# RELATIVE MAXIMA OF DIAMETER AND BASAL AREA

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It has often been observed that maximum dbh growth occurs at an earlier age than maximum individual tree basal area growth. This can be deduced from the geometry of the tree stem, by observing that a dbh increment at a given radius will be associated with a larger basal area increment than an equal dbh increment occurring at a shorter radius from the stem center. Thus basal area increment continues to increase after dbh increment has culminated. Nevertheless it is of interest to prove mathematically that the age of maximum basal area growth occurs later than the age of maximum dbh increment for a broad range of continuous functions that could be used to model diameter and basal area growth. This can be done using differential calculus for continuous functions representing diameter and basal area increment.

Tree cross-sectional growth at breast height can be characterized by diameter growth or by basal area growth functions. West (1980) compared individual tree diameter growth functions to individual tree basal area growth functions for use in forest growth simulators. West (1980) found either function to be equally effective. Borowski (1972) used finite differences to argue that when cross-sectional area is decreasing, the change in the diameter growth rate is always negative, indicating that the age of maximum diameter growth rate has passed.

Here we use differential calculus to show that the age of maximum basal area growth always occurs later than the age of diameter growth. Differentiating basal area growth, we obtain a function of diameter growth and acceleration. Using this function it is easily shown that at the age of maximum basal area growth, acceleration in diameter is negative. Assuming that diameter and basal area follow classic sigmoid curve forms, diameter growth will attain a maximum and then decline. Thus a negative acceleration for diameter indicates that maximum diameter growth has already occurred.

Finally we wish to consider the age of maximum individual volume increment in relation to the ages of maximum basal area increment and diameter increment. It is demonstrated that if the second partial derivative of individual tree volume with respect to basal area is strictly positive, then the age of maximum individual tree volume growth will be higher than the age of maximum individual tree basal area growth.

These results could be of interest to those who use continuous, differentiable functions to model basal area and diameter growth. An advantage of modeling basal area growth might be that it has a longer period of monotonically increasing with age. On the other hand, it might be desired to model diameter growth including a maximum at a younger age.

## LITERATURE CITED

**Borowski, M.** 1972. Influence of biosocial status of a tree on the increment of cross-sectional area and volume. Folia Forestalia Polonica Series A, z 19 (in Polish).

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