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Effects of stochastic interest rates in decision making under risk: A Markov decision process model for forest management

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A B S T R A C T

Most economic studies of forest decision making under risk assume a fixed interest rate. This paper investigated some implications of this stochastic nature of interest rates. Markov decision process (MDP) models, used previously to integrate stochastic stand growth and prices, can be extended to include variable interest rates as well. This method was applied to Douglas-fir/western hemlock forests in the Pacific Northwest of the United States. An MDP model was used to find the harvest decisions that maximized the forest value of a stand in a particular state, given the price level and interest rate. This optimal policy was compared with the policy that would hold in the same context but with an interest rate fixed at its historical average. The results showed that when the interest rate was lower than its historical average, the best decision differed for 52 of the 192 possible combinations of stand state and price level. Assuming a fixed interest rate underestimated the forest value by 4% to 16% depending on the initial condition. However, applying the harvest policy derived with a fixed interest rate led to a loss of no more than 7% depending on the initial condition. Taking explicit account of the variability of interest rate in setting harvest policies had some unexpected ecological benefits.

1. Introduction

When evaluating the economic performance of forestry projects, it is a common practice to discount future monetary benefits and costs with a fixed real interest rate (e.g. Bettinger et al., 2009). It is gradually being accepted that future ecological benefits and costs should also be discounted in a similar manner (Howarth, 2009).

However, like forest growth and prices, interest rates are subject to random variations. For example, from 1939 to 2007 the real interest rate of AAA Corporate Bonds in the United States has varied between −13% and +10% per year (Fig. 1). Thus, interest rates can hardly be predicted, except in a probabilistic fashion. Much work has been devoted to this random process (e.g. James and Webber, 2000) and its implications for investments (e.g. Martellini et al., 2003). Forestry is a special long-term investment in which the main cost is the interest-dependent capital cost embodied in the standing trees.

Other kinds of risk have been dealt with extensively in the literature with various methods: biological risk in forest growth (e.g. Hof et al., 1996; Pickens et al., 1991; Clarke and Reed, 1989), fluctuation of prices (e.g. Forboseh et al., 1995; Thomson, 1992), and catastrophes such as fire (e.g. Hurteau et al., 2009). Zhou and Buongiorno (2006) integrate biological, price, and catastrophic risk in MDP models, one of the few feasible methods for multi-dimensional risk optimization (Insley and Rollins, 2005). But most studies have discounted future costs and returns with a fixed interest rate.

Among the few that recognized stochastic interest rates, Liang et al. (2006) use stochastic simulation and trend surface analysis to compare management regimes. Alvarez and Koskela (2003) show that interest rate variability increases the optimal rotation and improves the value of the optimal policy, in a Wicksellian single-rotation context. In a similar context, Alvarez and Koskela (2006) study the effect of a stochastic interest rate on the optimal harvest and rotation when the landowners are risk averse. They find that interest rate volatility increases the optimal harvesting threshold while risk aversion decreases it. There is still a need to develop practical optimization methods to handle multi-dimensional risk, including interest rate variations.

Buongiorno and Zhou (2009) extend the classical MDP model with discounted returns to incorporate interest rate risk. Their MDP model is a generalization of Faustmann's formula (1849). Faustmann's formula gives the value of land or forests in a steady-state, deterministic environment, with an infinite planning horizon. The MDP model does the same in a stochastic environment described by Markov chains. Due to their generality and standard form, MDP models can be readily solved by linear or dynamic programming and the results are expressed in practical decision tables connecting best current action to the observed stand state, price level, and interest rate.

The objective of this paper was to investigate some practical consequences of acknowledging the variability of interest rate in forest

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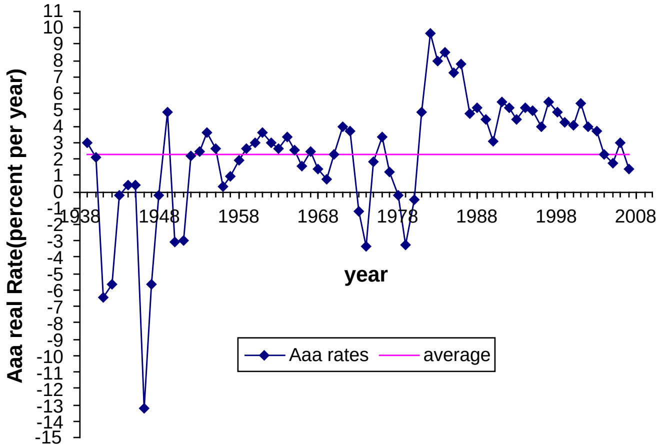

Fig. 1. Annual real rate of return on Aaa corporate bonds from 1938 to 2007 (Economic Report of the President 2009 http://www.gpoaccess.gov/eop/tables09.html).

management. First, how important is it for forest valuation? How does the introduction of a stochastic interest rate change the forest value, inclusive of land and initial growing stock? Second, what is the cost, in terms of forest value foregone, of adopting a policy that assumes a constant, average, interest rate, compared to one that recognizes explicitly its variations? The effect of interest rate variability on non-economic criteria such as landscape diversity, tree size, old growth area, and wildlife habitat, will also be briefly examined.

We addressed these issues for Douglas-fir/western hemlock (*Pseudotsuga menziesii*/*Tsuga heterophylla*) forests in the Pacific Northwest of the United States. Rich in timber and biological diversity (Franklin, 1988), these forests are under constant scrutiny, and precise financial methods and data are an important part of any assessment of their future management and conservation. The next section of the paper describes the Markov chain models of stand growth, stumpage price and interest rate, followed by linear-programming formulations of the optimization problem with fixed or stochastic interest rate. This is followed by the empirical results of an application to maximize expected net present value in Douglas-fir/western hemlock forests, with a fixed and stochastic interest rate, respectively. The conclusion stresses the importance of recognizing the variability of interest rate variation for forest valuation.

2. Methods and data

2.1. Markov chain models of stand growth, price, and interest rate

The model to predict forest stand growth was built from data on 14,794 plots in Oregon and Washington (Zhou et al., 2008). At any yearly decision time, a stand could be in one of 64 possible states. A state was defined by the basal area of shade-intolerant (mostly Douglas-fir) and shade tolerant (mostly western hemlock) trees in three diameter sizes: *small* (10 cm ≤ dbh < 25 cm), *medium* (25 cm ≤ dbh < 41 cm), and *large* (dbh ≥ 41 cm). For each species-size class the basal area was *low* (indicated by 0) if it was less than the average basal area over the plots used to build the model, or *high* (indicated by 1) otherwise. Thus, each stand state was represented by a string of six digits, such as 100,011. The first three digits referred to the basal area of shade-intolerant trees in the *small*, *medium*, and *large* trees, while the last three digits referred to the same for the shade-tolerant trees. Table 1 shows the expected basal area and volume per ha by stand state, and the frequency of the plots in each state over the region considered.

Stand growth was described by a matrix giving the probability that a stand would move from one state to another in one year (Table 2).

Table 1
Stand state definition, frequency of Forest Inventory and Analysis (FIA) plots, expected volume, and basal area.

Stand state #	Basal area by tree species and size[a]	Frequency of FIA plots by stand state	Expected volume (m^3 ha^{-1})	Expected basal area (m^2 ha^{-1})	Stand state #	Basal area by tree species and size[a]	Frequency of FIA plots by stand state	Expected volume (m^3 ha^{-1})	Expected basal area (m^2 ha^{-1})
1	000,000	0.066	234.2	18.2	33	100,000	0.055	248.2	21.1
2	000,001	0.002	385.6	50.4	34	100,001	0.001	399.5	53.5
3	000,010	0.011	351.9	28.7	35	100,010	0.005	365.9	31.1
4	000,011	0.011	403.3	50.2	36	100,011	0.004	417.2	53.7
5	000,100	0.031	245.3	20.9	37	100,100	0.016	259.3	26.3
6	000,101	0.002	396.7	54.0	38	100,101	0.001	410.7	58.7
7	000,110	0.025	363.1	30.3	39	100,110	0.012	377.0	34.0
8	000,111	0.027	414.4	53.7	40	100,111	0.011	428.4	56.4
9	001,000	0.004	375.3	49.3	41	101,000	0.002	389.3	55.6
10	001,001	0.024	426.7	63.6	42	101,001	0.013	440.6	66.5
11	001,010	0.003	393.0	49.8	43	101,010	0.001	407.0	54.3
12	001,011	0.021	444.4	63.5	44	101,011	0.011	458.3	65.3
13	001,100	0.004	386.4	52.2	45	101,100	0.002	400.4	57.7
14	001,101	0.024	437.8	65.0	46	101,101	0.017	451.7	67.3
15	001,110	0.005	404.2	52.3	47	101,110	0.003	418.1	54.7
16	001,111	0.043	455.5	63.9	48	101,111	0.028	469.5	66.8
17	010,000	0.015	353.8	32.3	49	110,000	0.041	367.8	35.3
18	010,001	0.002	405.2	50.4	50	110,001	0.003	419.1	57.4
19	010,010	0.007	371.5	37.9	51	110,010	0.013	385.5	40.7
20	010,011	0.007	422.9	52.9	52	110,011	0.010	436.9	56.6
21	010,100	0.008	365.0	35.9	53	110,100	0.024	378.9	38.7
22	010,101	0.002	416.3	53.8	54	110,101	0.004	430.3	61.9
23	010,110	0.012	382.7	39.7	55	110,110	0.026	396.6	44.0
24	010,111	0.016	434.0	53.8	56	110,111	0.025	448.0	58.9
25	011,000	0.013	394.9	50.9	57	111,000	0.022	408.9	55.1
26	011,001	0.018	446.3	62.4	58	111,001	0.025	460.2	65.0
27	011,010	0.008	412.6	51.4	59	111,010	0.011	426.6	56.1
28	011,011	0.018	464.0	62.0	60	111,011	0.015	477.9	64.3
29	011,100	0.011	406.0	53.8	61	111,100	0.019	420.0	57.9
30	011,101	0.017	457.4	64.5	62	111,101	0.023	471.4	66.6
31	011,110	0.016	423.8	54.8	63	111,110	0.022	437.7	58.5
32	011,111	0.032	475.1	63.1	64	111,111	0.037	489.1	65.8

[a] The first three digits refer to small, medium or large trees of the shade-intolerant species. The last three digits refer to the same for shade-tolerant species. 0 = low basal area, 1 = high basal area.

Table 2
Transition probabilities between stand states.

Stand state # at time t[a]	Stand state # at $t+1$ year and its probability()[b]
1	1(0.79), 3(0.01), 5(0.06), 17(0.01), 33(0.11)
2	2(0.78), 4(0.03), 6(0.07), 10(0.02), 14(0.01), 18(0.04), 34(0.04)
3	1(0.02), 3(0.80), 4(0.02), 7(0.07), 11(0.01), 19(0.03), 35(0.03)
4	2(0.01), 4(0.88), 8(0.05), 12(0.01), 20(0.02), 36(0.02)
5	1(0.04), 5(0.82), 7(0.07), 21(0.02), 37(0.04)
6	2(0.03), 4(0.01), 6(0.82), 8(0.04), 10(0.01), 14(0.02), 22(0.03), 34(0.01), 38(0.04)
7	3(0.03), 5(0.01), 7(0.84), 8(0.03), 15(0.01), 23(0.03), 39(0.04)
8	4(0.02), 8(0.91), 16(0.01), 24(0.02), 40(0.03)
9	9(0.83), 10(0.04), 11(0.03), 12(0.01), 13(0.03), 25(0.02), 41(0.03)
10	10(0.88), 12(0.02), 14(0.04), 26(0.02), 42(0.03)
11	9(0.04), 11(0.76), 12(0.07), 15(0.02), 27(0.05), 43(0.03)
12	10(0.04), 12(0.84), 16(0.04), 28(0.02), 44(0.04)
13	9(0.02), 13(0.83), 14(0.03), 15(0.04), 29(0.04), 45(0.03)
14	10(0.03), 14(0.88), 16(0.03), 30(0.02), 46(0.04)
15	11(0.02), 13(0.01), 15(0.83), 16(0.08), 31(0.02), 47(0.03)
16	12(0.02), 14(0.02), 16(0.90), 32(0.02), 48(0.04)
17	1(0.02), 17(0.81), 19(0.03), 21(0.03), 25(0.04), 49(0.06)
18	2(0.04), 18(0.76), 20(0.05), 22(0.04), 26(0.04), 50(0.05)
19	3(0.03), 17(0.03), 19(0.76), 20(0.02), 23(0.04), 27(0.05), 51(0.04)
20	4(0.04), 18(0.02), 20(0.81), 24(0.04), 28(0.03), 52(0.05)
21	5(0.03), 17(0.03), 21(0.77), 23(0.07), 29(0.03), 53(0.05), 55(0.01)
22	6(0.04), 18(0.04), 22(0.77), 24(0.05), 26(0.01), 30(0.04), 54(0.05)
23	7(0.04), 19(0.03), 21(0.01), 23(0.78), 24(0.03), 31(0.05), 55(0.05)
24	8(0.04), 20(0.02), 22(0.01), 24(0.85), 32(0.02), 56(0.04)
25	9(0.02), 25(0.84), 26(0.03), 27(0.02), 29(0.04), 57(0.04)
26	10(0.05), 26(0.84), 28(0.02), 30(0.04), 58(0.04)
27	11(0.02), 25(0.03), 27(0.80), 28(0.06), 31(0.04), 59(0.03)
28	12(0.05), 26(0.04), 28(0.82), 32(0.04), 60(0.03)
29	13(0.03), 25(0.03), 29(0.82), 30(0.03), 31(0.03), 61(0.04)
30	14(0.06), 26(0.03), 30(0.84), 32(0.03), 62(0.04)
31	15(0.01), 27(0.02), 29(0.01), 31(0.85), 32(0.05), 63(0.04)
32	16(0.05), 28(0.03), 30(0.02), 32(0.85), 64(0.04)
33	1(0.04), 33(0.85), 35(0.01), 37(0.03), 49(0.06)
34	2(0.05), 6(0.01), 18(0.01), 34(0.74), 36(0.05), 38(0.06), 42(0.01), 50(0.08)
35	3(0.02), 19(0.01), 33(0.03), 35(0.75), 36(0.02), 39(0.04), 51(0.10)
36	4(0.05), 34(0.03), 36(0.80), 40(0.05), 44(0.02), 52(0.05)
37	5(0.03), 33(0.03), 37(0.78), 39(0.05), 53(0.08)
38	6(0.07), 8(0.01), 34(0.01), 36(0.01), 38(0.75), 40(0.06), 46(0.03), 48(0.01), 54(0.05), 56(0.01)
39	7(0.04), 35(0.02), 37(0.01), 39(0.80), 40(0.02), 47(0.01), 55(0.09)
40	8(0.05), 24(0.01), 36(0.02), 38(0.01), 40(0.85), 48(0.01), 56(0.04)
41	9(0.04), 41(0.73), 42(0.07), 43(0.02), 45(0.03), 57(0.08), 59(0.01)
42	10(0.05), 42(0.86), 46(0.04), 58(0.02)
43	11(0.03), 12(0.01), 41(0.04), 42(0.01), 43(0.73), 44(0.08), 47(0.05), 59(0.02)
44	12(0.06), 42(0.04), 44(0.81), 46(0.01), 48(0.04), 60(0.02)
45	13(0.04), 15(0.01), 41(0.04), 45(0.77), 46(0.05), 47(0.03), 61(0.06)
46	14(0.05), 42(0.03), 46(0.88), 48(0.02), 62(0.02)
47	15(0.02), 16(0.01), 43(0.03), 45(0.01), 47(0.77), 48(0.07), 61(0.01), 63(0.05)
48	16(0.04), 44(0.02), 46(0.02), 48(0.88), 64(0.02)
49	17(0.03), 33(0.01), 49(0.87), 51(0.02), 53(0.04), 57(0.02)
50	18(0.03), 34(0.01), 50(0.82), 52(0.03), 54(0.05), 58(0.04)
51	19(0.03), 35(0.01), 49(0.04), 51(0.81), 52(0.02), 55(0.04), 59(0.03)
52	20(0.04), 36(0.02), 50(0.02), 52(0.86), 56(0.04), 60(0.02)
53	21(0.02), 37(0.01), 49(0.03), 53(0.84), 55(0.06), 61(0.03)
54	22(0.02), 38(0.01), 50(0.02), 54(0.87), 56(0.03), 62(0.04)
55	23(0.03), 39(0.01), 51(0.02), 53(0.01), 55(0.85), 56(0.03), 63(0.04)
56	24(0.03), 40(0.02), 52(0.02), 54(0.01), 56(0.90), 64(0.02)
57	25(0.03), 41(0.01), 57(0.87), 58(0.03), 59(0.02), 61(0.04)
58	26(0.03), 42(0.02), 58(0.88), 60(0.02), 62(0.04)
59	27(0.03), 57(0.04), 59(0.83), 60(0.04), 63(0.04)
60	26(0.01), 28(0.05), 44(0.02), 58(0.04), 60(0.81), 64(0.06)
61	29(0.03), 57(0.04), 61(0.85), 62(0.02), 63(0.04)
62	30(0.03), 46(0.03), 58(0.04), 62(0.87), 64(0.02)
63	31(0.04), 47(0.01), 59(0.02), 61(0.03), 63(0.85), 64(0.04)
64	32(0.04), 48(0.03), 60(0.02), 62(0.02), 64(0.88)

[a] Stand states are defined as in Table 1.
[b] Probabilities in parentheses may not add to 1 in some rows due to rounding.

Table 3
Transition probabilities between price levels, and expected price at each level for stumpage Douglas-fir high grade logs, in 2005 dollars.

Price level at t	Expected price ($\$\,m^{-3}$)	Price level at $t+1$ year		
		Low	Medium	High
		Probability		
Low ($\leq\$539\ m^{-3}$)	352.4	0.928	0.067	0.004
Medium ($>\$539\ m^{-3}$, $\leq\$824\ m^{-3}$)	682.9	0.072	0.866	0.062
High ($>\$824\ m^{-3}$)	1094.5	0.004	0.057	0.939

For example, over one year, a stand in state 1 would stay in state 1 with a probability of 0.79, move to state 3 with probability 0.01, move to state 5 with probability 0.06, and so on. The transition probabilities were obtained by bootstrap simulation with a nonlinear matrix growth model (Liang et al., 2006).

Yearly price movements were also modeled with a Markov chain, as in Zhou et al. (2008). Table 3 shows the definitions of the three price levels: *low*, *medium*, and *high*, the expected value at each level and the yearly probability of transition between price levels. The price of other species and grades was tied to this indicator price as in Liang et al. (2006).

We chose the real rate of interest on AAA Corporate bonds in the United States from 1939 to 2007 to represent the interest rate (Fig. 1). The interest rate could be in two states: *high* if it was above the historical average of 2.24%, *low* otherwise. Table 4 shows the transition probabilities between stand states observed in the data, and the expected value of the interest rate at each level. The strong serial correlation of the interest rate apparent in Fig. 1 translates in the high probability (0.692) of a *low* interest rate next year given a currently *low* interest rate, and the probability (0.786) of a *high* interest rate next year given a currently *high* interest rate.

Combining the 64 stand states with the three price levels, and the *high* or *low* interest rate gave $64\times3\times2=384$ possible system states. As there was no apparent correlation between stand state, price, and interest rate, the 384×384 matrix of transition probabilities between system states, $p(i,j)$ for i and $j=1,\ldots,384$ was the product of the transition probabilities of the stand, price, and interest rate between the relevant states.

2.2. Decisions and immediate returns

A decision meant harvesting a stand and thus changing its state. For example, for stand state #4 (000,011) a decision could be to do nothing, or to thin the large shade-tolerant trees to low basal area and thus move to state #3 (000,010), or to thin the medium shade-tolerant trees and move to state #2 (000,001), or to thin the medium and large shade-tolerant trees and thus move to state #1 (000,000).

After the decision, the residual stand would change state over one year according to the transition probabilities in Table 2. Meanwhile, the price and interest rate would change independently so that the entire system would move from pre-decision state i to the state j with a probability $p(i,j|k)$ that depends on the decision, k. Each decision results in an immediate return, R_{ik}, which depends on the system state i when the decision is made and on the decision, k. The interest rate, however,

Table 4
Transition probability between interest rate levels, and expected interest rate at each level, based on data in Fig. 1.

Interest rate at t	Expected interest rate	Interest rate at $t+1$ year	
		Low	High
		Probability	
Low ($\leq$2.24% $year^{-1}$)	$-1.02\%\ year^{-1}$	0.692	0.308
High($>$2.24% $year^{-1}$)	$4.35\%\ year^{-1}$	0.214	0.786

must be treated differently from the price level because it does not affect the immediate returns but the discounted value of future returns.

2.3. Optimization with a fixed interest rate

Most expositions of MDP models in or outside forestry that maximize expected net present value over an infinite horizon use a fixed interest rate (see Kaya and Buongiorno, 1987; Lin and Buongiorno, 1998, in forestry, and Stigter and van Langevelde, 2004, in range management). With a fixed interest, the application considered here would involve only 64 stand states and three price levels, i.e. 192 system states. The corresponding solution, i.e. the best decision in each state, could be found with various algorithms (Bertsekas, 1995). Mortan (1973) gives sufficient conditions for the optimal control of stationary MDP models, while an elegant and practical approach, which uses linear programming, is by d'Epenoux (1963):

$$\max NPV = \sum_i \sum_k R_{ik} y_{ik}$$

subject to:

$$\sum_k y_{jk} - a \sum_i \sum_k y_{ik} p(j|i,k) = \pi_j \quad j = 1, \ldots, 192 \tag{1}$$

$$y_{ik} \geq 0 \quad \forall i, k.$$

NPV is the expected present value of returns over an infinite horizon, i.e. the forest value inclusive of land and initial trees, given initial state probabilities. The variables, y_{ik}, are the total discounted time of being in state i and making decision k. The discount factor is $\alpha = 1/(1+r)$ where r is the yearly interest rate. π_j is the probability that the stand-price system is initially in state j. Given the optimal solution, the best policy is given by:

$$X_{ik} = \frac{y_{ik}}{\sum_k y_{ik}} \tag{2}$$

where X_{ik} is the probability of making decision k in state i. The optimal policy is stationary, depending only on the current state, and deterministic ($X_{ik} = 0$ or 1): for each state there is only one best decision, with probability 1. Consequently, the best policy is independent on the initial system state, π_j.

2.4. Optimization with stochastic interest rate

As observed above, assuming a variable interest rate with two levels leads to our application to a system with 384 states, each a combination of a particular stand state, price level, and interest rate level. In contrast with Eq. (1), at decision time the discount factor for the next period is unknown. It acquires a particular value according to the probability $p(i,j|k)$ of ending in state j, given initial state i and decision k. Thus, model (1) becomes (Buongiorno and Zhou, 2009):

$$\max NPV' = \sum_i \sum_k R_{ik} y_{ik}$$

subject to:

$$\sum_k y_{jk} - a_j \sum_i \sum_k y_{ik} p(j|i,k) = \pi_j \quad j = 1, \ldots, 384 \tag{3}$$

$$y_{ik} \geq 0 \quad \forall i, k$$

Table 5
Forest value (10^3 \$ ha^{-1}) with a stochastic interest rate, by initial stand state, interest rate level, and price level.

Stand state[a]	Interest rate						Stand state[a]	Interest rate					
	Low			High				Low			High		
	Price			Price				Price			Price		
	Low	Med	High	Low	Med	High		Low	Med	High	Low	Med	High
1	16	18	19	16	17	18	33	17	19	21	16	18	20
2	22	24	27	21	23	26	34	22	25	29	21	24	28
3	19	20	22	18	19	21	35	20	22	24	19	21	23
4	23	26	29	22	24	28	36	23	27	31	22	26	30
5	17	19	21	16	18	20	37	18	20	23	18	19	22
6	22	25	28	21	24	27	38	23	26	30	22	25	29
7	19	21	23	18	20	22	39	20	23	26	19	22	25
8	23	27	31	22	25	30	40	24	28	33	23	27	32
9	22	24	27	21	23	26	41	24	27	30	23	25	29
10	25	29	35	24	28	33	42	26	31	37	25	30	36
11	24	26	30	22	25	28	43	25	28	33	24	27	31
12	26	31	37	25	30	36	44	27	33	40	26	32	39
13	23	25	29	22	24	27	45	24	28	32	23	26	30
14	25	30	36	24	29	35	46	27	32	39	26	31	38
15	24	27	31	23	26	30	47	25	29	34	24	28	33
16	27	32	39	25	31	37	48	28	34	42	26	33	40
17	19	21	22	18	20	21	49	20	22	25	19	21	24
18	23	26	30	22	25	29	50	24	27	32	23	26	31
19	21	23	25	20	22	24	51	21	24	27	20	23	26
20	24	28	33	23	27	32	52	25	29	35	23	28	34
21	20	22	24	19	21	23	53	20	23	26	20	22	25
22	24	27	32	22	26	30	54	24	28	34	23	27	33
23	21	24	27	20	23	26	55	22	25	29	21	24	28
24	24	29	34	23	28	33	56	25	30	36	24	29	35
25	23	27	30	22	25	29	57	25	29	33	24	27	32
26	26	31	38	25	30	37	58	27	33	41	26	32	39
27	25	28	33	24	27	32	59	26	30	36	25	29	35
28	27	33	40	26	32	39	60	28	35	43	27	34	42
29	24	28	32	23	26	31	61	25	30	35	24	28	34
30	26	32	39	25	31	38	62	28	34	42	27	33	41
31	25	29	35	24	28	33	63	26	31	38	25	30	36
32	28	34	42	27	33	41	64	29	36	45	28	35	44

[a] Stand states defined in Table 1.

where a_j is the discount factor in state j. The only change with respect to Eq. (1) is in the number of variables and constraints, and in the recognition that the discount factor may vary according to each end state, j. Everything else stays the same as in the model (1) with a fixed interest rate. As for the case of the fixed interest rate, the solution obtained with Eq. (2) is stationary and deterministic, thus the decisions are independent of the initial stand state, price level, and interest rate. They depend only on their values at decision time.

However, as in the case of the fixed interest rate, the *NPV* depends very much on the initial condition. The expected NPV_s for a particular initial system state, s, was found by setting the right hand side of the constraints in the linear program (1) or (3) at $\pi_s = 1$ for $j = s$ and $\pi_j = 0$ for $j \neq s$.

It may be worth noting that while the constant interest rate in Eq. (1) must be positive for a solution to exist, lest the NPV becomes infinite; in formulation (3), the interest rate may be negative in some states, as long as its long-term value is positive. In particular, in this application the interest rate had an expected value of −1.02 when it was *low*, but its long-run value according to the probabilities in Table 4 was 2.14%, leading to a well-defined solution.

2.5. Effects of stochastic interest rates

To assess the importance of recognizing the variability of the interest rate in forest valuation we computed the best forest value with models (1) and (3), for every initial system state, s, combining stand state, price level, and interest rate, and the attendant policies.

In addition, we determined the effect on the *NPV* of applying a policy that assumed a fixed interest rate when in fact the interest rate varied. With a fixed interest rate, a policy sets a particular decision, k_0, for each system state, i, combining a stand state and a price level. For a particular policy the transition probability between system states i and j is $p(j|i,k_0)$. With a fixed interest rate and a particular policy, the net present value by initial system state, V_i, satisfies the following system of equations (Hillier and Lieberman, P 981, 2005):

$$V_i = R_{i,k_0} + \alpha \sum_j p(j|i,k_0)V_j \quad i = 1, \ldots, 192. \tag{4}$$

With a stochastic interest rate, the rate at which future returns must be discounted is unknown and its value varies according to the probabilities in Table 4. Accordingly, Eq. (4) becomes (Buongiorno and Zhou, 2009):

$$V_i = R_{i,k_0} + \sum_j \alpha_j p(j|i,k_0)V_j \quad j = 1, \ldots, 384. \tag{5}$$

Solving the system of Eq. (5) with the policy that ignored the variation in interest rate gave, for each initial system state, i, i.e. each combination of stand state, price level, and interest rate, the expected *NPV* in an environment where, in fact, the interest rate varied.

The long-run probability of each system state, p_j, for a particular policy was obtained by solving the steady-state equations:

$$\begin{aligned} &p_j = \sum_i p(j|i,k_0)p_i \qquad j = 1, \ldots, 384 \\ &\sum_j p_j = 1 \\ &p_j \geq 0. \end{aligned} \tag{6}$$

Table 6
Underestimation of forest value ($\$\,\mathrm{ha}^{-1}$) estimated with a fixed interest rate, relative to the forest value estimated with a stochastic interest rate, by stand state, interest rate level, and price level.

Stand state[a]	Interest rate						Stand state[a]	Interest rate					
	Low			High				Low			High		
	Price			Price				Price			Price		
	Low	Med	High	Low	Med	High		Low	Med	High	Low	Med	High
1	16%	15%	14%	10%	9%	8%	33	15%	14%	12%	10%	9%	7%
2	14%	12%	11%	8%	7%	6%	34	13%	12%	10%	8%	7%	5%
3	15%	14%	13%	9%	9%	8%	35	14%	13%	12%	9%	8%	7%
4	13%	12%	10%	8%	7%	6%	36	13%	11%	9%	8%	6%	5%
5	15%	14%	13%	10%	9%	8%	37	15%	14%	11%	9%	8%	7%
6	13%	12%	10%	8%	7%	5%	38	13%	11%	9%	8%	6%	5%
7	15%	13%	12%	9%	8%	7%	39	14%	13%	11%	9%	8%	6%
8	13%	11%	9%	8%	6%	5%	40	12%	11%	9%	7%	6%	5%
9	14%	13%	12%	8%	8%	7%	41	13%	12%	11%	8%	7%	6%
10	12%	11%	9%	7%	6%	5%	42	12%	10%	9%	7%	6%	5%
11	13%	12%	11%	8%	7%	6%	43	13%	11%	10%	7%	6%	5%
12	12%	10%	8%	7%	6%	5%	44	12%	10%	8%	7%	5%	4%
13	13%	12%	11%	8%	7%	6%	45	13%	12%	11%	8%	7%	6%
14	12%	10%	9%	7%	6%	5%	46	12%	10%	8%	7%	6%	5%
15	13%	12%	10%	8%	7%	5%	47	13%	11%	10%	7%	6%	5%
16	12%	10%	8%	7%	6%	4%	48	11%	9%	8%	7%	5%	4%
17	15%	14%	12%	9%	8%	7%	49	14%	13%	11%	9%	8%	6%
18	13%	11%	10%	8%	7%	5%	50	13%	11%	9%	7%	6%	5%
19	14%	13%	11%	9%	8%	7%	51	14%	12%	10%	8%	7%	6%
20	13%	11%	9%	7%	6%	5%	52	12%	10%	8%	7%	6%	5%
21	14%	13%	11%	9%	8%	6%	53	14%	12%	10%	9%	7%	6%
22	13%	11%	9%	8%	6%	5%	54	12%	10%	8%	7%	6%	5%
23	14%	12%	11%	8%	7%	6%	55	13%	12%	10%	8%	7%	6%
24	12%	10%	8%	7%	6%	5%	56	12%	10%	8%	7%	6%	4%
25	13%	12%	10%	8%	7%	6%	57	13%	11%	10%	7%	6%	5%
26	12%	10%	8%	7%	6%	5%	58	11%	9%	8%	7%	5%	4%
27	13%	11%	9%	7%	6%	5%	59	12%	11%	9%	7%	6%	5%
28	11%	9%	8%	7%	5%	4%	60	11%	9%	8%	6%	5%	4%
29	13%	11%	10%	8%	7%	5%	61	12%	11%	9%	7%	6%	5%
30	12%	9%	8%	7%	6%	4%	62	11%	9%	8%	7%	5%	4%
31	12%	11%	9%	7%	6%	5%	63	12%	10%	9%	7%	6%	5%
32	11%	9%	7%	7%	5%	4%	64	11%	9%	7%	6%	5%	4%

[a] Stand states defined in Table 1.

The probability of a particular state j and decision k_0 given this policy was:

$$p_{jk_0} = p_j p(k_0|j) \tag{7}$$

from which we then derived the long-run probability of each stand state given a policy, and the corresponding expected standing basal area and volume, annual harvest, basal area of large trees (diameter at breast height of at least 41 cm (16 in.)), and probability of stand states with late-seral and spotted-owl habitat characteristics (Hummel and Caulkin, 2005; Zhou et al., 2008). Eq. (7) also gave the probability of a harvest given a policy, and its inverse, the expected interval between harvests, i.e. the expected cutting cycle (Kaya and Buongiorno, 1987).

3. Results

3.1. Forest value with variable or fixed interest rate

Table 5 shows the optimal forest values derived with Eq. (3) for all combinations of stand state, price level, and interest rate. The highest forest value was achieved for a forest stand in state #64 with *high* basal area in all species and size classes, when the interest rate was *low*, and the price was *high*. It was lowest for a forest stand in state #1, with *low* basal area in all tree species and size classes, when the interest rate was *high*, and the price was *low*.

For the same initial stand state and interest rate, the forest value was higher with a *high* initial price. For example, in state #64 which had the highest basal area, and at *low* interest rate, the forest value was 56% higher with a *high* initial price than with a *low* price. But, the difference was only 16% for a stand in state #1, with the lowest basal area.

For any given initial stand state and price level, the forest value with a *low* initial interest rate was higher than with a *high* rate. This is consistent with the strong serial correlation of the interest rate. The probability that the interest rate would end up in the same state in a year was 0.69 from *low* to *low*, and 0.79 from *high* to *high* (Table 4). The present value of future harvests depends on the series of interest rates from now to harvest time. A *low* initial interest rate is more likely to result in a *low* interest rate in the near future, and thus to a higher present value of future harvests.

Table 6 shows that the forest value obtained with a fixed interest rate, set at the historical average of 2.24%, underestimated the forest value that recognized the variation in interest rate. The difference ranged from 5% to 16% depending on the initial stand state, price level, and interest rate. Other things being equal, the underestimation of forest value was relatively higher when price and interest rate were both *low*.

3.2. Best decisions with variable or fixed interest rate

Table 7 gives the best decision by stand state, price level, and interest rate, taking into account the variability of interest rate. As expected, for a given stand state and interest rate, a higher price tended to call for a stand with lower basal area in some species-size categories, i.e. for more harvest.

At *low* price the best decision was to cut more when the interest rate was *high* in nine stand states. At *medium* price, the decision was the same for all 64 stand states regardless of interest rate. At *high*

Table 7
Decisions that maximized forest value with a stochastic interest rate, by stand state, interest rate, and price level.

Stand state	Interest rate						Stand state[b]	Interest rate					
	Low			High				Low			High		
	Price			Price				Price			Price		
	Low	Med	High	Low	Med	High		Low	Med	High	Low	Med	High
1	1[a]	1	1	1	1	1	33	33	1	1	33	1	1
2	2	2	2	2	2	1	34	2	2	2	2	2	1
3	3	3	3	3	3	3	35	35	35	3	35	35	3
4	2	2	2	2	2	3	36	2	2	2	2	2	3
5	5	1	1	5	1	1	37	37	37	1	37	37	1
6	2	2	2	2	2	1	38	38	2	2	38	2	1
7	7	3	3	7	3	3	39	39	35	3	39	35	3
8	2	2	2	2	2	3	40	38	2	2	38	2	3
9	9	9	9	9	9	9	41	41	41	41	41	41	41
10	10	9	9	9	9	9	42	41	41	41	41	41	41
11	11	11	9	11	11	9	43	43	41	41	43	41	41
12	11	11	9	11	11	9	44	43	41	41	43	41	41
13	13	9	9	9	9	9	45	41	41	41	41	41	41
14	14	9	9	9	9	9	46	41	41	41	41	41	41
15	15	11	9	15	11	9	47	47	41	41	47	41	41
16	15	11	9	15	11	9	48	47	41	41	47	41	41
17	17	17	17	17	17	1	49	17	17	17	17	17	1
18	18	2	2	18	2	1	50	50	2	2	18	2	1
19	19	19	19	19	19	3	51	19	19	19	19	19	3
20	18	2	2	18	2	3	52	50	2	2	18	2	3
21	21	21	17	21	17	1	53	21	21	17	21	17	1
22	18	2	2	18	2	1	54	50	2	2	18	2	1
23	23	19	19	23	19	3	55	23	19	19	23	19	3
24	18	2	2	18	2	3	56	50	2	2	18	2	3
25	9	9	9	9	9	9	57	41	41	41	41	41	41
26	26	9	9	9	9	9	58	41	41	41	41	41	41
27	11	11	9	11	11	9	59	43	41	41	43	41	41
28	11	11	9	11	11	9	60	43	41	41	43	41	41
29	13	9	9	9	9	9	61	41	41	41	41	41	41
30	14	9	9	9	9	9	62	41	41	41	41	41	41
31	15	11	9	15	11	9	63	47	41	41	47	41	41
32	15	11	9	15	11	9	64	47	41	41	47	41	41

[a] Stand state number, defined in Table 1, that results from a harvest decision.
[b] Stand states defined in Table 1.

Table 8
Differences between the decisions obtained with a stochastic interest rate and those obtained with a constant average interest rate (in parentheses).

Stand state	Interest rate						Stand state[b]	Interest rate					
	Low			High				Low			High		
	Price			Price				Price			Price		
	Low	Med	High	Low	Med	High		Low	Med	High	Low	Med	High
1							33						
2			2[a](1[a])				34			2(1)			
3							35						
4			2(3)				36			2(3)			
5							37						
6			2(1)				38			2(1)			
7							39						
8			2(3)				40			2(3)			
9			9(1)			9(1)	41						
10	10(9)		9(1)			9(1)	42						
11			9(3)			9(3)	43						
12			9(3)			9(3)	44						
13	13(9)		9(1)			9(1)	45						
14	14(9)		9(1)			9(1)	46						
15			9(3)			9(3)	47						
16			9(3)			9(3)	48						
17			17(1)				49			17(1)			
18			2(1)				50			2(1)			
19			19(3)				51			19(3)			
20			2(3)				52	50(18)		2(3)			
21		21(17)	17(1)				53		21(17)	17(1)			
22			2(1)				54	50(18)		2(1)			
23			19(3)				55	50(23)		19(3)			
24			2(3)				56	50(18)		2(3)			
25			9(1)			9(1)	57						
26	26(9)		9(1)			9(1)	58						
27			9(3)			9(3)	59						
28			9(3)			9(3)	60						
29	13(9)		9(1)			9(1)	61						
30	14(9)		9(1)			9(1)	62						
31			9(3)			9(3)	63						
32			9(3)			9(3)	64						

[a] Stand state number, defined in Table 1, that results from a harvest decision.
[b] Stand states defined in Table 1.

price, a *high* interest rate called for more harvest in 23 stand states. This suggests that overall more would be harvested with a *high* interest rate.

The differences in decisions recognizing variations of interest rate compared to assuming a fixed average interest rate of 2.24% are shown in Table 8. The decisions were different for 68 of the 384 combinations of stand state, interest rate, and price level. At *low* interest rate, the decision differed for 10 stand states at *low* price, 2 stand states at *medium* price, and 40 stand states at *high* price. At high interest rate, the decisions were the same at *low* and *medium* prices regardless of stand state, and differed for 16 stand states at *high* price.

Where the decisions were different the decision derived with a fixed interest rate always harvested more than the decision that recognized the variability of interest rate.

3.3. Financial gain from a policy adaptive to interest rate variations

It was shown above that ignoring the variability of interest rate substantially underestimates the forest value, i.e. the value of a stand of trees and the land on which they grow (Table 6). Another issue is how much present value would be foregone by applying a policy that assumed a fixed interest rate, when in fact the interest rate varies.

The policy assuming a fixed interest rate was defined by Table 7, modified by the decisions in parentheses in Table 8. Applying Eq. (5) then gave the present value by the initial stand state, price level, and level of interest. The results, compared with those in Table 5 show the financial gain of the interest-rate-adaptive policy (Table 9). The gain was relatively small, ranging from 0 to 7%.

3.4. Landscape and other effects of a policy adapted to interest rate variations

Table 10 shows the steady-state probability of each stand state with a policy based on a fixed or on a stochastic interest rate. This probability can also be interpreted as the fraction of the entire forest area that would be observed in each stand state over a long period of time. With a policy that recognized the variability of the interest rate, 41% of the steady-state landscape would be in state #9 with *high* basal area in the largest shade-tolerant trees and 29% would be in state #41 with *high* basal area in the *small* and *large* shade-tolerant trees. Both states had *high* basal areas of *large* shade-intolerant trees and *low* basal areas of shade-tolerant trees.

In contrast, with a fixed interest rate policy, 28% of the landscape would be occupied by stands of type #1 with *low* basal area in all species and size classes, and about 12% would be in state #3, also with *low* basal area except in the *medium* sized shade-intolerant trees (see Table 1). This result is consistent with the decisions shown in Table 8, which imply at least equal and often higher harvest with the fixed interest rate.

Other long-term ecological and economic effects of the policies that maximized forest value, assuming a fixed or variable interest rate are summarized in Table 11. Given the initial distribution of stands in Table 1 and assuming initial average price and interest rate, the stochastic-rate policy gave a forest value of 3049 $/ha (2005 dollar), or 8.5% more than the

Table 9
Loss of forest value ($\$\,ha^{-1}$) by applying decisions that would be optimal with a fixed interest rate, instead of the best decisions obtained with a stochastic interest rate, by initial stand state, price level, and interest rate.

Stand state[a]	Interest rate						Stand state[a]	Interest rate					
	Low			High				Low			High		
	Price			Price				Price			Price		
	Low	Med	High	Low	Med	High		Low	Med	High	Low	Med	High
1	1%	0%	1%	1%	0%	1%	33	1%	0%	1%	1%	0%	1%
2	3%	2%	4%	3%	2%	3%	34	3%	2%	1%	3%	2%	0%
3	2%	2%	2%	2%	2%	2%	35	2%	2%	2%	2%	2%	1%
4	3%	2%	2%	2%	2%	2%	36	3%	2%	1%	2%	1%	1%
5	1%	0%	1%	2%	1%	1%	37	1%	0%	1%	2%	1%	0%
6	3%	2%	4%	3%	2%	3%	38	3%	2%	1%	3%	1%	0%
7	2%	2%	2%	2%	2%	1%	39	2%	1%	2%	2%	1%	1%
8	3%	2%	2%	3%	1%	2%	40	3%	2%	1%	3%	1%	1%
9	2%	3%	7%	3%	3%	3%	41	2%	3%	3%	3%	3%	3%
10	2%	2%	5%	3%	3%	2%	42	2%	2%	2%	3%	3%	2%
11	3%	3%	5%	3%	3%	3%	43	3%	3%	2%	3%	3%	3%
12	3%	2%	4%	3%	3%	2%	44	3%	2%	2%	3%	3%	2%
13	2%	3%	6%	3%	3%	3%	45	2%	2%	3%	3%	3%	3%
14	3%	2%	5%	3%	3%	2%	46	2%	2%	2%	3%	3%	2%
15	2%	3%	4%	3%	3%	3%	47	2%	3%	2%	3%	3%	3%
16	2%	2%	4%	3%	3%	2%	48	2%	2%	2%	3%	3%	2%
17	2%	2%	2%	2%	1%	2%	49	2%	2%	1%	2%	1%	0%
18	3%	2%	4%	2%	2%	3%	50	3%	2%	1%	2%	1%	0%
19	2%	2%	3%	2%	2%	2%	51	2%	2%	2%	2%	2%	1%
20	2%	2%	2%	2%	1%	2%	52	2%	2%	1%	2%	1%	1%
21	2%	2%	2%	2%	2%	1%	53	2%	2%	1%	2%	1%	0%
22	2%	2%	3%	2%	1%	3%	54	2%	2%	1%	2%	1%	0%
23	2%	2%	2%	2%	2%	2%	55	2%	2%	2%	2%	2%	1%
24	2%	2%	2%	2%	1%	2%	56	2%	2%	1%	2%	1%	1%
25	2%	3%	6%	3%	3%	3%	57	2%	2%	3%	3%	3%	3%
26	2%	2%	5%	3%	3%	2%	58	2%	2%	2%	3%	3%	2%
27	3%	3%	4%	3%	3%	3%	59	3%	3%	2%	3%	3%	2%
28	3%	2%	3%	3%	2%	2%	60	3%	2%	2%	3%	2%	2%
29	2%	2%	5%	3%	3%	3%	61	2%	2%	3%	3%	3%	3%
30	3%	2%	4%	3%	3%	2%	62	2%	2%	2%	3%	2%	2%
31	2%	3%	4%	3%	3%	2%	63	2%	2%	2%	3%	3%	2%
32	2%	2%	3%	2%	2%	2%	64	2%	2%	2%	2%	2%	2%

[a] Stand states are defined in Table 1.

fixed-rate policy. This was achieved with a slightly longer average cutting cycle (interval between harvests) and a higher average annual harvest.[1] With the variable interest rate policy this higher production could be maintained with higher standing basal area and volume, and higher basal area of trees of at least 41 cm (16 in.) in diameter at breast height. The total basal area and basal area of large trees would even be higher under the stochastic interest rate policy than in the current forest. However, the fixed-interest rate policy and the variable interest rate had similar negative effects on the percentage of area in late seral forest and with nesting habitat for spotted owls, according to the criteria of Hummel and Caulkin (2005) compared with the current condition.

4. Conclusion

The objective of this study was to investigate some consequences of recognizing the interest rate risk in setting harvest policies, in concert with the risk inherent in forest growth and timber prices.

To this end, the MDP model proposed in Buongiorno and Zhou (2009) was adapted to the data related to the Douglas-fir/western hemlock forest type in the Pacific Northwest of the United States. The results showed that the forest value (land and trees) obtained with a fixed interest rate underestimated the forest value obtained by recognizing the variation in interest rate by 4 to 16% depending on the initial stand state, price level, and interest rate.

However, the loss of present value with a fixed interest rate policy applied to an environment where in fact the interest rate varied was more modest, 0 to 7% depending on the initial conditions. Thus, recognizing the variability of interest rate is relatively more important for forest valuation, and to compare forestry with other investments, than for choosing a harvest policy.

Taking explicit account of the variability of interest rate in setting harvest policies also had some unexpected ecological benefits. By harvesting less in most combinations of stand state, price, and interest rate, it kept more standing basal area and volume, and more large trees, and yet produced more annually.

Application of the results in the field seems straightforward as they consist of decision tables indicating the harvest for every combination of stand state, price, and interest rate. However, financial return is only one of the many objectives of forest management and it may be necessary to introduce constraints to tailor these decision tables to each owner's objectives. The linear programming solution adopted here does facilitate such constraints, as shown for example in Zhou and Buongiorno (2006).

It is worth noting that the optimal policy from the MDP model used in this paper assumes risk neutrality — the decision maker is only interested in the expected value and indifferent to the variation of the income over time. Ruszczynski (2009) proposes a way to handle risk aversion within an MDP framework which develops a Markov risk measure and solves the MDP problem by dynamic programming. This

[1] With a fixed interest rate policy, 28% of the forest will be in state #1, and 12% in state #3 (Table 10), both with low basal areas (Table 1). With a stochastic interest rate policy, 41% of the forest will be in state #9 with much higher basal area. So although the policy with a fixed rate harvests more frequently and more or the same in every state (Table 8), the forest in the steady state will be largely composed of small and medium trees, resulting in a lower annual harvest and lower stand volume.

Table 10
Long-term probability of each stand state with a policy based on a fixed or on a stochastic interest rate.

Stand state #	Probability		Stand state #	Probability	
	Fixed interest	Stochastic interest		Fixed interest	Stochastic interest
1	0.280	0.001	33	0.057	0.000
2	0.029	0.000	34	0.002	0.000
3	0.123	0.000	35	0.011	0.000
4	0.004	0.000	36	0.000	0.000
5	0.034	0.000	37	0.007	0.000
6	0.003	0.000	38	0.000	0.000
7	0.020	0.000	39	0.004	0.000
8	0.001	0.000	40	0.000	0.000
9	0.103	0.410	41	0.098	0.292
10	0.006	0.022	42	0.010	0.030
11	0.048	0.082	43	0.015	0.031
12	0.005	0.009	44	0.001	0.002
13	0.004	0.017	45	0.005	0.014
14	0.000	0.001	46	0.000	0.000
15	0.013	0.018	47	0.007	0.013
16	0.001	0.002	48	0.001	0.001
17	0.031	0.000	49	0.004	0.000
18	0.008	0.000	50	0.000	0.000
19	0.022	0.000	51	0.002	0.000
20	0.001	0.000	52	0.000	0.000
21	0.006	0.000	53	0.001	0.000
22	0.000	0.000	54	0.000	0.000
23	0.007	0.000	55	0.001	0.000
24	0.000	0.000	56	0.000	0.000
25	0.004	0.012	57	0.010	0.031
26	0.001	0.001	58	0.000	0.000
27	0.004	0.005	59	0.001	0.003
28	0.000	0.000	60	0.000	0.000
29	0.000	0.000	61	0.000	0.000
30	0.000	0.000	62	0.000	0.000
31	0.001	0.000	63	0.000	0.001
32	0.000	0.000	64	0.000	0.000

Table 11
Long-term effects on economic and ecological criteria of policies that maximized present value based on a fixed or stochastic interest rate.

		Fixed interest	Stochastic interest	Current
Forest value[a]	\$/ha	35,835	38,884	–
Cutting cycle	Year	5.8	6.0	–
Annual harvest	m^3/ha	3.1	4.1	–
Standing BA	m^2/ha	34.0	52.0	49.4
Standing volume	m^3/ha	321.1	384.2	398.3
BA of standing large trees[b]	m^2/ha	20.7	36.7	31.3
Late seral area[c]	%	0.0	0.3	5.4
Owl habitat area[c]	%	0.0	0.2	4.5

[a] Given the initial distribution of stand states in Table 1, and average initial price and interest rate.
[b] Trees with a diameter at breast height of 41 cm (16 in.) and above.
[c] According to the criteria of Hummel and Caulkin (2005).

offers a promising way to handle risk aversion, which will be investigated in future research in the multi-dimensional risk context addressed in this paper.

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