Evaluating Forest Vegetation Simulator Predictions for Southern Appalachian Upland Hardwoods with a Modified Mortality Model

Philip J. Radtke, Nathan D. Herring, David L. Loftis, and Chad E. Keyser

ABSTRACT

Prediction accuracy for projected basal area and trees per acre was assessed for the growth and yield model of the Forest Vegetation Simulator Southern Variant (FVS-Sn). Data for comparison with FVS-Sn predictions were compiled from a collection of $n = 1{,}780$ permanent inventory plots from mixed-species upland hardwood forests in the Southern Appalachian Mountains. Over a 5-year projection interval, baseline FVS-Sn predictions fell within 15% of observed values in over 88% of the test plots. Several modifications to FVS-Sn were pursued, including a refitting of the background mortality equation by logistic regression. Following the modifications, FVS-Sn accuracy statistics increased to 91 and 94% for basal area and trees per acre, respectively. In plots with high initial stand densities, notable gains in accuracy were achieved by relaxing thresholds that activated a density-dependent mortality algorithm in FVS-Sn. Detailed accuracy results for forest types of the region were generated. Twenty-five-year projection results show size-density trajectories consistent with the concept of maximum stand density index.

Keywords: Forest Inventory and Analysis (FIA), uncertainty assessment, model testing, validation, calibration

The causes of tree mortality in forests of the southeastern United States range from abiotic factors, such as wind, lightning, fire, and stresses caused by drought or flooding, to biotic causes, including consumption by animals or insects, diseases, an inability to compete for limited resources among trees, and age-related senescence (Franklin et al. 1987). Growth and yield projection systems must account for tree mortality to be accurate and useful for management-related applications (Rauscher et al. 2000). Mortality is a key factor in the prediction of stand volume, basal area, and number of stems in growth and yield modeling systems. Trees that die during a projection interval directly reduce volume and basal area accretion, with the surviving number of stems per unit area correspondingly reduced.

Mortality in growth and yield models can be accounted for through parameters that distinguish between different types or categories of mortality. This is the approach taken in the forest vegetation modeling system called Forest Vegetation Simulator (FVS) (Crookston and Dixon 2005). Mortality in FVS is generally classified as originating from one of three possible sources: (1) external factors, such as pathogens, fire, or herbivory; (2) density-dependent factors, mainly intertree competition for growing space and associated resources; and (3) noncatastrophic, occasional incidence of tree death that is not readily attributable to either external or competition-related factors. In FVS, these three types of mortality are typically classified as (1) event-driven, or catastrophic, (2) density-dependent, and (3) attributable to background causes (Crookston and Dixon 2005, Dixon 2010). Event-driven mortality in FVS is addressed through specialized algorithms that simulate the effects of external forces on forest stands, including fire, insects and diseases, and silviculture. The subject of this work is limited to mortality unrelated to external causes. Here, we considered both density-dependent and background mortality, both of which are typically predicted in management-oriented forest growth and yield modeling systems, including the default mortality algorithms in FVS.

The FVS system's Southern Variant (FVS-Sn) was designed to predict forest growth and yield across the US South, a region that includes the Southern Appalachian Mountains from central Alabama to northern Virginia (Figure 1). Both background and density-dependent mortality are modeled in FVS-Sn when run in its default mode, minus extensions that account for external causes of mortality. Background mortality is estimated for individual trees using a logistic regression equation to calculate each tree's probability of mortality in prediction cycles for which estimates of forest growth and yield are generated. Density-dependent mortality is predicted only when the stand density—measured as either stand density index (SDI) (Reineke 1933) or basal area—approaches an upper level assumed to limit basal area accretion. In practice, FVS specifies density-dependent mortality according to two SDI thresholds. One is the threshold for moderate density-dependent mortality (SDI_{mod}), set by default at 55% of maximum possible SDI (SDI_{max}). The other is a threshold for severe density-dependent mortality (SDI_{sev}), set at 85% of SDI_{max}. When a stand's observed SDI is less than 55% of SDI_{max}, mortality is computed by summing the background mortality probabilities of individual trees. When SDI exceeds

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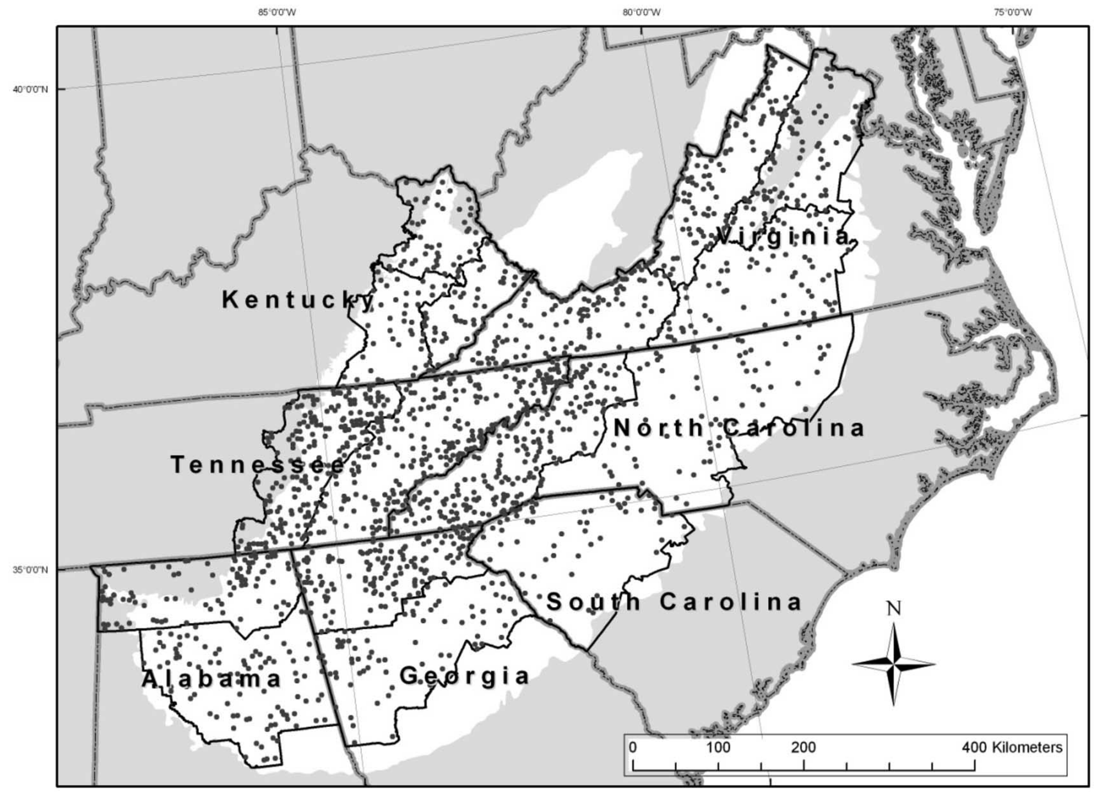

Figure 1. Forest Inventory and Analysis survey unit boundaries (Southern Forest Resource Assessment 2004) and approximate locations of plots remeasured since 1998 (Forest Inventory and Analysis 2009) in seven states corresponding to Southern Appalachian physiographic provinces. Unshaded area indicates the extent of ecological units that make up the study area (McNab et al. 2007).

55% of SDI_{max}, mortality is increased above the background rate to reduce basal area accretion so that it follows a trajectory approaching 85% of SDI_{max}. In cases where SDI reaches or exceeds 85% of the maximum, additional mortality is simulated to reduce SDI to 85% and prevent it from rising above the SDI_{sev} threshold. Because FVS-Sn operates at a resolution that accounts for individual trees, its density-dependent mortality algorithm assigns mortality to individual tree records on the basis of their relative size (percentiles in the basal area distribution) and shade tolerance (Crookston and Dixon 2005).

The background mortality equation in FVS-Sn depends on the species and dbh (diameter at 4.5 ft), where the probability of a tree dying in a 1-year interval is

$$P(Mort) = \frac{0.5}{[1 + exp\{\widehat{b_0} + \widehat{b_1}dbh\}]} \tag{1}$$

where dbh is measured in inches and $\widehat{b_0}$ and $\widehat{b_1}$ are coefficients estimated by regression analysis (Keyser 2011).

Species are assigned to one of four sets of species-group regression coefficients for Equation 1 to predict background mortality (Table 1). Group 1 is a large group including mainly deciduous, broadleaved species, with many diffuse porous species, including maples (*Acer* spp.), tupelo (*Nyssa* spp.), sourwood (*Oxydendrum arboreum* L.), and sweetgum (*Liquidambar styraciflua* L.). Group 2 consists mainly of softwoods, including the genus *Pinus*. Group 3 includes other deciduous, broadleaved species, including oaks (*Quercus* spp.), hickories (*Carya* spp.), and yellow-poplar (*Liriodendron tulipifera* L.). Eastern redcedar (*Juniperus virginiana* L.) is the only species belonging to group 4.

Table 1. Species groups and regression coefficients for the Forest Vegetation Simulator (FVS) Southern Variant mortality equation (Equation 1).

Group	FVS species[a]	$\widehat{b}_0$	$\widehat{b}_1$
1	AB, AE, AH, AS, BE, BG, BK, BU, BW, DW, EL, FM, FR, GA, HA, HH, HM, HY, LB, LK, MB, MG, ML, MS, PI, PS, RA, RD, RL, RM, SB, SD, SM, SR, SS, SV, WE, WI	5.1677	−0.00777
2	BY, LL, LP, OS, PC, PD, PP, PU, SA, SP, TM, VP, WP	5.5877	−0.00535
3	AP, BA, BB, BC, BJ, BN, BO, BT, CA, CB, CK, CO, CT, CW, HB, HI, HL, LO, MV, OH, OT, OV, PO, QS, RO, SK, SN, SO, SU, SY, TO, TS, WA, WK, WN, WO, WT, YP	5.9617	−0.03401
4	JU	9.6943	−0.01273

[a] See Keyser (2011) for species code definitions. A partial list is shown in Table 4.

Typically, prediction equations incorporated into simulation models for growth and yield projection are developed and evaluated for accuracy using regression analysis and data sets compiled to match the populations intended for future modeling applications. For example, the equations developed for FVS-Sn to predict diameter increment in large trees (dbh > 3.0 in) were fitted to an extensive data set of permanent inventory plots across 13 states in the southern United States using widely accepted regression analysis tools (Keyser 2011). Despite agreement between observed data and regression model predictions, additional testing is usually warranted when numerous prediction equations are implemented together in computer simulation models. In such systems, the original accuracy of component equations is not necessarily preserved because deviations can become amplified or errors propagated as predictions from one equation are given as inputs to another (Gertner 1987, Mowrer 1991, Kangas 1997). Furthermore, because interest in growth and yield modeling applications often involves stand-level predictions of timber-related attributes, assessment of the performance of individual-tree equations can be augmented by tests that assess accuracy for stand attributes of interest (Rauscher et al. 2000). To date, no extensive accuracy assessment of FVS-Sn predictions has been reported in published literature; however, unrealistic FVS-Sn size-density trajectories for longleaf pine (*Pinus palustris* Mill.) have been reported by DeRose et al. (2008). Shaw (2009) points out that extensive new data sets have been collected in the past decade by the US Forest Service Southern Research Station's Forest Inventory and Analysis (FIA) program that could readily be used to calibrate and validate FVS-Sn predictions.

The goal of this study was to assess and improve the accuracy of FVS-Sn projections of per-acre basal area (BA) ($ft^2\ ac^{-1}$) and stem density (trees per acre [TPA] [trees ac^{-1}]) in broadleaved mixed-species upland forests of the Southern Appalachian Mountains. Accuracy assessments were aimed at characterizing both bias and precision of model projections using observed values from permanent inventory data collected in Southern Appalachian hardwood forests independent of the data used to develop FVS-Sn. Specific tasks to be accomplished in pursuit of these goals included the following:

- Use recently collected data from FIA to establish baseline accuracy statistics for FVS-Sn 5-year growth projections in upland hardwood forest types in the Southern Appalachians.
- Modify existing FVS-Sn background mortality model coefficients and parameters that specify transitions between background and density-dependent mortality.
- Characterize gains in accuracy following modification, examining results for subpopulations, including forest type, that may be relevant to model users.
- Examine long-range projections in relation to theoretical limits of stand density as a means of testing model logic.

Materials and Methods

Data

Remeasured forest inventory plot and tree records obtained from the FIA program served as the database for model accuracy assessment. To ensure independence from data used to develop the FVS-Sn (Keyser 2011), only growth observations made since 1998 were used. Records were obtained from the FIA online database (Forest Inventory and Analysis 2009a) for physiographic provinces within the Southern Appalachian Mountains that included portions of seven states in the US Forest Service's Southern Region: Alabama, Georgia, Kentucky, North Carolina, South Carolina, Tennessee, and Virginia (Figure 1). The most recent survey data included in the downloaded tables were collected in 2006 for Kentucky and North Carolina, 2007 for South Carolina and Tennessee, and 2008 for Alabama, Georgia, and Virginia.

The geographic scope of the model evaluation spanned three ecological provinces that characterize forests of the Southern Appalachian Mountains from central Alabama to northern Virginia. Nine ecological sections, following McNab et al. (2007), encompassed most of the study region, from the Northern Cumberland Plateau in east central Kentucky and Tennessee to the Southern and Central Appalachian Piedmont sections of Alabama, Georgia, the Carolinas, and Virginia (Figure 1, unshaded area). Although FVS-Sn operates at the province level, ecological sections or subsections of provinces outside the Southern Region were excluded because they did not lie within the geographic region of interest. Because FIA survey units corresponded well to the ecological sections of interest, they served as a geographic extent for identifying plots to be used for model evaluation (Figure 1). Sixteen survey units were identified as making up the region of interest: four in Virginia; three in Kentucky; two each in Alabama, Georgia, North Carolina, and Tennessee; and one in South Carolina. Only plots remeasured since 1998 were included in the evaluation data set, corresponding with the date at which FIA's annual inventory data collection design was first implemented (McRoberts et al. 2005).

Numerous criteria were established to limit the data set to the population of interest, namely those mixed-species upland natural forests that had remained relatively unaffected by human or natural disturbances over the period of observation. Plot records were limited to those with soil drainage classes ranging from xeric to moist-mesic. Event-driven catastrophic mortality was avoided by excluding plots with ≥25% mortality from insect, disease, fire, weather, or human activity, along with plots having any disturbance code recorded by field crews. Only remeasured plots having a single uniform stand condition over the entire field plot were selected, a necessary condition when pooling data from FIA subplots into a single input tree list to represent a forest stand of interest (Shaw 2009). The analysis was limited to trees with dbh ≥5 in., a threshold that establishes which trees are measured in FIA phase II field plots (Forest Inventory and Analysis 2007). Plots visited over a remeasurement interval <1.0 year were excluded because of the likelihood that measurement error would overwhelm diameter growth observations over such short time periods.

Eleven forest types were identified as broadleaved deciduous forests of the region (Eyre 1980), five comprising upland oaks, three upland hardwoods, two northern hardwoods, and one lowland hardwood forest types. The sweetgum-yellow-poplar type was included despite its status as a lowland forest type because yellow-poplar is generally the major species in this forest type and an important commercial species in southeastern deciduous forests (Olson 1969). On selection for the population attributes of interest, the evaluation data set consisted of 1,780 plots, with over 73% of the data corresponding to upland oak forest types.

Baseline FVS-Sn Accuracy Assessment

Initial BA was tabulated for each tree record on an area basis using the reciprocal of FIA plot size. Because only plots with uniform stand conditions were selected, the plot sizes were equal for all records at 0.166 ac; thus, each tree record represented 6.018 trees ac^{-1}. Live trees measured at the initial measurement period were specified as input to the FVS-Sn software in the form of text-coded tree lists, one for each FIA plot. Primary tree attributes entered included species, dbh, and height, where available. Missing tree heights were not seen as a source of error because tree height is not directly used in BA or tree mortality projections in FVS-Sn. Basal area and number of trees per acre were chosen as the primary attributes of interest to avoid the complications associated with volume estimation, errors in tree height measurements or estimates, and because basal area is highly correlated with volume and biomass (Grosenbaugh 1967, Packard and Radtke 2007). Site index and the site tree species from FIA records were entered as initial conditions for each plot. Optional attributes were provided as inputs to the

FVS-Sn simulations where available from the FIA database, even though they are not directly used by FVS-Sn in projecting BA or TPA: latitude and longitude, ecological section, stand age, slope aspect, percent slope, and elevation.

Information on baseline accuracy was obtained by comparing FIA data from remeasured plots to FVS-Sn projections on the same plots. To match the objectives of typical applications for growth and yield modeling, accuracy was assessed for area-based, or per acre, attributes. A summary statistic denoting the percentage of predictions accurate to within ±15% of the observed value (PA-15) was calculated from FVS-Sn predictions and observed growth from remeasured FIA plots (Rykiel 1996, Rauscher et al. 2000). Predicted values were based on 5-year FVS-Sn projections to match the prediction interval of the log-transformed BA increment equation embedded within FVS-Sn (Wykoff 1990). Observed values from the FIA database were standardized to the 5-year interval length.

$$BA_2' = BA_1 + \frac{5(BA_2 BA_1)}{t_2 - t_1} \tag{2}$$

where BA_2' = basal area (ft^2 ac^{-1}) at end of a standardized 5-year growth interval; BA_1 = FIA plot basal area observed at start of growth interval; BA_2 = FIA plot basal area observed at end of growth interval; $t_2 - t_1$ = growth interval measurement period (number of growing seasons) for FIA plot.

Measurement intervals denoted $t_2 - t_1$ in Equation 2 were recorded in the FIA database for each field plot to the nearest 0.1 year, based on field survey dates for the first and second measurements and the number of growing seasons between measurements. The lengths of measurement intervals ranged from 1.0 to 8.8, with an average interval of 4.1 growing seasons. Because Equation 2 uses linear interpolation to standardize the observed growth intervals for all remeasured FIA plots to 5 years, it was assumed that this step would cause some errors in plots having measurement intervals much different from 5 years. To determine the impact of such errors, analyses were conducted separately for plots grouped by 1-year remeasurement interval classes.

In addition to PA-15, model accuracy and bias were characterized by root mean squared error (RMSE) and mean bias statistics for the FIA remeasurement data. Negative values of mean bias from Equation 4 indicate that model predictions, on average, overestimate observed values, whereas positive values indicate model underprediction.

$$RMSE = \sqrt{\frac{1}{(n-1)}\sum(BA_2' - \widehat{BA}_2)^2} \tag{3}$$

$$\overline{BIAS} = \frac{1}{n}\sum(BA_2' - \widehat{BA}_2) \tag{4}$$

where BA_2' = FIA observed basal area (ft^2 ac^{-1}) at end of a standardized 5-year growth interval; $\widehat{BA}_2$ = FVS-Sn predicted basal area at the end of a 5-year growth interval; n = number of FIA plots on which BA_2' and $\widehat{BA}_2$ were observed.

Modifying FVS-Sn Mortality Model Coefficients

Following baseline accuracy assessment, regression analysis was used to estimate species-group-specific coefficients for the FVS-Sn mortality Equation 5 from individual tree records in the FIA data set. Noting that Equation 1 represented a modification of the standard logistic model, the constant 0.5 was dropped in favor of Equation 5 (Hamilton 1986). The four species groups listed in Table 1 were used for this step. In addition, species-specific regression coefficients for Equation 5 were estimated for species with sufficient numbers of observations in the data set.

$$P(Mort) = \frac{1}{[1 + exp\{\widehat{b_0} + \widehat{b_1}dbh\}]} \tag{5}$$

Other than a functional dependency of *P*(*Mort*) on tree dbh, it was hypothesized that one or more plot attributes would affect tree mortality. Two measures of stand density, BA (ft^2 ac^{-1}) and SDI (10-in. trees ac^{-1}) were considered, along with the quadratic mean dbh (QMD) (inches), a measure of average tree size, in Equation 6.

$$P(Mort) = \frac{1}{[1 + exp\{\widehat{b_0} + \widehat{b_1}dbh + \widehat{b_2}BA + \widehat{b_3}SDI + \widehat{b_4}QMD\}]} \tag{6}$$

Separate sets of group and species-specific coefficients were estimated for Equations 5 and 6 by maximum likelihood using logistic regression analysis and forward selection with $\alpha = 0.05$. Tree records from one FIA plot were omitted for each of 1,780 replicated data sets used in regression fitting. The purpose of the "leave-one-out" approach was to generate a set of 1,780 data sets that would each be independent of the omitted FIA plot, similar to a jackknife procedure (Harrell 2001, p. 93–94) for use in model evaluation. Mortality model coefficients b_0 and b_1 in the FVS-Sn background mortality model were modified according to the newly fitted regression Equation 5, one set of coefficients for each replicated data set. Modification of the *P*(Mort) function in FVS-Sn was made to accommodate any of the additional predictors $b_2 - b_4$ found to be statistically significant in logistic regression analyses based on Equation 6.

Observations of survival or mortality were coded as a binary [0, 1] variable, based on a 1-year time interval, so that *P*(Mort) could be estimated by logistic regression. To standardize measurement intervals to an annual basis for estimating *P*(Mort), tree records were replicated r_t times each, where r_i was the integer value of the remeasurement interval, rounded to the nearest integer, for the *i*th plot. For trees that survived the entire measurement interval, all replicated records were coded as 0, indicating no mortality. For trees that died between measurements, r_i was set to match the number of years between the initial measurement and the year of death, which was recorded by field crews based on FIA protocols for estimating year of death in periodic plot examinations (Forest Inventory and Analysis 2007). Also, for trees that died, only the final replicated record was coded as 1, indicating that mortality occurred in the final "annual" observation.

Because the average remeasurement interval was 4.1 years, the number of replicated annual observations of mortality available for regression model fitting was about 4 times the number of tree records from the FIA evaluation database. Mortality coefficient estimates were also needed for species not adequately represented in the FIA data set. To meet this need data were aggregated according to the FVS-Sn mortality species groups (Table 1), and sets of regression coefficients for each group were estimated for Equations 5 and 6. The fitting procedure was replicated for each of the leave-one-out

Table 2. Accuracy statistics for Forest Vegetation Simulator Southern Variant 5-year predictions for basal area (BA) and trees per acre (TPA) compared with 1,780 Forest Inventory and Analysis plots, under alternative scenarios for modifying the background mortality model. SDI_{mod}, stand density index threshold for moderate density-dependent mortality; SDI_{sev}, stand density index threshold for severe density-dependent mortality; RMSE, root mean squared error; Mort, mortality; PA-15, percentage of predictions accurate to within ±15% of observed values.

			Bias		PA-15		RMSE	
Scenario	SDI_{mod}	SDI_{sev}	BA (ft^2ac^{-1})	TPA	BA	TPA	BA (ft^2ac^{-1})	TPA
					......(%)......			
A. Baseline	55	85	−3.4	−6.0	88.4	88.0	9.5	11.5
B. *P*(*Mort*) × 2	55	85	−2.9	−4.7	88.8	89.6	9.5	11.4
C. Equation 5	55	85	−1.5	−1.2	90.4	92.4	9.5	11.1
D. Equation 6	55	85	−1.4	−1.1	90.6	92.4	9.4	11.2
E. Equation 5	95	99	−1.6	−1.1	90.9	93.7	9.1	10.3
F. Equation 6	95	99	−1.1	−0.4	91.3	93.9	9.0	10.3

data sets so that 1,780 sets of species-group-specific regression coefficients for Equations 5 and 6 were generated, in addition to the 1,780 species-specific coefficient sets.

Following leave-one-out refitting of the background mortality regression Equations 5 and 6, SDI_{mod} and SDI_{sev} were systematically changed in ±5% steps, rerunning FVS-Sn at each step. The constraint $SDI_{mod} < SDI_{sev}$ was imposed to prevent illogical scenarios. Lower (20%) and upper (99%) limits for both parameters were set to avoid either the application of density-dependent mortality at very low stand densities or its exclusion in FVS-Sn altogether. Results were examined to determine what effects modifying the density-dependent mortality threshold parameters had on accuracy statistics or patterns in residuals.

Characterizing Gains in Accuracy Following Modification

To assess the accuracy of FVS-Sn predictions that implemented newly developed regression coefficients, plot-based projections $\hat{Y}_{(-i)}$ were generated for each of the FIA evaluation plots $\{i = 1, 2, \ldots, 1{,}780\}$, where $\hat{Y}_{(-i)}$ denotes an FVS-Sn projection for an attribute of interest Y, where the mortality model coefficients used to predict $\hat{Y}_{(-i)}$ were estimated with data from the ith plot left out. Basal area and number of trees per acre were examined using this convention, and PA-15 statistics were computed for both attributes. Accuracy statistics were then formulated as in Equations 3 and 4, but with $\widehat{BA}_2$ replaced by $\hat{Y}_{(-i)}$ from each of the FIA plots.

Final Mortality Estimates

Following the leave-one-out accuracy assessment for FVS-Sn projections using new regression coefficients from Equation 6, coefficients were estimated from all 1,780 plots in a final model fitting. Species-specific coefficient sets for Equation 6 were developed for all species recorded on ≥27 plots the FIA data, so long as logistic regression results showed significant statistical evidence ($\alpha = 0.05$) to include at least one predictor in the mortality model. Final models were also developed for species groups (Table 1) for cases where species-specific equations could not be developed because of a paucity of observations or lack of significant predictors. Final model results were implemented in a modified version of FVS-Sn, and 25-year projections for each of the FIA field plots were generated. For comparison, 25-year projection results were generated using the unmodified FVS-Sn model for the same plots. Projection results were plotted graphically and compared with maximum SDI values from Keyser (2011).

Results

Baseline Accuracy

Baseline FVS-Sn predictions for standardized 5-year growth intervals for either BA or TPA were within ±15% of observed values for 88% of the FIA field plots in mixed hardwood forest types in the Southern Appalachian Mountains (Table 2, scenario A). Overprediction biases were noted for both BA and TPA. Root mean squared errors for baseline predictions of BA and TPA were 9.8 and 8.0%, respectively, of FIA-observed means at the end of the 5-year projection intervals ($\overline{BA} = 97.3$ ft^2 ac^{-1} and $\overline{TPA} = 144$ trees ac^{-1}).

Modifying Mortality Coefficients

Replacing the coefficient 0.5 in the FVS-Sn background mortality Equation 1 with a constant 1.0 to match the standard logistic model in Equation 5 resulted in slight gains in accuracy for both BA and TPA (Table 2, scenario B). This doubling of *P*(Mort) from the baseline model reduced mean bias by 15% for BA predictions and 21% for TPA. A slight decrease in RMSE reflected the reduced bias; however, model precision did not increase appreciably by doubling background mortality probabilities.

Using the leave-one-out approach and refitting Equation 5 to the 1,780 replicated data sets, accuracy statistics showed a reduction of overestimation biases by 56 and 80% for BA and TPA, respectively, compared with the baseline FVS-Sn results (Table 2, scenario C). The 15% prediction accuracy increased by several percentage points when the refitted mortality equations were used. Following the mortality model refitting, scatter plots of leave-one-out residuals ($TPA_2' - \widehat{TPA}_2$) showed that the distribution of prediction errors was negatively skewed over a range of initial BA values up to about 150 ft^2 ac^{-1} (Figure 2A), a result consistent with model overprediction of TPA. At initial BA values > 150 ft^2 ac^{-1}, underprediction errors in both BA (not shown) and TPA were noted (Figure 2A).

Modification of the background mortality equation to include additional predictors BA, SDI, and QMD from Equation 6 required slightly more involved modification of FVS-Sn than simply refitting the coefficients for Equation 5 to FIA observed mortality data; however, it also led to small additional gains in accuracy (Table 2, scenario D). Patterns in residuals (not shown here) for modification scenario D were similar to those shown for scenario C in Figure 2A.

Reducing the SDI_{mod} and SDI_{sev} thresholds below the default values of 55 and 85% of SDI_{max} in some cases decreased overall prediction bias, but generally increased RMSE and exacerbated the

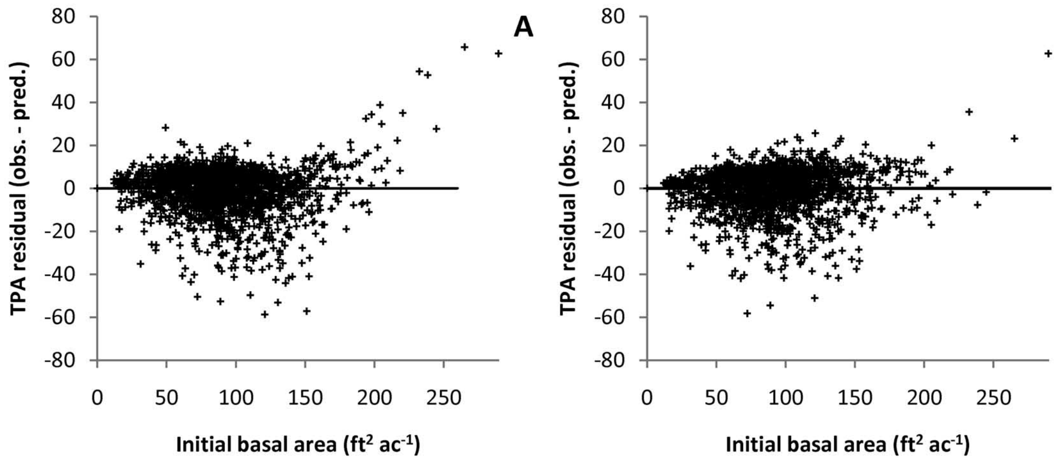

Figure 2. Trees per acre leave-one-out residuals for 1,780 Forest Inventory and Analysis plots predicted by the Forest Vegetation Simulator Southern Variant. A, Following modification of background mortality coefficients with default stand density index thresholds (scenario C; Table 2). B, Setting the SDI thresholds for moderate and severe density-dependent mortality to 95 and 99%, respectively (scenario E; Table 2).

Table 3. Accuracy statistics by forest type for Forest Vegetation Simulator Southern Variant (FVS-Sn) 5-year projections for basal area (BA) and trees per acre (TPA) compared with 1,780 Forest Inventory and Analysis plots in leave-one-out error assessment following modifications to the FVS-Sn mortality prediction equations. PA-15, percentage of predictions accurate to within ±15% of observed values.

		Bias		PA-15		RMSE	
Forest type	Plots (no.)	BA	TPA	BA	TPA	BA	TPA
				......(%)......			
Post-blackjack oak	42	−1.9	−1.4	90.5	97.6	5.6	9.1
Chestnut oak	202	−1.8	0.2	89.6	95.5	9.7	10.1
White oak–red oak–hickory	627	−1.5	−1.5	91.9	92.7	7.7	10.5
White oak	67	−2.4	−0.7	92.5	97.0	5.5	8.7
Yellow poplar–white oak–red oak	206	0.4	0.5	93.7	95.6	6.8	8.7
Sweetgum–yellow poplar	60	−0.7	−0.9	90.0	93.3	6.5	9.9
Yellow poplar	33	−1.4	−1.8	90.9	93.9	12.0	10.9
Chestnut–black–scarlet oak	369	−0.7	1.4	92.4	94.6	9.3	10.4
Mixed upland hardwoods	77	−0.7	−3.0	81.8	88.3	9.2	12.9
Sugar maple–beech–yellow birch	70	−0.6	1.9	91.4	92.9	19.0	11.8
Hard maple–basswood	27	−2.5	−2.8	85.2	92.6	10.7	7.5

residual pattern shown in Figure 2A. Increasing the thresholds resulted in small gains in PA-15 over and above the gains realized by refitting mortality Equation 5 or 6 (Table 2, scenarios E and F). It also noticeably reduced the apparent underestimation biases in both BA (not shown) and TPA in high-density plots (Figure 2B).

Characterizing Gains in Accuracy following Modification

Several detailed examinations of accuracy statistics were made following the modification of the FVS-Sn mortality model based on Equation 6 and adjustments that set $SDI_{mod} = 95\%$ and $SDI_{sev} = 99\%$ (Table 2, scenario F). Fifteen percent prediction accuracy varied with the length of the FIA plots' remeasurement intervals. For the plots used here to evaluate FVS-Sn accuracy, PA-15 was generally lowest for those having shortest measurement intervals, although relatively few plots having 8- and 9-year remeasurement intervals were available for testing. Bias and RMSE were generally highest for plots with short remeasurement intervals, with a pattern of model overprediction (negative bias) ranging between 0 and 2% evident across most measurement periods. Prediction accuracies for 4–9-year plot remeasurement lengths were consistent with those computed across all plots and reported in Table 2, scenarios E and F.

The 15% prediction accuracy for FVS-Sn following modifications exceeded 89% for all forest types except for mixed upland hardwoods and hard maple-basswood forest types, which together made up only about 6% of the study plots (Table 3). Biases as large as -2.8 ft^2 ac^{-1} for BA and -2.5 trees ac^{-1} were observed, both in the hard maple-basswood forest type. Predictions typically exceeded observed values of BA and TPA, with the exception being the yellow-poplar-white oak-red oak forest type. Accuracy statistics for the 16 individual FIA survey units also showed that biases did not exceed 2 ft^2 ac^{-1} for BA or 3 trees ac^{-1} in absolute value, values generally in the range of ±2.2% of survey unit means.

Final Mortality Estimates

Forty-four species were observed on at least 27 FIA plots (Table 4), making them candidates for species-specific logistic regression mortality models. At least one predictor showed a significant relationship to mortality in 22 of the species and all four species groups, which included a separate group for eastern redcedar (Table 5). For 15 species and three of the four species groups, dbh was identified as a significant predictor of mortality. In all but 3 of these, the positive sign on dbh coefficients indicated decreasing P(Mort) with increasing dbh. The three species having negative signs on dbh coefficients had P values >0.01, an indication of somewhat modest evidence for including dbh as predictors in this model. The next most commonly identified

Table 4. Forest Vegetation Simulator Southern Variant (FVS-Sn) species codes and common names of 44 species fitted to the logistic regression equation (Equation 6).

FVS-Sn			FVS-Sn		
No.	Code	Species common name	No.	Code	Species common name
86	AE	American elm	77	PO	Post oak
26	AH	American hornbeam	10	PP	Pitch pine
24	BB	Birch spp.	32	PS	Common persimmon
62	BC	Black cherry	30	RD	Eastern redbud
54	BG	Blackgum, black tupelo	87	RL	Slippery elm
70	BJ	Blackjack oak	20	RM	Red maple
80	BK	Black locust	75	RO	Northern red oak
78	BO	Black oak	57	SD	Sourwood
23	BU	Buckeye, horsechestnut	65	SK	Southern red oak
83	BW	Basswood	22	SM	Sugar maple
72	CK	Chinkapin oak	64	SO	Scarlet oak
74	CO	Chestnut oak	5	SP	Shortleaf pine
47	CT	Cucumbertree	82	SS	Sassafras
31	DW	Flowering dogwood	44	SU	Sweetgum
37	GA	Green ash	14	VP	Virginia pine
40	HA	Silverbell	35	WA	White ash, American ash
56	HH	Eastern hophornbeam	85	WE	Winged elm
27	HI	Hickory spp.	73	WK	Water oak
17	HM	Hemlock	43	WN	Black walnut
2	JU	Eastern redcedar	63	WO	White oak
13	LP	Loblolly pine	12	WP	Eastern white pine
90	OT	Unknown or not listed	45	YP	Yellow-poplar

Table 5. Mortality prediction equation (Equation 6) coefficients for 24 species and 3 Forest Vegetation Simulator Southern Variant species groups for upland hardwood forests in the Southern Appalachian Mountains. BA, basal area; SDI, stand density index; QMD, quadratic mean dbh.

Species/group	Plots (no.)	Trees	n	Intercept	dbh	BA	SDI	QMD
AH	48	68	241	5.7019	−0.4140[a]	—[b]	—[b]	—[b]
BJ	36	93	319	8.2744	—[b]	—[b]		−0.5845[a]
BO	642	1,484	6,091	4.4860	0.1197	—[b]	—[b]	−0.1609[a]
DW	235	363	1,168	3.4302	—[b]	—[b]	−0.0063	—[b]
GA	94	195	805	7.1898	—[b]	−0.0235[a]	—[b]	—[b]
HA	27	81	326	6.2714	−0.1722[a]	—[b]	—[b]	—[b]
HI	1,159	4,035	16,465	3.7797	0.1472	—[b]	—[b]	—[b]
JU	118	321	1,342	2.4715	—[b]	—[b]	0.0125[a]	—[b]
LP	114	261	987	7.2222	0.2023[a]	—[b]	−0.0323	—[b]
PP	73	142	565	7.7426	—[b]	—[b]	−0.0243	—[b]
RM	1,080	4,423	17,829	6.7526	—[b]	—[b]	—[b]	−0.1656
RO	689	1,590	6,538	3.2001	0.1191	—[b]	—[b]	—[b]
SD	825	2,535	10,041	2.4996	0.3599	—[b]	—[b]	—[b]
SK	306	806	3,000	2.8172	0.1755	—[b]	—[b]	—[b]
SM	327	1,197	5,130	7.1651	—[b]	—[b]	—[b]	−0.1982[a]
SO	626	1,912	7,809	4.7115	0.1083	—[b]	—[b]	−0.1519[a]
SP	252	561	1,990	4.6652	—[b]	—[b]	−0.0092	—[b]
SS	171	318	1,309	2.0890	0.1809[a]	—[b]	—[b]	—[b]
VP	276	670	2,514	3.5501	0.1096	−0.0129	—[b]	—[b]
WA	267	611	2,622	3.0163	0.1581[a]	—[b]	—[b]	—[b]
WE	89	179	634	6.4660	−0.2859[a]	—[b]	—[b]	—[b]
WO	983	4,115	16,752	6.7723	0.1520	−0.0079[a]	—[b]	−0.2422
YP	944	3,982	16,255	4.5045	0.2144	—[b]	—[b]	−0.1563
Group 1	1,692	12,653	50,358	4.8088	0.0603	—[b]	0.0033	−0.1401
Group 2	690	2,143	8,134	4.2577	0.0511[a]	—[b]	−0.0069	—[b]
Group 3	1,782	29,427	119,023	4.7899	0.1109	—[b]	—[b]	−0.1076

[a] Coefficient estimates with P values between 0.01 and 0.05. Others resulted in P values of <0.01.
[b] Dashes indicate a lack of statistical evidence ($\alpha = 0.05$) for nonzero coefficients.

predictor was QMD, followed by SDI and stand BA. All but two of the estimated coefficients for QMD, SDI, and BA were negative in sign, meaning that mortality rates generally increased along with measures of stand density or mean tree size.

SDI remained below SDI_{max} for all predictions, including a small number of plots close to SDI_{max} at the starts of their growth intervals that subsequently dropped farther below the SDI_{max} boundary (Figure 3) over a 25-year projection. Slopes for modified model trajectories were steeper than the unmodified model trajectories, on average, by −0.49 units (logarithmic scale). For plots that started out close to SDI_{max}, trajectories were similar for both the modified and unmodified models. Steeper, longer trajectories for plots having large initial values of QMD reflected the increased mortality rates in the modified model because of negative QMD coefficients in the regression Equation 6 for hardwood species groups 1 and 3, and several species, e.g., sugar

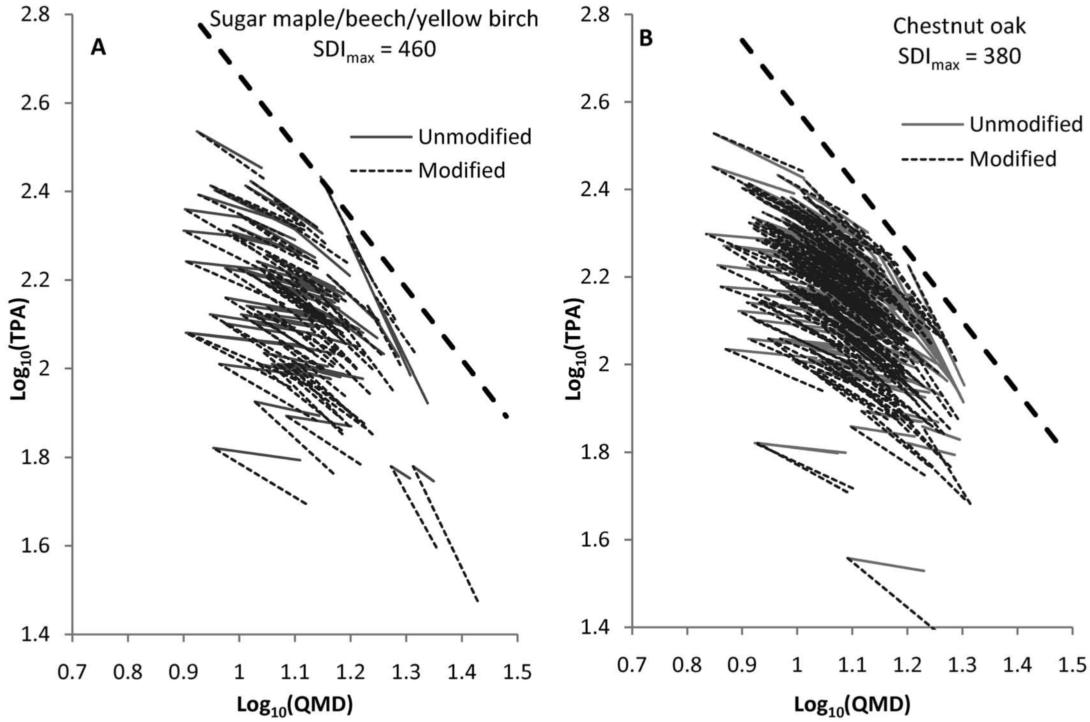

Figure 3. Size-density trajectories (log-log scale) of two forest types for 25-year growth simulations in the Forest Vegetation Simulator Southern Variant, with and without modification of the mortality model. Heavy dashed lines correspond to forest type maximum possible SDI (SDI_{max}). QMD, quadratic mean dbh.

maple (*Acer saccharum* Marshall), having their own coefficient sets (Table 5).

Discussion and Conclusions

Baseline projections for FVS-Sn showed that accuracy within ±15% of observed values was obtained on slightly more than 88% of remeasured FIA plots from upland mixed hardwoods in the Southern Appalachians (Table 2). In addition, baseline FVS-Sn projections were biased, on average, by 3.5% for BA and 4.1% for TPA. These results are in line with other models that predict growth of upland hardwoods in the southern United States (Rauscher et al. 2000). Prediction biases are not uncommon in tests of growth and yield models, and those noted here seem to be fairly typical (Holdaway and Brand 1986, Kowalski and Gertner 1989).

Relatively simple modifications to FVS-Sn were proposed that led to improved results in precision, bias, or both. Modifying the mortality function by replacing the constant 0.5 in Equation 1 with 1.0 as in Equation 5 led to modest reductions in prediction biases and slight gains in accuracy. Updating the mortality equation coefficients based on logistic regression analysis of FIA data from the region led to additional gains in accuracy, including the reduction of model biases by more than one-half (Table 2). Small additional gains were realized by including one or more stand-level predictors in the *P*(Mort) regression model Equation 6, either in addition to or in place of the tree-level predictor dbh.

A goal of this work was to seek improvements that could be implemented in FVS-Sn without requiring major changes to its program structure; as such, we did not include random effects in the mortality models tested. Doing so would likely lead to increased accuracy for model applications having suitable calibration data (Chen et al. 2008). The gains from such an approach are well documented, and mixed-effects models have become increasingly common in growth and yield modeling (Lappi 1991, Calama and Montero 2005, Budhathoki et al. 2008). The FVS system gives users a way to improve prediction accuracy by calibration when appropriate data are available based on growth and mortality multipliers, but its statistical basis is not as robust as mixed-effects modeling (Holdaway 1985, Hamilton 1994). Even though the present study was not aimed at making use of application-specific calibration data sets, the development and implementation of a system for calibrating FVS predictions using mixed-effects modeling is a goal worth pursuing.

Accuracy assessment based solely on TPA and BA ignores the contribution of height growth to forest production. Here, comparison of FVS-Sn volume predictions to FIA volume estimates was avoided because of possible complications related to errors in tree height measurements or discrepancies caused by the use of different volume equations, merchantability standards, or volume units. Bailes and Brooks (2004) reported gains of 7–17% in coefficients of determination when tree heights were used to predict sawtimber volumes, compared with predictions based on BA alone. Movement toward national systems of volume and biomass equations in recent years is a positive step toward closing gaps between FIA and FVS regarding methods of estimating tree volumes (Shaw 2009, Forest Management Service Center 2011).

Accuracy rates reported in Table 2 apply to 4–9-year FVS-Sn projections, with slightly lower accuracy noted in projections shorter than 4 years. It should be noted that more sophisticated analytical procedures exist for interpolating observations from one measurement interval to that of another (McDill and Amateis 1993, Cao 2000). Here, the 5-year projections from FVS-Sn were interpolated to match measurement interval lengths of 2–9 years using the same method implemented in FVS by its "cycle at" (CYCLEAT) keyword to generate projections different than a variant's default increment length (Dixon 2010, p. 31–32). Prediction accuracy generally decreases as projection intervals increase (Shortt and Burkhart 1986). Because the FIA data available for this study were limited to short (<10-year) intervals, it was not possible to determine the degree to which accuracy declines with projection intervals longer than this. The need remains for testing FVS-Sn predictions over longer projection intervals (Shaw 2009). Furthermore, because the evaluation conducted here was aimed at upland mixed-species hardwood forest types in the Southern Appalachians, additional research may be needed to evaluate FVS-Sn prediction accuracies for other species, forest types and ecological sections across the US South.

Allowing stand-level predictors in the *P*(Mort) model Equation 6 allowed for the characterization of empirical relationships where mortality was greater in plots having large QMD, high BA, or high SDI. Notable exceptions included species group 1 and eastern redcedar (JU), which exhibited lower mortality in dense plots (Table 5). Previous research has observed higher mortality in dense stands of eastern redcedar compared with open stands (Quinn and Meiners 2004), a finding not supported here. Eastern redcedar trees have been observed to grow taller in closed stands than in open ones, a characteristic atypical of most eastern tree species (Arend and Collins 1949). Although this growth characteristic may be related to the low mortality rates in dense stands observed here, logistic regression analyses showed somewhat modest evidence ($0.01 < P < 0.05$) to reject a null hypothesis H_0: $b_3 = 0$ (Table 5). Given the modest evidence for inference found here and the fact that multiple hypotheses were tested from results in Table 5, the possibility that a Type I statistical error occurred should not be disregarded. Background mortality coefficients estimated here generally resulted in higher predictions of individual-tree background mortality than the baseline FVS-Sn regression coefficients (Table 1), a finding consistent with DeRose et al. (2008), who observed higher mortality rates in longleaf pine (*Pinus palustrus* Mill.) than were predicted by FVS-Sn

Thresholds for SDI_{mod} and SDI_{sev} were adjusted to levels that effectively bypassed the density-dependent mortality algorithm in FVS-Sn, except in plots above 95% of maximum SDI. Although the justification for such modifications was not overwhelming from simply examining overall accuracy statistics, examination of residuals for the unmodified FVS-Sn model across the range of stand densities pointed to the potential for reducing prediction errors in heavily stocked stands (Figure 2). Twenty-five-year size-density trajectories indicated that the modifications to Equation 6 effectively compensated for the increase in SDI_{mod} and SDI_{sev} parameters, reducing biases in model predictions for dense stands. Either of the modifications involving updated regression coefficients (Table 2, scenarios E or F) seems justified by the overall gains in accuracy that will result when implemented in FVS-Sn modeling applications for Southern Appalachian mixed-species hardwood forests. The adoption of changes made for scenario F seemed justified over those of scenario E on the basis of additional heuristic considerations. Among them are the fact that scenario F generally accounts for higher mortality rates in dense plots and those having larger mean-tree diameters, both of which are indicative of high competition levels between trees (Lorimer 1983, Holmes and Reed 1991). The utility of the SDI-based mortality model may merit further attention in applications that will use FVS-Sn to make predictions outside of the range of conditions observed in the evaluation and fitting data sets, in particular for stands having many small (<5-in. dbh) trees (Hamilton 1990). If desired, FVS users can easily adjust the SDI threshold parameters from the default values of 55 and 85% using the system's "SDIMAX" keyword (Van Dyck and Smith-Mateja 2010)

Models that project growth and yield for multiple species generally perform better for some species than others (Rauscher et al. 2000, Lessard et al. 2000). Single-species models, such as the yellow-poplar model YPOP (Knoebel et al. 1986), may outperform others that predict for mixed-species conditions, such as the upland hardwoods model G-HAT (Harrison et al. 1986). Results for FVS-Sn showed that the model was more accurate for some forest types than others, but it was not necessarily most accurate for types dominated by one or two species (Table 3).

FVS-Sn prediction accuracies were initially found to be in line with other operational growth and yield models for Southern Appalachian mixed-species forests (Rauscher et al. 2000). In modifying the background mortality prediction equation using FIA data from mixed-species upland hardwood forests of the region, PA-15 was increased to 90% or more for both BA and TPA. Model users can expect to realize these accuracy rates in applications of FVS-Sn for Southern Appalachian mixed-species forests over 4–9 year projections when external mortality agents are assumed to be absent. With remeasurement data from FIA increasingly available across the South and other parts of the United States (Forest Inventory and Analysis 2009b), modelers have greater ability to test the accuracy of FVS-Sn and other growth and yield models and ultimately to improve their prediction accuracies.

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