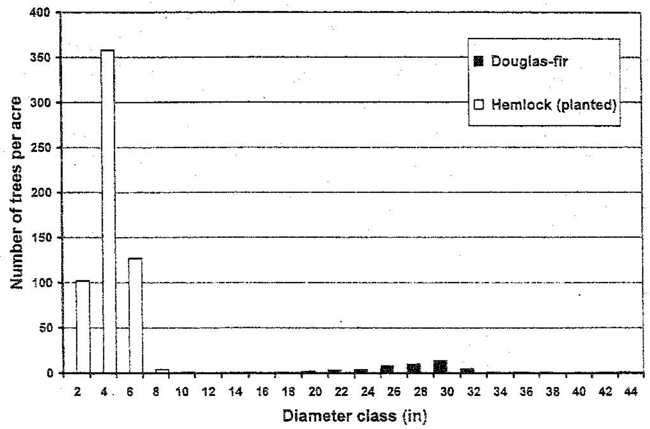

Figure 4. Diameter distribution in 1990 on Black Rock plot 31, age 81.

Table 3. Comparison of values for Black Rock plot 31, age 81.

Measure	Douglas-fir	Trees ≥1.6 in	Trees ≥D40/4 = 8.48 in.
Basal area (ft^2/ac)	225.6	280.2	226.4
TPA	51.0	690.0	52.0
QMD (in.)	28.48	8.63	28.25
RD_{qmd}	42.3	95.4	42.6
RD_{sum}	41.9	68.5	42.0
SDI_{qmd}	272	545	274
SDI_{sum}	270	411	271

Abbreviations are as defined in Table 1.

total (11 %) than are the hemlock values of any of the several forms of RD and SDI. Values of RD or SDI for a given stand represent averages over all trees in the stand, and these averages are proportional to stand averages of the approximately 1.5-1.6 power of diameters. Additivity of the summation forms SDI_{sum} and RD_{sum} does not necessarily justify interpretation of values of $d_i^{1.5}$ or $d_i^{1.6}$ for individual trees, or of $Ed_i^{1.5}$ or $Ed_i^{1.6}$ for segments of a diameter distribution, as measures of the growing space occupancy or competitive influence of that tree or segment of the distribution.

Case 3: Selected Examples of Multicohort Stands

The following sections use data from a series of 113 plots established by the Washington State Adaptive Management Program in a study titled *Validation of the Western Washington Riparian Desired Future Condition Performance Targets in the Washington State Forest Practice Rules with Data from Mature, Unmanaged, Conifer-dominated Riparian Stands,* under the Washington Department of Natural Resources Cooperative Monitoring Evaluation and Research program (not published). These data are hereafter referred to as the desired future condition (DFC) data.

The plots were selected as unmanaged stands near fish-bearing streams, in the 80-200-year age range, that represented conditions considered desirable from a stream and fisheries protection standpoint. Plot sizes ranged from 0.23 to 0.56 ac, depending on estimated site class and on stream width. There was no evidence of cutting since stand establishment. The stands include a wide range of stand structures and species mixtures and therefore provide an opportunity to examine the behavior of stand density measures when applied to conditions other than the even-aged, single-species condition from which they were derived.

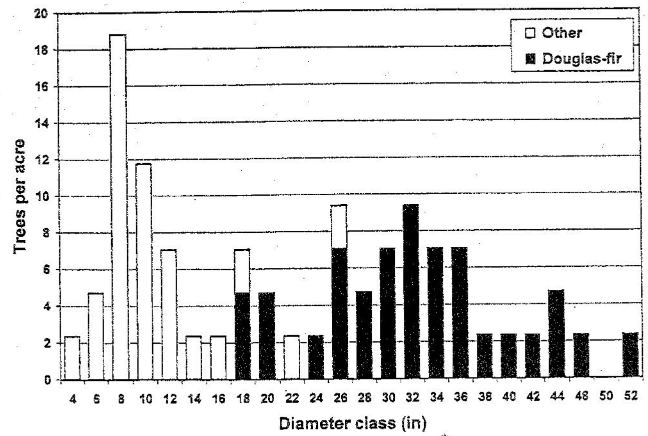

Figure 5. Diameter distribution in DFC plot 17851.

Table 4. Comparison of values for DFC plot 17851.

Measure	Trees ≥4.0 in.	Suppressed excluded	Trees ≥D40/4 = 9.5 in.
Basal area (ft^2/ac)	477.6	469.0	470.0
TPA	126.9	101.1	101.1
QMD (in.)	26.3	29.2	29.2
RD_{qmd}	93.2	86.8	86.9
RD_{sum}	85.0	82.0	82.2
SDI_{qmd}	595	560	561
SDI_{sum}	551	534	535

Abbreviations are as defined in Table 1.

The lower limit of diameter measurement was 4 in. Trees were classified by crown class. Understory was not explicitly recognized as a category, but inspection of the tree lists suggests that the "suppressed" category is more or less equivalent.

Graphs of the diameter distributions were made for each of the 113 plots. The two examples shown below were selected as examples of older stands with markedly different structures. Some comparisons were then made using the full set of 113 plots.

Example: DFC Plot 17851

This stand is composed of an even-aged cohort of Douglas-fir of about age 130 and younger hemlock and redcedar of highly variable ages (Figure 5). It represents the common condition of an initially even-aged Douglas-fir stand with a younger uneven-aged component of shade-tolerant species.

As expected, the summation method gives consistently lower values than. the traditional method based on Q}.\1D (Table 4). *As* in the previous example, values calcul~ted by conventional methods after excluding suppressed trees or excluding trees smaller than *D40/4* are dose to each other and dose to the values obtained by summation without exclusions.

Example: DFC Plot 4122

This has a J-shaped diameter distribution implying a more or less all-aged structure, and is primarily composed of western hemlock with a substantial component of Pacific silver fir *(Abies amabilis* Dougl. ex Forbes) including some large trees (Figure 6). Calculated value of D40 was 25.54 in. The oldest tree bored was -130 years old, but others were quite variable and usually much younger. The diameter distribution suggests a very prolonged regeneration period following a stand replacement disturbance.

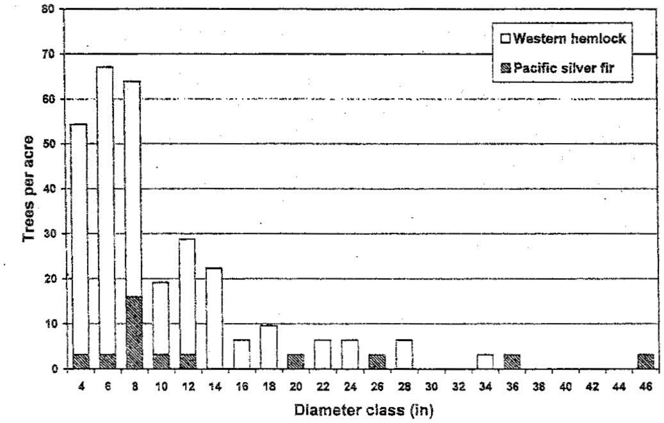

Figure 6. Diameter distribution in DFC plot 4122.

Table 5. Comparison of values for DFC plot 4122.

Measure	Trees ≥4.0 in.	Suppressed excluded	Trees ≥D40/4 = 6.4 in.
Basal area (ft^2/ac)	285.7	279.2	278.6
TPA	306.8	230.1	226.9
QMD (in.)	13.1	14.9	15.0
RD_{qmd}	79.0	72.2	71.9
RD_{sum}	67.5	63.9	63.7
SDI_{qmd}	470	435	434
SDI_{sum}	411	392	391

Abbreviations are as defined in Table 1.

As expected, the summation method gives consistently lower values than the conventional method based on QMD (Table 5). Values calculated from QMD after excluding either suppressed trees or trees smaller than *D40/4* are close to each other and to the values calculated by summation without exclusions.

Case 4: Comparisons of Means for the 113 Unmanaged Plots in the DFC Data Set

Alternative calculations were done for each of 113 plots in the DFC data set. As before, values calculated by summation are substantially lower than those calculated by the conventional formulas using QMD (Table 6). On average, results from the summation procedure are nearly the same as those obtained by deleting suppressed trees and then applying the conventional formulas. For comparison, values of RD_{sum} calculated from the original data used as the basis for McArdle et al. (1961) have a mean of 62 with SD 8.8. Values of RD_{qmd} for the same data are 64 with SD 9.0.

Case 5: Washington Department of Natural Resources Stand Inventory Data

Following the above work, some comparisons were made using data for stands with apparently unreasonable values of SDI, taken from Washington Department of Natural Resources stand inventory data. These data differed from the above examples in that all trees with measurable dbh values (>0.1 in) were included. The example given below illustrates the effect of minimum measurement diameter on relative density measures.

Example: Washington Department of Natural Resources Stand 103461

This is an.older, more or less all-aged stand, mainly composed of hemlock, redcedar, Pacific silver fir, and some Sitka spruce *(Picea sitchensis* [Bong. J Carr.), It has a declining j-shaped diameter distribution similar to DFC 4122 above, with some large trees and very large numbers of small trees (Table 7). Inclusion of all trees produces preposterous values of RD_{qmd} and SDI_{qmd}, and unlikely values of RD_{sum} and SDI_{sum}. Values of RD_{sum} and SDI_{sum} for trees 2:1.6 in. seem entirely reasonable. As minimum measurement diameter increases above 1.6 in., RD_{sum} and SDI_{sum} decline only slightly. In contrast, values of RD_{qmd} and SDI_{qmd} decline sharply with increasing minimum diameter. Values of RD_{qmd} and SDI_{qmd} calculated for trees larger than *D40/4* are virtually the same as the values of RD_{sum} and SDI_{sum} calculated for all trees 1.6 in. and larger.

Discussion

SDI_{qmd} has a long history of successful use in even-aged stands throughout North America, as.a guide to stand density control, as a predictor variable in growth equations, and as a measure of the intensity of within-stand competition. More recently, RD_{qmd} has also had extensive use in Douglas-fir for similar purposes.

Both measures were originally derived from even-aged unimodal stands with one principal species. In such stands, SDI_{sum} and RD_{sum} have nearly the same values as SDI_{qmd} and RD_{qmd}. Their values can differ markedly when applied to uneven-aged or multilayer stand structures. It seems plausible to assume that any generaliy satisfactory form applied to such stands should yield values consistent with the range of values found in even-aged stands of similar species composition.

Figure 4 in Reineke (1933) indicates maximum SDI_{qmd} for Douglas-fir as about 600 (number of stems at 10-in. QMD), which corresponds to an RD_{qmd} of about 100. A number of more recent authors have also given estimates, those for Douglas-fir being usually slightly lower than Reineke's, whereas those for hemlock are somewhat higher than for Douglas-fit (Long 1985). Experience with untreated even-aged Douglas-fir on small research plots (usually selected for uniformity) with appreciable suppression mortality indicates maximum RD values in the range of 70-85, but with occasional extremely dense stands having values up to 100. Averages for stands of substantial area would generally be somewhat lower.

This study examined examples of seemingly unreasonable results, in an effort to identify how extreme values arise and to consider whether minor procedural changes that are consistent with the underlying rationale of relative density measures would reduce or eliminate implausible values. For relative density measures to be useful as guides to stand management, consistent procedures must be followed in deriving guides and in applying them in stand management.

The most common practical application of RD or SDr is as a guide to density control in young even-aged stands, often plantations. A common problem in application of RDqmd and SDlqrnd arises from the presence of a shade-tolerant understory in stands that are essentially even-aged. Many even-aged Douglas-fir stands, particularly those that have been thinned, develop an abundant understory of small stems of shade-tolerant species, such as hemlock and redcedar. These small understory stems contribute little to stand basal area and volume or to basal area growth and volume growth, and appear to have little effect on competition and on growth of the main stand. In general, their inclusion will produce a negligible increase in basal area but a large increase in TPA and a corresponding decrease.in QMD, which increases RDqrndand SDlqmd in a way that may have little relation to the overall stand competition level. If

Table 6. Comparison of means, standard deviations, and maximum and minimum values of RD and SDI calculated by alternative procedures for the 113 plots in the DFC data set.

	RD_{qmd}, trees ≥4 in.	RD_{qmd} without suppressed	RD_{qmd}, trees ≥D40/4	RD_{sum}, trees ≥4 in.	RD_{sum}, without suppressed	RD_{sum}, trees ≥D40/4	SDI_{qmd}, trees ≥4 in.	SDI_{qmd} without suppressed	SDI_{qmd}, trees ≥D40/4	SDI_{sum}, trees ≥4 in.	SDI_{sum}, without suppressed	SDI_{sum}, trees ≥D40/4
Mean	78.6	69.0	72.6	70.9	64.7	67.5	485	433	453	444	408	425
SD	17.5	15.1	15.8	15.9	14.2	15.3	107	95	100	98	91	95
Maximum	126.4	106.8	117.0	114.1	99.62	110.8	772	673	744	674	648	656
Minimum	35.4	30.4	34.3	31.8	28.76	31.4	218	191	213	200	182	197

Abbreviations are as defined in Table 1.

Table 7. Comparison of values based on alternative minimum measurement diameters, for Department of Natural Resources stand 103461.

Measure	Trees ≥0.1 in.	Trees ≥1.6 in.	Trees ≥3.6 in.	Trees ≥ D40/4 = 7.6 in.
Basal area (ft^2/acre)	361.7	356.0	352.1	345.4
TPA	6002.7	370.2	188.5	134.0
QMD (in.)	3.32	13.3	18.5	21.7
RD_{qmd}	198.5	97.7	81.8	74.1
RD_{sum}	105.3	74.6	71.8	68.8
SDI_{qmd}	1028	583	505	464
SDI_{sum}	607	466	452	436

Abbreviations are as defined in Table 1.

RD_{qmd} or SDI_{qmd} is used as a guide to thinning such stands, understory should be excluded. In relatively young stands with an even-aged main cohort, the distinction between main stand and understory is often self-evident; understory stems are small and often below the measurement limit used in stand examinations. Hence, this is not usually a problem in such stands.

In older stands, the distinction between main stand and understory often becomes less clear, and it can be meaningless for uneven-aged multilayered stands. There may be a place for some alternative rules for stems to be excluded from density calculations.

For diameter distributions other than the more or less symmetrical unimodal distribution of the idealized even-aged stand, values of measures calculated by the summation procedures are consistently less than those calculated using QMD, corresponding to the difference between conventional QMD and Zeide's (1983) D_R. This difference decreases markedly if the diameter distribution is truncated to eliminate the smallest trees (Ducey 2009), as shown in Tables 1-7.

There is no objective test applicable to one-time measurements such as these that can lead to a clear conclusion that one particular method is best in all situations. However, one can make some judgments based on apparent reasonableness and on consistency of values with the ranges and limits of values observed in the even-aged condition from which these measures were originally derived.

These trials suggest that exclusion of stems smaller than *D40/4*. will usually give values of RD_{qmd} and SDI_{qmd} that are consistent with those from subjective exclusion of understory, and consistent with RD_{sum} and SDI_{sum} values based on trees 1.6 in. and larger. This may not always be a feasible or meaningful procedure with highly irregular stands that have very large trees of an older cohort, as will become increasingly common with the practice of green tree retention.

Mean values obtained for the DFC data using either the summation procedure, or the QMD procedure after elimination of trees smaller than *D40/4* or of trees classed as "suppressed," appear generally consistent with mean values observed elsewhere for well-

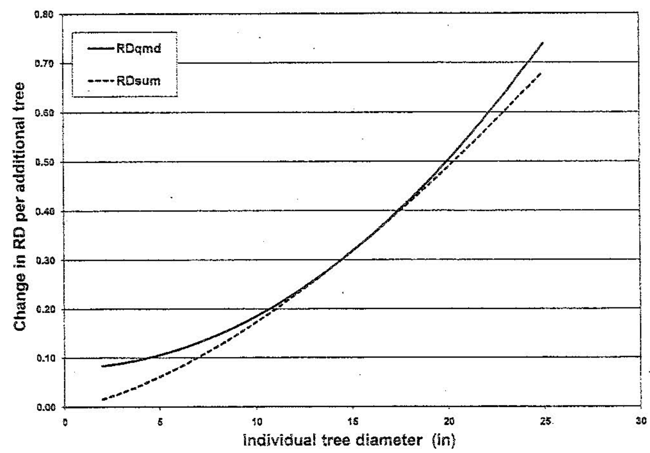

Figure 7. Relative effect of extreme diameters on estimates of quadratic mean diameter of Curtis relative density (RD_{qmd}) and sum of Curtis relative density (RD_{sum}).

stocked even-aged stands and have smaller means and smaller standard deviations than the corresponding values calculated using all trees and QMD. Although the relatively small size of plots relative to the large trees present may increase variability, the overall means of RD_{sum} and SDI_{sum} given for the DFC data (Table 6) are consistent with recognized "normal" stand values, The general results of these empirical comparisons using actual plot data are consistent with results reported by Ducey (2009) for comparisons using theoretical diameter distributions.

There is little practical difference between relative density measures calculated with the 1.5 versus 1.55 versus the 1.6 power of diameter. There are also near-perfect correlations (R = 0.99) between either $d_i^{1.6}$ or $d_i^{1.5}$ and maximum crown areas estimated from the Paine and Hann (1982) maximum crown width equation for Douglas-fir, which is the basis for crown competition factor (CCF) values for Douglas-fir used in the Pacific Northwest Coast and West Cascades variants of the Forest Vegetation Simulator (Dixon 2009). CCF is essentially the same as RD_{sum} and SDI_{sum}. except for scale factors.

Although the DFC data include a considerable range in stand structures (and skewness) and species composition, a regression of the form $\ln(TPA) = a + b \times \ln(QMD)$ fit to the pooled DFC data produced an estimated slope coefficient of $b = -1.6$, consistent with that expected as an average of even-aged stands. The differences between the QMD and summation methods of calculation are illustrated by Figure 7, which shows the change in the estimate that results from the addition of one tree of a given diameter. Here, the curve for RD_{sum} shows the contribution of one additional tree of diameter d_i, based on Equation 4 above. The curve RD qmd gives the corresponding change calculated for a stand with QMD of 15 in.

and TPA of 175 using Equation 1. The estimates are equal for a tree of diameter QMD, but the change in RD based on QMD for one additional tree increases relative to that for RD_{sum} as the difference between d_i and QMD increases. Consequently, RD_{qmd} values are more influenced by the extreme diameters found in skewed diameter distributions and are generally higher than are RD_{sum} values.

Conclusions

The conclusions of this study are as follows.

1. The minimum diameter-limit of measurement used can have major effects on numerical values of QMD-based relative density measures. This effect becomes important when QMD-based relative density measures are calculated for stands with diameter distributions markedly different from the symmetrical unimodal distribution of the idealized even-aged stand.
2. In general, trees smaller than 1.6 in. dbh should be excluded from computation of relative density measures. This is consistent with common (though not universal) practice on the research plots from which reference values are often derived and will tend to reduce problems associated with the presence of numerous very small trees in the understory.
3. When an understory of numerous small stems is clearly distinguishable from a main, basically even-aged stand (a common situation in young thinned Douglas-fir stands), the understory should be excluded from computation of RD_{qmd} or SDI_{qmd}. The differences between RD_{qmd} and RD_{sum} and between SDI_{qmd} and SDI_{sum} will then be much reduced and usually inconsequential.
4. In older stands, a similar result can be obtained by a suitable choice of minimum diameter, although this becomes a subjective judgment for the individual stand. In general, *D40/4* appears to a be a reasonable break point for specifying stems to be excluded from computation and an objective basis that could be incorporated in procedures for processing stand inventory data in quantity.
5. The summation forms RD_{sum} and SDI_{sum} are less affected by presence of a "tail" of numerous small understory trees, or by other departures from the more or less symmetrical frequency distribution of even-aged stands, than are the conventional forms RD_{qmd} and SDI_{qmd}. The summation forms are therefore generally preferable, and will give results comparable to the QMD-based forms when applied to truly even-aged stands.
6. The principal argument for continuing use of RD_{qmd} and SDI_{qmd} is simply that QMD and BA are conventional statistics produced by most summary programs and readily available from existing records, whereas RD_{sum} and SDI_{sum} are not. It would be desirable to provide for computation of D40, RD_{sum}, and SDI_{sum} in standard summary programs.
7. For Douglas-fir, there is little practical difference between stand density measures calculated with the 1.5 versus 1.55 versus 1.6 powers of diameter, other than scale factors.

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