

METHODOLOGY FOR CALCULATING SHEAR STRESS IN A MEANDERING CHANNEL

Prepared for the

U. S. Department of the Interior
Bureau of Reclamation
Albuquerque Area Office
555 Broadway N.E., Suite 100
Albuquerque, New Mexico 87102-2352

*This research was supported in part by funds provided by the
Rocky Mountain Research Station, Forest Service, U. S. Department of Agriculture.*



Prepared by

Kyung-Seop Sin, Christopher I. Thornton, Amanda L. Cox, and Steven R. Abt

June 2012

Colorado State University
Daryl B. Simons Building at the
Engineering Research Center
Fort Collins, Colorado 80523



METHODOLOGY FOR CALCULATING SHEAR STRESS IN A MEANDERING CHANNEL

Prepared for the

U. S. Department of the Interior
Bureau of Reclamation
Albuquerque Area Office
555 Broadway N.E., Suite 100
Albuquerque, New Mexico 87102-2352

*This research was supported in part by funds provided by the
Rocky Mountain Research Station, Forest Service, U. S. Department of Agriculture.*

Prepared by

Kyung-Seop Sin, Christopher I. Thornton, Amanda L. Cox, and Steven R. Abt

June 2012

Colorado State University
Daryl B. Simons Building *at the*
Engineering Research Center
Fort Collins, Colorado 80523



EXECUTIVE SUMMARY

Shear stress in meandering channels is the key parameter to predict bank erosion and bend migration. A representative study reach of the Rio Grande River in central New Mexico has been modeled in the Hydraulics Laboratory at Colorado State University. To determine the shear stress distribution in a meandering channel, a large-scale (1:12) physical modeling study was conducted in the following phases: 1) model construction, 2) data collection, 3) data analysis, and 4) conclusions and technical recommendations. Flow depth, flow velocity in three velocity components (V_x , V_y , and V_z), and bed shear stress data were collected in the laboratory. Bed shear stresses were directly measured using a Preston tube.

According to the laboratory data analysis, shear stress from a Preston tube is the most appropriate shear stress calculation method. In the case of the Preston tube, data collection was performed directly on the surface of the channel. Other shear stress calculation methods were based on Acoustic Doppler Velocity (ADV) data that were not collected directly on the bed surface. Therefore, the shear stress determined from ADV measurements was underestimated. Additionally, ratio of maximum shear stress to average shear stress (K_b) plots were generated. Finally, the envelope equation for K_b from the Preston tube measurements was selected as the most appropriate equation to design meandering channels.

TABLE OF CONTENTS

EXECUTIVE SUMMARY	i
LIST OF FIGURES	iv
LIST OF TABLES	viii
LIST OF SYMBOLS, UNITS OF MEASURE, AND ABBREVIATIONS	x
1 INTRODUCTION.....	1
1.1 Background.....	1
1.2 Project Background	1
1.3 Research Objectives.....	2
2 LITERATURE REVIEW	3
2.1 Introduction.....	3
2.2 Open Channel Flow in Meandering Bends.....	4
2.2.1 Meandering Channel Geometry	4
2.2.2 Flow Patterns in Meandering Channels.....	6
2.2.2.1 Stream Morphology	6
2.2.2.2 Transverse Velocity Distribution in Bends.....	10
2.2.2.3 Super Elevation of the Water Surface against the Outside of the Bank	11
2.2.3 Erosion Patterns in Meandering Rivers.....	12
2.2.4 Channel Stability	14
2.3 Shear Stress Calculation Methods	15
2.3.1 Preston Tube Method	15
2.3.1.1 Preston (1954)	16
2.3.1.2 Ippen and Drinker (1962).....	19
2.3.1.3 Heintz (2002)	21
2.3.1.4 Sciafani (2008)	22
2.3.2 Linear Regression of the Flow Velocity Profile.....	23
2.3.2.1 Clauser (1956).....	23
2.3.2.2 Schlichting (1987).....	25
2.3.2.3 Smart (1999)	26
2.3.3 Rozovskii (1961) Method.....	27
2.3.3.1 Laboratory Study for Validation of Theory	29
2.3.3.2 Field Observation for Validation of Theory	29
2.3.3.3 Summary	32
2.3.4 Reynolds Shear Stress Extrapolation.....	32
2.3.4.1 Wahl (2000)	33
2.3.4.2 Baird (2004)	35
2.4 Summary.....	37
3 DATA ANALYSIS	38
3.1 Introduction.....	38

3.2 Shear Stress Calculation Results	38
3.2.1 Preston Tube Method	39
3.2.2 Linear Regression of the Flow Velocity Profile	44
3.2.3 Rozovskii Method	47
3.2.4 Reynolds Shear Stress Extrapolation.....	52
3.3 Summary.....	59
4 COMPARISONS OF SHEAR STRESS CALCULATION METHODS.....	60
4.1 Comparison of Shear Stress from the Preston Tube Method and Linear Regression of the Velocity Profile for the Longitudinal Direction	61
4.2 Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile for the Lateral Direction (τ_y) and Rozovskii Method	64
4.3 Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation.....	65
4.4 Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile for the Longitudinal Direction (τ_x) and Reynolds Shear Stress Extrapolation	69
4.5 Comprehensive Comparisons	73
4.6 Averaged Shear Stress from HEC-RAS and K_b Value.....	76
5 CONCLUSIONS AND RECOMMENDATIONS.....	80
5.1 Overview.....	80
5.2 Conclusions.....	80
5.3 Recommendations for Future Research.....	82
5.4 Limitations of this Study	83
6 REFERENCES.....	84
APPENDIX A PHYSICAL MODEL DESCRIPTION	87
APPENDIX B PRESTON TUBE CALIBRATION.....	115
APPENDIX C LINEAR REGRESSION OF THE FLOW VELOCITY PROFILE	120
APPENDIX D SHEAR STRESS FROM ROZOVSKII (1961) METHOD	131
APPENDIX E REYNOLDS SHEAR STRESS EXTRAPOLATION	142
APPENDIX F COMPARISON OF SHEAR STRESS FROM LINEAR REGRESSION OF THE FLOW VELOCITY PROFILE FOR THE LATERAL DIRECTION (τ_y) AND ROZOVSKII METHOD	153
APPENDIX G COMPARISON OF SHEAR STRESS MAXIMUM VALUES	156

LIST OF FIGURES

Figure 2.1: Typical Meandering Channel (from Watson <i>et al.</i> (2005)).....	3
Figure 2.2: Geometry of Meandering Channel (from Watson <i>et al.</i> (2005)).....	5
Figure 2.3: Description of Sine-generated Curve Function (from Knighton (1998)).....	6
Figure 2.4: Graphic Description of a Straight Channel and a Meandering Channel (from the Federal Interagency Stream Restoration Working Group (FISRWG) (1998))	7
Figure 2.5: Typical Meandering Stream with Point Bars (from Watson <i>et al.</i> (2005)).....	7
Figure 2.6: Typical Middle Bar (from Watson <i>et al.</i> (2005))	8
Figure 2.7: Typical Alternate Bars (from Watson <i>et al.</i> (2005))	9
Figure 2.8: Typical Braid Bars (from Watson <i>et al.</i> (2005))	9
Figure 2.9: Schematic Descriptions of Flow in a Meandering Channel (from FHWA (2001))	10
Figure 2.10: Sketch of the Flow around a Bend (from FHWA (2001))	11
Figure 2.11: Geomorphic Change of Straight Channel into Meandering Channel (from Knighton (1998))	13
Figure 2.12: Relationship of Relative Migration Rate to Bend Curvature for Eighteen Rivers (from Knighton (1998))	13
Figure 2.13: Seven Modes of Meander Loop Development (from FHWA (2001)).....	14
Figure 2.14: Lane's Balance (from Rosgen (1996))	15
Figure 2.15: Preston Data-collection Facility (from Preston (1954))	17
Figure 2.16: Actual Observations of Round Pitot on Wall of Circular Pipe (from Preston (1954))	18
Figure 2.17: Plot of Non-dimensional Results of Preston (1954) Experiments	19
Figure 2.18: Laboratory Flumes (from Ippen and Drinker (1962)).....	20
Figure 2.19: Calibration Instrumentation for Shear Stress Measurement (from Heintz (2002))	22
Figure 2.20: Flume at Colorado State University (Sclafani 2008)	23
Figure 2.21: Relationship between Log Profile Parameters and Shear Velocity Head (from Smart (1999))	27
Figure 2.22: Velocity Distribution from the Experiment of Polygonal Section Test Results (from Rozovskii (1961))	29
Figure 2.23: Description of the Desna River Study Reach (from Rozovskii (1961)).....	30
Figure 2.24: Plan View of the Snov River (from Rozovskii (1961)).....	31
Figure 2.25: Distribution of Flow Velocity of Snov River (from Rozovskii (1961)).....	31
Figure 2.26: Surface Stresses (from Julien (1998))	32
Figure 2.27: Description of Turbulence in Flow (from Julien (1998)).....	33

Figure 2.28: Reynolds Shear Stress Profile (from Baird (2004))	36
Figure 3.1: Shear Stress Distribution (τ_x) from the Preston Tube for 8 cfs (psf).....	40
Figure 3.2: Shear Stress Distribution (τ_x) from the Preston Tube for 12 cfs (psf).....	41
Figure 3.3: Shear Stress Distribution (τ_x) from the Preston Tube for 16 cfs (psf).....	42
Figure 3.4: Shear Stress Distribution (τ_x) from the Preston Tube for 20 cfs (psf).....	43
Figure 3.5: Shear Stress Distribution (τ_x) by Linear Regression for 20 cfs (psf)	45
Figure 3.6: Shear Stress Distribution (τ_y) by Linear Regression for 20 cfs (psf)	46
Figure 3.7: Diagram of Distance between Piezo a and Piezo g	47
Figure 3.8: Idealized Flow Velocity Profile	49
Figure 3.9: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) for 20 cfs (psf)	50
Figure 3.10: Water Surface Slope Distribution for 20 cfs	51
Figure 3.11: Reynolds Shear Stress Profile of τ_{xz} for 20 cfs, Cross Section 8, Piezo e.....	53
Figure 3.12: Reynolds Shear Stress Profile of τ_{yz} for 20 cfs, Cross Section 8, Piezo e.....	53
Figure 3.13: Contour Map of τ_{bed} from the Reynolds Shear Stress for 8 cfs (psf)	55
Figure 3.14: Contour Map of τ_{bed} from the Reynolds Shear Stress for 12 cfs (psf)	56
Figure 3.15: Contour Map of τ_{bed} from the Reynolds Shear Stress for 16 cfs (psf)	57
Figure 3.16: Contour Map of τ_{bed} from the Reynolds Shear Stress for 20 cfs (psf)	58
Figure 4.1: Shear Stress Calculation Methods Comparison Schematic.....	60
Figure 4.2: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 8 cfs	61
Figure 4.3: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 12 cfs	62
Figure 4.4: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 16 cfs	62
Figure 4.5: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 20 cfs	63
Figure 4.6: Averaged Percentage Difference between the Preston Tube and Linear Regression with Change of Discharge	63
Figure 4.7: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 8 cfs	66
Figure 4.8: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 12 cfs	66
Figure 4.9: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 16 cfs	67
Figure 4.10: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 20 cfs	67
Figure 4.11: Averaged Percentage Difference between the Preston Tube and Reynolds Shear Stress Extrapolation with Change of Discharge.....	68

Figure 4.12: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 8 cfs.....	70
Figure 4.13: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 12 cfs.....	70
Figure 4.14: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 16 cfs.....	71
Figure 4.15: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 20 cfs.....	71
Figure 4.16: Averaged Percentage Difference between Reynolds Shear Stress Extrapolation and Linear Regression of the Flow Velocity Profile with Change of Discharge	72
Figure 4.17: Percentage Difference of Maximum Shear Stress with the Preston Tube for 8 cfs	74
Figure 4.18: Percentage Difference of Maximum Shear Stress with the Preston Tube for 12 cfs	74
Figure 4.19: Percentage Difference of Maximum Shear Stress with the Preston Tube for 16 cfs	75
Figure 4.20: Percentage Difference of Maximum Shear Stress with the Preston Tube for 20 cfs	75
Figure 4.21: K_b Plots from the Ratio of Maximum Shear Stress from the Preston Tube to Cross-section Averaged Shear Stress from HEC-RAS	78
Figure 4.22: K_b Plots from the Ratio of Maximum Shear Stress from the Reynolds Shear Stress to Cross-section Averaged Shear Stress from HEC-RAS.....	79
Figure A.1: Physical Model in the Hydromachinery Laboratory at Colorado State University	88
Figure A.2: Types of Bends of the Study Reach (from Heintz (2002)).....	90
Figure A.3: Main Channel Schematic.....	91
Figure A.4: Hydromachinery Laboratory Schematic.....	91
Figure A.5: Four-foot High Walls of the Model.....	92
Figure A.6: Skeleton and Filling Material of the Main Channel	93
Figure A.7: Leveling of Fill Sand of the Main Channel	93
Figure A.8: Installation of Steel Flashing.....	94
Figure A.9: Pouring of Concrete for Model Construction.....	95
Figure A.10: Concrete Broom Finish for Desired Channel Roughness.....	95
Figure A.11: Flow Meter Instrumentation (from Kinzli (2005)).....	96
Figure A.12: Point-gauge Installation for Flow Depth Measurement	97
Figure A.13: Piezometer Locations for Type 1 and Type 3 Bends (from Heintz (2002)).....	98
Figure A.14: Installation of Piezometer for Flow Depth Measurement	98
Figure A.15: Stilling Wells for Water Surface Elevation Measurements.....	99

Figure A.16: Photograph of the ADV (from Heintz (2002)).....	100
Figure A.17: Installation of the ADV in the Data-collection Cart.....	101
Figure A.18: Preston Tube for Shear Stress Measurement (from Sclafani (2008)).....	102
Figure A.19: Stop-log Installation of the Model.....	103
Figure A.20: Stop Log for Calibration of the Model.....	103
Figure A.21: Baseline Condition Test Matrix.....	104
Figure C.1: Shear Stress Distribution (τ_x) by Linear Regression for 8 cfs (psf).....	125
Figure C.2: Shear Stress Distribution (τ_x) by Linear Regression for 12 cfs (psf).....	126
Figure C.3: Shear Stress Distribution (τ_x) by Linear Regression for 16 cfs (psf).....	127
Figure C.4: Shear Stress Distribution (τ_y) by Linear Regression for 8 cfs (psf).....	128
Figure C.5: Shear Stress Distribution (τ_y) by Linear Regression for 12 cfs (psf).....	129
Figure C.6: Shear Stress Distribution (τ_y) by Linear Regression for 16 cfs (psf).....	130
Figure D.1: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) Method for 8 cfs (psf).....	136
Figure D.2: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) Method for 12 cfs (psf).....	137
Figure D.3: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) Method for 16 cfs (psf).....	138
Figure D.4: Water Surface Slope Distribution for 8 cfs	139
Figure D.5: Water Surface Slope Distribution for 12 cfs	140
Figure D.6: Water Surface Slope Distribution for 16 cfs	141
Figure F.1: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 8 cfs.....	154
Figure F.2: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 12 cfs.....	154
Figure F.3: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 16 cfs.....	155
Figure F.4: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 20 cfs.....	155
Figure G.1: Comparison of Shear Stress Maximum Values (8 cfs Upstream).....	157
Figure G.2: Comparison of Shear Stress Maximum Values (8 cfs Downstream).....	158
Figure G.3: Comparison of Shear Stress Maximum Values (12 cfs Upstream).....	158
Figure G.4: Comparison of Shear Stress Maximum Values (12 cfs Downstream).....	159
Figure G.5: Comparison of Shear Stress Maximum Values (16 cfs Upstream).....	159
Figure G.6: Comparison of Shear Stress Maximum Values (16 cfs Downstream).....	160
Figure G.7: Comparison of Shear Stress Maximum Values (20 cfs Upstream).....	160
Figure G.8: Comparison of Shear Stress Maximum Values (20 cfs Downstream).....	161

LIST OF TABLES

Table 2.1: Dimension of the Pitot Tubes (from Preston (1954))	17
Table 2.2: Geometric Information of Flume (from Ippen and Drinker (1962))	19
Table 2.3: Summary of Experiment Results of Flow in Bend (from Rozovskii (1961)).....	28
Table 4.1: Calculation Results of Percentage Difference between the Preston Tube and Linear Regression of the Flow Velocity Profile	64
Table 4.2: Calculation Results of Percentage Difference between Linear Regression of the Flow Velocity Profile and Rozovskii Method	65
Table 4.3: Calculation Results of Percentage Difference between the Preston Tube and Reynolds Shear Stress Extrapolation.....	69
Table 4.4: Calculation Results of Percentage Difference between Linear Regression of the Flow Velocity Profile for the Longitudinal Direction and Reynolds Shear Stress Extrapolation.....	73
Table 4.5: HEC-RAS Output for Shear Stress for Upstream Bend (psf).....	76
Table 4.6: HEC-RAS Output for Shear Stress for Downstream Bend (psf).....	76
Table 4.7: K_b Value from Maximum Shear Stress of the Reynolds Shear Stress Extrapolation	77
Table 4.8: K_b Value from Maximum Shear Stress of the Preston Tube Method.....	77
Table A.1: Geometry of the Prototype Study Reach (from Heintz (2002)).....	89
Table A.2: Geometry Information of the Physical Model (from Heintz (2002))	89
Table B.1: Shear Stress from the Preston Tube Calibration for 8 cfs.....	116
Table B.2: Shear Stress from the Preston Tube Calibration for 12 cfs.....	117
Table B.3: Shear Stress from the Preston Tube Calibration for 16 cfs.....	118
Table B.4: Shear Stress from the Preston Tube Calibration for 20 cfs.....	119
Table C.1: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 8 cfs	121
Table C.2: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 12 cfs	122
Table C.3: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 16 cfs	123
Table C.4: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 20 cfs	124
Table D.1: Shear stress (τ_y) from Rozovskii (1961) Method for 8 cfs	132
Table D.2: Shear stress (τ_y) from Rozovskii (1961) Method for 12 cfs	133
Table D.3: Shear stress (τ_y) from Rozovskii (1961) Method for 16 cfs	134
Table D.4: Shear stress (τ_y) from Rozovskii (1961) Method for 20 cfs	135
Table E.1: Reynolds Shear Stress Calculation Summary for 8 cfs (psf)	144

Table E.2: Reynolds Shear Stress Calculation Summary of Upstream Bend for 12 cfs (psf).....	145
Table E.3: Reynolds Shear Stress Calculation Summary of Downstream Bend for 12 cfs (psf).....	146
Table E.4: Reynolds Shear Stress Calculation Summary of Upstream Bend for 16 cfs (psf).....	147
Table E.5: Reynolds Shear Stress Calculation Summary of Cross section 10 through Cross section 15 for 16 cfs (psf).....	148
Table E.6: Reynolds Shear Stress Calculation Summary of Cross Section 16 and Cross Section 17 for 16 cfs (psf)	149
Table E.7: Reynolds Shear Stress Calculation Summary of Upstream Bend for 20 cfs (psf).....	150
Table E.8: Reynolds Shear Stress Calculation Summary of Cross Section 10 through Cross Section 15 for 20 cfs (psf)	151
Table E.9: Reynolds Shear Stress Calculation Summary of Cross Section 16 and Cross Section 17 for 20 cfs (psf)	152

LIST OF SYMBOLS, UNITS OF MEASURE, AND ABBREVIATIONS

α	=	slope of the linear regression [dimensionless]
α	=	shear stress by the Preston tube [$\text{ML}^{-1}\text{T}^{-2}$]
α_1	=	value of shear stress by linear regression of the flow velocity profile for the lateral direction (V_y) [$\text{ML}^{-1}\text{T}^{-2}$]
α_2	=	value of shear stress calculated from the Preston tube [$\text{ML}^{-1}\text{T}^{-2}$]
α_3	=	value of shear stress by Reynolds shear stress extrapolation [$\text{ML}^{-1}\text{T}^{-2}$]
b	=	intercept of the regression line of Reynolds shear stress extrapolation [dimensionless]
B	=	intercept of the linear regression [dimensionless]
β	=	shear stress from linear regression of the flow velocity profile [$\text{ML}^{-1}\text{T}^{-2}$]
β_1	=	shear stress from the Rozovskii method [$\text{ML}^{-1}\text{T}^{-2}$]
β_2	=	shear stress from Reynolds shear stress extrapolation [$\text{ML}^{-1}\text{T}^{-2}$]
β_3	=	shear stress from linear regression [$\text{ML}^{-1}\text{T}^{-2}$]
COV	=	covariance
$COV-XY$	=	covariance of two variables (V_x and V_y) [dimensionless]
$COV-YX$	=	covariance of two variables (V_y and V_x) [dimensionless]
$COV-XZ$	=	covariance of two variables (V_x and V_z) [dimensionless]
$COV-ZX$	=	covariance of two variables (V_z and V_x) [dimensionless]
$COV-ZY$	=	covariance of two variables (V_z and V_y) [dimensionless]
$COV-YZ$	=	covariance of two variables (V_y and V_z) [dimensionless]
d	=	outer diameter of Pitot tube [L]
d	=	Pitot tube diameter taken as a representative length [L]
d_1	=	three times of the difference of height of Piezo a and Piezo d [L]
d_2	=	channel bottom width [L]
d_3	=	three times of the difference of height of Piezo g and Piezo d [L]
d_{50}	=	grain size for 50% of the material weight is finer [L]
dp	=	differential pressure [L]
dz	=	super elevation of the water surface [L]
D	=	horizontal distance between Piezo a and Piezo g [L]
D	=	internal diameter of pipe [L]
D_{50}	=	median size of bed material [L]
δ	=	percentage difference of shear stress between the Preston tube and linear regression of the flow velocity profile

δ_1	= percentage difference between the Preston tube and Rozovskii methods
δ_2	= percentage difference between the Preston tube and Reynolds shear stress extrapolation
δ_3	= percentage difference between linear regression of the flow velocity profile for the longitudinal direction and Reynolds shear stress extrapolation
∂p	= differential pressure [$\text{ML}^{-1}\text{T}^{-2}$]
∂r	= differential radius of curvature [L]
Δp	= dynamic pressure in feet of water [L]
ΔZ	= height of super elevation [L]
$\Delta \theta$	= differential change of flow direction along the reach (Knighton 1998)
F	= representative linear function
F_c	= centrifugal force [MLT^{-2}]
g	= acceleration of gravity [LT^{-2}]
γ	= specific gravity of water [$\text{ML}^{-2}\text{T}^{-2}$]
γdz	= differential hydrostatic force [L]
h	= flow depth [L]
h	= difference of water surface elevation between Piezo a and Piezo g [L]
H	= flow depth [L]
θ	= arc angle (Knighton 1998)
θ	= channel direction
k	= ratio of 2π to the meander wavelength
k_s	= grain roughness [dimensionless]
K_b	= ratio of maximum shear stress to averaged shear stress [dimensionless]
κ	= constant of von Kármán [dimensionless]
L	= meander wavelength [L] (Knighton 1998)
L	= length of test section [L]
L_c	= channel length [L]
L_v	= straight-line valley length of a channel [L]
m	= mass of the moving object [M]
m	= regression slope of linear regression equation
M	= coefficient that relates shear velocity head to hydraulic roughness [dimensionless]
M_A	= meander amplitude
M_L	= meander arc length [L]
n	= Manning's roughness value

n	= number of samples that collected velocity data [dimensionless]
p	= dynamic pressure [$ML^{-1}T^{-2}$]
p_1	= pressure at the beginning of length (L) [$ML^{-1}T^{-2}$]
p_2	= pressure at the end of length (L) [$ML^{-1}T^{-2}$]
p_o	= static pressure [$ML^{-1}T^{-2}$]
p_t	= impact pressure [$ML^{-1}T^{-2}$]
P	= dynamic pressure [$ML^{-1}T^{-2}$]
Q	= water flow rate [L^3/T^1]
Q_s	= sediment flow rate [MT^{-1}]
r	= radius of curvature of the channel bend [L]
r_c	= radius of curvature (Knighton 1998)
r_{ce}	= center radius of curvature in bend [L]
r_i	= inner radius of curvature in bend [L]
r_o	= outer radius of curvature in bend [L]
R	= radius of curvature of the channel bend [L]
R_c	= radius of curvature of meandering channel [L]
ρ	= mass density of water [ML^{-3}]
S	= slope [dimensionless]
S_r	= water surface slope [dimensionless]
T_w	= top width of channel [L]
τ	= shear stress [$ML^{-1}T^{-2}$]
τ_{bed}	= the turbulent shear stress in longitudinal direction that applies on the channel bed [$ML^{-1}T^{-2}$]
τ_o	= bed (or boundary) shear stress [$ML^{-1}T^{-2}$]
τ_r	= radial shear stress [$ML^{-1}T^{-2}$]
τ_x	= boundary shear stress in the x-direction (longitudinal direction) [$ML^{-1}T^{-2}$]
τ_y	= boundary shear stress in the y-direction (lateral direction) [$ML^{-1}T^{-2}$]
τ_{xy}	= turbulent shear stress that applies in the y-axis and caused by V_x [$ML^{-1}T^{-2}$]
τ_{xz}	= turbulent shear stress that applies in the z-axis and caused by V_x [$ML^{-1}T^{-2}$]
τ_{yx}	= turbulent shear stress that applies in the x-axis and caused by V_y [$ML^{-1}T^{-2}$]
τ_{yz}	= turbulent shear stress that applies in the z-axis and caused by V_y [$ML^{-1}T^{-2}$]
τ_{zx}	= turbulent shear stress that applies in the x-axis caused by V_z [$ML^{-1}T^{-2}$]

τ_{zy}	=	turbulent shear stress that applies in the y-axis and caused by V_z [$ML^{-1}T^{-2}$]
u	=	mean point velocity in the longitudinal direction [LT^{-1}]
u_*	=	shear velocity [LT^{-1}]
u'	=	instantaneous velocity fluctuations about the mean in the longitudinal direction [LT^{-1}]
$\bar{U}$	=	vertical averaged stream-wise velocity [LT^{-1}]
U^*	=	velocity scaling parameter for log law, identical with shear velocity (u_*) for uniform two-dimensional flow [LT^{-1}]
v	=	moving velocity of the object [LT^{-1}]
v'	=	instantaneous velocity fluctuations about the mean in the vertical direction [LT^{-1}]
v_θ	=	flow velocity along the bend of the channel [LT^{-1}]
v_o	=	flow velocity at the channel bed equal to zero [LT^{-1}]
v_1	=	flow velocity value that locates in the lowest data-collection point [LT^{-1}]
v_i	=	flow velocity data-collection point located within the vertical column where i ranges from 1 to number of data point in the column (n) [LT^{-1}]
v_n	=	flow velocity located at the highest data-collection point with a vertical column [LT^{-1}];
V	=	flow velocity [LT^{-1}]
V_{max}	=	maximum flow velocity [LT^{-1}]
V_x	=	flow velocity along x-axis [LT^{-1}]
V_y	=	flow velocity along y-axis [LT^{-1}]
V_z	=	flow velocity along z-axis [LT^{-1}]
ν (nu)	=	kinematic viscosity of water [L^2T^{-1}]
w	=	average width at bankfull discharge [L]
x	=	flow distance along the reach [L]
x, y, z	=	axes
y	=	distance normal to the plane of the channel bottom [L]
$\frac{y}{D}$	=	relative depth [dimensionless]
z	=	vertical distance the bottom of the channel [L]
z_1	=	elevation of lowest data collection point in a vertical column [L]
z_o	=	elevation where velocity is zero [L]
z_i	=	elevation of flow velocity data-collection points where i ranges from 1 to number of data point in the column (n) [L]

z_{Bed}	=	bed elevation of the channel [L]
z_n	=	elevation of highest data-collection point in a vertical column [L]
z_{WSE}	=	water-surface elevation of the channel [L]
ω	=	maximum angle between a channel segment and the mean down valley axis

Units of Measure

cfs	cubic feet per second
cm	centimeter(s)
cm/s	centimeter(s) per second
°	degree(s)
ft	foot or feet
ft/ft	feet per foot
ft/s	feet per second
ft/s ²	feet per second squared
ft ² /s	square feet per second
ft ³	cubic feet
ft ³ /s	cubic feet per second
Hz	Hertz
in.	inch(es)
L	length dimension
lb(s)/ft ³	pound(s) per cubic foot
m	meter(s)
M	mass dimension
%	percent
psf	pound(s) per square foot
sec	second(s)
T	time dimension

Abbreviations

1 – 18	cross sections in physical model
2-D	two-dimensional
3-D	three-dimensional
a – g	Piezometer locations at each cross section
A – G	modes of the meander loop development (FHWA 2001)

ADV	Acoustic Doppler Velocity
CSU	Colorado State University
ERC	Engineering Research Center
D/S	downstream
FHWA	Federal Highway Administration
FISRWG	Federal Interagency Stream Restoration Working Group
H:V	Horizontal:Vertical
HEC-RAS	Hydrologic Engineering Center – River Analysis System
out	tabular data where shear stress profile does not show similar trend with figure
PVC	polyvinyl chloride
U/S	upstream
USBR	U. S. Bureau of Reclamation
V:H	Vertical:Horizontal

1 INTRODUCTION

1.1 Background

Natural channels never stop changing their geomorphic characteristics. Natural alluvial streams are similar to living creatures because they generate water flow, develop point bars, alter bed profile, scour the bed, erode the bank, and cause other phenomena in the stream system. The geomorphic changes in a natural system lead to a wide array of research worldwide, because methods of predicting channel change are limited. Bank erosion due to meandering of the channel is a critical issue in modern river engineering. In the United States, the severity of the problem was recognized when Congress enacted the Streambank Erosion Control Evaluation and Demonstration Act of 1974 and authorized research in the field (Odgaard 1986). The increased demand for bank erosion research resulted in many theoretical and experimental studies to predict geomorphic changes in natural channels. Julien (2002) described the magnitude and orientation of applied shear stress as one of the main factors of bank erosion. Julien (2002) also describes the damping of shear stress by erosion control.

1.2 Project Background

The 29-mile long study reach of Rio Grande River was located in central New Mexico. The study reach was a straight braided system with a shifting sand bed and shallow banks (Heintz 2002). The high supply of sediment caused channelization of the river. Cochiti Dam was constructed by the U. S. Army Corps of Engineers for the purposes of flood and sediment transport control for the Albuquerque area in 1973 (Schmidt 2005). Subsequently, the channel changed from an aggrading to degrading system. As a result, the planform geometry of the reach has changed from a straight braided to an active meandering system (Heintz 2002). Channel meandering resulted in bank erosion and lateral channel migration. Bank erosion and lateral channel migration occurred to satisfy quasi-equilibrium conditions of the stream system. Fish

habitat, vegetation, and infrastructure around the stream and farming lands were threatened due to bank erosion.

Understanding shear stress distribution in the study reach is necessary because shear stress is the key parameter to predict bank erosion and lateral channel migration. Shear stress distribution was investigated through physical modeling conducted by Colorado State University (CSU). For testing purposes, a 1:12 Froude-scale model was constructed and tested in the Hydromachinery Laboratory that is located at the Engineering Research Center (ERC) at CSU. Appendix A provides details of the physical model construction, instrumentation, model calibration, and test program. The purpose of the physical modeling was to simulate conditions in the reach and to predict the shear stress distribution at four target flow rates: 8, 12, 16, and 20 cubic feet per second (cfs). Laboratory tests were performed in baseline test conditions without installing hydraulic structures in the main channel.

1.3 Research Objectives

The focus of this study was shear stress distribution analysis in meandering channels using the following methods: 1) Preston tube method, 2) linear regression of the flow velocity profile, 3) Rozovskii method, and 4) Reynolds shear stress extrapolation.

The objectives of this study were to:

- understand in detail the shear stress distribution mechanism for meandering channels by performing a physical model study;
- calculate shear stress by using four different methods;
- compare the different shear stress values; and
- recommend the most appropriate shear stress calculation method for the establishment of a design guide for channels in the field.

The scope of this research was defined to satisfy the following objectives:

- conduct a comprehensive literature review of previous studies on shear stress calculation methods in meandering channels;
- analyze collected data to calculate shear stress distribution in meandering channels;
- compare and discuss the different shear stress calculation methods; and
- recommend the most appropriate shear stress analysis method and direction of future research.

2 LITERATURE REVIEW

2.1 Introduction

Flow in a meandering channel results in geomorphic changes. A photograph of a typical meandering channel is presented in Figure 2.1. Flow in meandering channels results in geomorphic changes, such as bank erosion. Implementation of bank protection to prevent erosion is a significant economic expense. Therefore, development of a tool for implementation of bank protection is important. This tool would be used to predict shear stress, which is the most important aspect of bank erosion. Since the beginning of the 20th century, many researchers have developed methods for the analysis of flow in bends. The literature review provides an explanation of the basic concepts of fluvial geomorphology; and the following boundary shear stress analysis methods: 1) Preston tube method, 2) linear regression of the flow velocity profile, 3) Rozovskii method, and 4) Reynolds shear stress extrapolation.



Figure 2.1: Typical Meandering Channel (from Watson *et al.* (2005))

2.2 Open Channel Flow in Meandering Bends

To study flow characteristics in a meandering alluvial channel, a basic understanding of a meandering channel is necessary. This section reports the background of meandering channel geometry, flow patterns in bends, erosion patterns, and channel stability.

2.2.1 Meandering Channel Geometry

Sinuosity is used to describe a meandering channel. When sinuosity is greater than 1.5, the channel is classified as meandering (Knighton 1998). Equation 2.1 describes the method used for calculating channel sinuosity:

$$\text{Sinuosity} = \frac{L_c}{L_v} \quad \text{Equation 2.1}$$

where

L_c = channel length [L]; and

L_v = straight-line valley length [L].

There are two methods available to analyze meander geometry (Knighton 1998). The first method focused on the individual bend statistics: meander wavelength (L) and radius of curvature (r_c), which are averaged over a series of bends. The second method is a series approach method that spans sequences of bends and treats the stream trace as a spatial series of flow direction (arc angle θ) or differential change of flow direction ($\Delta\theta$) along the reach (Knighton 1998). For meandering channels, Knighton (1998) stated that the second method provides more flexibility for flow characteristics analysis and theoretical model development. Figure 2.2 is a diagram of the geometry of a meandering channel.

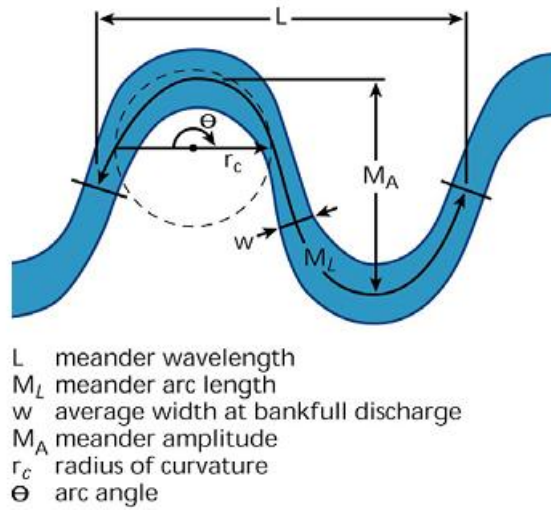


Figure 2.2: Geometry of Meandering Channel (from Watson *et al.* (2005))

A sine-generated curve is used to develop a simple model of a meandering channel, as described by Equation 2.2:

$$\theta = \omega \sin kx \quad \text{Equation 2.2}$$

where

- θ = channel direction;
- ω = maximum angle between a channel segment and the mean down valley axis;
- k = ratio of 2π to the meander wavelength; and
- x = flow distance along the reach $[L]$.

A sketch of the sine-generated curve is provided in Figure 2.3.

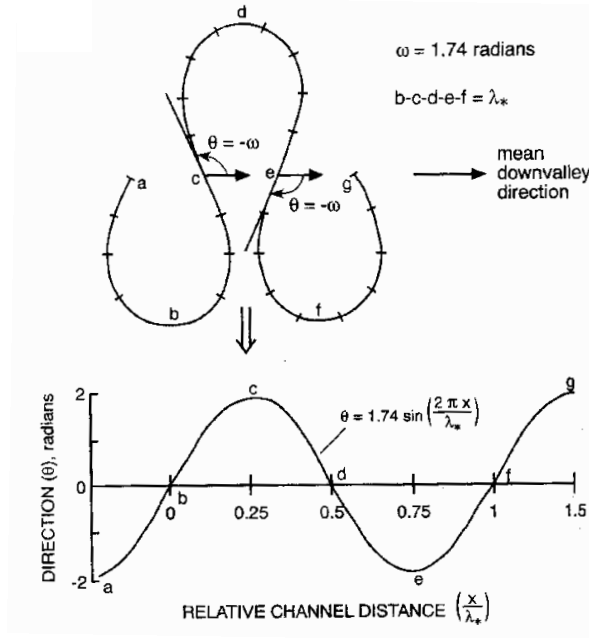


Figure 2.3: Description of Sine-generated Curve Function (from Knighton (1998))

2.2.2 Flow Patterns in Meandering Channels

The description of flow patterns in a meandering channel has three components: 1) stream morphology, 2) transverse velocity distribution in bends, and 3) super elevation of the water surface against the outside of the bank. The following sub-sections explain these three main components of flow patterns in meandering channels.

2.2.2.1 Stream Morphology

Drawings of a straight and a meandering channel are presented in Figure 2.4. Pool-riffle sequences are the characteristics of cobble, gravel, and mixed-load rivers of moderate gradient (smaller than 5%) (Sear 1996). Topographic high points are defined as riffles and low points are defined as pools (Watson *et al.* 2005). The grain sizes found in riffles are larger than those found in pools. Keller (1971) explained that the reason for the difference in grain size is caused by the sorting process. In addition, Keller (1971) explained that fine sediments are removed from riffles during low flows due to flow velocity and shear stress. These sediments are then deposited in pools. Normally, the outside bank is deeper than the inside bank, forming a point bar at the inside bank.

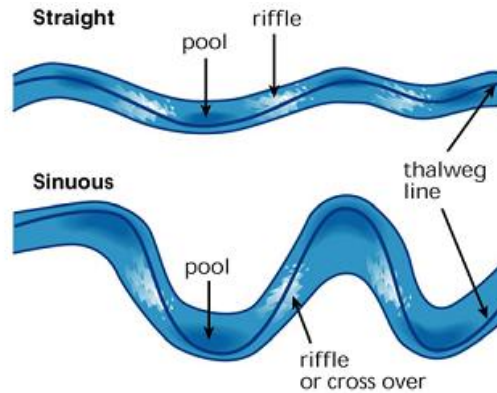


Figure 2.4: Graphic Description of a Straight Channel and a Meandering Channel (from the Federal Interagency Stream Restoration Working Group (FISRWG) (1998))

In addition to pools and riffles, the following features are also observed in natural streams: point bars, middle bars, alternate bars, and braiding. In a typical meandering stream, point bars form at the inside bank of a bend as seen in Figure 2.5. The factors that influence the formation of point bars are flow characteristics, degree of sinuosity, and sediment transportation. Reduction of sediment transport capacity at the inside of the bank, transverse flow velocity distribution and secondary currents in bends result in the formation of point bars (Watson *et al.* 2005).



Figure 2.5: Typical Meandering Stream with Point Bars (from Watson *et al.* (2005))

A middle bar is a depositional feature located within the channel, but not connected to the banks. A typical middle bar is shown in Figure 2.6.



Figure 2.6: Typical Middle Bar (from Watson *et al.* (2005))

Alternate bars are regularly-spaced depositional features that are located in the opposite sides of a straight or slightly sinuous stream (Watson *et al.* 2005). Figure 2.7 presents the typical shape of alternate bars. Formation of alternate bars is a good indicator of the initiation of a meander or braided planform (Watson *et al.* 2005). Figure 2.8 shows a braided river, which is composed of a straight channel with multi-threads.

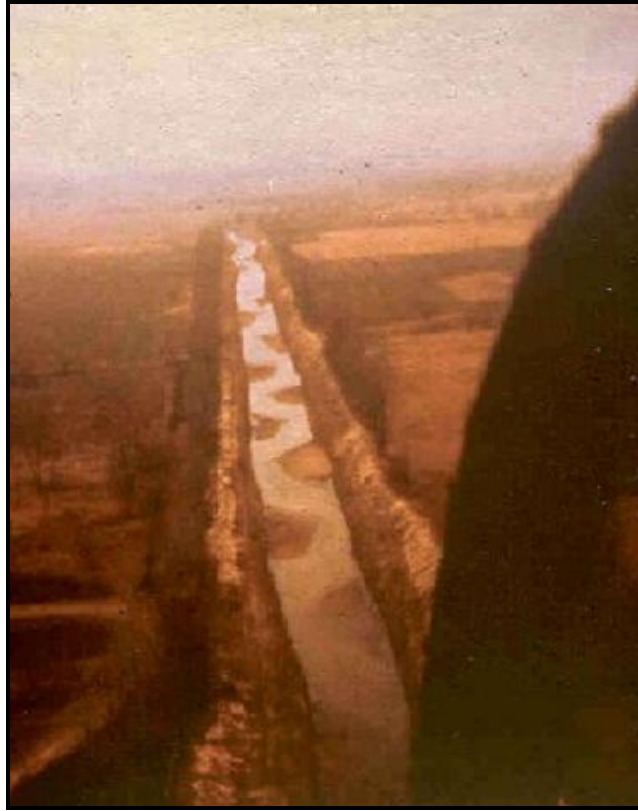


Figure 2.7: Typical Alternate Bars (from Watson *et al.* (2005))



Figure 2.8: Typical Braid Bars (from Watson *et al.* (2005))

2.2.2.2 Transverse Velocity Distribution in Bends

Imbalance in radial pressure around the bend causes the transverse velocity distribution in a meandering channel (Federal Highway Administration (FHWA) 2001). Figure 2.9 represents a typical cross section within a bend and the vertical distribution of longitudinal velocity. The radial forces that act on the shaded control volume (Figure 2.9(a)) are the centrifugal force that was presented in Equation 2.3 (FHWA 2001):

$$F_c = \frac{mv^2}{r} \quad \text{Equation 2.3}$$

where

- F_c = centrifugal force [MLT⁻²];
- m = mass of the moving object [M];
- v = moving velocity of the object [LT⁻¹]; and
- r = value of the radius of curvature [L].

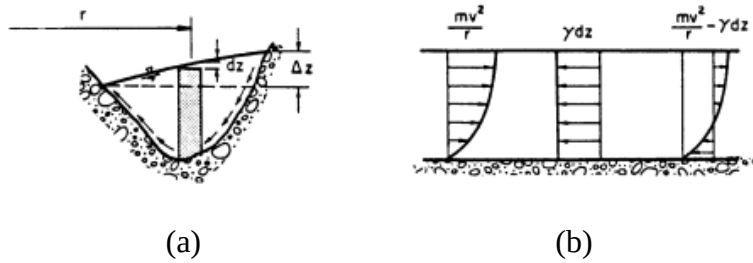


**Figure 2.9: Schematic Descriptions of Flow in a Meandering Channel
(from FHWA (2001))**

In addition, super elevation of the water surface (dz) results in the differential hydrostatic force (γdz). Therefore, the centrifugal force is greater near the surface where the flow velocity is greater and less at the bed where the flow velocity is small (FHWA 2001). The differential hydrostatic force is constant throughout the depth of the control volume. Figure 2.9(b) explains that the combination of centrifugal force and hydrostatic force causes a secondary flow in the bend (FHWA 2001).

2.2.2.3 Super Elevation of the Water Surface against the Outside of the Bank

Due to the centrifugal force of flow in the bend, there is a super elevation of the water surface in the bend (FHWA 2001). The water surface elevation at the outside bank is higher than at the inside bank. The differential pressure in the radial direction results from the radial acceleration and can be expressed by Equation 2.4:

$$\frac{1}{\rho} \times \frac{\partial p}{\partial r} = \frac{v_{\theta}^2}{r} \quad \text{Equation 2.4}$$

where

- ρ = mass density of water [ML^{-3}];
- ∂p = differential pressure [$\text{ML}^{-1}\text{T}^{-2}$];
- ∂r = differential radius of curvature [L];
- v_{θ} = flow velocity along the bend of a channel [LT^{-1}]; and
- r = radius of curvature [L].

A sketch of the flow around a bend is provided in Figure 2.10.

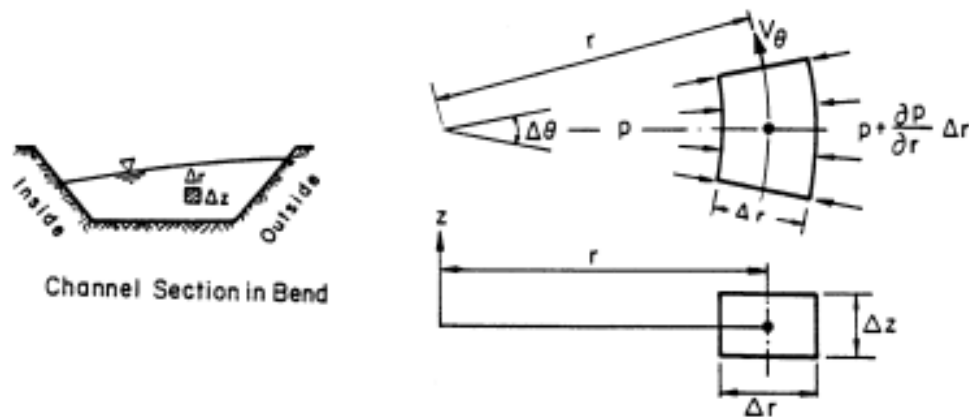


Figure 2.10: Sketch of the Flow around a Bend (from FHWA (2001))

For the calculation of the super elevation, two assumptions should be made:

- 1) the magnitudes of radial and vertical velocities are smaller than tangential velocities;
and
- 2) distribution of pressure in the bend is hydrostatic.

If the two assumptions are satisfied, it is possible to calculate the super elevation by using Equation 2.5:

$$\Delta Z = \frac{V_{max}}{2g} \left(2 - \left(\frac{r_i}{r_{ce}} \right)^2 - \left(\frac{r_{ce}}{r_o} \right)^2 \right) \quad \text{Equation 2.5}$$

where

- ΔZ = height of super elevation [L];
- V_{max} = maximum flow velocity [LT^{-1}];
- g = acceleration of gravity [LT^{-2}];
- r_i = inner radius of curvature in bend [L];
- r_{ce} = center radius of curvature in bend [L]; and
- r_o = outer radius of curvature in bend [L].

2.2.3 Erosion Patterns in Meandering Rivers

There are four stages to change a straight channel into a meandering channel (after Keller (1972)):

- 1) Stage 1 – alternating bars dominant;
- 2) Stage 2 – development of incipient pools and riffles;
- 3) Stage 3 – development of well-developed pools and riffles with a mean spacing of 5 to 7 channel widths; and
- 4) Stage 4 – development of meandering channel with riffles at inflection points and pools at bend apices where bank erosion is concentrated.

Figure 2.11 shows the geomorphic change of a straight channel into a meandering channel.

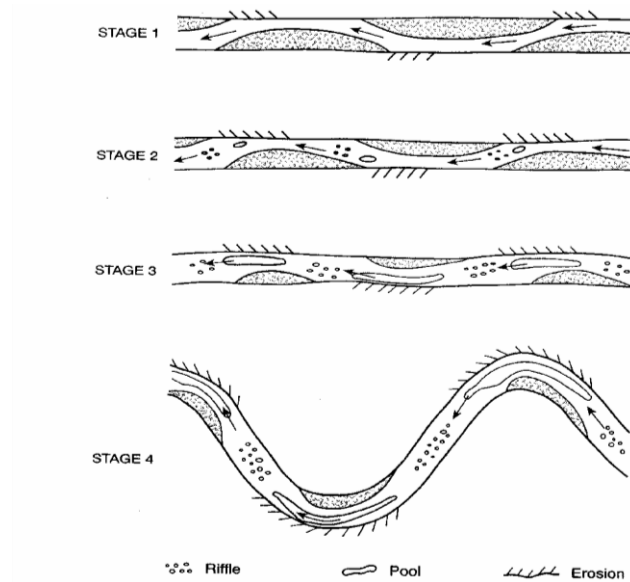


Figure 2.11: Geomorphic Change of Straight Channel into Meandering Channel (from Knighton (1998))

Knighton (1998) explained that the relationship between flow and geomorphic form has a link with the radius of the curvature versus width relationship, which can alter flow patterns in meandering channels. Bagnold (1960) reported that if the ratio of radius of curvature and channel width is smaller than 2, water filaments begin to separate from the inner bank. Hickin and Nanson (1975) suggested that there is a corresponding reduction in the rate of erosion at the concave bank. Hickin and Nanson (1986) plotted the relationship of relative migration rate to bend curvature for eighteen rivers (Figure 2.12).

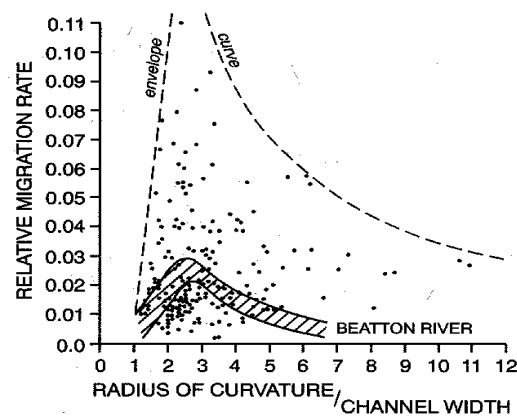


Figure 2.12: Relationship of Relative Migration Rate to Bend Curvature for Eighteen Rivers (from Knighton (1998))

It is hard to predict the planform of the channel, but the FHWA (2001) suggested the modes of meander loop development will aid in planform prediction. Figure 2.13 presents the seven modes of the meander loop development, these modes are: 1) A – Extension, 2) B – Translation, 3) C – Rotation, 4) D – Conversion to a compound loop, 5) E – Neck cutoff by closure, 6) F – Diagonal cutoff by chute, and 7) G – Neck cutoff by chute (FHWA 2001).

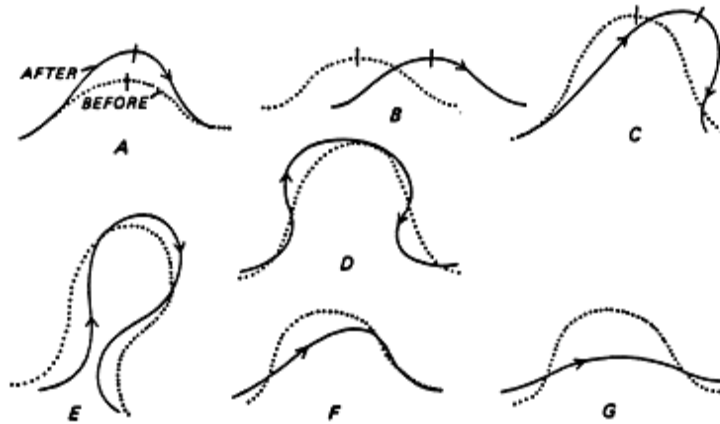


Figure 2.13: Seven Modes of Meander Loop Development (from FHWA (2001))

2.2.4 Channel Stability

It is possible to classify the stream system as stable or unstable. The criterion of the classification depends on whether the stream has adjusted or is still adjusting to the flow and sediment regimes (FHWA 2001). A graded stream is defined as a stream that is not necessarily static or fixed and may exhibit temporary morphological changes in response to the impacts of extreme events. The main attitude of a graded stream is that fluvial processes operating under formative flows tend to restore stream morphology to the graded condition (FHWA 2001). This type of stability is described as dynamic equilibrium. To explain the concept of dynamic equilibrium, the concept of Lane's (1955) balance is commonly used. A diagram of Lane's balance is provided in Figure 2.14. Lane's balance shows the change of the following four variables: Q , S , Q_s , and D_{50} . If one or more of these variables change, the river will respond by causing a change in the other variables to restore equilibrium.

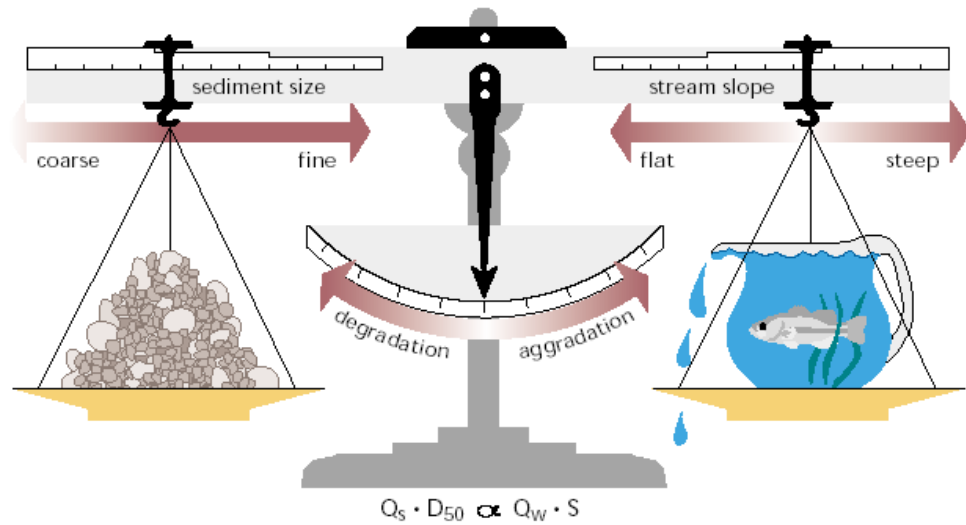


Figure 2.14: Lane's Balance (from Rosgen (1996))

The following equation describes Lane's balance:

$$QS \sim Q_s D_{50} \quad \text{Equation 2.6}$$

where

- Q = water flow rate [L^3/T^{-1}];
- S = slope [dimensionless];
- Q_s = sediment flow rate [MT^{-1}]; and
- D_{50} = median size of bed material [L].

2.3 Shear Stress Calculation Methods

Calculation methods of shear stress in meandering channels have been suggested by many researchers. In this section, the techniques for calculating shear stress using the: 1) Preston tube method, 2) linear regression of the flow velocity profile, 3) Rozovskii method, and 4) Reynolds shear stress extrapolation are discussed.

2.3.1 Preston Tube Method

The Preston tube method computes shear stress by positioning a Preston tube directly on the channel bed at the point of measurement. After data collection, calibration was performed to

convert data from an electronic signal into bed shear stress. Relevant studies of the Preston tube shear stress measurement method are presented in the following sub-sections.

2.3.1.1 Preston (1954)

Preston (1954) performed a laboratory study to determine turbulent skin friction by using Pitot tubes. The purpose of the laboratory study was to calculate skin friction that applies on the surface. Before Preston (1954), Stephens and Haslam (1938) suggested the method to determine local skin friction by reading local static pressure. The relationship between local skin friction and local static pressure is presented in Equation 2.7:

$$\frac{(P - p_o) \times d^2}{\rho v^2} = F \left(\frac{\tau_o d^2}{\rho v^2} \right) \quad \text{Equation 2.7}$$

where

- P = dynamic pressure [$\text{ML}^{-1}\text{T}^{-2}$];
- p_o = static pressure [$\text{ML}^{-1}\text{T}^{-2}$];
- d = Pitot tube diameter taken as a representative length [L];
- ρ = mass density of water [ML^{-3}];
- ν = kinematic viscosity of water [L^2T^{-1}];
- F = representative linear function; and
- τ_o = boundary shear stress [$\text{ML}^{-1}\text{T}^{-2}$].

For circular pipes, Preston (1954) explained shear stress, τ_o can be calculated from Equation 2.8:

$$\tau_o = (p_1 - p_2) \frac{D}{4L} \quad \text{Equation 2.8}$$

where

- τ_o = boundary shear stress [$\text{ML}^{-1}\text{T}^{-2}$];
- p_1 = pressure at the beginning of length (L) [$\text{ML}^{-1}\text{T}^{-2}$];
- p_2 = pressure at the end of length (L) [$\text{ML}^{-1}\text{T}^{-2}$];
- D = internal diameter of pipe [L]; and
- L = length of test section [L].

The Preston data-collection instrumentation is presented in Figure 2.15. For data collection, four different Pitot tubes were used. The dimensions of the Pitot tubes are presented in Table 2.1.

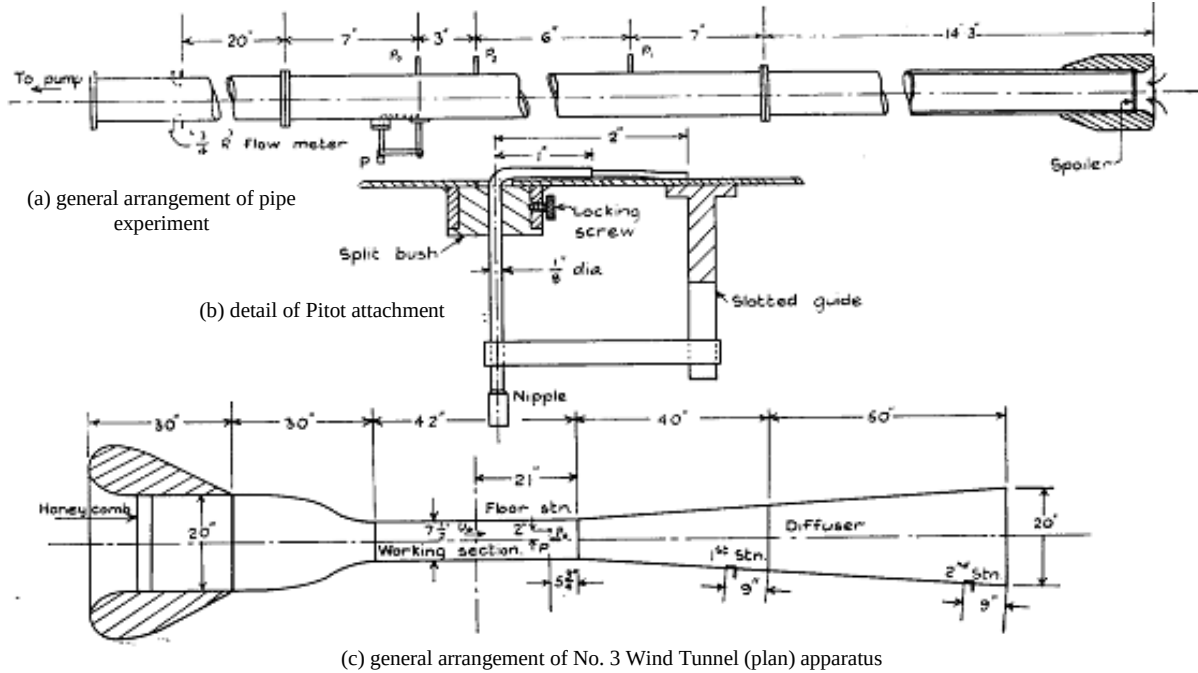


Figure 2.15: Preston Data-collection Facility (from Preston (1954))

Table 2.1: Dimension of the Pitot Tubes (from Preston (1954))

	Pitot Tube Number			
	1	2	3	4
External Diameter (in.)	0.02915	0.0544	0.0907	0.1214
Ratio: (Internal/External) Diameter	0.602	0.603	0.599	0.598

The actual observations from laboratory experiments are plotted in Figure 2.16. Preston (1954) also presents two different equations by expressing the relationship between $\log_{10} \frac{\tau_o d^2}{4\rho v^2}$ and $\frac{(p - p_o)d^2}{4\rho v^2}$ in the non-dimensional form as shown in Figure 2.17. For the condition of

$\log \frac{(p_t - p_o)^2}{4\rho v^2} > 5$ (where p_t is impact pressure), the equation is expressed in Equation 2.9 and the expression of the relationship under the condition of viscous flow is presented in Equation 2.10:

$$\log_{10} \frac{\tau_o d^2}{4\rho v^2} = 2.604 + \frac{7}{8} \times \log_{10} \frac{(p - p_o) d^2}{4\rho v^2} \quad \text{Equation 2.9}$$

$$\log_{10} \frac{\tau_o d^2}{4\rho v^2} = \frac{1}{2} \left(\log_{10} 2 + \log_{10} \frac{(p - p_o)^2}{4\rho v^2} \right) \quad \text{Equation 2.10}$$

where

- τ_o = boundary shear stress [$\text{ML}^{-1}\text{T}^{-2}$];
- d = Pitot tube diameter taken as a representative length [L];
- ρ = mass density of water [ML^{-3}];
- ν = kinematic viscosity of water [L^2T^{-1}];
- p = dynamic pressure [$\text{ML}^{-1}\text{T}^{-2}$]; and
- p_o = static pressure [$\text{ML}^{-1}\text{T}^{-2}$].

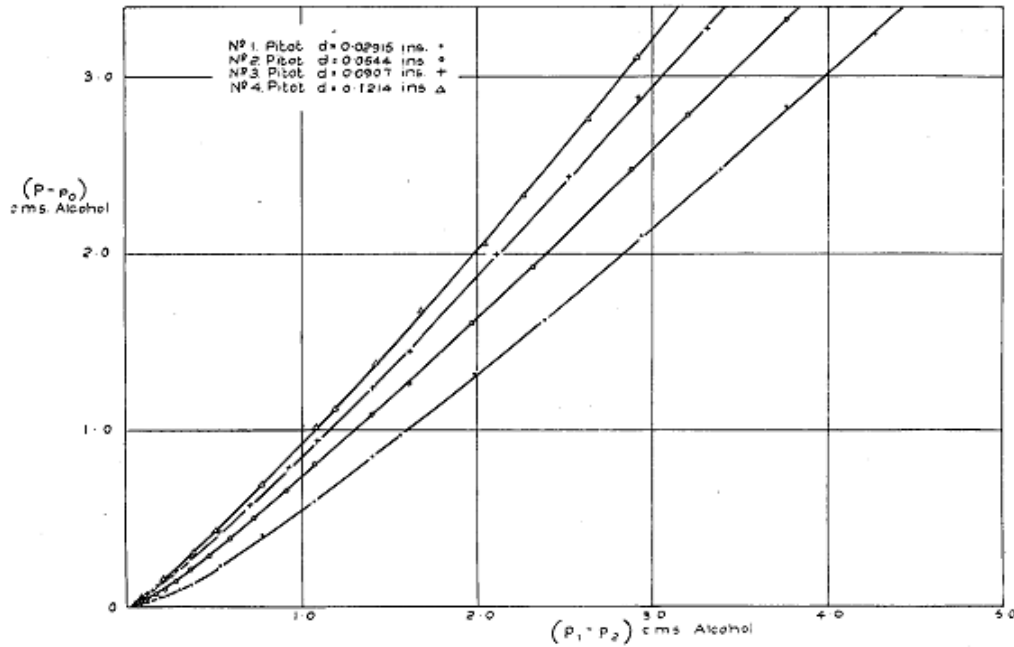


Figure 2.16: Actual Observations of Round Pitot on Wall of Circular Pipe (from Preston (1954))

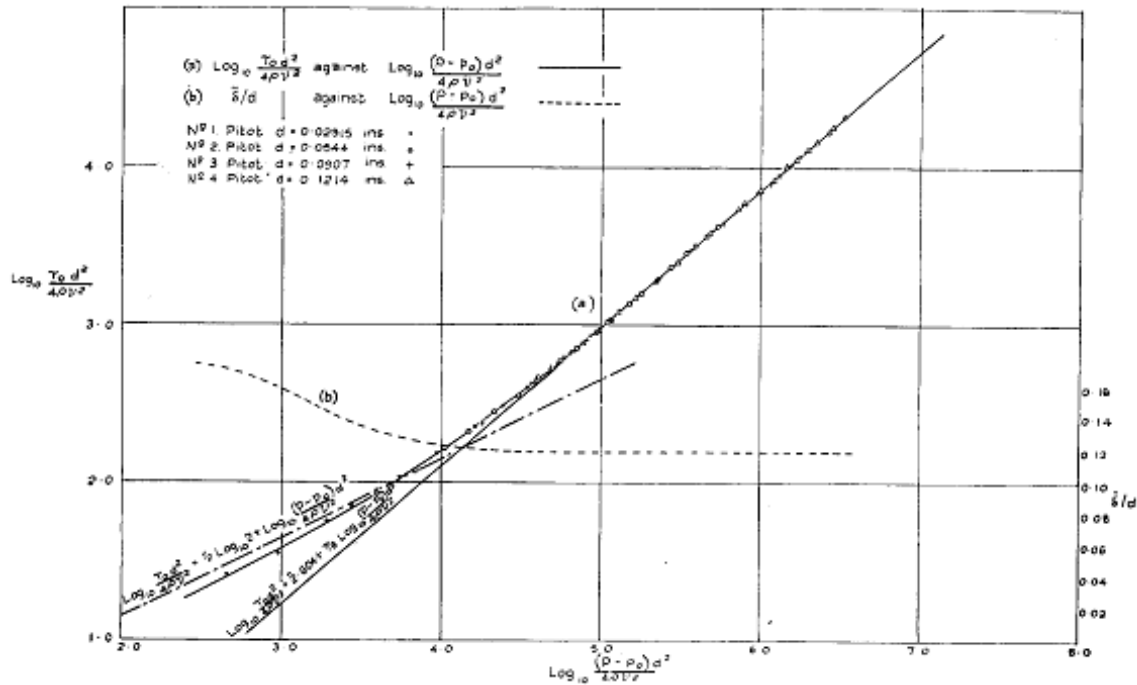


Figure 2.17: Plot of Non-dimensional Results of Preston (1954) Experiments

2.3.1.2 Ippen and Drinker (1962)

Ippen and Drinker (1962) performed laboratory experiments to correlate the impacts of shear stress on meandering channel bank erosion by using a Preston tube. Two flumes with trapezoidal geometry were constructed for data collection. Table 2.2 presents the geometric information of these two flumes. Figure 2.18 is a diagram of the flumes in the laboratory.

Table 2.2: Geometric Information of Flume (from Ippen and Drinker (1962))

	Shape of Cross Section	Side Slope (Horizontal:Vertical)	Rotation Angle (°)	Bottom Width (in.)	Radius of curvature (in.)
Larger Flume	Trapezoidal	2H:1V	60	24	60
Smaller Flume	Trapezoidal	2H:1V	60	12	70

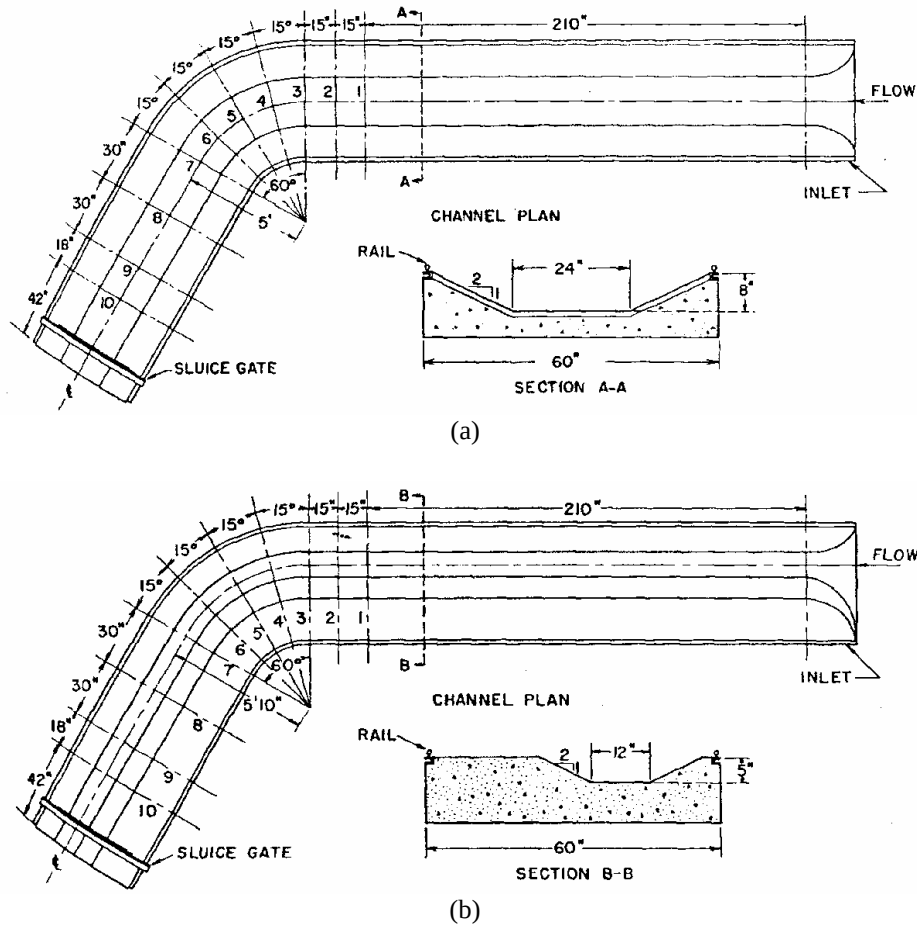


Figure 2.18: Laboratory Flumes (from Ippen and Drinker (1962))

For flow depth adjustment at the curve entrance, a sluice gate was installed at the exit section to control downstream flow depth. At the beginning of the experiment, the Manning's roughness coefficient (n) was assumed as 0.009. As the experiment progressed, calculation of the roughness coefficient was conducted and the result was 0.010. To measure boundary shear stress, Ippen and Drinker (1962) selected the Preston (1954) method presented in Equation 2.11. This method computes shear stress on a smooth boundary by the pressure differential between static and dynamic Pitot tubes resting on the bed surface. Ippen and Drinker (1962) calibrated the pressure head recorded in shear stress measurements in pounds per square feet (psf) by using Equation 2.11:

$$\log \frac{\tau_o d^2}{4\rho v^2} = -1.396 + 0.875 \times \log \frac{(p_t - p_o)^2}{4\rho v^2} \quad \text{Equation 2.11}$$

where

τ_o = local boundary shear stress [$\text{ML}^{-1}\text{T}^{-2}$];

d = outer diameter of Pitot tube [L];

ρ = mass density of water [ML^{-3}];

ν = kinematic viscosity of water [L^2T^{-1}];

p_t = impact pressure [$\text{ML}^{-1}\text{T}^{-2}$]; and

p_o = static pressure [$\text{ML}^{-1}\text{T}^{-2}$].

After experimental investigation of flow in meandering channels by using a Preston tube, Ippen and Drinker (1962) drew the following conclusions:

- At present, the maximum shear stress value that resulted from the curved flow is not possible to predict by theoretically or using empirical coefficient of head loss in curved flow.
- During the laboratory experiments, in the case of large curvatures, a high value of shear was observed near the inside bank in the curve and near the outside bank below the curve exit.
- The laboratory test proved that boundary shear stress distribution is an important parameter in understanding erosion and deposition processes in meandering channels.

2.3.1.3 Heintz (2002)

Heintz (2002) performed the Preston tube calibration to measure shear stress in a curved channel constructed at the Engineering Research Center located at Colorado State University. A pressure transducer was used for the collection of pressure data in a trapezoidal meandering channel. Figure 2.19 presents the Preston tube and pressure transducer that were used in the calibration. Equation 2.12 was obtained after Preston tube calibration:

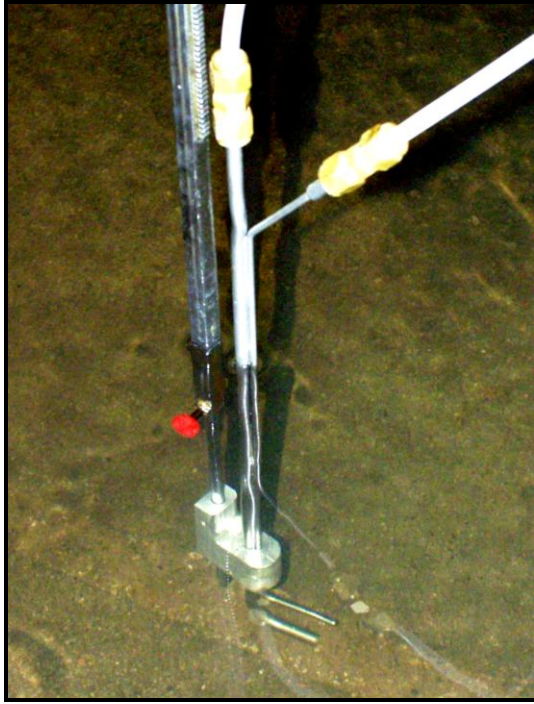
$$\tau = 0.02788 \times \gamma \times \Delta p \quad \text{Equation 2.12}$$

where

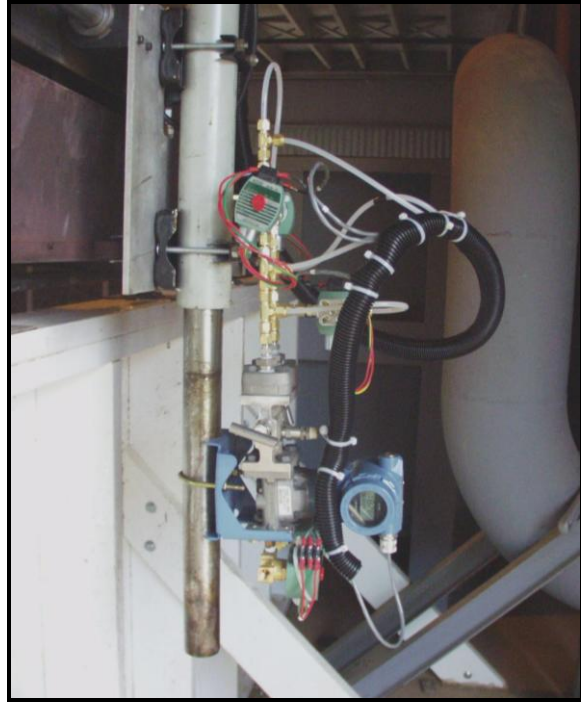
τ = shear stress (psf);

γ = specific gravity of water (lbs/ft^3); and

Δp = dynamic pressure in feet of water (ft).



(a) Preston tube



(b) pressure transducer

Figure 2.19: Calibration Instrumentation for Shear Stress Measurement (from Heintz (2002))

2.3.1.4 Sclafani (2008)

Sclafani (2008) performed another Preston tube calibration at the Engineering Research Center at Colorado State University. Sclafani (2008) explained that the purpose of the second Preston tube calibration was to overcome the limitation of the meandering channel geometry and to develop uniform flow completely in the flume. The Preston tube was installed and calibrated in a 60-ft long, 4-ft wide, and 2.5-ft tall flume (Sclafani 2008). Figure 2.20 provides a sketch of the flume used by Sclafani (2008) to calibrate the Preston tube.

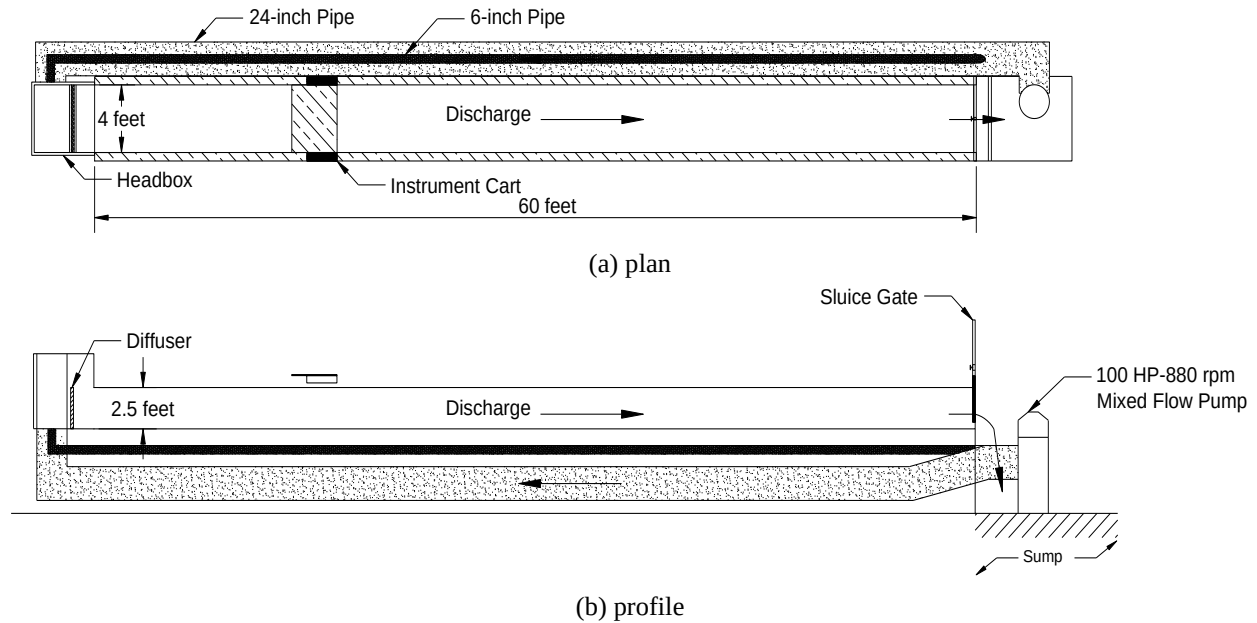


Figure 2.20: Flume at Colorado State University (Sclafani 2008)

After the second Preston tube calibration, Sclafani (2008) obtained a close relationship between differential pressure and bed shear stress presented in Equation 2.13:

$$\tau = 0.1644 \times dp \quad \text{Equation 2.13}$$

where

τ = shear stress (psf); and

dp = differential pressure (in.).

2.3.2 Linear Regression of the Flow Velocity Profile

The linear regression of the flow velocity profile method for shear stress calculation is based on the relationship between the logarithmic profile of flow depth and flow velocity. Relevant studies of this method are presented in the following sub-sections.

2.3.2.1 Clauser (1956)

Shear velocity can be calculated using Equation 2.14. If Equation 2.14 is rearranged, it is possible to obtain shear stress:

$$u_* = \sqrt{\frac{\tau_o}{\rho}} \quad \text{Equation 2.14}$$

where

u_* = shear velocity [LT^{-1}];

τ_o = bed shear stress [$\text{ML}^{-1}\text{T}^{-2}$]; and

ρ = mass density of water [ML^{-3}].

According to Clauser (1956), shear velocity can be obtained by taking linear regression between the mean point velocity (u) and natural log of $\left[\left(y + \frac{y + 0.2d_{50}}{d_{50}} \right) \right] + B$:

$$u = \alpha \times \ln \left[\left(y + \frac{y + 0.2d_{50}}{d_{50}} \right) \right] + B \quad \text{Equation 2.15}$$

where

α = slope of the linear regression [dimensionless];

y = distance normal to the plane of the channel bottom [L];

d_{50} = grain size for 50% of the material weight is finer [L]; and

B = intercept of the linear regression [dimensionless].

Also, Afzalimehr and Ancil (2000) explained that the combination of law of the wall with Equation 2.15 generates Equation 2.16 to estimate shear velocity. The result of this combination is presented in Equation 2.16:

$$\frac{u}{u_*} = \frac{1}{\kappa} \times \ln \left[\left(\frac{y + 0.2 \times d_{50}}{k_s} \right) \right] + B \quad \text{Equation 2.16}$$

where

u = mean point velocity in the longitudinal direction [LT^{-1}];

u_* = shear velocity [LT^{-1}];

κ = von Kármán coefficient [dimensionless];

y = distance normal to the plane of the channel bottom [L];

d_{50} = grain size for 50% of the material weight is finer [L];

k_s = grain roughness [dimensionless]; and
 B = intercept of Equation 2.16 [dimensionless].

2.3.2.2 Schlichting (1987)

Schlichting (1987) suggested a bed shear stress calculation method by applying a linear regression between the logarithmic profile of flow depth and flow velocity. The shear stress calculation method is explained in Equation 2.17 and Equation 2.18:

$$V = \frac{u_*}{\kappa} \ln \left(\frac{z}{z_o} \right) \quad \text{Equation 2.17}$$

$$V = \frac{u_*}{\kappa} \ln z + C \quad \text{Equation 2.18}$$

$$C \text{ is the constant: } = -\frac{u_*}{\kappa} \ln(z_o) \quad \text{Equation 2.19}$$

where

V = flow velocity [LT^{-1}];

$\frac{u_*}{\kappa}$ = slope of linear regression equation, where u_* is shear velocity [LT^{-1}] and κ is the von Kármán coefficient [dimensionless];

z = vertical distance to the bottom of the channel [L]; and

z_o = elevation where velocity is zero [L].

As a result, the shear stress (τ) can be computed by Equation 2.20:

$$\tau = \rho \times (\kappa \times S)^2 \quad \text{Equation 2.20}$$

where

ρ = mass density of water [ML^{-3}];

κ = von Kármán coefficient, the value of 0.4 was assumed; and

S = slope [dimensionless] of Equation 2.19.

2.3.2.3 Smart (1999)

Smart (1999) published a theory of turbulent velocity profiles and boundary shear by analyzing field data. Smart (1999) collected velocity data at six rivers in New Zealand by using an Acoustic Doppler Velocity (ADV) meter. Three rivers have medium flow conditions: Waiho, North Ashburton, and Rangitata Rivers. The other three rivers have flood conditions: Waitaki, Hurunui, and Rangitata Rivers. Smart (1999) obtained the flow resistance equation presented in Equation 2.21:

$$\frac{\bar{U}}{U^*} = 2.5 \times \left[\ln \left(M \frac{gH}{U^{*2}} \right) - 1 \right] \quad \text{Equation 2.21}$$

where

$\bar{U}$ = vertical averaged stream-wise velocity [LT^{-1}];

U^* = velocity scaling parameter for log law, identical with shear stress (u_*) for uniform two-dimensional (2-D) flow [LT^{-1}];

M = coefficient that relates shear velocity head to hydraulic roughness [dimensionless];

g = acceleration of gravity [LT^{-2}]; and

H = flow depth [L].

To determine the M value in Equation 2.21, the relationship between shear velocity head and hydraulic roughness was plotted in Figure 2.21.

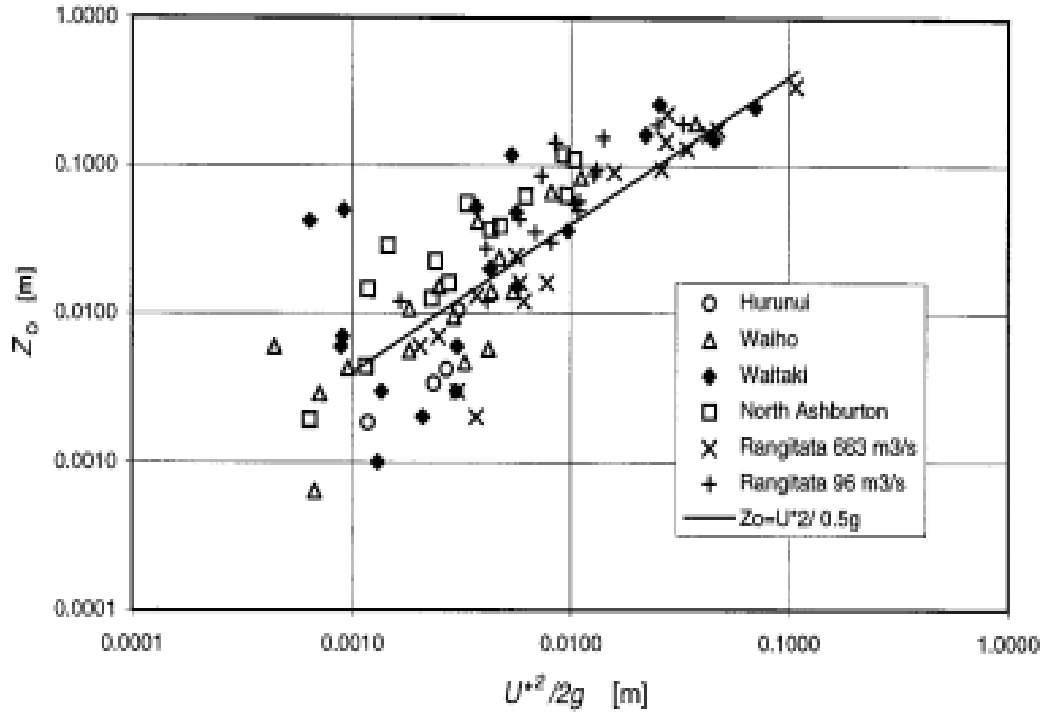


Figure 2.21: Relationship between Log Profile Parameters and Shear Velocity Head (from Smart (1999))

Smart (1999) concluded that the collected field data indicate that the grain roughness is proportional to the log profile parameter U^* under the condition of mobile-bed channels. In addition, Smart (1999) reported that bed shear stress (τ_o) is linearly proportional to the grain roughness in the case of uniform flow conditions.

2.3.3 Rozovskii (1961) Method

The Rozovskii method is for computation of radial shear stress by using equations that were derived from Cartesian coordinates. Rozovskii published a paper titled *Flow of Water in Bends of Open Channels* in 1957, which was translated into English in 1961 at Academy of the Science of the Ukrainian SSR, Institute of Hydrology and Hydraulic Engineering. Rozovskii introduced the equations for analyzing the flow characteristics in a meandering channel. In addition, Rozovskii (1961) provided a comparison and discussion of other researchers' studies since the introduction of the radial shear stress calculation method in meandering channels. Rozovskii's radial shear stress equation in meandering channel is provided in Equation 2.22:

$$\tau_r = [\rho g S_r H] - \left[\rho \times \int_0^H \frac{v_\theta^2}{R} dz \right] \quad \text{Equation 2.22}$$

where

- τ_r = radial shear stress [ML⁻¹T⁻²];
 ρ = mass density of the fluid [ML⁻³];
 g = acceleration of gravity [LT⁻²];
 S_r = water surface slope [dimensionless];
 H = flow depth [L];
 v_θ = flow velocity along the bend of the channel [LT⁻¹];
 R = radius of curvature of the channel bend [L]; and
 dz = super elevation of the water surface [L].

Before Rozovskii (1961) suggested the method for radial shear stress computation, several researchers conducted laboratory tests to determine the flow characteristics in meandering channels. Table 2.3 presents the summary of the laboratory tests.

Table 2.3: Summary of Experiment Results of Flow in Bend (from Rozovskii (1961))

Author	Year of Experiments	Rotation Angle (°)	Channel Width (cm)	Radius of Curvature (cm)	Flow Depth (cm)	Mean Flow Velocity (cm/s)	h/b	h/r	Channel Form	Subject of Measurement
Milovich	1914	180	24	28	0.45-13.8	0.66-5	-	-	Rectangular	Trajectories of bottom particles with the aid of coloring material
Daneliya	1936	180	40	60	12	18	0.3	0.2	Rectangular	Distribution of longitudinal and transverse velocity components
Kozhevnikov	1940-1941	180	80	160	2-10	22-50	0.025-0.125	0.0125-0.067	Rectangular triangular parabolic	Trajectories of bottom and surface floats, topography of free surface
Mockmore	1943	2 x 180	45	52.5	9-12	15-25	0.2-0.27	0.17-0.23	Rectangular	Longitudinal velocity components topography of free surface
Shuckry	1949-1950	45-180	30	15-90	18-36	90-78	0.6-1.2	0.4-1.2	Rectangular	Distribution of velocities, topography of free surface
Fidman	1949	3 x 117	100	225	11.5-16	8.7-3.6	0.12-0.16	0.051-0.071	Rectangular	Topography of free surface
Terasvatsatyan	1950	180	38.4	58	-15	30-40	0.39	0.26	Rectangular	Distribution of velocities, with aid of ball nozzle
Same	1950	67	38.4	209	-11	-	0.29	0.048	Rectangular	Distribution of velocities, with aid of ball nozzle
Ananyan	1953	2 x 180	50	250	15-18	26-33	0.3-0.36	0.06-0.72	Rectangular	Distribution of velocities, with aid of ball nozzle
Makkareev Romanenko	1940	180	25	50	3-12	20-72	0.1-0.5	-	Rectangular of sinusoidal configuration in plane	Velocity distributions

2.3.3.1 Laboratory Study for Validation of Theory

For the validation of the Rozovskii theory, a physical model was constructed by the Hydraulic Laboratory of the Academy of the Science of the Ukrainian SSR, Institute of Hydrology and Hydraulic Engineering. The rotation angle was 90° , radius of curvature was 16.4 ft, maximum flow depth was 0.459 ft, and bed slope was 0.001 ft/ft. A hydrometric tube was used for the velocity measurement. To obtain the velocity profile, flow velocity measurements were recorded. The depth increment was 0.0656 ft. In the case of the bend section that starts at Cross section 14, a nozzle ball was used for the velocity measurement. Figure 2.22 shows the test results for flow velocity distribution.

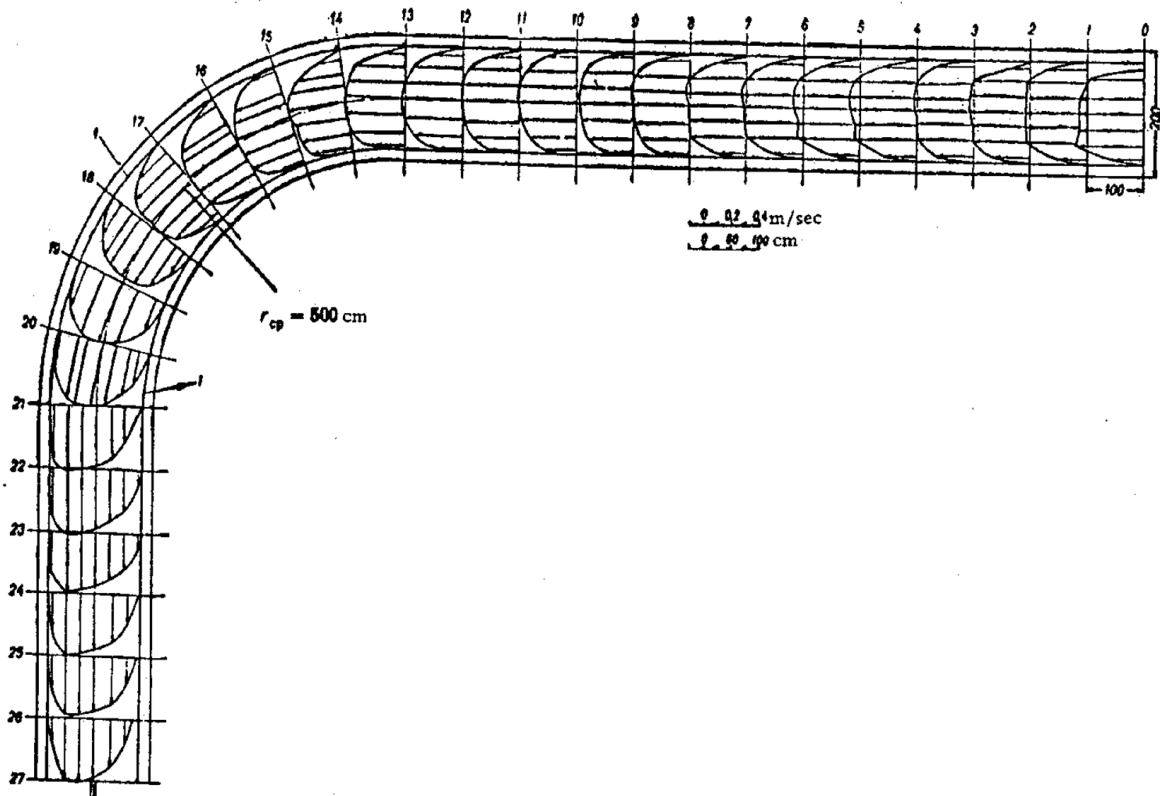


Figure 2.22: Velocity Distribution from the Experiment of Polygonal Section Test Results
(from Rozovskii (1961))

2.3.3.2 Field Observation for Validation of Theory

Field observations were also performed by the Hydraulic Laboratory of the Academy of the Science of the Ukrainian SSR, Institute of Hydrology and Hydraulic Engineering between

1951 and 1952. The locations of the field observations were the Desna River and Snov River. The rotation angle of the Desna River study reach was 180° and radius of curvature was 400 m. Figure 2.23 shows the study reach of the river. During the field observation, field surveying was performed to determine the water surface elevation and flow depth. According to the surveying results, the general cross section was rectangular. The flow depth was especially deep at Cross section 4, the location of contraction of the channel width to 110 m. A rotary flow meter with an elongated vane was used for flow velocity measurement.

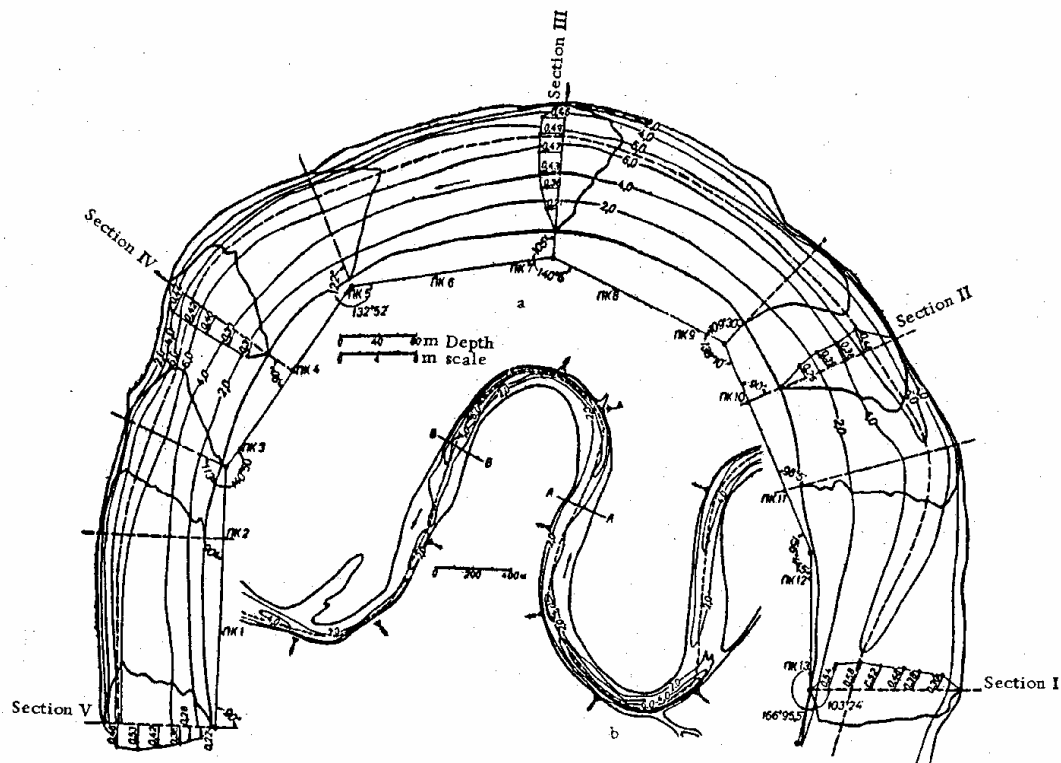


Figure 2.23: Description of the Desna River Study Reach (from Rozovskii (1961))

Another field observation was performed on the Snov River by the Hydraulic Laboratory of the Academy of the Science of the Ukrainian SSR, Institute of Hydrology and Hydraulic Engineering. Figure 2.24 shows the plan view and Figure 2.25 shows the flow velocity distribution of the Snov River. The geometry of the Snov River was:

- averaged channel width was 82.02 ft;
- radius of curvature was 279 ft;
- maximum flow velocity was 2.624 ft/s;

- discharge of Snov River was about $2,119 \text{ ft}^3/\text{s}$; and
- bed slope was 0.000382 ft/ft .

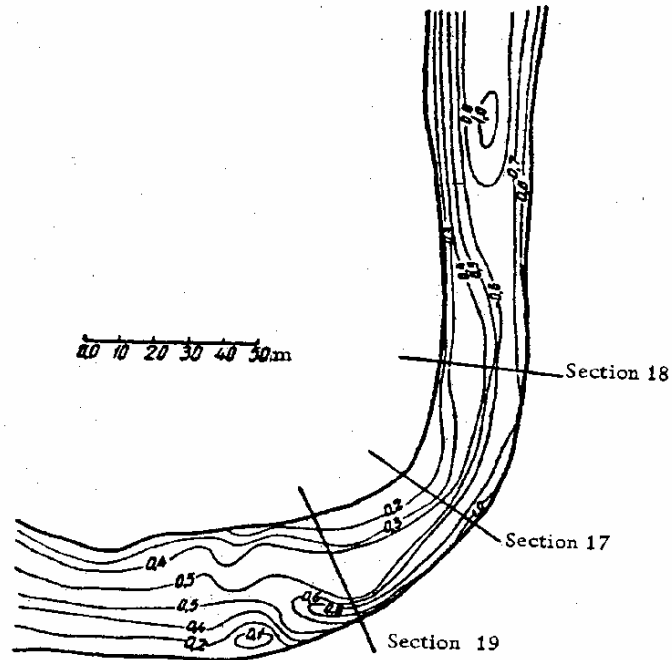


Figure 2.24: Plan View of the Snov River (from Rozovskii (1961))

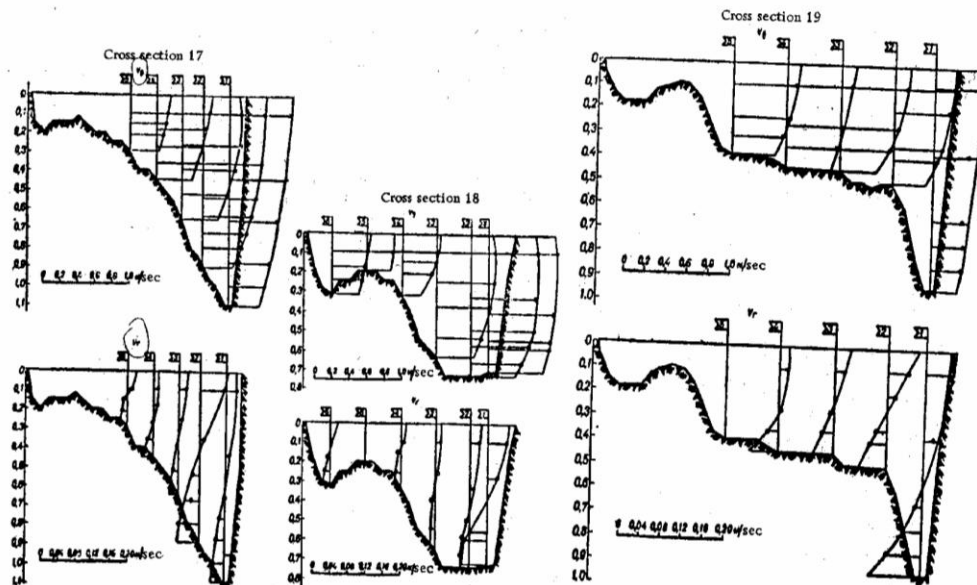


Figure 2.25: Distribution of Flow Velocity of Snov River (from Rozovskii (1961))

2.3.3.3 Summary

According to the results of the laboratory study and field observations, Rozovskii (1961) concluded that there was good agreement between field observation and the desired theoretical equation. Following Rozovskii's research, several people such as Odgaard (1986) conducted mathematical modeling to determine flow characteristics for a meandering channel.

2.3.4 Reynolds Shear Stress Extrapolation

Julien (1998) explained the Cartesian element of fluid composed with two different forces: 1) internal force and 2) external force. Internal forces apply at the center of the mass (Julien 1998). In the case of external forces, stress can be classified as two other components: 1) normal stress and 2) tangential stress. The study focuses on the calculation of the tangential stress that corresponds with the Reynolds shear stress comprised of six different components: $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, and $\tau_{yz} = \tau_{zy}$. A diagram of these surface stresses is presented in Figure 2.26.

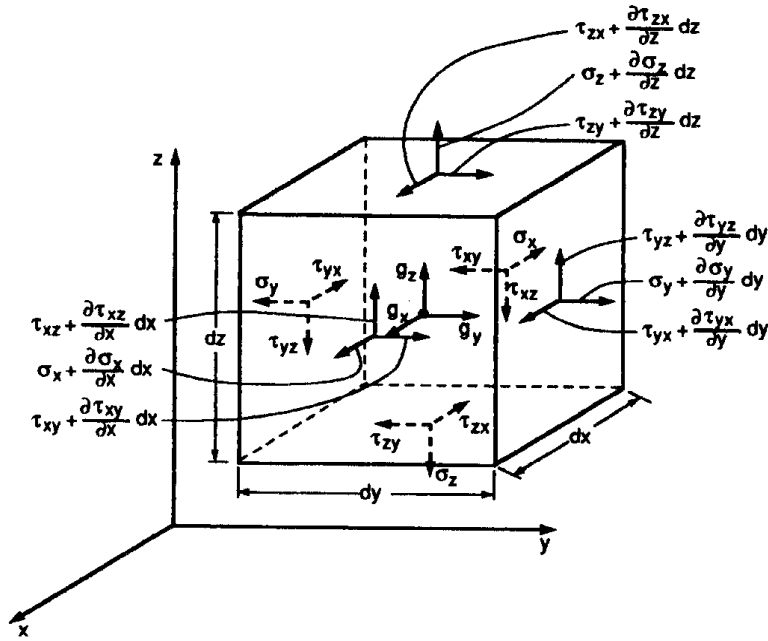


Figure 2.26: Surface Stresses (from Julien (1998))

2.3.4.1 Wahl (2000)

To calculate six components of the Reynolds shear stress, an ADV meter was used in this study. The representative flow data from the ADV are covariance (COV) because they are directly used to obtain the Reynolds shear stress. As shown in Figure 2.27, flow velocity is not always identical with the averaged flow velocity due to the turbulence in the flow. The basic concept of covariance in this study is the correlation of two components of flow velocity out of three flow velocity components: V_x , V_y , and V_z .

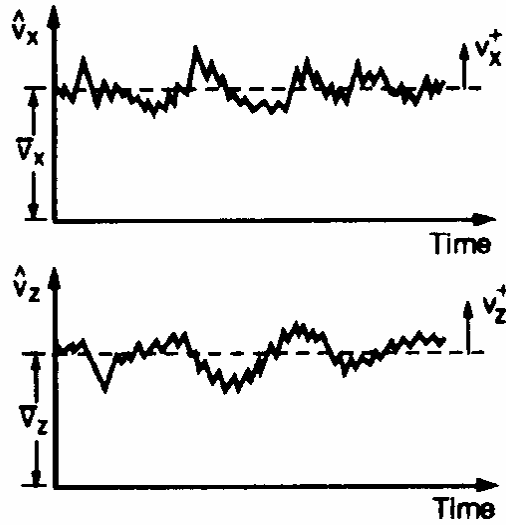


Figure 2.27: Description of Turbulence in Flow (from Julien (1998))

Equation 2.23 through Equation 2.28 show the calculation method of Reynolds shear stress extrapolation using ADV data (Wahl 1996):

$$\tau_{xy} = \tau_{yx} = -\rho \times (COV-XY) \quad \text{Equation 2.23}$$

where

τ_{xy} = turbulent shear stress that applies along the y-axis and caused by flow velocity fluctuation in the direction of the x-axis [$ML^{-1}T^{-2}$];

τ_{yx} = turbulent shear stress that applies along the x-axis and caused by flow velocity fluctuation in the direction of the y-axis [$ML^{-1}T^{-2}$];

ρ = mass density of water [ML^{-3}]; and

$COV-XY$ = covariance of two variables.

COV-XY is the covariance of two variables (V_x and V_y). It can be computed by using Equation 2.24:

$$COV-XY = \frac{\sum V_x V_y}{n-1} - \frac{\sum V_x \sum V_y}{n(n-1)} \quad \text{Equation 2.24}$$

where

V_x = flow velocity along the x-axis [LT^{-1}];

V_y = flow velocity along the y-axis [LT^{-1}]; and

n = number of samples that collected velocity data [dimensionless].

Equation 2.25 provides the equation for computing the turbulent shear stress that applies along the x-axis and caused by flow velocity fluctuation in the direction of the z-axis (τ_{zx}) [$ML^{-1}T^{-2}$]:

$$\tau_{zx} = \tau_{xz} = -\rho \times (COV-XZ) \quad \text{Equation 2.25}$$

where

τ_{xz} = turbulent shear stress that applies along the z-axis and caused by flow velocity fluctuation in the direction of the x-axis [$ML^{-1}T^{-2}$];

ρ = mass density of water [ML^{-3}]; and

COV-XZ = covariance of two variables.

COV-XZ is the covariance of two variables (V_x and V_z). It can be computed by using Equation 2.26:

$$COV-XZ = \frac{\sum V_z V_x}{n-1} - \frac{\sum V_z \sum V_x}{n(n-1)} \quad \text{Equation 2.26}$$

where

V_z = flow velocity along the z-axis [LT^{-1}];

V_x = flow velocity along the x-axis [LT^{-1}]; and

n = number of samples that collected velocity data [dimensionless].

Equation 2.27 provides the equation for computing the turbulent shear stress that applies along the y-axis and caused by flow velocity fluctuation in the direction of the z-axis (τ_{zy}) [$ML^{-1}T^{-2}$]:

$$\tau_{zy} = \tau_{yz} = -\rho \times (COV-YZ) \quad \text{Equation 2.27}$$

where

τ_{yz} = turbulent shear stress that applies along the z -axis and caused by flow velocity fluctuation in the direction of the y -axis [$ML^{-1}T^{-2}$];

ρ = mass density of water [ML^{-3}]; and

$COV-YZ$ = covariance of two variables.

$COV-YZ$ is the covariance of two variables (V_y and V_z). It can be computed by using Equation 2.28:

$$COV-YZ = \frac{\sum V_y V_z}{n-1} - \frac{\sum V_y \sum V_z}{n(n-1)} \quad \text{Equation 2.28}$$

where

V_y = flow velocity along the y -axis [LT^{-1}];

V_z = flow velocity along the z -axis [LT^{-1}]; and

n = number of samples that collected velocity data [dimensionless].

2.3.4.2 Baird (2004)

Baird (2004) suggested linear extrapolation of the Reynolds shear stress profile to obtain bed shear stress using ADV data. An example of the Reynolds shear stress profile is presented in Figure 2.28. Due to the existence of the roughness sublayer, the Reynolds shear stress decreases to the near-bed area just after a maximum value of the Reynolds shear stress (Raupach 1991, Nikora and Gohring 2000). As a result, Montes (1998) suggested linear extrapolation of the linear portion of the Reynolds shear stress profile as a method of bed shear stress calculation. Actually, Baird (2004) stated “*Reynolds shear stress can be calculated by extrapolating the slope of the linear portion of the shear stress profile to the bed.*” In addition, Montes (1998) explained that finding the point of interception with the x -axis where the relative depth is zero (Figure 2.28), is a good method of estimation of bed shear stress.

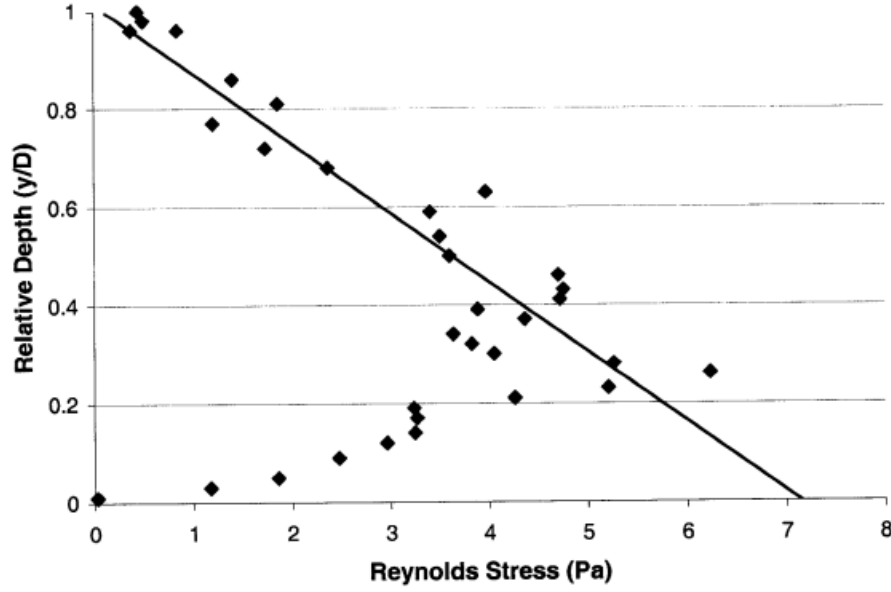


Figure 2.28: Reynolds Shear Stress Profile (from Baird (2004))

Baird (2004) rearranged Equation 2.29 (cited from Muste and Patel (1997) and Montes (1998)) into Equation 2.30 to obtain the linear regression equation from the linear portion of the Reynolds shear stress profile that was presented in Figure 2.28:

$$\frac{\tau}{\rho} = -\overline{u'v'} = u_*^2 \left(1 - \frac{y}{D} \right) \quad \text{Equation 2.29}$$

where

τ = shear stress [$\text{ML}^{-1}\text{T}^{-2}$];

ρ = mass density of water [ML^{-3}];

u' = instantaneous velocity fluctuations about the mean in the longitudinal direction [LT^{-1}];

v' = instantaneous velocity fluctuations about the mean in the vertical direction [LT^{-1}];

u_* = shear velocity [LT^{-1}]; and

$\frac{y}{D}$ = relative depth [dimensionless].

$$\frac{y}{D} = m \times (\tau) + b \quad \text{Equation 2.30}$$

where

$\frac{y}{D}$ = relative depth [dimensionless];

m = regression slope of linear regression equation;

τ = shear stress that equals to $-\rho \bar{u'v'}$ at depth $\frac{y}{D}$ [$\text{ML}^{-1}\text{T}^{-2}$]; and

b = intercept of the regression line [dimensionless].

2.4 Summary

Numerous researchers have performed studies to quantify shear stress distribution in meandering channels. Mathematical modeling and laboratory experiments have been implemented for the validation of theories. The basic mechanisms of flow in meandering channels were introduced: parameters to determine meandering channel geometry, stream morphology, concept of secondary flow in bend, effect of super elevation, and Lane's balance to explain the dynamic equilibrium in a stream system. In Section 2.3.1 through Section 2.3.4, the methods of shear stress calculation were introduced for data analysis of the study. These calculation methods are: 1) Preston tube method, 2) linear regression of the flow velocity profile, 3) Rozovskii method, and 4) Reynolds shear stress extrapolation by using ADV data.

3 DATA ANALYSIS

3.1 Introduction

Data analysis was conducted to determine the distribution of shear stress. Measurements of flow velocity, flow depth, and boundary shear stress were performed during the data-collection procedure. In accordance with the procedure presented in Section 2.3, four methods were used for the data analysis: 1) Preston tube method, 2) linear regression of the flow velocity profile, 3) Rozovskii method, and 4) Reynolds shear stress extrapolation. In this chapter, the shear stress calculation results and the value of differences between each shear stress calculation method are presented.

3.2 Shear Stress Calculation Results

Four calculation methods were used for data analysis. Entering and exiting flow in the model results in overestimation of shear stress. As a result, shear stress data of Cross section 1 through Cross section 3, and Cross section 18 were excluded from analysis. The Preston tube method is the shear stress calculation using a Pitot tube. Logarithmic velocity profile is the technique of shear stress calculation by obtaining a linear regression after plotting the relationship between flow depth and logarithmic flow velocity profile. The Rozovskii method was derived from the equation of motion in Cartesian coordinates. Reynolds shear stress is calculated from ADV data based on the covariance of two variables of flow velocity. The two variables originated from two of three flow velocity components: V_x , V_y , and V_z . In this section, the results and discussion of the data analysis for the four different shear stress calculation methods are presented.

3.2.1 Preston Tube Method

The Preston tube calibration was conducted in a flume by Sclafani (2008). Boundary shear stress in the x -direction (τ_x) was computed by converting Preston differential pressure data (inch of head) into shear stress (psf) using the Preston tube calibration that was exhibited as Equation 2.13. These tabulated shear stress data are reported in Appendix B. The shear stress distribution calculations from the Preston tube method are presented in Figure 3.1 through Figure 3.4. The range of τ_x values by the Preston tube were between 0.0075 and 0.072 psf. Clearly, the area with a higher value of τ_x was located at the inner side of the downstream bend of the channel. Also, proportionality of the τ_x value with increasing flow rate was observed except for in the case of 16 cfs. With the increases of the constant rate, the measured data consistently moved into a higher level of τ_x values.

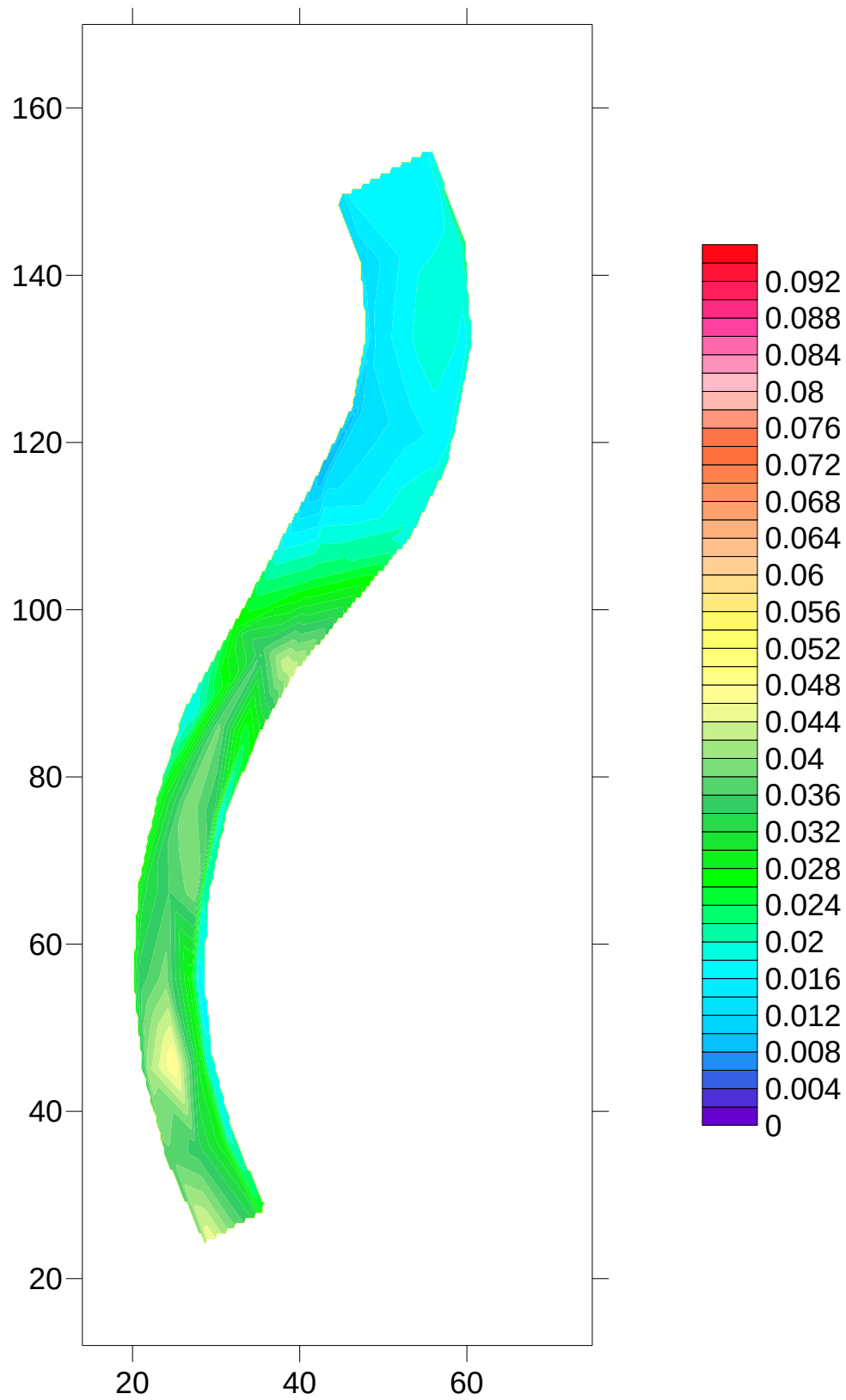


Figure 3.1: Shear Stress Distribution (τ_x) from the Preston Tube for 8 cfs (psf)

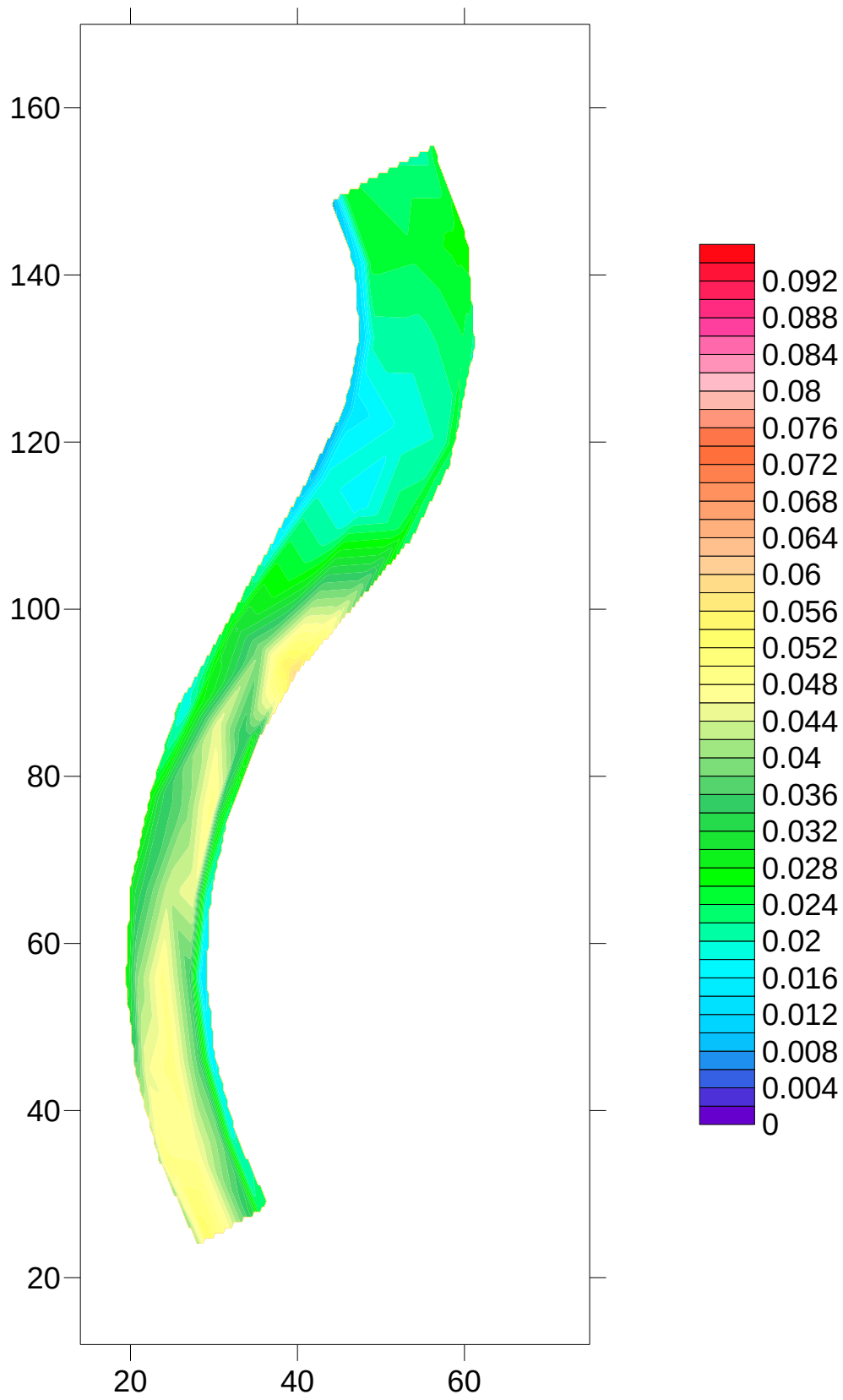


Figure 3.2: Shear Stress Distribution (τ_x) from the Preston Tube for 12 cfs (psf)

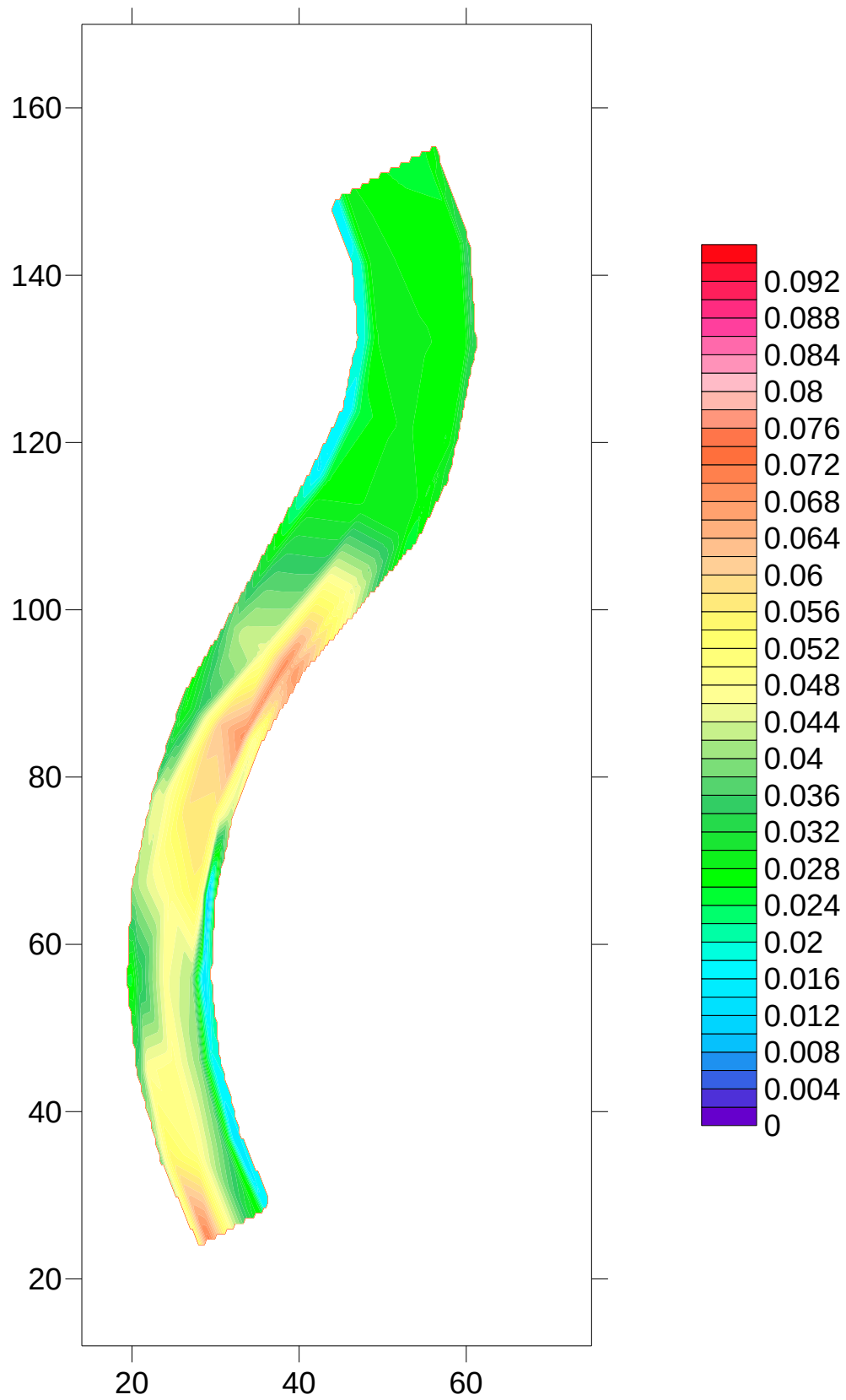


Figure 3.3: Shear Stress Distribution (τ_x) from the Preston Tube for 16 cfs (psf)

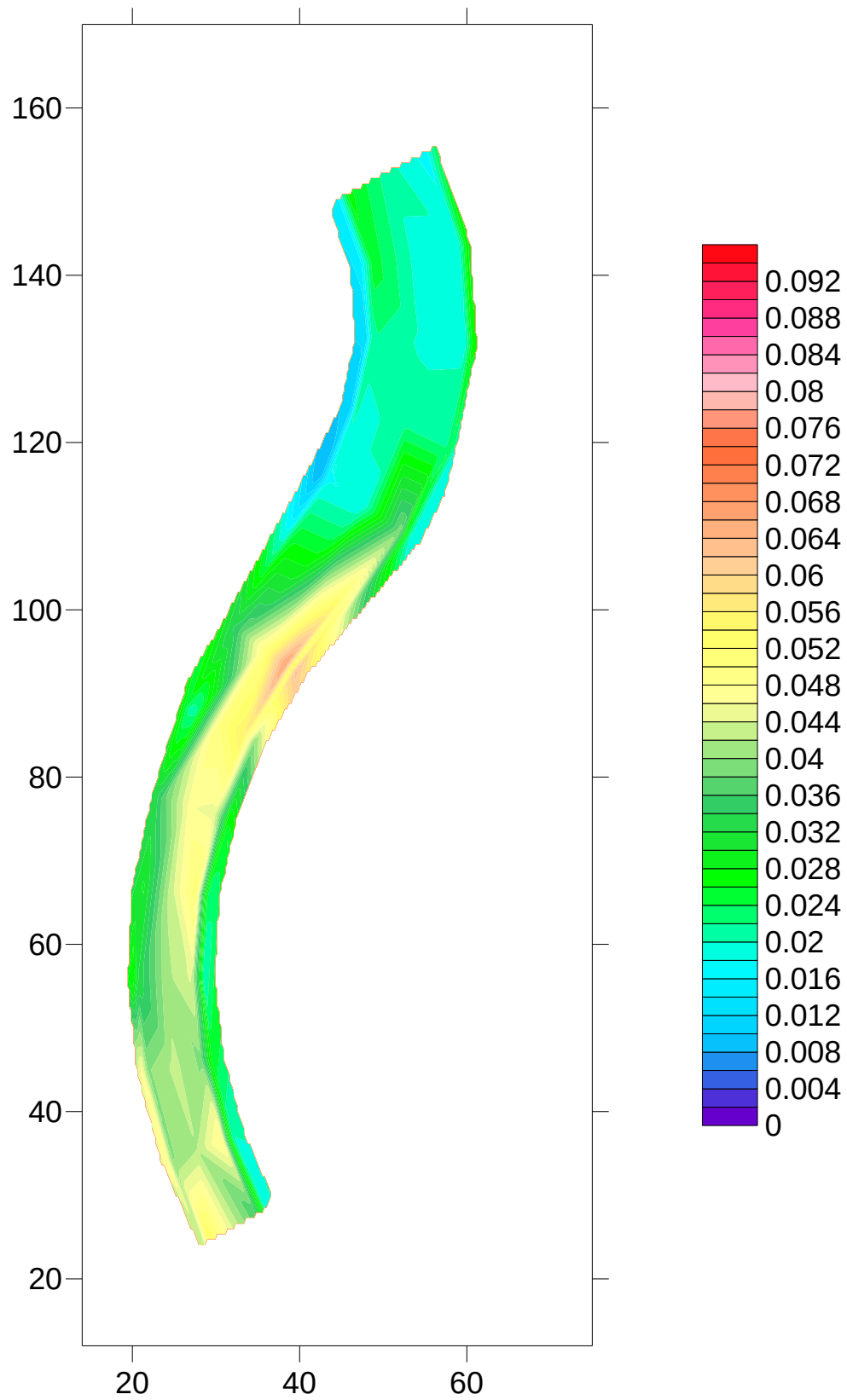


Figure 3.4: Shear Stress Distribution (τ_x) from the Preston Tube for 20 cfs (psf)

3.2.2 Linear Regression of the Flow Velocity Profile

Equation 2.17 through Equation 2.20 described the method of linear regression of the flow velocity profile. Ideally, the τ_x value that was obtained from this method is identical with the value of τ_x obtained by the Preston tube method. Figure 3.5 presents the distribution of turbulent shear stress (τ_x) for 20 cfs. The value of τ_x ranged between 0 and 0.076 psf. Just as in the case of τ_x by the Preston tube, a higher value of τ_x was observed at the inside of the downstream bend of the channel. Proportionality of flow rate and shear stress also was observed. In addition, lateral shear stress (τ_y) was calculated using the same method to obtain τ_x . The result is presented in Figure 3.6. The tabulated shear stress (τ_x) data for 8, 12, 16, and 20 cfs, along with the shear stress (τ_x and τ_y) distribution plots for 8, 12, and 16 cfs are presented in Appendix C.

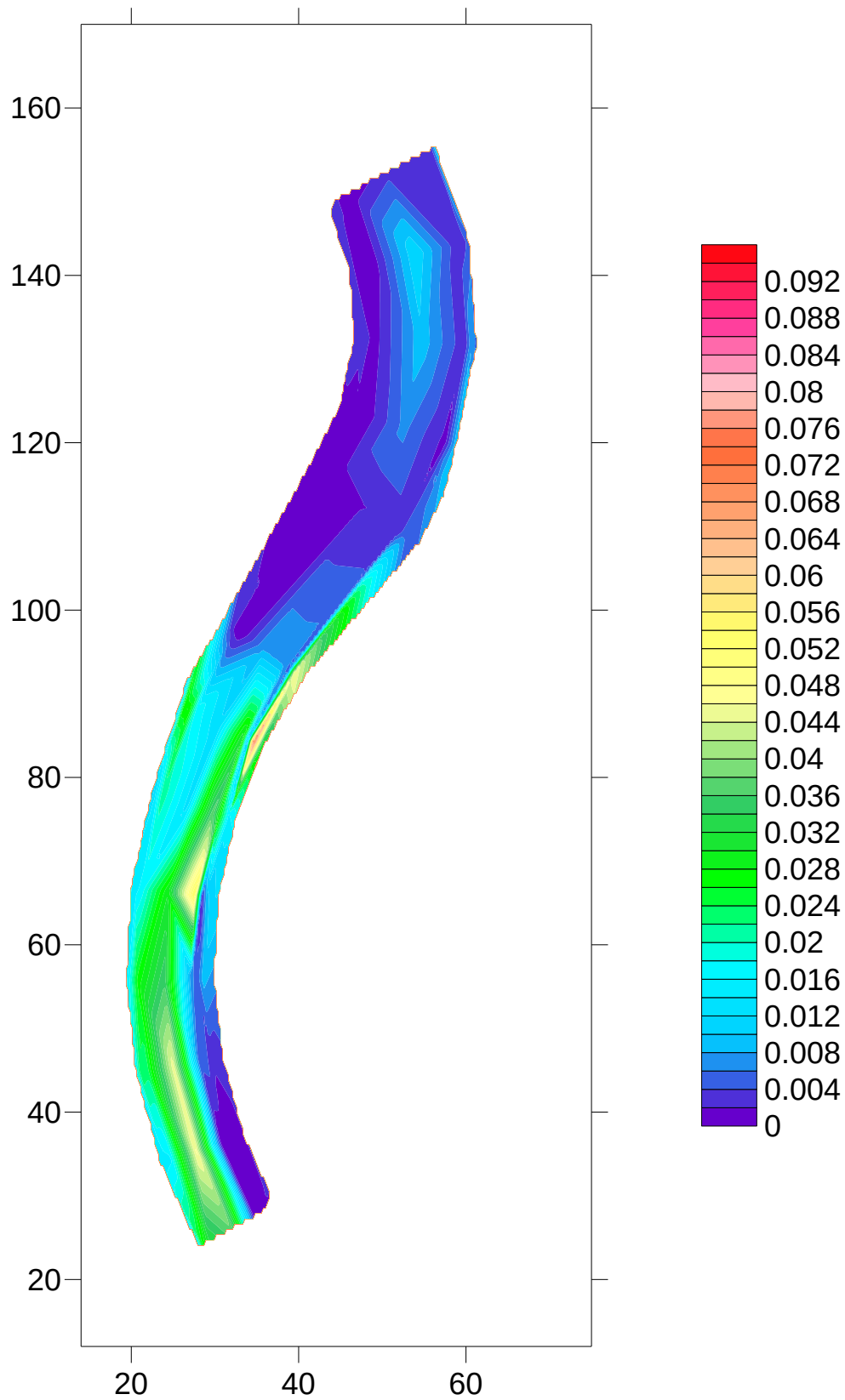


Figure 3.5: Shear Stress Distribution (τ_x) by Linear Regression for 20 cfs (psf)

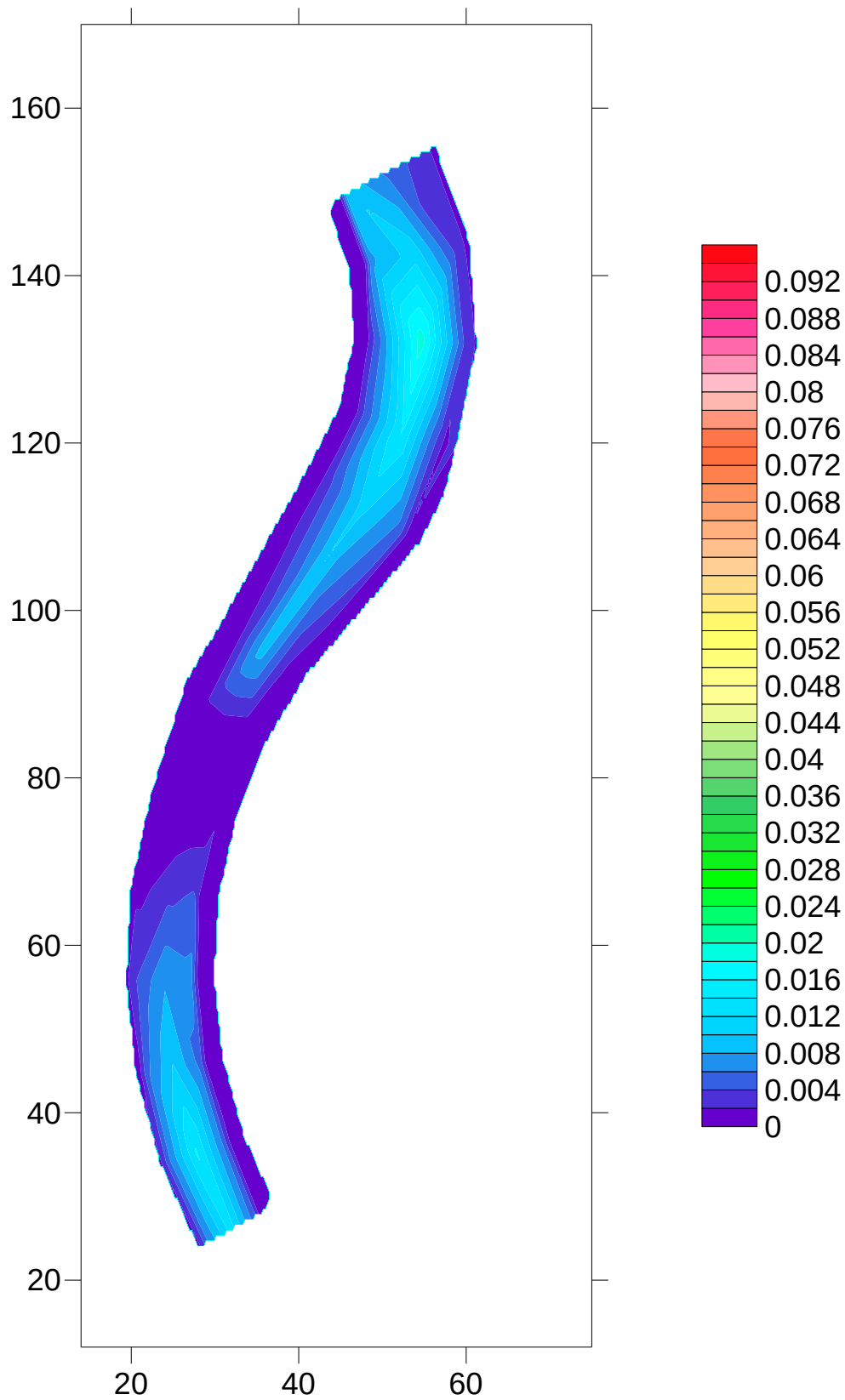


Figure 3.6: Shear Stress Distribution (τ_v) by Linear Regression for 20 cfs (psf)

3.2.3 Rozovskii Method

The shear stress calculation method presented by Rozovskii (1961) computed the radial shear stress (τ_r) in a meandering channel, by using Equation 2.22. A detailed calculation procedure for the water surface slope (S_r) and the second bracket in the right-hand side of Equation 2.22 is presented in this section. The water surface slope can be expressed by Equation 3.1:

$$S_r = \frac{h}{D} \quad \text{Equation 3.1}$$

where

S_r = water surface slope [dimensionless];

h = difference of water surface elevation between Piezo a and Piezo g [L]; and

D = horizontal distance between Piezo a and Piezo g [L].

Equation 3.2 shows the detailed calculation method for the distance between Piezo a and Piezo g:

$$D = d_1 + d_2 + d_3 \quad \text{Equation 3.2}$$

where

D = horizontal distance between Piezo a and Piezo g [L];

d_1 = three times of the difference of height of Piezo a and Piezo d [L];

d_2 = channel bottom width [L]; and

d_3 = three times of the difference of height of Piezo g and Piezo d [L].

The multiplying factor of 3 comes from the 3H:1V (Horizontal:Vertical) side slope of the trapezoidal channel. Figure 3.7 illustrates the distance between Piezo a and Piezo g.

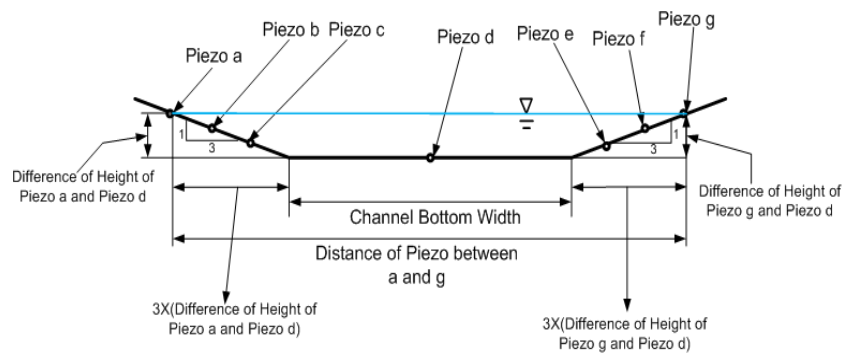


Figure 3.7: Diagram of Distance between Piezo a and Piezo g

For the calculation of the second bracket in the right-hand side of Equation 2.22, a manual integration method was used. Figure 3.8 illustrates the idealized flow velocity profile.

Manual integration of the term $\int_0^H \frac{v_\theta^2}{r} dz$ is described by Equation 3.3:

$$\frac{1}{r} \left\{ \left(\frac{v_1 + v_o}{2} \right)^2 (z_1 - z_{Bed}) + \dots + \left(\frac{v_i + v_{i+1}}{2} \right)^2 \times (z_{i+1} - z_i) + v_n^2 \times (z_{WSE} - z_n) \right\} \quad \text{Equation 3.3}$$

where

- r = radius of curvature of the channel bend [L];
- v_1 = flow velocity value that locates in the lowest data-collection point [LT^{-1}];
- v_o = flow velocity at the channel bed equal to zero [LT^{-1}];
- z_1 = elevation of lowest data collection point in a vertical column [L];
- z_{Bed} = bed elevation of the channel [L];
- v_i = flow velocity data-collection point located within the vertical column where i ranges from 1 to number of data point in the column (n) [LT^{-1}];
- z_i = elevation of flow velocity data-collection points where i ranges from 1 to number of data point in the column (n) [L];
- v_n = flow velocity located at the highest data-collection point with a vertical column [LT^{-1}];
- z_n = elevation of highest data-collection point in a vertical column [L]; and
- z_{WSE} = water-surface elevation of the channel [L].

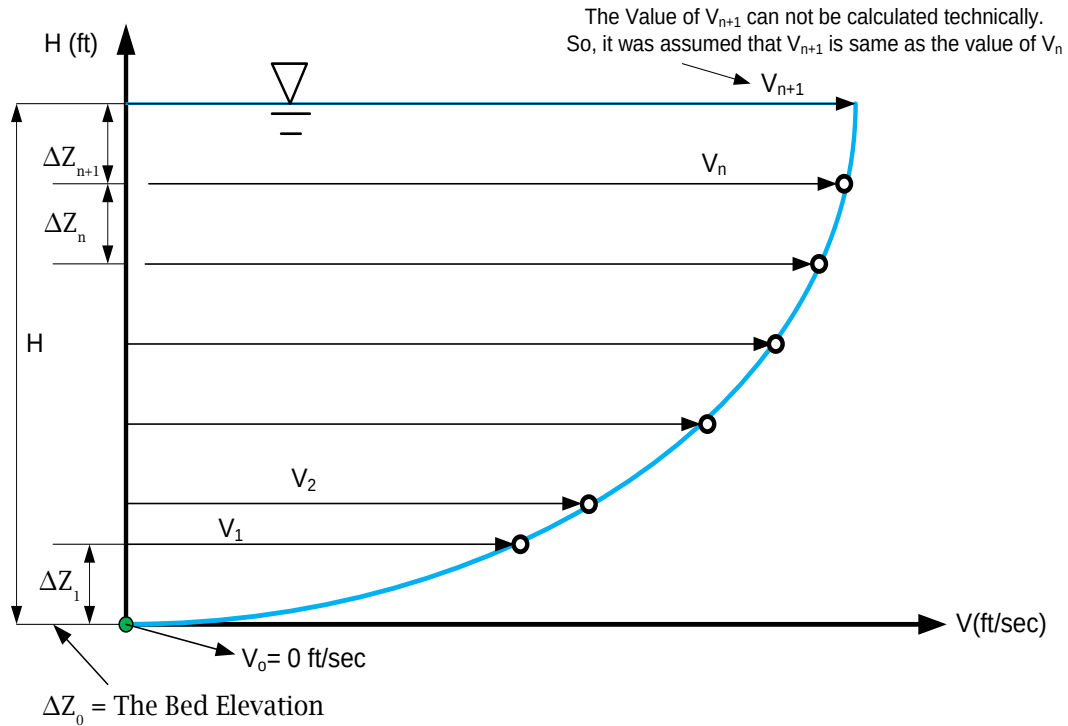


Figure 3.8: Idealized Flow Velocity Profile

The shear stress distribution from the Rozovskii method is presented in Figure 3.9 for 20 cfs. The range of the values of τ_r were between -0.048 and 0.13 psf. Different from the Preston tube method and linear regression of the flow velocity profile, shear stress by the Rozovskii method did not show proportionality with increase of the flow rate. Also, negative shear stress values were observed: the point of negative shear stress has a less steep water surface slope. That means the S_r value is the dominant parameter of shear stress from the Rozovskii method. To validate the observation, the water surface slope distribution for each flow rate is presented in Figure 3.10 for 20 cfs. The plots show good agreement between the change of water surface slope and shear stress distribution from Rozovskii. The tabulated shear stress data for 8, 12, 16, and 20 cfs, radial shear stress (τ_r) distribution plots for 8, 12, and 16 cfs, along with the water surface slope distribution plots for 8, 12, and 16 cfs are presented in Appendix D.

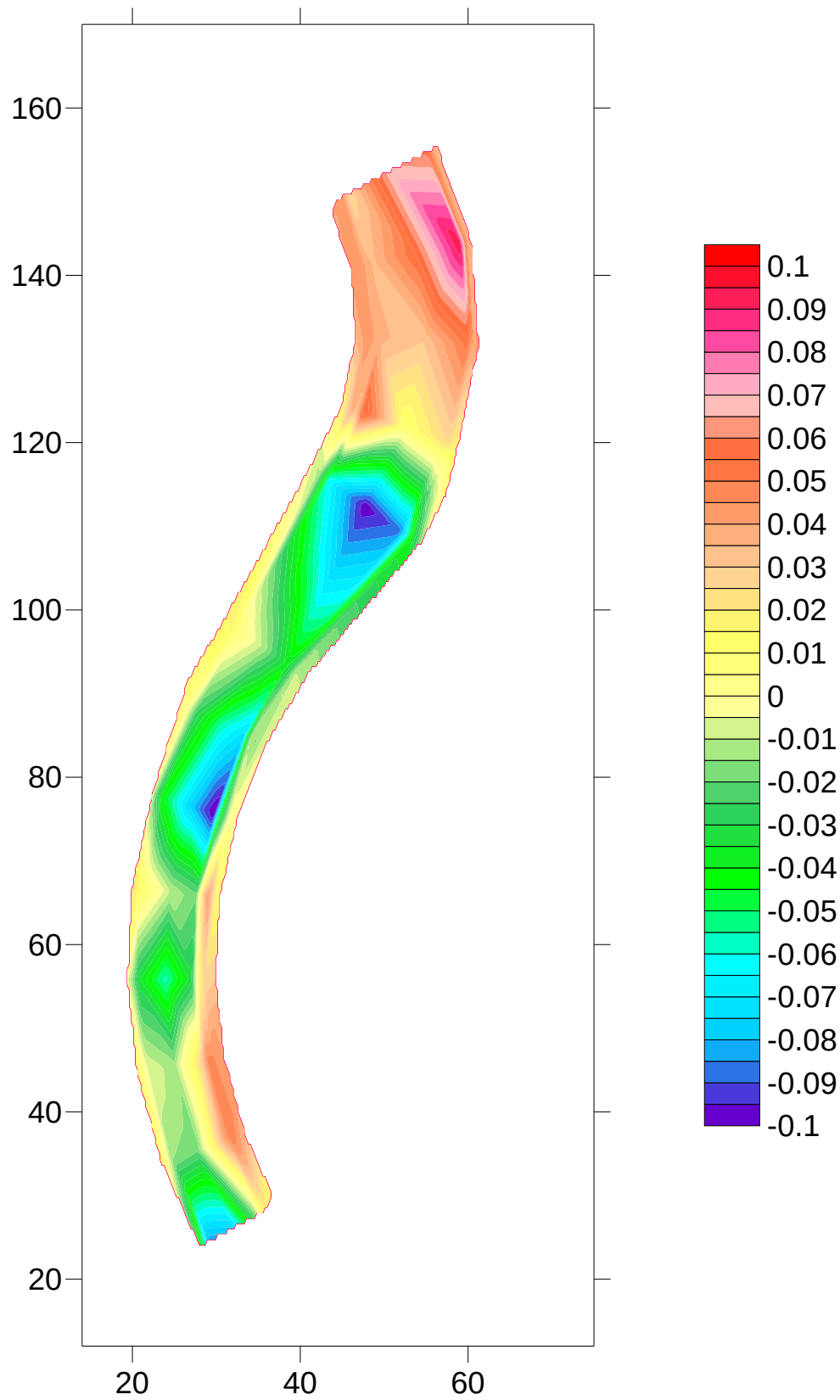


Figure 3.9: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) for 20 cfs (psf)

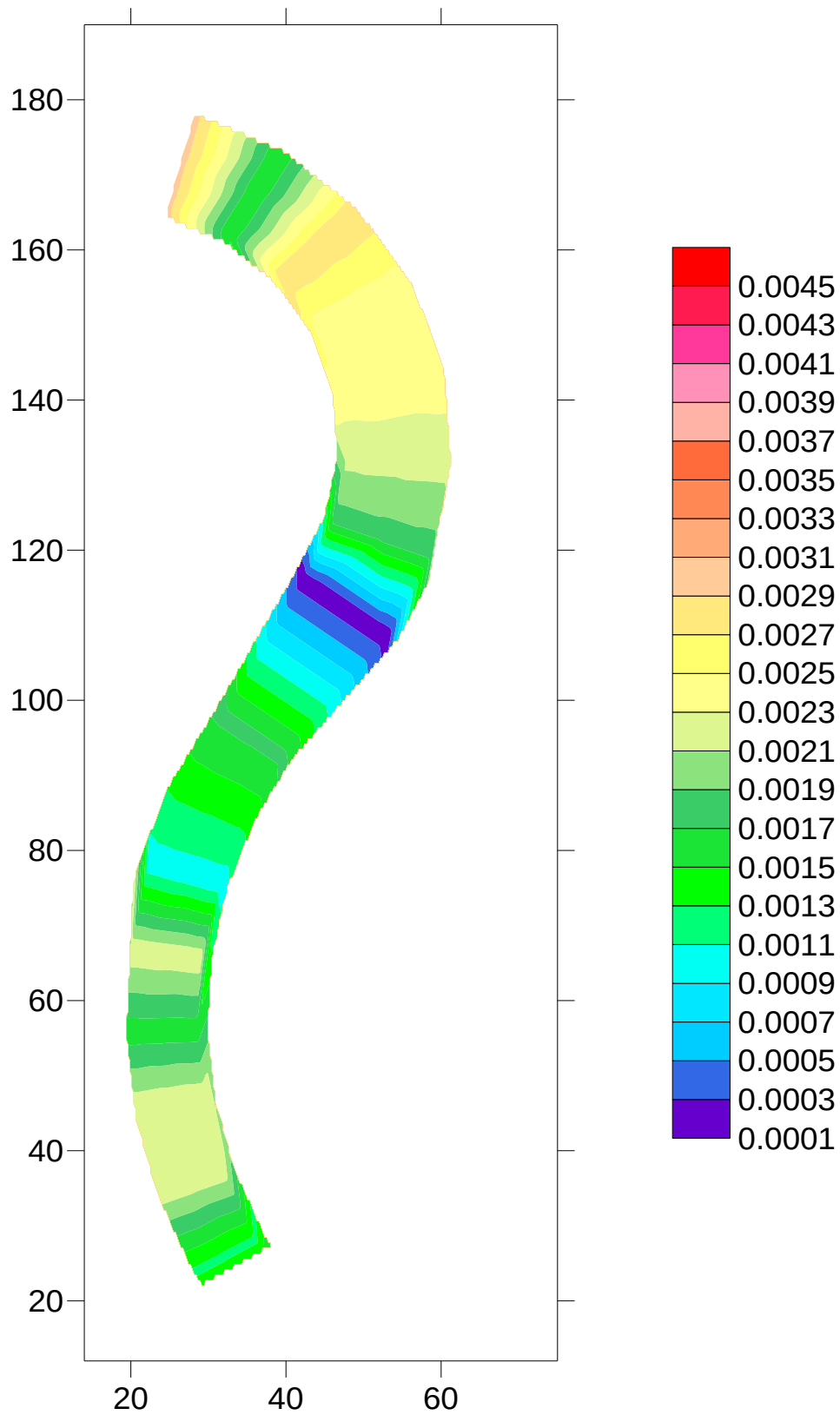


Figure 3.10: Water Surface Slope Distribution for 20 cfs

3.2.4 Reynolds Shear Stress Extrapolation

Reynolds shear stress extrapolation is based on the ADV meter data of each piezometer and each cross section of the model. As suggested by Montes (1998), the Reynolds shear stress profile was plotted to take a linear regression of the linear portion of the Reynolds shear stress profile. The x -axis of the linear regression equation is the Reynolds shear stress and the y -axis is the relative depth. The non-linear portion of the Reynolds shear stress profile, that is shown as pink squares in Figure 3.11 and Figure 3.12, was removed for the linear regression. In addition, Wahl (2000) explained that when a data point of correlation is below 70 and the signal-to-noise ratio is below 15 there is a high possibility of low accuracy. As a result, ADV data point covariance below 70 and signal-to-noise ratio below 15 were excluded from the plotting of the Reynolds shear stress profile. Montes (1998) explained the point where the linear regression equation intercepts the x -axis is the bed shear stress. Shear stress in the longitudinal direction that applies on the bed of the channel (τ_{bed}) can be computed from Equation 3.4:

$$\tau_{bed} = \sqrt{(\tau_{zx})^2 + (\tau_{zy})^2} \quad \text{Equation 3.4}$$

where

τ_{bed} = turbulent shear stress in longitudinal direction that applies on the bed of the channel [$\text{ML}^{-1}\text{T}^{-2}$];

τ_{zx} = turbulent shear stress that applies in the x -axis and is caused by V_z [$\text{ML}^{-1}\text{T}^{-2}$]; and

τ_{zy} = turbulent shear stress that applies in the y -axis and is caused by V_z [$\text{ML}^{-1}\text{T}^{-2}$].

Summaries of the calculation results from Reynolds shear stress extrapolation are presented in Appendix E.

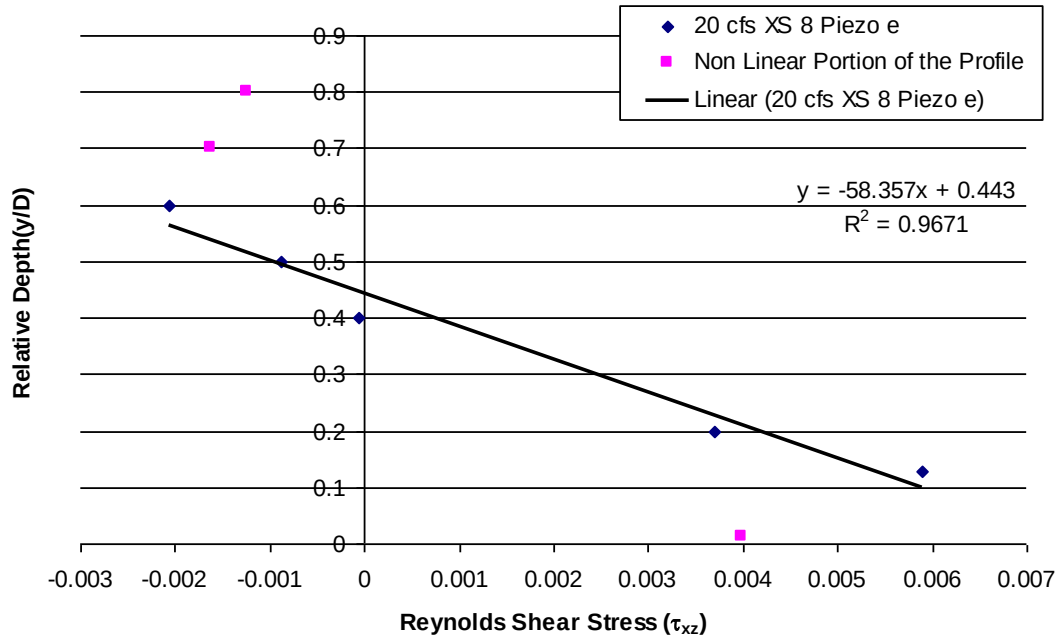


Figure 3.11: Reynolds Shear Stress Profile of τ_{xz} for 20 cfs, Cross Section 8, Piezo e

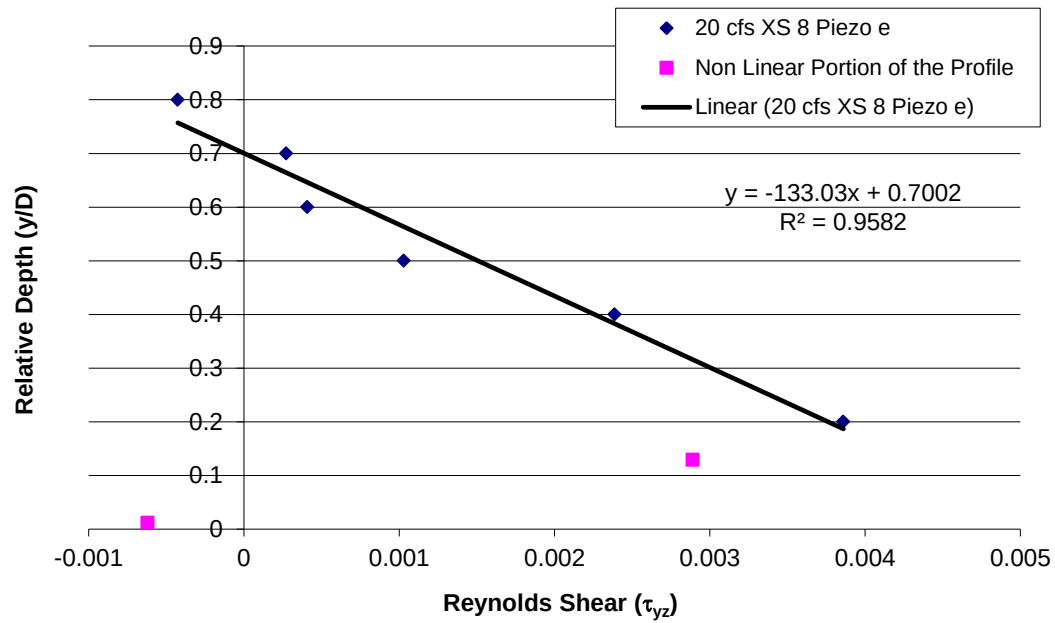


Figure 3.12: Reynolds Shear Stress Profile of τ_{yz} for 20 cfs, Cross Section 8, Piezo e

Figure 3.13 through Figure 3.16 present the shear stress distribution for the overall flow rates of 8, 12, 16, and 20 cfs. The lowest value of the Reynolds shear stress is indicated by a violet color and the highest value of turbulent shear stress is indicated by a red color. The value of τ_{bed} ranged from 0.002 to 0.058 psf. The location of the maximum value of τ_{bed} was observed at the near apex of the downstream bend.

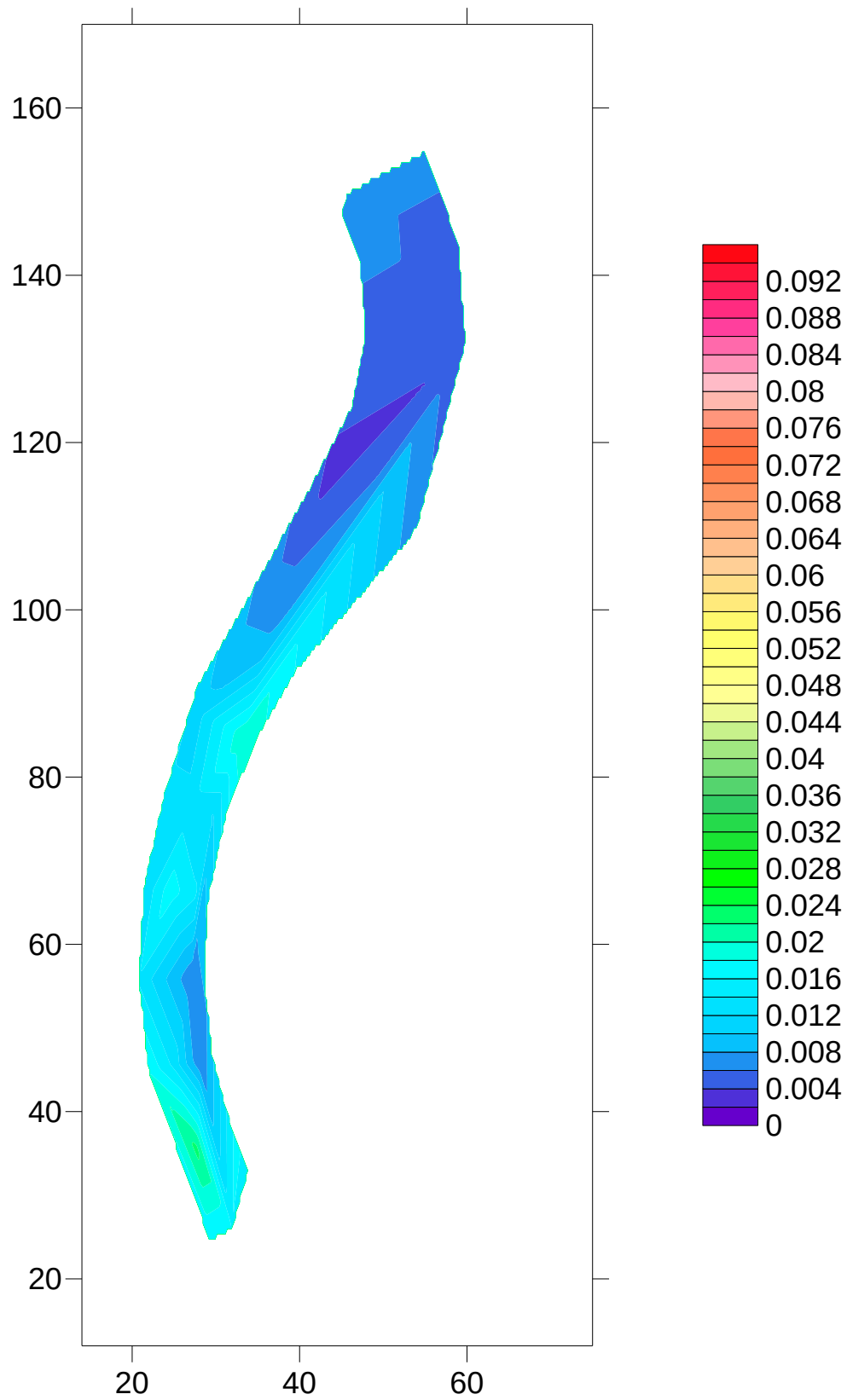


Figure 3.13: Contour Map of τ_{bed} from the Reynolds Shear Stress for 8 cfs (psf)

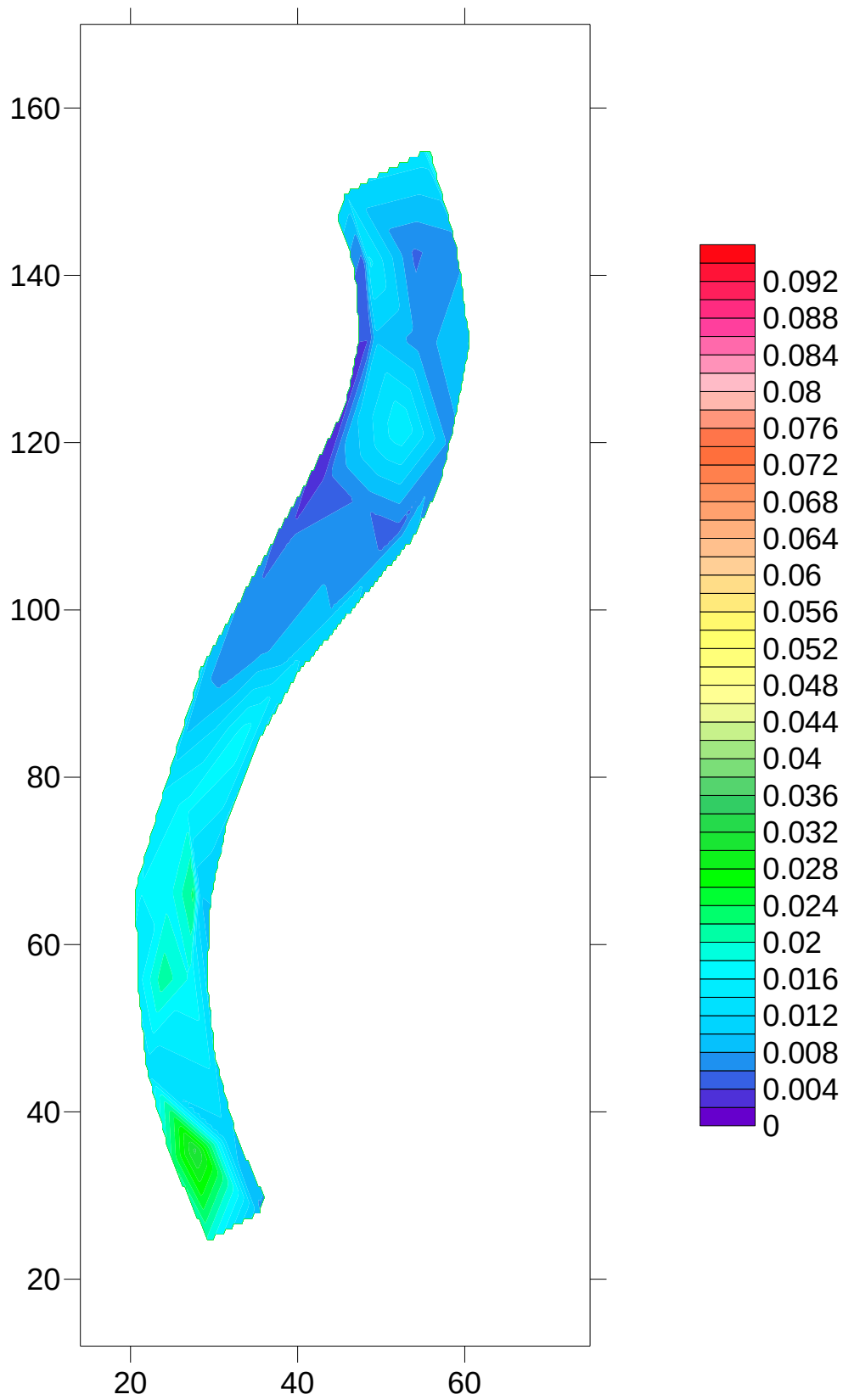


Figure 3.14: Contour Map of τ_{bed} from the Reynolds Shear Stress for 12 cfs (psf)

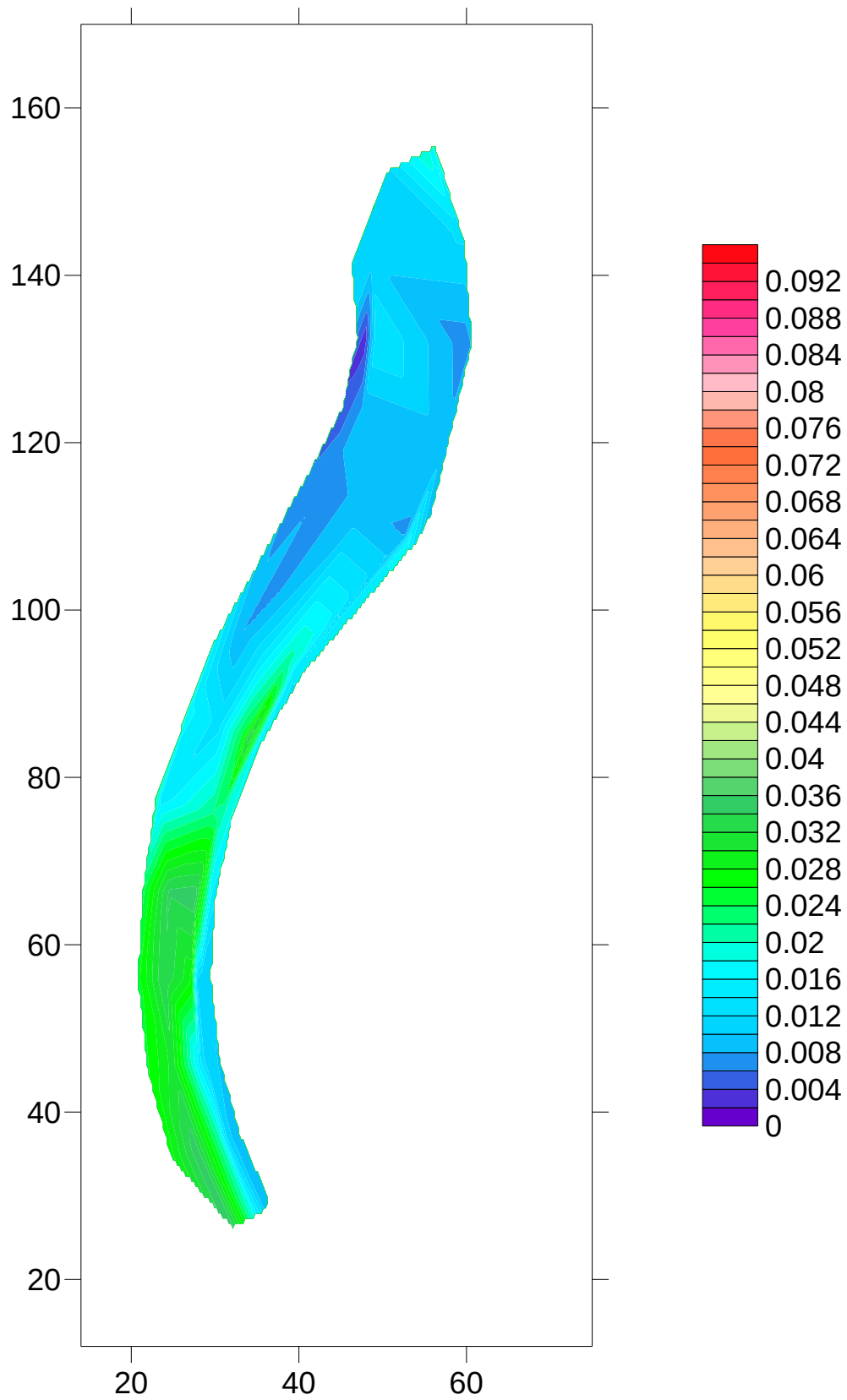


Figure 3.15: Contour Map of τ_{bed} from the Reynolds Shear Stress for 16 cfs (psf)

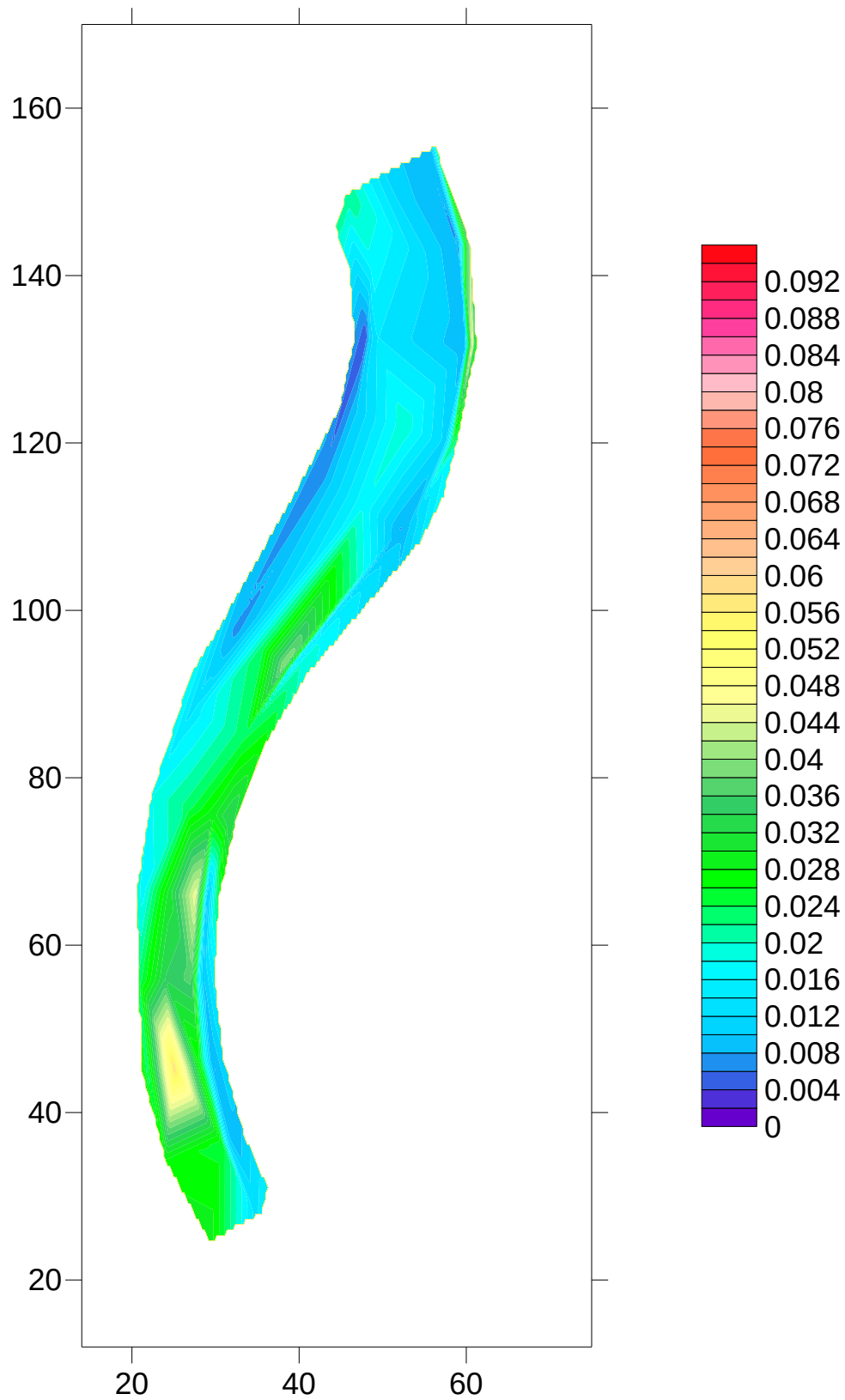


Figure 3.16: Contour Map of τ_{bed} from the Reynolds Shear Stress for 20 cfs (psf)

3.3 Summary

During this study, data analysis was conducted to calculate shear stress by using the following methods: 1) Preston tube method, 2) linear regression of the flow velocity profile, 3) Rozovskii method, and 4) Reynolds shear stress extrapolation. Generally, shear stress from the Preston tube method showed higher values than any other shear stress from different methods. According to the results of the data analysis, shear stress calculation from the Preston tube method and Reynolds shear stress extrapolation showed the best agreement because both methods showed proportionality of τ_x with the increase of flow rate and showed similar distribution of the value of τ_x . Specifically, both methods showed that the similar trend of the τ_x value indicates reinforced bank protection is required at the inside of the downstream bend in the channel. In addition, the agreements indicated that the flow velocity from the ADV meter is a good parameter for predicting the shear stress distribution.

4 COMPARISONS OF SHEAR STRESS CALCULATION METHODS

In Chapter 3, there were differences of shear stress calculation results due to the choice of different shear stress calculation methods. In this chapter, the differences in these shear stress calculations will be described. The method of comparison is presented in Figure 4.1.

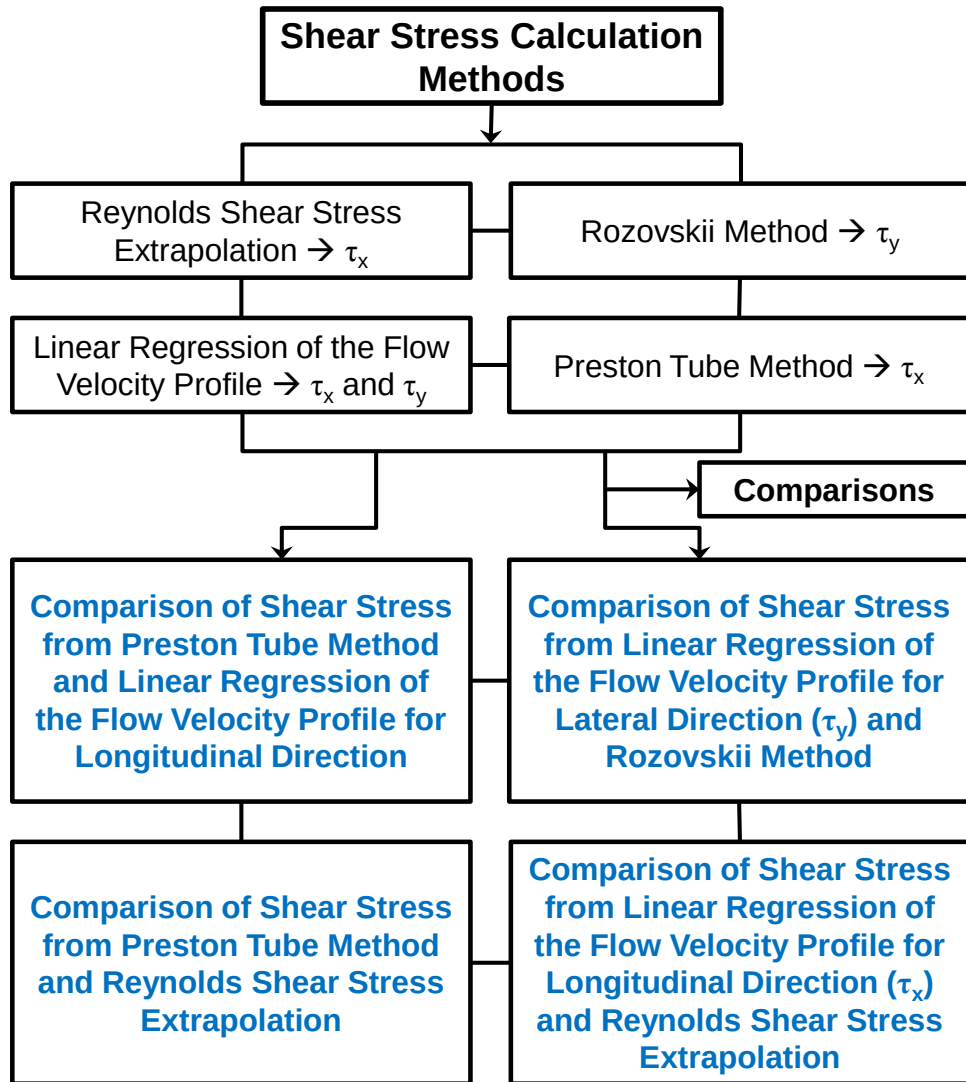


Figure 4.1: Shear Stress Calculation Methods Comparison Schematic

4.1 Comparison of Shear Stress from the Preston Tube Method and Linear Regression of the Velocity Profile for the Longitudinal Direction

Plots to observe the difference of the two methods are presented in Figure 4.2 through Figure 4.5. The x-axis is shear stress from linear regression of the flow velocity profile and the y-axis is shear stress from the Preston tube, respectively. If the points perfectly fit the $y=x$ line, then the two methods have perfect agreement. Figure 4.2 through Figure 4.5 indicate that most of the points are concentrated in the vicinity of the y-axis, which means shear stress calculated from the Preston tube is much greater. The green line in the plots indicates the best-fit line from different shear stress calculation methods. The most persuasive reason for the difference of the two methods is the difference of measurement method. The Preston tube was directly installed on the bed; however, the ADV could not be located on the bed because the ADV cannot recognize the flow velocity on the bed. Therefore, shear stress from a linear regression of the flow velocity profile underestimates bed shear stress. Additionally, the averaged percentage difference of the two methods with change of discharge is presented in Figure 4.6.

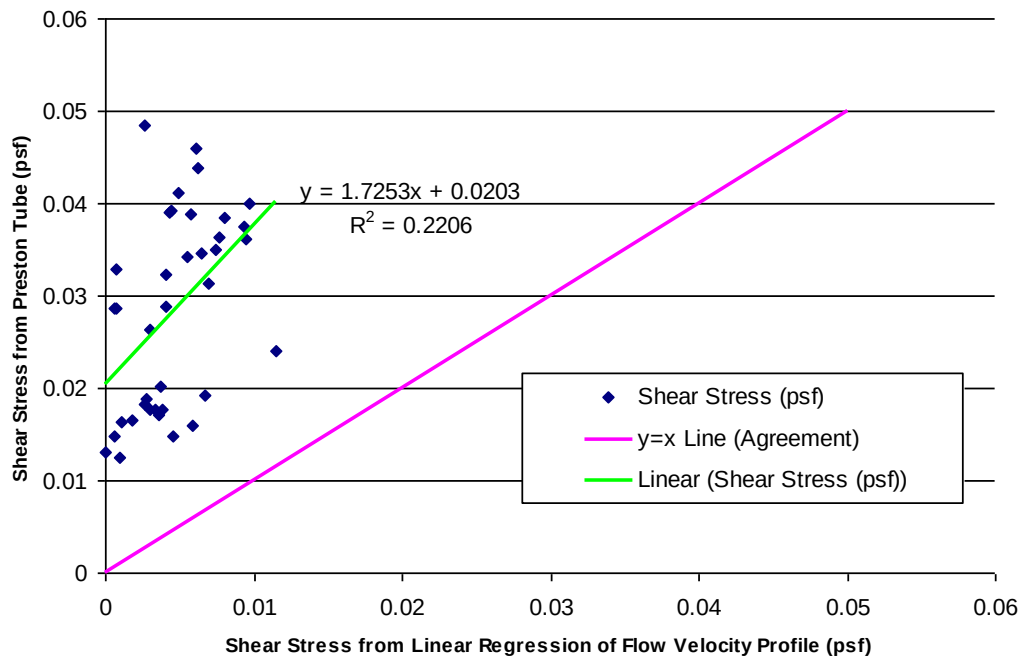


Figure 4.2: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 8 cfs

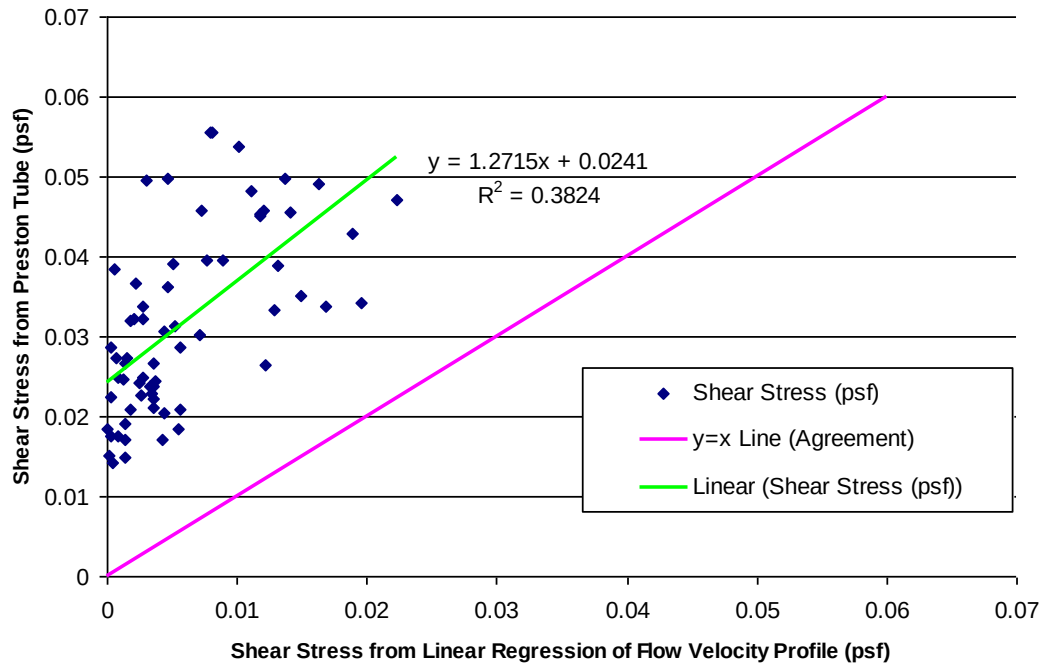


Figure 4.3: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 12 cfs

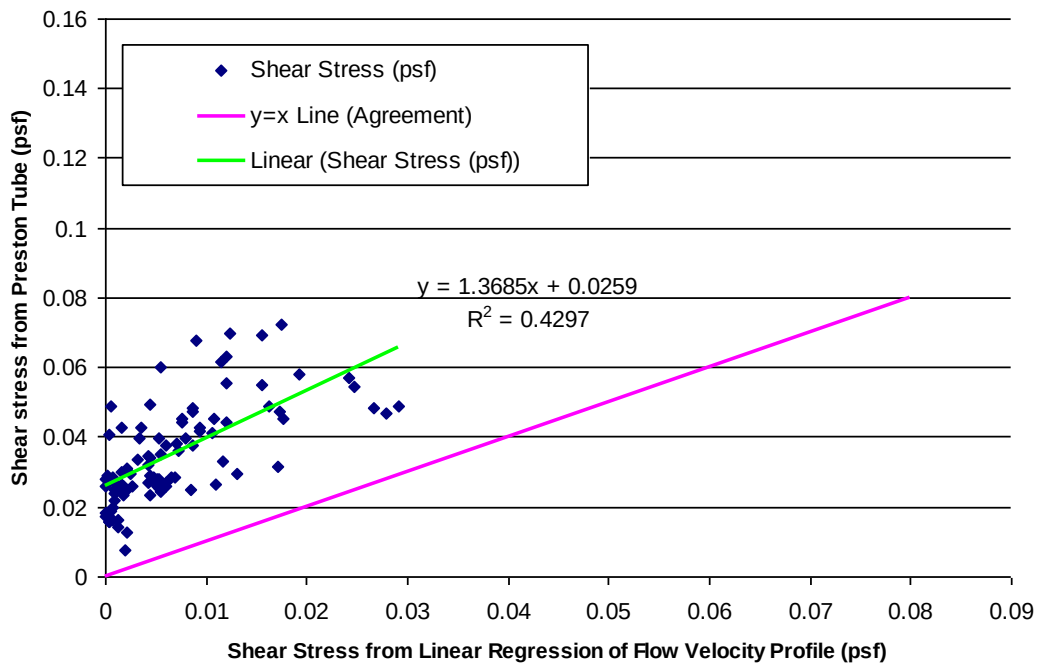


Figure 4.4: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 16 cfs

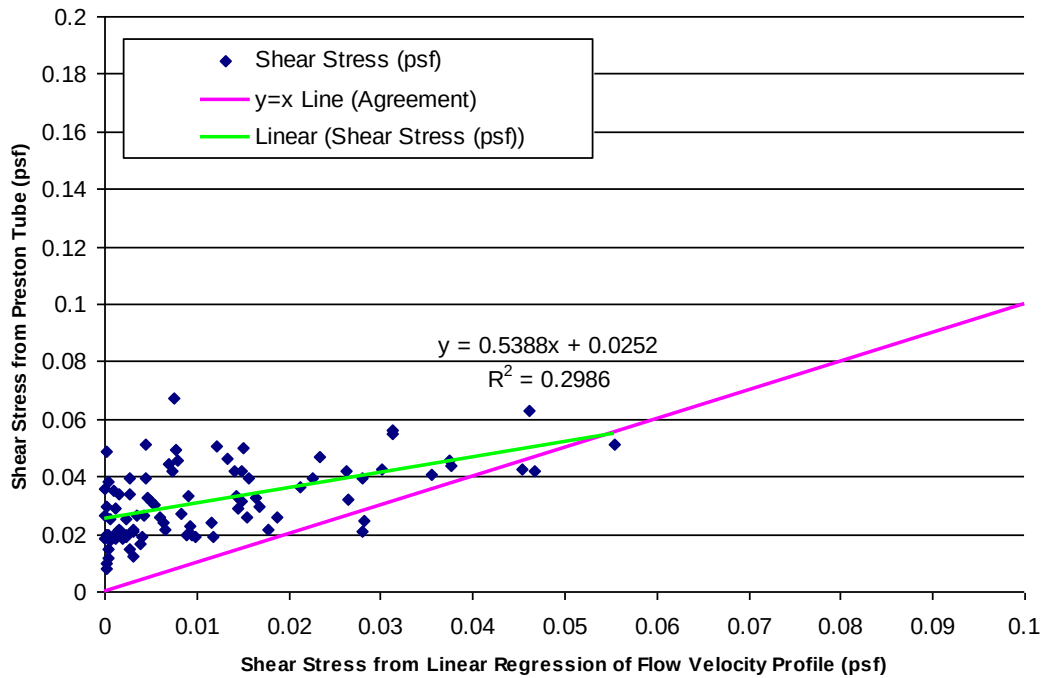


Figure 4.5: Comparison of Shear Stress from the Preston Tube and Linear Regression of the Flow Velocity Profile for 20 cfs

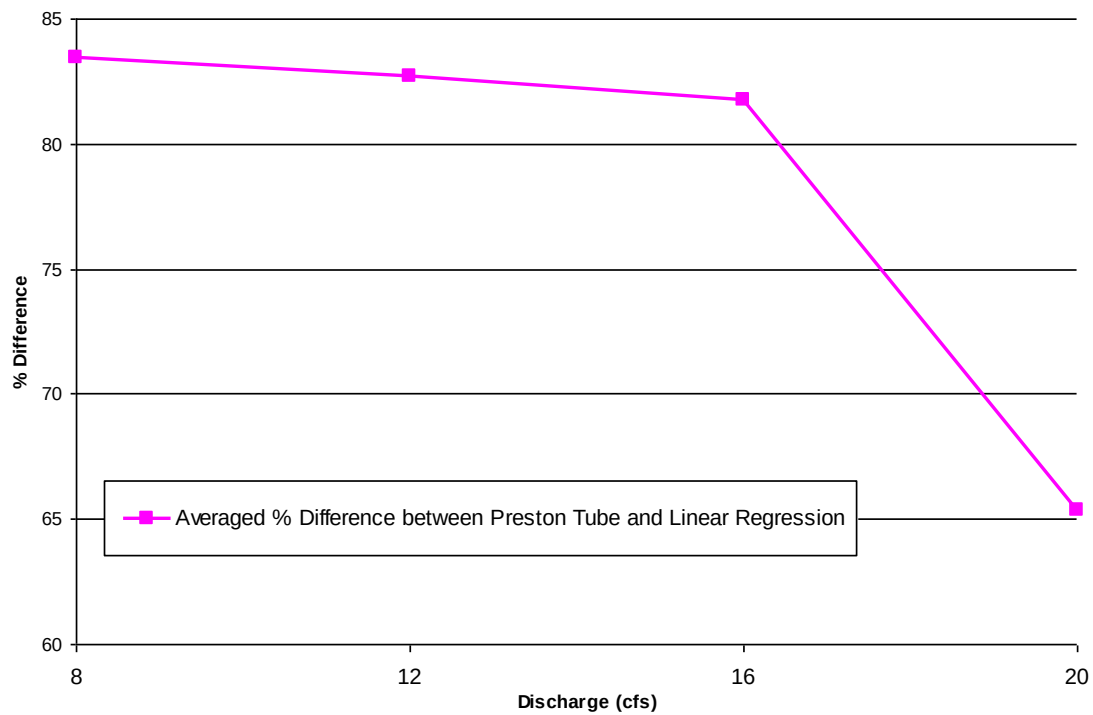


Figure 4.6: Averaged Percentage Difference between the Preston Tube and Linear Regression with Change of Discharge

The percentage difference value of shear stress from the Preston tube and linear regression of the flow velocity profile was computed by using Equation 4.1:

$$\delta = \left(\frac{\alpha - \beta}{\alpha} \right) \times 100 \quad \text{Equation 4.1}$$

where

δ = shear stress percentage difference between the Preston tube and linear regression of the flow velocity profile (%);

α = value of shear stress by the Preston tube [$\text{ML}^{-1}\text{T}^{-2}$]; and

β = shear stress from linear regression of the flow velocity profile [$\text{ML}^{-1}\text{T}^{-2}$].

A summary table of the maximum and minimum of the percentage difference of these two methods is presented in Table 4.1.

Table 4.1: Calculation Results of Percentage Difference between the Preston Tube and Linear Regression of the Flow Velocity Profile

Flow Rate (cfs)	Maximum Value (%)	Minimum Value (%)	Averaged Difference (%)
8	99.8	52.0	83.45
12	99.8	43.0	82.69
16	100	40.4	81.75
20	100	-6.7	67.42

4.2 Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile for the Lateral Direction (τ_y) and Rozovskii Method

For a detailed analysis of the differences between the two methods, plots are presented in Appendix F to show the difference of the two methods. The x-axis is shear stress from the Rozovskii method and the y-axis is shear stress from linear regression of the flow velocity profile for the lateral direction (V_y). The plots indicate the points are mostly distributed horizontally because of the negative shear stress from the Rozovskii method. The negative shear stress from the Rozovskii method resulted from a lower water surface slope. As presented in Equation 2.22, the low water surface slope caused the right-hand side bracket of Equation 2.22 to be greater. As a result, negative shear stress was observed from the Rozovskii method. The percentage difference between the shear stress from linear regression of the flow velocity profile for the lateral direction and the Rozovskii method was calculated by using Equation 4.2:

$$\delta_1 = \left(\frac{\alpha_1 - \beta_1}{\alpha_1} \right) \times 100 \quad \text{Equation 4.2}$$

where

δ_1 = percentage difference between the Preston tube and Rozovskii methods (%);

α_1 = value of shear stress by linear regression of the flow velocity profile for the lateral direction (V_y) [$\text{ML}^{-1}\text{T}^{-2}$]; and

β_1 = value of shear stress from the Rozovskii method [$\text{ML}^{-1}\text{T}^{-2}$].

The summary of the comparison of these two methods is shown in Table 4.2.

Table 4.2: Calculation Results of Percentage Difference between Linear Regression of the Flow Velocity Profile and Rozovskii Method

Flow Rate (cfs)	Maximum Value (%)	Minimum Value (%)	Averaged Difference (%)
8	-123.17	-84.64	-99.20
12	434.76	-19.95	-92.10
16	-757.42	-45.01	-104.78
20	-205.69	-2.83	-99.65

4.3 Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation

Plots to compare the difference of shear stress from the Preston tube and Reynolds shear stress extrapolation are presented in Figure 4.7 through Figure 4.10. In addition, the averaged percentage difference of the two methods with change of discharge is presented in Figure 4.11. The x-axis is shear stress from Reynolds shear stress extrapolation obtained from Equation 4.3 and the y-axis is shear stress from the Preston tube. The green line in the plots indicates the best-fit line from different shear stress calculation methods. The plots indicate that the points are generally located between the y-axis and the $y=x$ curve. This means the shear stress from the Preston tube is higher than shear stress from Reynolds shear stress extrapolation because the Preston tube directly touches on the bed surface of the flume; however, Reynolds shear stress from the ADV does not touch the bed surface during data collection.

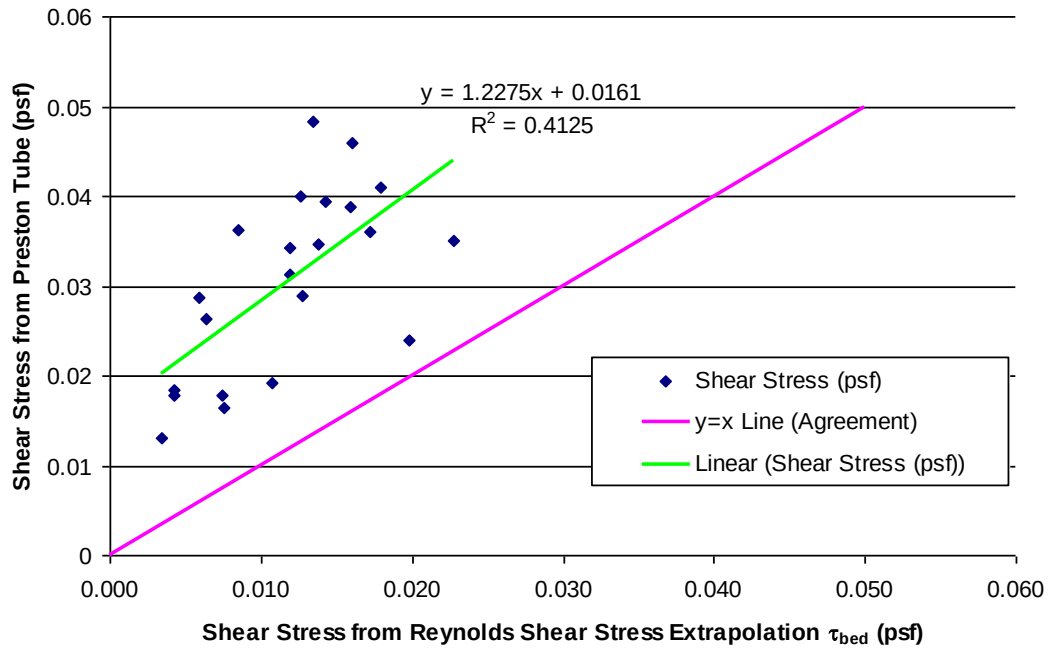


Figure 4.7: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 8 cfs

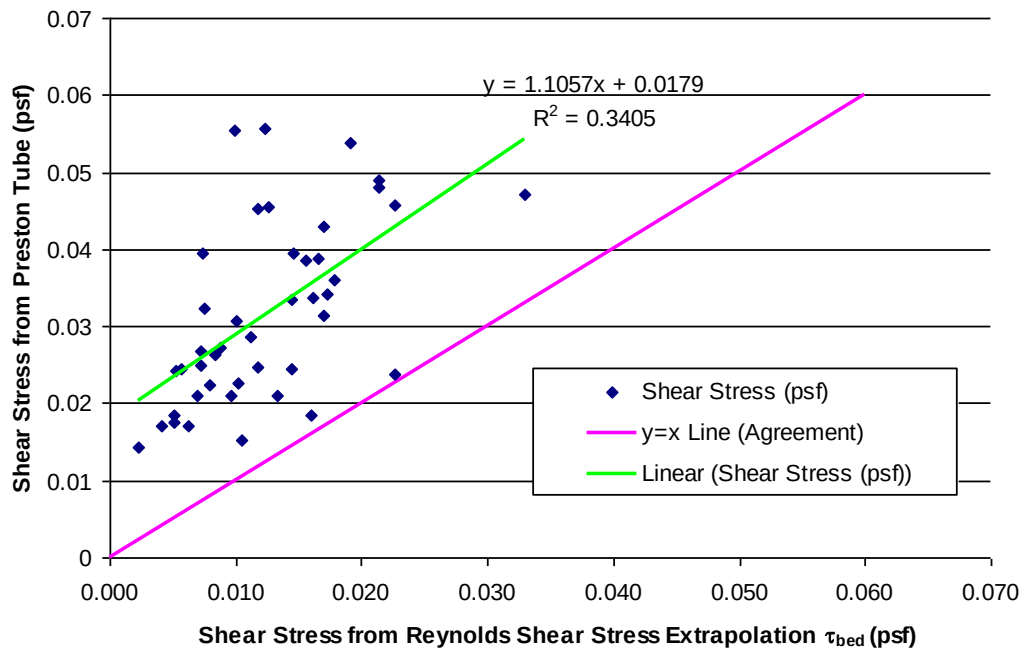


Figure 4.8: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 12 cfs

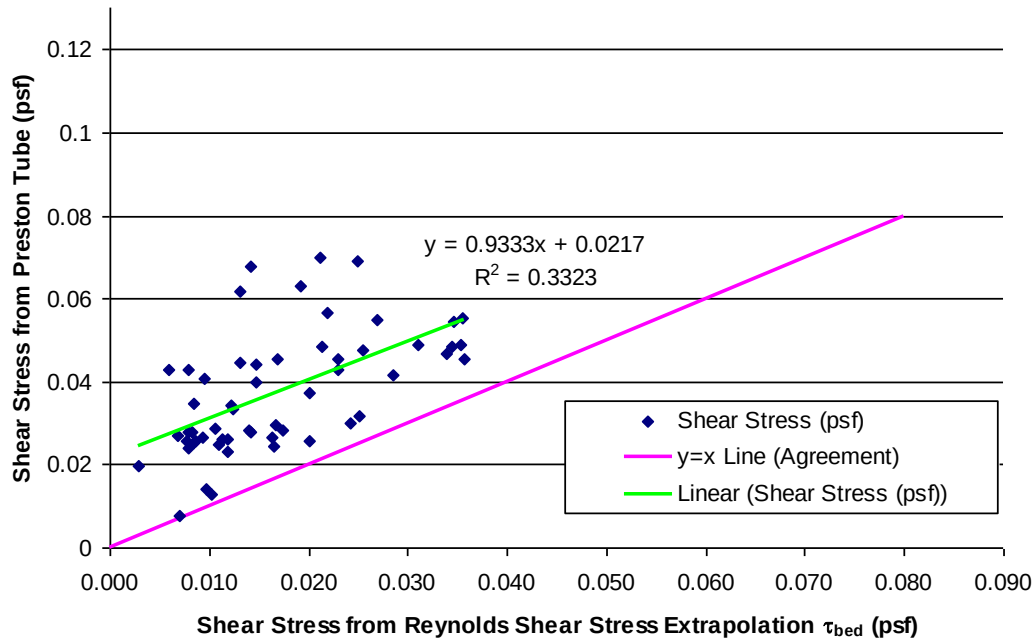


Figure 4.9: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 16 cfs

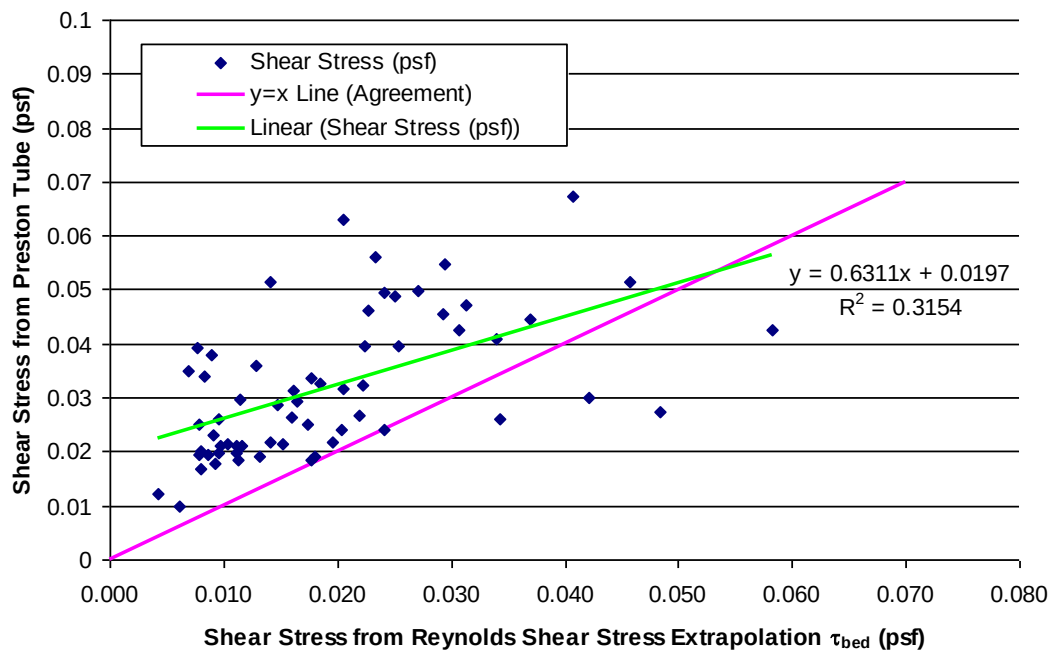


Figure 4.10: Comparison of Shear Stress from the Preston Tube and Reynolds Shear Stress Extrapolation for 20 cfs

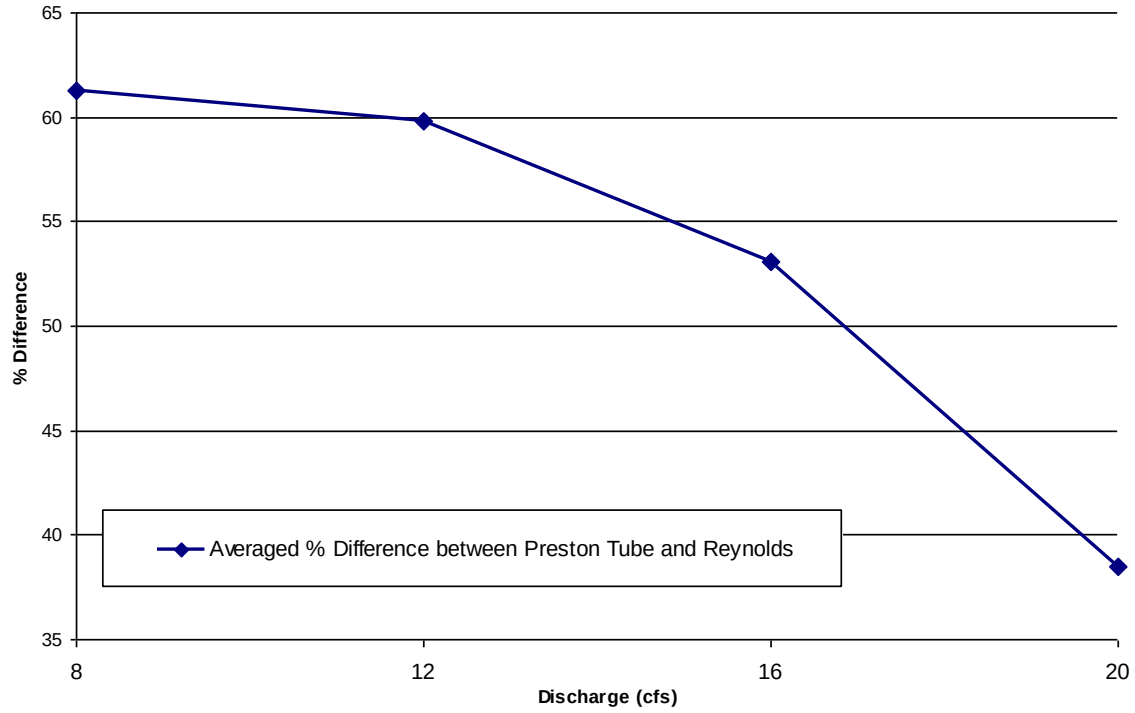


Figure 4.11: Averaged Percentage Difference between the Preston Tube and Reynolds Shear Stress Extrapolation with Change of Discharge

The percentage difference of shear stress from the Preston tube and Reynolds shear stress extrapolation method was obtained by using the Equation 4.3:

$$\delta_2 = \left(\frac{\alpha_2 - \beta_2}{\alpha_2} \right) \times 100 \quad \text{Equation 4.3}$$

where

δ_2 = shear stress percentage difference between the Preston tube and Reynolds shear stress extrapolation (%);

α_2 = value of shear stress calculated from the Preston tube [$\text{ML}^{-1}\text{T}^{-2}$]; and

β_2 = shear stress from Reynolds shear stress extrapolation [$\text{ML}^{-1}\text{T}^{-2}$].

Table 4.3 presents the difference of the calculation results.

Table 4.3: Calculation Results of Percentage Difference between the Preston Tube and Reynolds Shear Stress Extrapolation

Flow Rate (cfs)	Maximum Value (%)	Minimum Value (%)	Averaged Difference (%)
8	79.37	17.64	61.28
12	83.74	5.20	59.79
16	86.24	6.89	53.04
20	80.50	0.66	38.49

4.4 Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile for the Longitudinal Direction (τ_x) and Reynolds Shear Stress Extrapolation

Plots comparing differences of shear stress using a linear regression of the flow velocity profile along the x -axis and the Reynolds shear stress extrapolation are presented in Figure 4.12 through Figure 4.15. The x -axis is shear stress from linear regression of the flow velocity profile for the longitudinal direction and the y -axis is shear stress from Reynolds shear stress extrapolation. The plots indicate shear stress from Reynolds shear stress extrapolation is greater than shear stress from linear regression because Reynolds shear stress extrapolation was calculated by obtaining the x -axis intercept point of the linear portion of relative depth and shear stress, but shear stress from linear regression of the flow velocity profile for the longitudinal direction was obtained from the slope of the linear regression equation. The green line in the plots indicates the best-fit line from different shear stress calculation methods. Additionally, the averaged percentage difference of the two methods with change of discharge is presented in Figure 4.16.

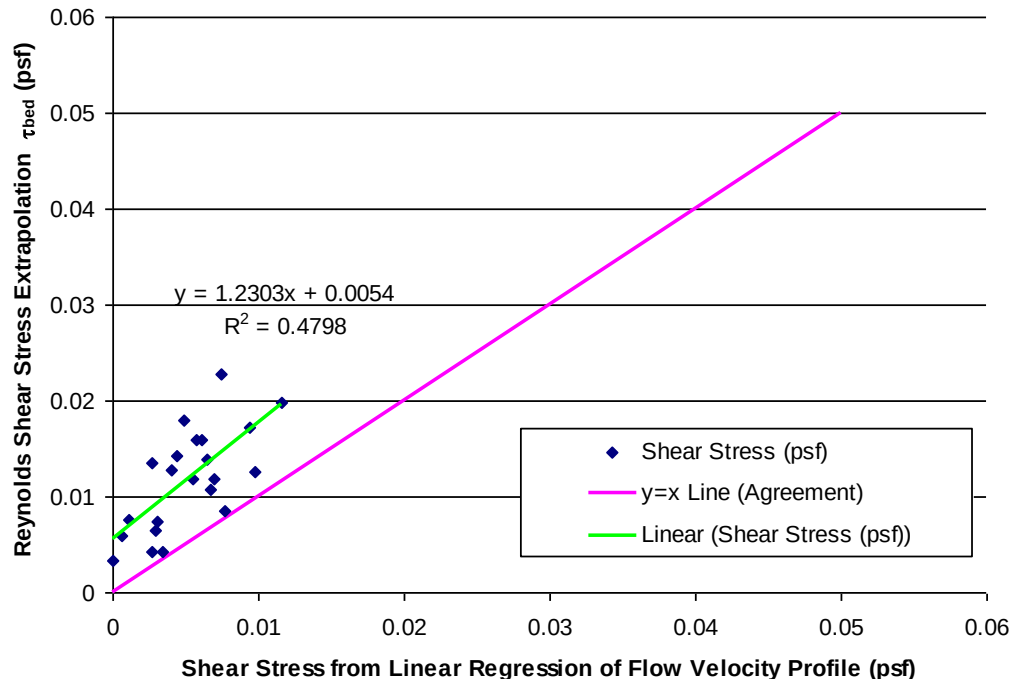


Figure 4.12: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 8 cfs

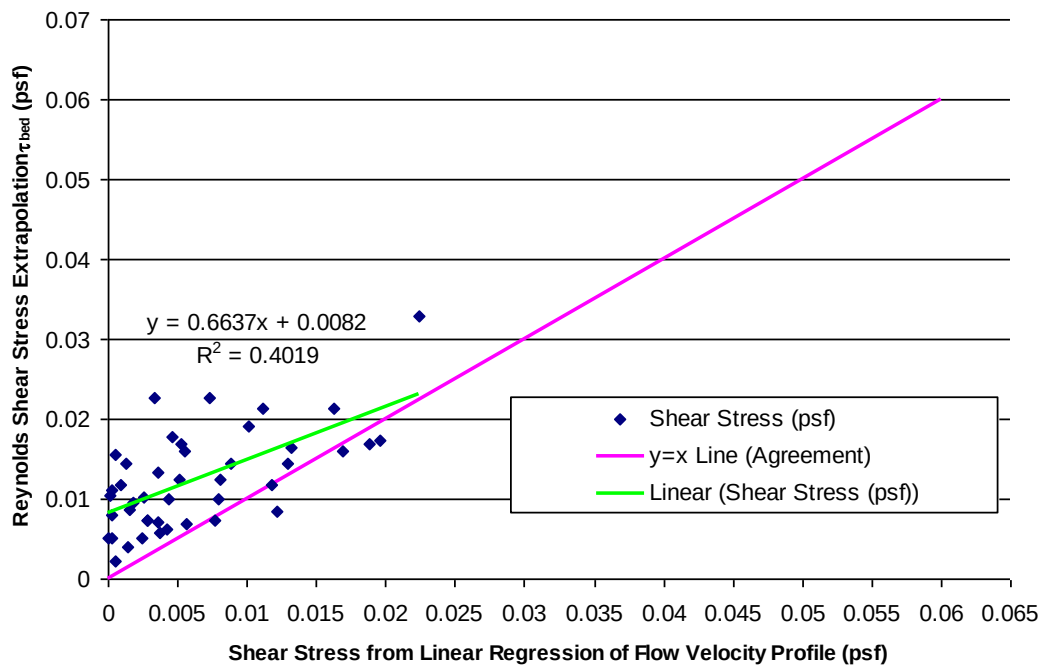


Figure 4.13: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 12 cfs

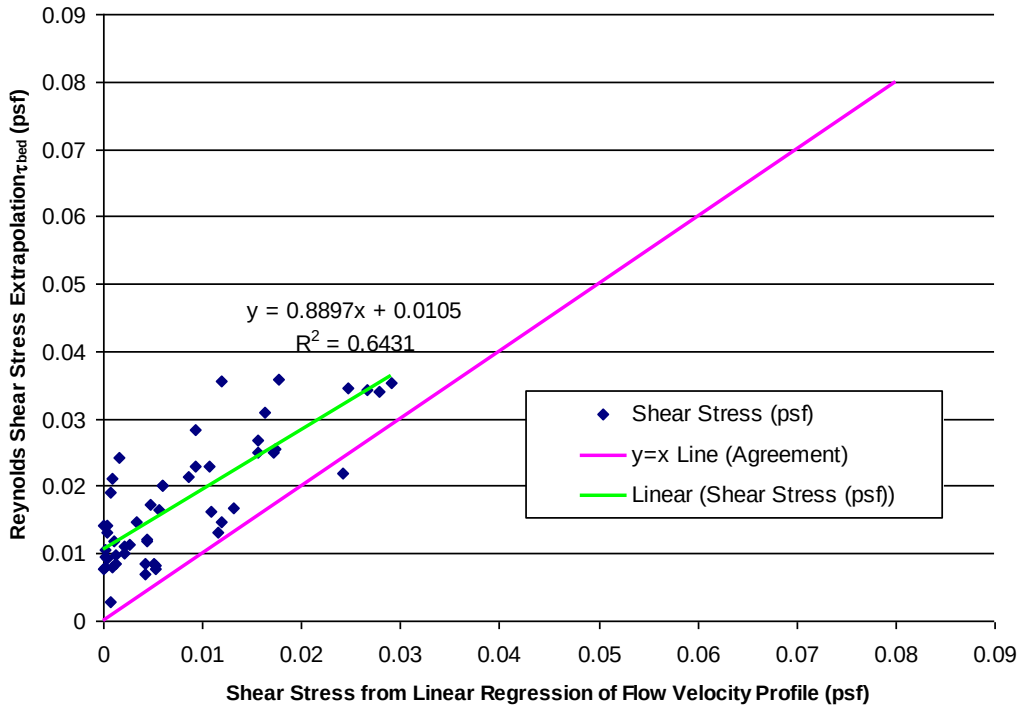


Figure 4.14: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 16 cfs

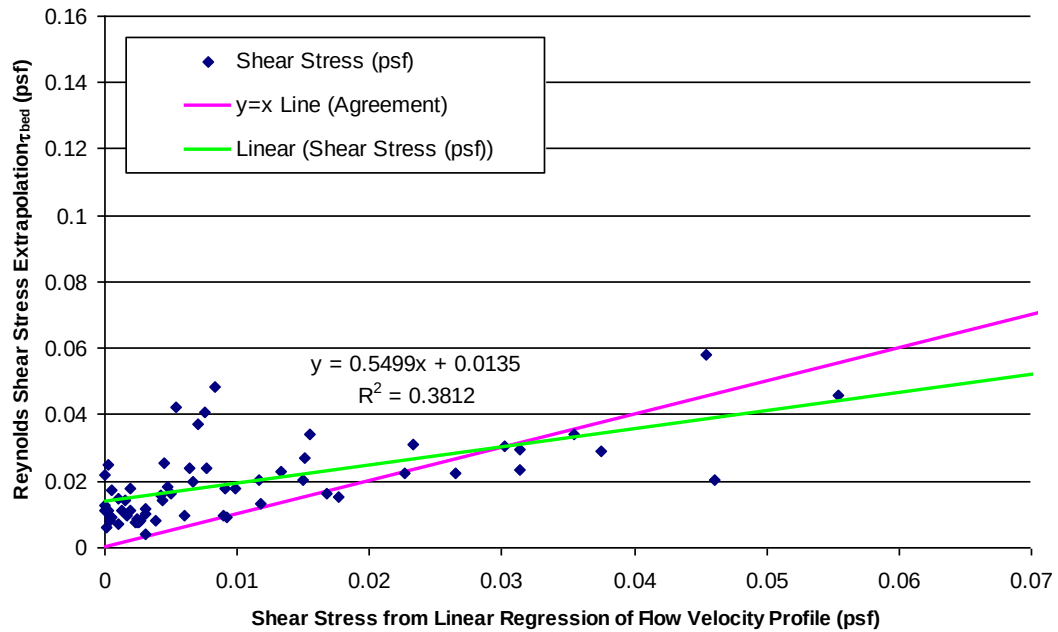


Figure 4.15: Comparison of Shear Stress from Linear Regression of the Flow Velocity Profile and Reynolds Shear Stress Extrapolation for 20 cfs

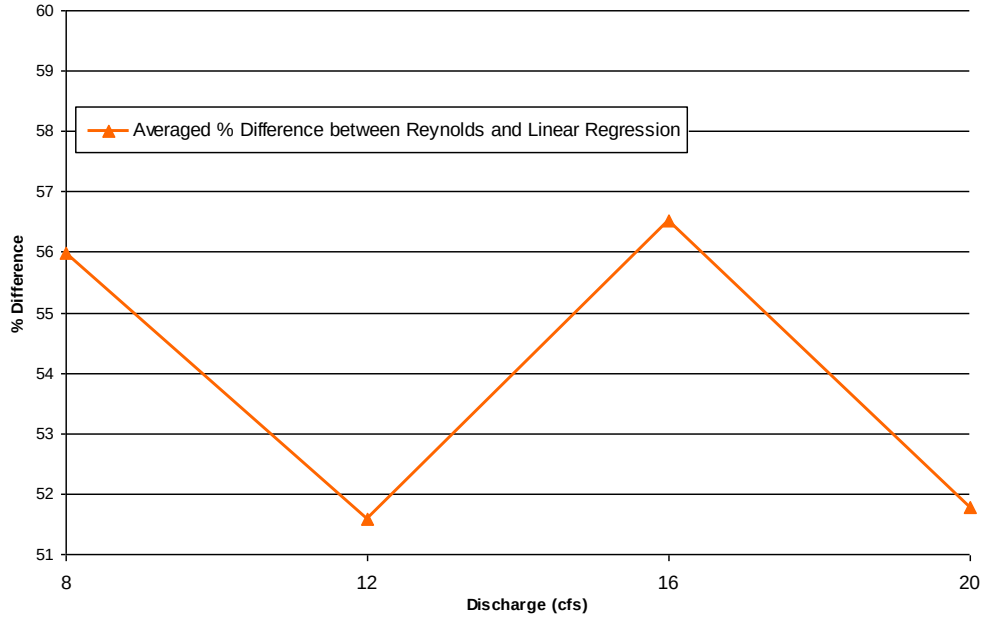


Figure 4.16: Averaged Percentage Difference between Reynolds Shear Stress Extrapolation and Linear Regression of the Flow Velocity Profile with Change of Discharge

Another way to compare shear stress calculation methods is comparing shear stress from linear regression of the x -direction flow velocity and shear stress by extrapolating Reynolds shear stress that applies in the bed (τ_{bed}). The percentage difference of shear stress calculated from linear regression of the flow velocity profile for the longitudinal direction and Reynolds shear stress extrapolation was obtained using Equation 4.4:

$$\delta_3 = \left(\frac{\alpha_3 - \beta_3}{\alpha_3} \right) \times 100 \quad \text{Equation 4.4}$$

where

δ_3 = shear stress percentage difference between linear regression of the flow velocity profile for the longitudinal direction and Reynolds shear stress extrapolation (%);

α_3 = value of shear stress by Reynolds shear stress extrapolation [$\text{ML}^{-1}\text{T}^{-2}$]; and

β_3 = shear stress from linear regression [$\text{ML}^{-1}\text{T}^{-2}$].

Table 4.4 presents the percentage difference of the calculation results.

Table 4.4: Calculation Results of Percentage Difference between Linear Regression of the Flow Velocity Profile for the Longitudinal Direction and Reynolds Shear Stress Extrapolation

Flow Rate (cfs)	Maximum Value (%)	Minimum Value (%)	Averaged Difference (%)
8	99.25	9.24	55.99
12	99.23	0.14	51.59
16	99.93	-10.07	56.53
20	-125.32	-1.1	51.77

4.5 Comprehensive Comparisons

Section 4.1 through Section 4.4 described the differences of shear stress calculation methods from: 1) Preston tube method, 2) linear regression of the flow velocity profile, 3) Rozovskii method, and 4) Reynolds shear stress extrapolation. This section reports the shear stress calculated from the Preston tube method is the most accurate method because the Preston tube directly touched the channel bed during data collection. In the cases of Reynolds shear stress extrapolation and linear regression of the flow velocity profile, these methods are based on ADV data, where the ADV does not directly touch the channel bed during measurements.

Bar charts are presented in Figure 4.17 through Figure 4.20 that show the percentage difference between maximum shear stress from the Preston tube and other maximum shear stress values by using Reynolds shear stress extrapolation, linear regression of the flow velocity profile, and Hydrologic Engineering Center – River Analysis System (HEC-RAS). Due to the low accuracy, shear stress from the Rozovskii method was excluded in the bar charts. The plots indicate that Reynolds shear stress extrapolation has relatively better agreement with shear stress from the Preston tube than a linear regression of the flow velocity profile.

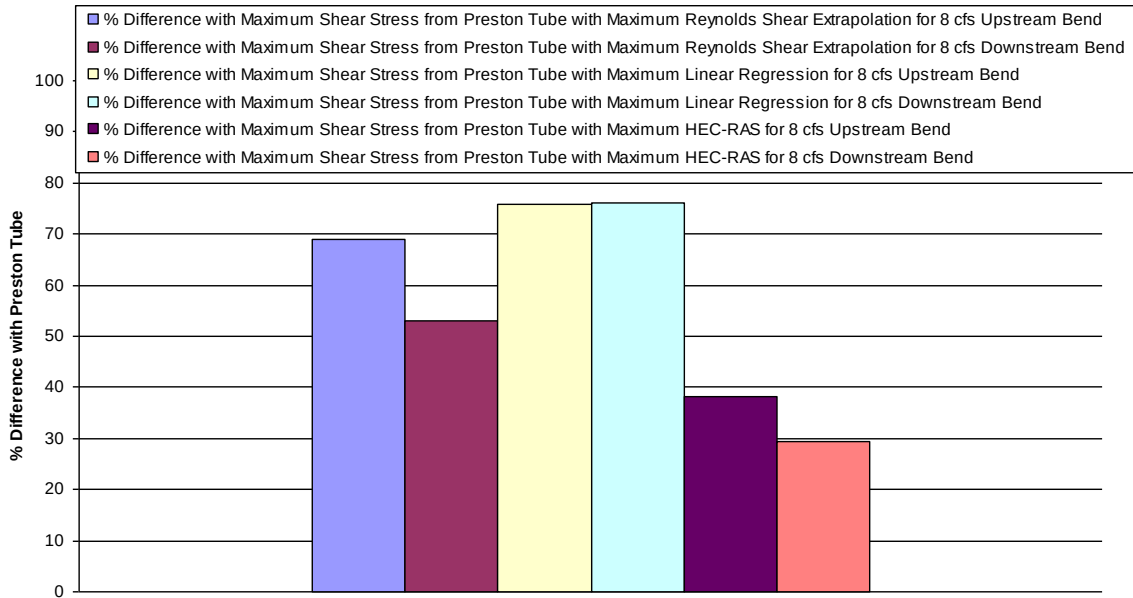


Figure 4.17: Percentage Difference of Maximum Shear Stress with the Preston Tube for 8 cfs

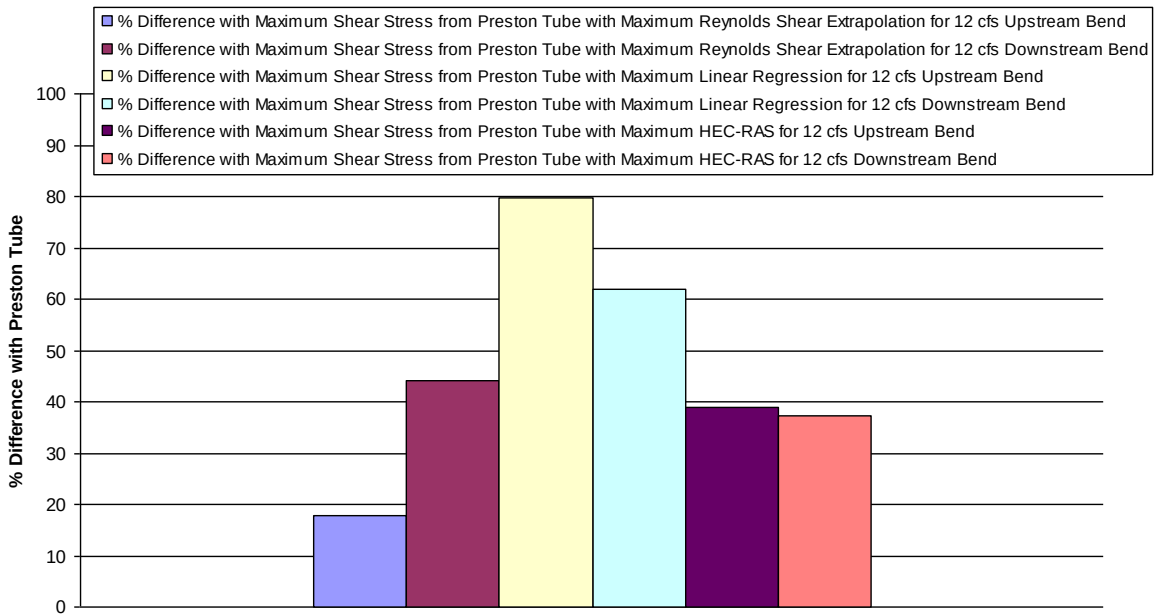


Figure 4.18: Percentage Difference of Maximum Shear Stress with the Preston Tube for 12 cfs

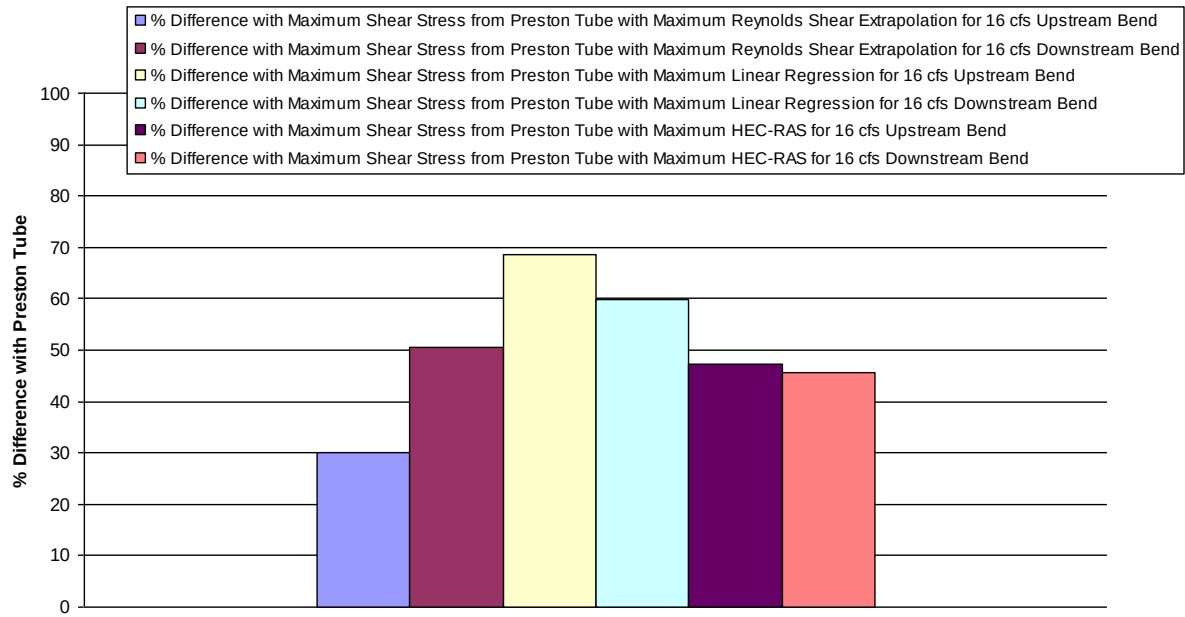


Figure 4.19: Percentage Difference of Maximum Shear Stress with the Preston Tube for 16 cfs

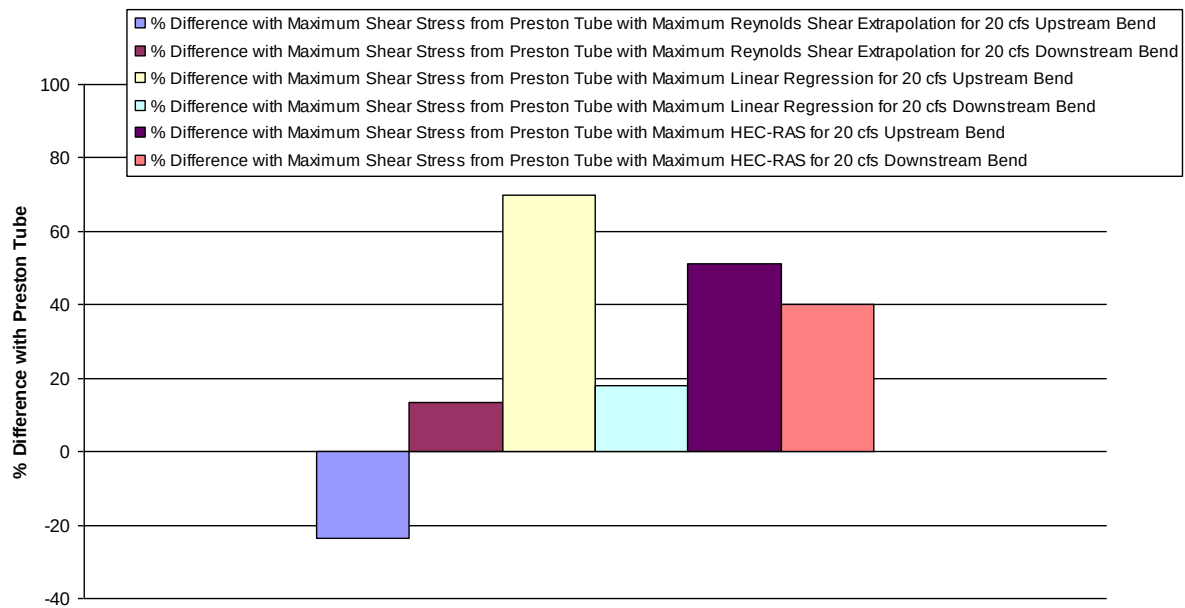


Figure 4.20: Percentage Difference of Maximum Shear Stress with the Preston Tube for 20 cfs

4.6 Averaged Shear Stress from HEC-RAS and K_b Value

Heintz (2002) explained that an adjustment factor (K_b) should be developed due to the different geometry conditions between the upstream and downstream bends. The definition of K_b is the ratio of maximum shear stress to averaged shear stress. In this study, averaged shear stress was calculated by taking the average HEC-RAS shear stress output of Cross section 1 and Cross section 2 for the upstream bend and taking the average output of Cross section 10 and Cross section 11 for the downstream bend to calculate cross-section averaged shear stress. HEC-RAS modeling output for shear stress is presented in Table 4.5 and Table 4.6. Two maximum shear stresses were selected for analysis: 1) from Reynolds shear stress extrapolation and 2) from the Preston tube method. The K_b value, computed by dividing maximum shear stress by cross-section averaged shear stress from HEC-RAS, is presented and summarized in Table 4.7 and Table 4.8. Plots comparing shear stress maximum values for upstream and downstream bends at 8, 12, 16, and 20 cfs are reported in Appendix G.

Table 4.5: HEC-RAS Output for Shear Stress for Upstream Bend (psf)

Cross Section	8 cfs	12 cfs	16 cfs	20 cfs
1	0.014551	0.016432	0.018744	0.02055
2	0.012438	0.014542	0.016896	0.018763
3	0.014991	0.016844	0.019166	0.020974
4	0.014289	0.016125	0.018418	0.020209
5	0.013597	0.01551	0.01782	0.019631
6	0.014445	0.016236	0.018536	0.020332
7	0.012989	0.015048	0.017457	0.019347
8	0.013057	0.015025	0.017375	0.019229

Table 4.6: HEC-RAS Output for Shear Stress for Downstream Bend (psf)

Cross Section	8 cfs	12 cfs	16 cfs	20 cfs
10	0.025227	0.029593	0.034569	0.038344
11	0.028996	0.032956	0.038042	0.041828
12	0.034162	0.036949	0.041766	0.045203
13	0.025574	0.029527	0.034338	0.037916
14	0.025627	0.029753	0.034675	0.038318
15	0.02509	0.029061	0.033865	0.037415
16	0.025559	0.029464	0.034246	0.037739
17	0.028973	0.032477	0.037302	0.040723
18	0.028534	0.031707	0.036329	0.039591

Table 4.7: K_b Value from Maximum Shear Stress of the Reynolds Shear Stress Extrapolation

	Flow Rate	Averaged Shear Stress (psf)	Maximum Shear Stress (psf)	R_c/T_w	K_b Value	Location
Upstream	8 cfs	0.0134945	0.007502549	2.788572251	0.55597	Cross section 4, Piezo e
Downstream	8 cfs	0.0271115	0.022733373	6.913463558	0.83851	Cross section 16, Piezo d
Upstream	12 cfs	0.015487	0.022623014	2.617889474	1.46077	Cross section 4, Piezo b
Downstream	12 cfs	0.0312745	0.032936826	6.202185792	1.05315	Cross section 16, Piezo d
Upstream	16 cfs	0.01782	0.024180063	2.476354806	1.35691	Cross section 4, Piezo a
Downstream	16 cfs	0.0363055	0.03574071	5.720368439	0.98444	Cross section 17, Piezo d
Upstream	20 cfs	0.0196565	0.048435716	2.41252646	2.46411	Cross section 6, Piezo b
Downstream	20 cfs	0.040086	0.058249589	5.442294974	1.45312	Cross section 15, Piezo d

Table 4.8: K_b Value from Maximum Shear Stress of the Preston Tube Method

	Flow Rate	Averaged Shear Stress (psf)	Maximum Shear Stress (psf)	R_c/T_w	K_b Value	Location
Upstream	8 cfs	0.0134945	0.024213	2.819001891	1.794286	Cross section 8, Piezo c
Downstream	8 cfs	0.0271115	0.048366069	6.814699793	1.78397	Cross section 15, Piezo d
Upstream	12 cfs	0.015487	0.027531493	2.620013523	1.77772	Cross section 5, Piezo a
Downstream	12 cfs	0.0312745	0.05886005	6.122581845	1.88205	Cross section 10, Piezo a
Upstream	16 cfs	0.01782	0.034551784	2.485886579	1.93893	Cross section 6, Piezo b
Downstream	16 cfs	0.0363055	0.072179902	5.720368439	1.98813	Cross section 17, Piezo e
Upstream	20 cfs	0.0196565	0.039212852	2.354478065	1.99491	Cross section 8, Piezo c
Downstream	20 cfs	0.040086	0.067410083	5.444995864	1.68164	Cross section 10, Piezo c

Before this study, Ippen and Drinker (1962), the U. S. Bureau of Reclamation (USBR; 1964), and Yen (1965) performed research to determine the empirical equations of K_b . After K_b values are obtained by using Reynolds shear stress extrapolation and the Preston tube method, the empirical equations and upper envelope curves of K_b were obtained by taking the regression of the power function, including the data from previous studies by Ippen and Drinker (1962), the USBR (1964), and Yen (1965). The plots are presented in Figure 4.21 and Figure 4.22.

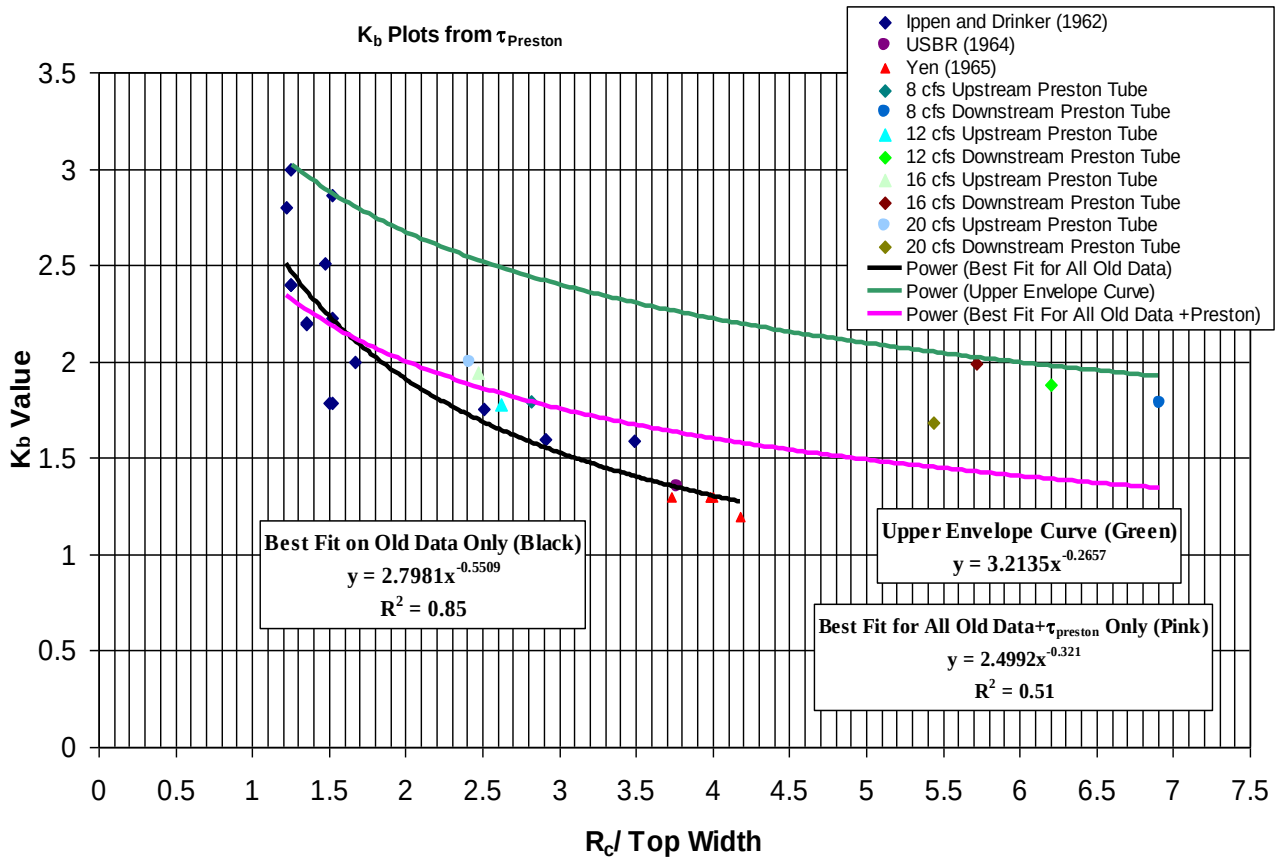


Figure 4.21: K_b Plots from the Ratio of Maximum Shear Stress from the Preston Tube to Cross-section Averaged Shear Stress from HEC-RAS

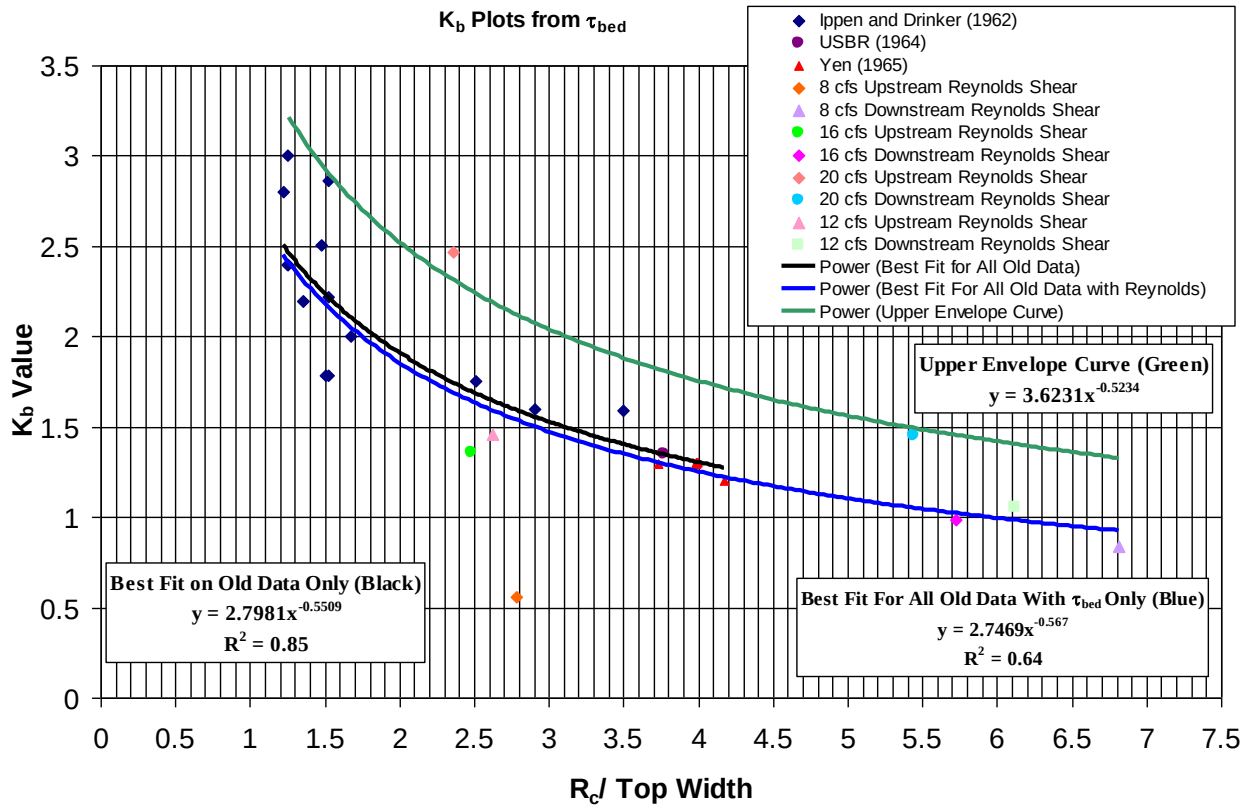


Figure 4.22: K_b Plots from the Ratio of Maximum Shear Stress from the Reynolds Shear Stress to Cross-section Averaged Shear Stress from HEC-RAS

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Overview

The physical model study provided the distribution of shear stress in a meandering channel. The previous research and laboratory experiments presented a guideline of shear stress analysis in a meandering channel. However, the previous studies on flow in bends have limitations because they focused on the distribution of shear stress in a single bend. This study presented data analysis results to determine shear stress in a meandering channel by performing laboratory tests. The flume is constructed as the combination of two types of bends: Type 1 and Type 3. The ratio of radius of curvature and top width of Type 1 and Type 3 bends in the flume was 2.02 and 4.39, respectively.

5.2 Conclusions

To determine the shear stress distribution in a meandering channel, the physical modeling study was conducted in the following phases:

- model construction and calibration;
- data collection of flow depth, flow velocity, and boundary shear stress; and
- data analysis.

After the completion of all phases of the physical modeling study, the following conclusions were drawn from this experimental study:

- The shear stress from the Rozovskii (1961) method that was presented in Equation 2.2 only shows the shear stress that applies in the lateral direction. On the other hand, the shear stresses from the Preston tube method and the Reynolds shear stress extrapolation show total shear stress. Therefore, the shear stress from Rozovskii (1961) was significantly smaller than the shear stress from the Preston tube and Reynolds shear stress extrapolation.

- The shear stress from the Preston tube is the most accurate value among those obtained by four different methods because the Preston tube collected data directly on the surface of the channel bed; however, the other methods used ADV data. In other words, the ADV has instrumentation limitations because it could not touch the bed of the channel bed.
- The shear stress from Reynolds shear stress extrapolation showed the best agreement with the shear stress from the Preston tube because Reynolds shear stress extrapolation considers the limitation of instrumentation using the ADV by extrapolating shear stress.
- This study obtained equations to estimate K_b values (the ratio of maximum shear stress to averaged shear stress) in meandering channels by using the Preston tube method and Reynolds shear stress extrapolation. The equations derived are: 1) the best-fit equation by taking regression of K_b points from previous studies (Ippen and Drinker 1962, USBR 1964, Yen 1965) and this study; and 2) an upper envelope equation that was obtained from the maximum values of K_b . The K_b values from the upper envelope curve using the Preston tube were 27% higher in average than the K_b values from the best-fit curve. Also, the K_b values from the upper envelope curve using Reynolds shear stress extrapolation was 28% higher in average than the K_b values from the best-fit curve. Therefore, it was concluded that the effect of enveloping in both methods was similar.
- In design situations, the upper envelope equation from the Preston tube is recommended for design because the upper envelope equation from the Preston tube considers the worst-case scenario. The best-fit and upper envelope equations for K_b values from the Preston tube are presented in Equation 5.1 and Equation 5.2, respectively.

$$K_b = 2.4992 \times \left(\frac{R_c}{T_w} \right)^{-0.321} \quad \text{Equation 5.1}$$

$$K_b = 3.2135 \times \left(\frac{R_c}{T_w} \right)^{-0.2657} \quad \text{Equation 5.2}$$

where

K_b = ratio of maximum shear stress and averaged shear stress from HEC-RAS modeling [dimensionless];

R_c = radius of curvature of meandering channel [L]; and

T_w = top width of channel [L].

- The upper envelope curve for the K_b value from the Preston tube showed that previous studies (Ippen and Drinker 1962, USBR 1964, Yen 1965) were underestimated by 46% when the ratio of radius of curvature and top width of channel was greater than 4. Therefore, cautious observation of the K_b value from the Preston tube is necessary for application of the upper envelope curve of K_b in real-world situations.
- Due to the limitation of the range of data points, use of the K_b chart (presented in Figure 4.21) is recommended when the ratio of the radius of curvature and top width of the channel ranged between 1 and 7.

5.3 Recommendations for Future Research

During the physical modeling study, model construction, data collection, and experimental data analysis were conducted. The shear stress distribution was calculated and plotted by selecting four different methods. For future research, several recommendations are provided:

- It is recommended to investigate shear stress distribution in the case of native topography geometry by using the same methods of this study that are applicable to hydraulic design in the real world.
- The number of data points (seven or eight) that were collected by the ADV in the velocity profile was not good enough to obtain an accurate result from a linear regression of the flow velocity profile. To increase the accuracy of the linear regression, the number of data points collected should be at least fifteen for a reasonable linear regression (about twice of the current number of data points).

5.4 Limitations of this Study

This experimental study introduced four different methods to calculate shear stress in a bend. Also, the study obtained equations to estimate how much shear stress in a bend is higher than in a straight channel, but this study is limited in the following points:

- this study was based on laboratory data from only four discharges: 8, 12, 16, and 20 cfs; and
- in the low flow-rate cases (8 and 12 cfs), data collection was limited because the ADV could not be submerged in the water due to low water surface elevation.

6 REFERENCES

- Afzalimehr, H. and Anctil, F. (2000). Accelerating shear velocity in gravel-bed channels. *Hydrological Sciences-Journal-des Sciences Hydrologiques*, 45(1):113-124.
- Bagnold, R.A. (1960). Some Aspects of the Shape of River Meanders. U. S. Geological Survey Professional Paper 282E, pp. 135-144.
- Baird, D.C. (2004). Turbulent and Suspended Sediment Transport in a Mobile, Sand Bed Channel with Riprap Side Slopes. Ph.D Dissertation, The University of New Mexico, Albuquerque, NM.
- Clauser, F. (1956). The turbulent boundary layer. *Advances in Applied Mechanics*, 4:1-51.
- Federal Highway Administration (FHWA) (2001). River Engineering for Highway Encroachments. Report No. FHWA NHI 01-004 HDS 6.
- Federal Interagency Stream Restoration Working Group (FISRWG) (1998). Stream Corridor Restoration: Principles, Processes, and Practices. By the FISRWG (fifteen Federal agencies of the U. S. government), GPO Item No. 0120-A, SuDocs No. A 57.6/2:EN 3/PT.653, ISBN-0-934213-59-3.
- Heintz, M.L. (2002). Investigation of Bendway Weir Spacing. M.S. Thesis, Colorado State University, Department of Civil Engineering, Fort Collins, CO.
- Hickin, E.J. and Nanson, G.C. (1975). The character of channel migration on the Beatton River, north-east British Columbia, Canada. *Bulletin of the Geological Society of America*, 86:487-494.
- Hickin, E.J. and Nanson, G.C. (1986). A statistical examination of bank erosion and channel migration in western Canada. *Bulletin of the Geological Society of America*, 97:497-504.
- Ippen, A.T. and Drinker, P.A. (1962). Boundary shear stresses in curved trapezoidal channels. *ASCE Journal of the Hydraulics Division*, 88(5):143-180.
- Julien, P.Y. (1998). Erosion and Sedimentation. Cambridge University Press, New York, NY.
- Julien, P.Y. (2002). River Mechanics. Cambridge University Press, New York, NY.

- Keller, E.A. (1971). Areal sorting of bed load material: the hypothesis of velocity reversal. *Bulletin of the Geological Society of America*, 82:753-756.
- Keller, E.A. (1972). Development of alluvial stream channels: a five-stage model. *Bulletin of the Geological Society of America*, 83:1531-1536.
- Kinzli, K. (2005). Effects of Bendway Weir Characteristics on Resulting Eddy and Channel Flow Conditions. M.S. Thesis, Colorado State University, Department of Civil Engineering, Fort Collins, CO.
- Knighton, D.A. (1998). Fluvial Forms and Processes. Oxford Press, (published under license from Arnold Publishing, London), New York, NY.
- Lane, E.W. (1955). Design of stable channels. *ASCE Transactions*, 120(2776):1234-1260.
- Montes, S. (1998). Hydraulics of Open Channel Flow. American Society of Civil Engineers Press, Reston, VA.
- Muste, M. and Patel, V.C. (1997). Velocity profiles for particles and liquid in open-channel flow with suspended sediment. *ASCE Journal of Hydraulic Engineering*, 123(9):742-751.
- Nikora, V. and Gohring, D. (2000). Flow turbulence over fixed and weakly mobile gravel beds. *ASCE Journal of Hydraulic Engineering*, 126(9):679-690.
- Odgaard, A.J. (1986). Meander flow model, I: development. *ASCE Journal of the Hydraulics Engineering*, 112:1117-1134.
- Preston, J.H. (1954). The determination of turbulent skin friction by means of Pitot tubes. *Journal of the Royal Aeronautical Society*, 58:109-121.
- Raupach, M.R. (1991). Saltation layers, vegetation canopies and roughness lengths. *Acta Mechanica*, 1:83-96 (from Nikora and Gohring (2000)).
- Rosgen, D. (1996). Applied River Morphology. Printed Media Companies, Minneapolis, MN.
- Rozovskii, I.L. (1961). Flow of water in bends of open channels. Paper, Academy of Sciences of the Ukrainian SSR, Israel Program for Scientific Translations; Washington, DC (available from the Office of Technical Services, U. S. Department of Commerce).
- Schlichting H. (1987). Boundary Layer Theory. 7th Edition, McGraw-Hill, New York, NY.
- Schmidt P.G. (2005). Effects of Bendway Weir Field Geometric Characteristics on Channel Flow Conditions. M.S. Thesis, Colorado State University, Department of Civil Engineering, Fort Collins, CO.

- Sclafani, P. (2008). Preston Tube Calibration. Open File Report, Colorado State University, Engineering Research Center, Fort Collins, CO.
- Sear, D.A. (1996). Sediment transport in pool-riffle sequences. *Earth Surface Processes and Landforms*, 21:241-262.
- Smart, G.M. (1999). Turbulent velocity profiles and boundary shear in gravel bed rivers. *ASCE Journal of Hydraulic Engineering*, 125(2):106-116.
- Stephens, A.V. and Haslam, J.A.G. (1938). Flight Experiments on Boundary Layer Transition in Relation to Profile Drag. Reports and Memoranda No. 1800, Aeronautical Research Council, Her Majesty's Stationary Office, August.
- U. S. Bureau of Reclamation (USBR) (1964). Progress Report I Boundary Shear Distribution around a Curve in a Laboratory Canal. Report No. Hyd-526, June.
- Wahl, T.L. (1996). User's Manual of WinADV. U. S. Bureau of Reclamation, Water Resources Research Laboratory.
- Wahl, T.L. (2000). Analyzing ADV Data Using WinADV. In: Proceedings of the 2000 Joint Conference on Water Resources Engineering and Water Resources Planning & Management, ASCE, July 30-August 2, Minneapolis, MN.
- Watson, C.C., Biedenharn, D.S., and Thorne, C.R. (2005). Stream Rehabilitation. Cottonwood Research, Fort Collins, CO.
- Yen, B.C. (1965). Characteristics of Subcritical Flow in a Meandering Channel. Ph.D. Dissertation, University of Iowa, Department of Mechanics and Hydraulics, Iowa City, IA, June.

APPENDIX A PHYSICAL MODEL DESCRIPTION

A.1 Introduction

In order to facilitate analysis of shear stress distribution in a meandering channel, physical modeling was conducted at the Colorado State University Engineering Research Center. In this appendix, the model geometry, model construction, instrumentation, methods of measurements, model calibration, and test matrix are discussed.

A.2 Model Description

The physical model is an undistorted, 1:12 Froude-scale model. The boundary of the channel is prismatic, which means the geometry of the channel is constant. The composition of the physical model is: two kinds of bends and one transitional area between two bends. A data-collection cart was mounted on top of the physical model with a point gage for the measurement of three-dimensional (3-D) flow velocity (V_x , V_y , and V_z), flow depth (h), and boundary shear stress (τ) (Heintz 2002). Figure A.1 is a photograph of the physical model in the Hydromachinery Laboratory at Colorado State University.



Figure A.1: Physical Model in the Hydromachinery Laboratory at Colorado State University

A.2.1 Model Geometry

It is impossible to construct a physical model that completely models the entire study reach because the prototype study reach length is 29 miles. Therefore, a representative section of the reach was selected to describe the flow in a meandering channel (Heintz 2002). The data of the reach geometry were obtained by using 1992 aerial photographs (Heintz 2002). The measured parameters of the study reach were: top width, radius of curvature, length of the bend, relative curvature, and the rotation angle of the bend. Table A.1 shows the geometry of the prototype study reach. Further, Table A.2 provides the geometry of the physical model. Figure A.2 describes the geometry measurement results of the study reach. To maintain the 1:12 Froude scale, Type 1 and Type 3 bends were selected for the physical model (Heintz 2002).

Table A.1: Geometry of the Prototype Study Reach (from Heintz (2002))

Type	Top Width ft (m)	Radius of Curvature ft (m)	Bend Angle (°)	Relative Curvature, r/b	Channel Length ft (m)
1	230.4 (70.2)	465 (141.73)	125	2.02	1014 (309)
3	180 (54.86)	789.96 (240.77)	73	4.39	1002 (305)

Table A.2: Geometry Information of the Physical Model (from Heintz (2002))

Type	Top Width ft (m)	Radius of Curvature ft (m)	Bend Angle (°)	Relative Curvature, r/b	Channel Length ft (m)
1	19.2 (5.9)	38.75 (11.81)	125	2.02	84.5 (25.8)
3	15 (4.6)	65.83 (20.06)	73	4.39	83.5 (25.5)

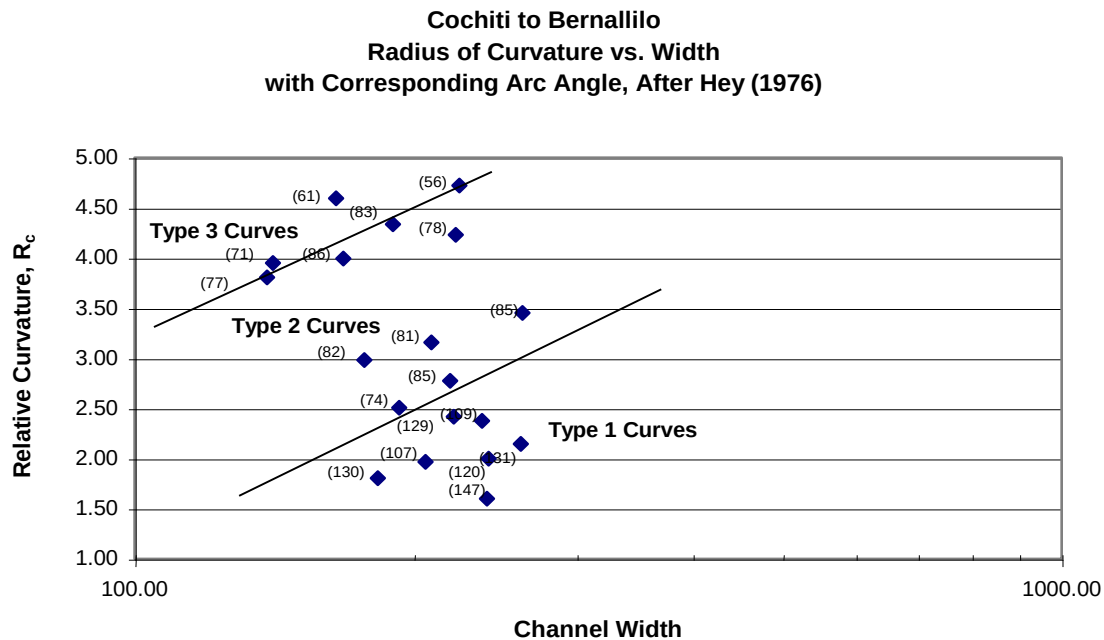


Figure A.2: Types of Bends of the Study Reach (from Heintz (2002))

The model shape is trapezoidal and the side slope of the channel is 1V:3H (Vertical: Horizontal). The channel depth of 1.5 ft (0.46 m) was selected to satisfy the data-collection flow rate (Heintz 2002). The model bed slope, 0.000863, was close enough as measured in the prototype reach. A straight transition area 10:1 for the smooth connection between upper bend and lower bend was installed (Heintz 2002). Figure A.3 is a schematic of the physical model.

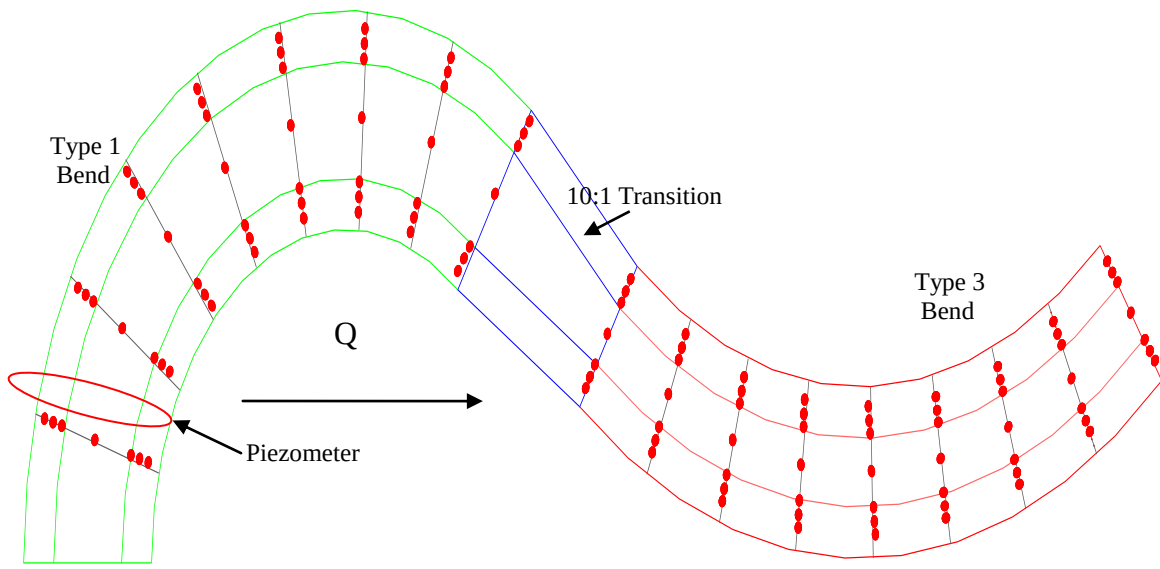


Figure A.3: Main Channel Schematic

A.2.2 Model Construction

Model construction began in early August 2000 and lasted about 8 months. During the construction phase the: headbox, main channel, and tailbox were constructed. Figure A.4 is a schematic of the Hydromachinery Laboratory. In the end of the physical model, the tailbox was constructed for energy dissipation and consistent flow conditions. The components of the headbox are: pipe manifold, a rock baffle, and side and bed transitional boundaries (Heintz 2002).

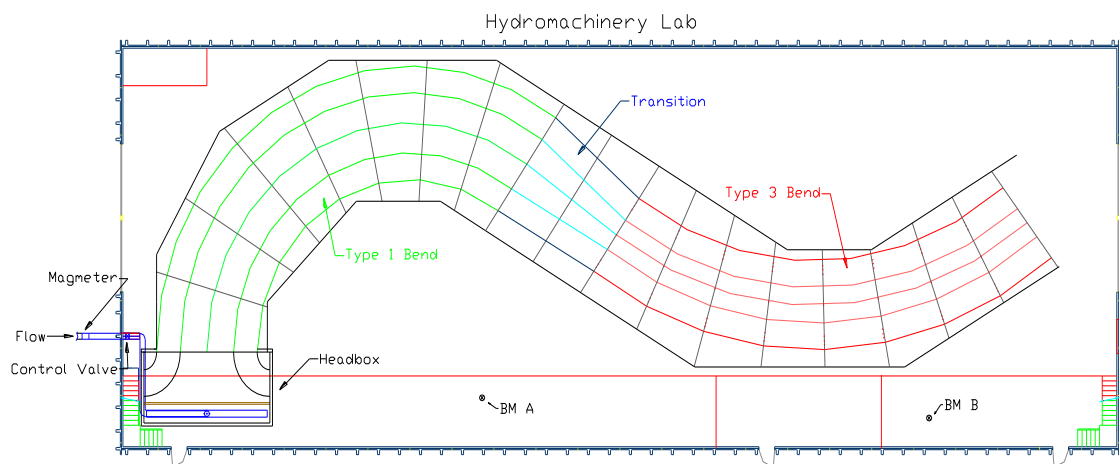


Figure A.4: Hydromachinery Laboratory Schematic

Horsetooth Reservoir (located west of the Engineering Research Center) is the source of water used for the experiment (Heintz 2002). A 12-in. pipe line is directly connected to the headbox of the model from the reservoir (Heintz 2002). For energy dissipation and flow condition maintenance, a rock baffle was installed in the headbox. The average size of the rock is 2 in. In addition, 4-ft high walls are installed on the laboratory floor as shown in Figure A.5 for reinforcement of the model (Heintz 2002). To construct rigid channel geometry, a plywood skeleton was installed in each cross section, where eight pieces of plywood were located in the upstream bend and the other eight pieces of plywood were located in the downstream bend. For the construction of the main channel, sand was filled, saturated, compacted, and leveled (Heintz 2002). Figure A.6 and Figure A.7 show the sand filling and leveling for main channel construction, respectively.



Figure A.5: Four-foot High Walls of the Model



Figure A.6: Skeleton and Filling Material of the Main Channel



Figure A.7: Leveling of Fill Sand of the Main Channel

To maintain a rigid geometry in the bend, the cross-section skeletons were connected to each other with 4-in. by 1/8th-in. steel flashing (Heintz 2002). The flashing was installed at the top bank points and channel toes in each cross section (Heintz 2002). Figure A.8 shows the installation of the steel flashing. After the completion of steel flashing and welding, a 2-in. thick concrete cap was placed over the top of the finished sand to reinforce the strength of the channel geometry. The next step of model construction was the pouring of the concrete. About 40 cubic yards of concrete and 600 tons of sand were used for the model construction. To obtain the roughness of the channel, a broom finish was applied to the concrete. Next, water sealant was sprayed on the main channel to prevent leaking. Figure A.9 and Figure A.10 show the pouring of concrete and broom finish applied for the desired channel roughness, respectively.

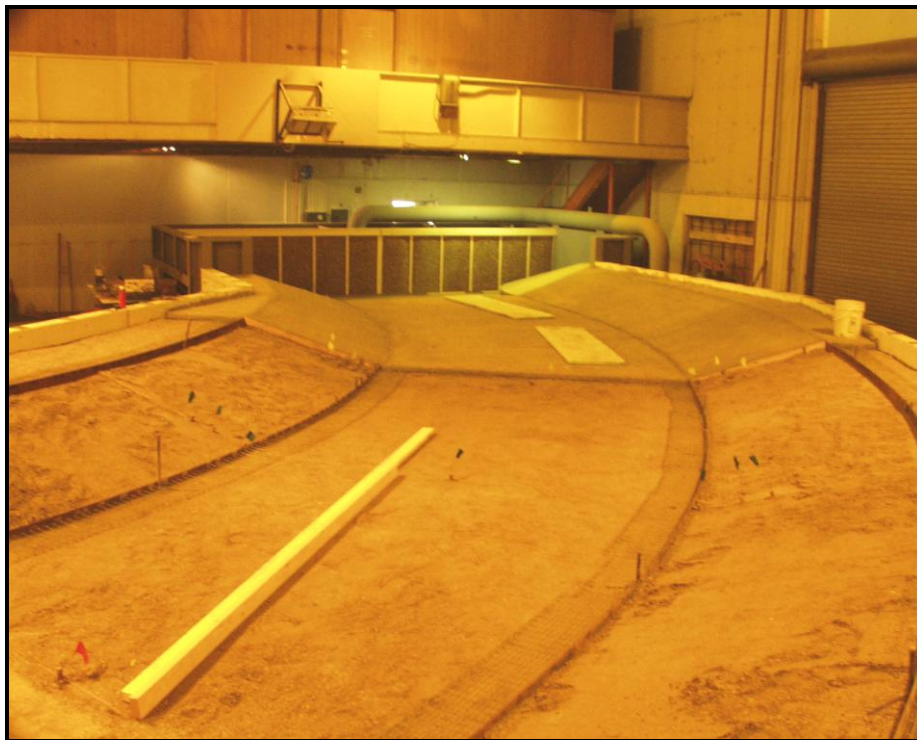


Figure A.8: Installation of Steel Flashing



Figure A.9: Pouring of Concrete for Model Construction



Figure A.10: Concrete Broom Finish for Desired Channel Roughness

In the downstream end of the model, a stop log was installed to develop backwater to generate the proper relationship between discharge and stage. Also, a 10-in. pipe and two 12-in. culverts were installed in the tailbox to convey water flow to College Lake (Heintz 2002).

A.3 Instrumentation

The physical model was constructed for the measurements of flow velocity, flow depth, flow rate, and shear stress. Data-collection instrumentation was placed on the cart and installed above the main channel. In the following sub-sections, the procedures and methods of data collection are discussed.

A.3.1 Flow Rate Measurement

For flow-rate measurements, a Georg Fischer[®] SIGNET 2550 Insertion Magmeter with an accuracy of $\pm 2\%$ was used. The pipeline that connects Horsetooth Reservoir and the model is constructed of 12-in. polyvinyl chloride (PVC) pipe. Figure A.11 are photographs of the flow meter instrumentation.



(a) SIGNET meters

(b) electronic display boxes

Figure A.11: Flow Meter Instrumentation (from Kinzli (2005))

A.3.2 Flow Depth Measurement

Two methods of flow depth measurement were used for data collection. The first method is point-gauge measurement, which is presented in Figure A.12. For this method, a point gauge was installed on the data-collection cart.

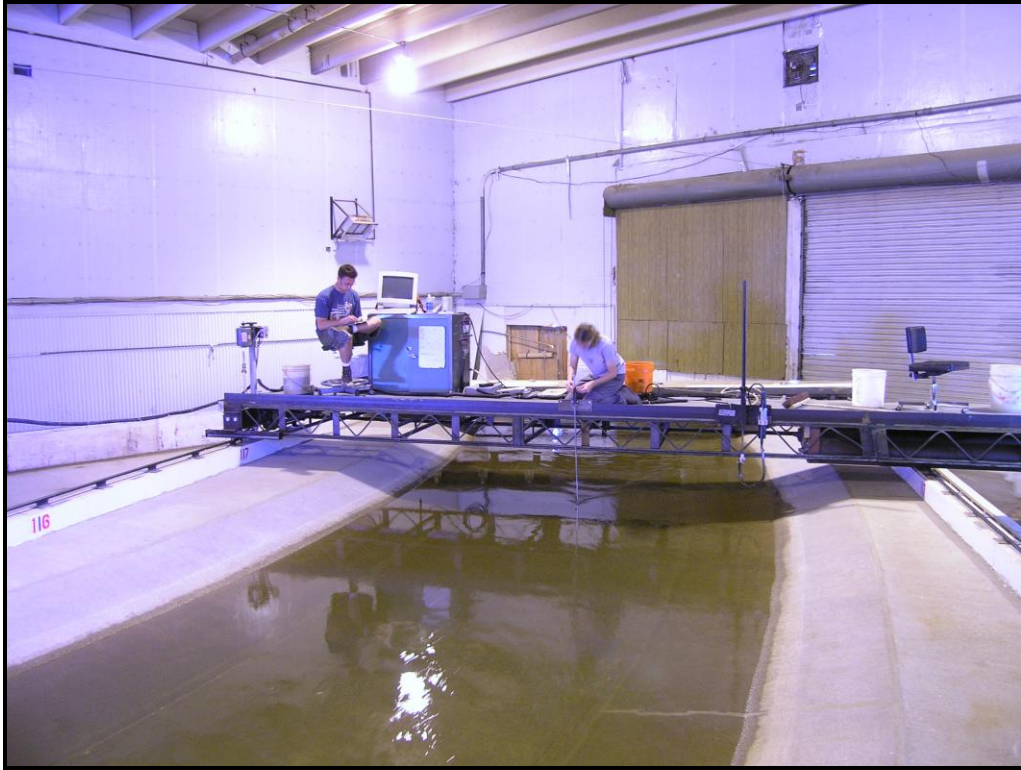


Figure A.12: Point-gauge Installation for Flow Depth Measurement

In addition to the point-gage method, piezometer taps were installed throughout the model. During the main channel construction, a piezometer was placed at the skeleton of the main channel. In each cross section, seven piezometers were installed. In total, 122 piezometers were installed for data collection. In each cross section, piezometer locations were labeled a through g, from left to right. Figure A.13 is a schematic of the piezometer locations and Figure A.14 shows the installation of a piezometer.

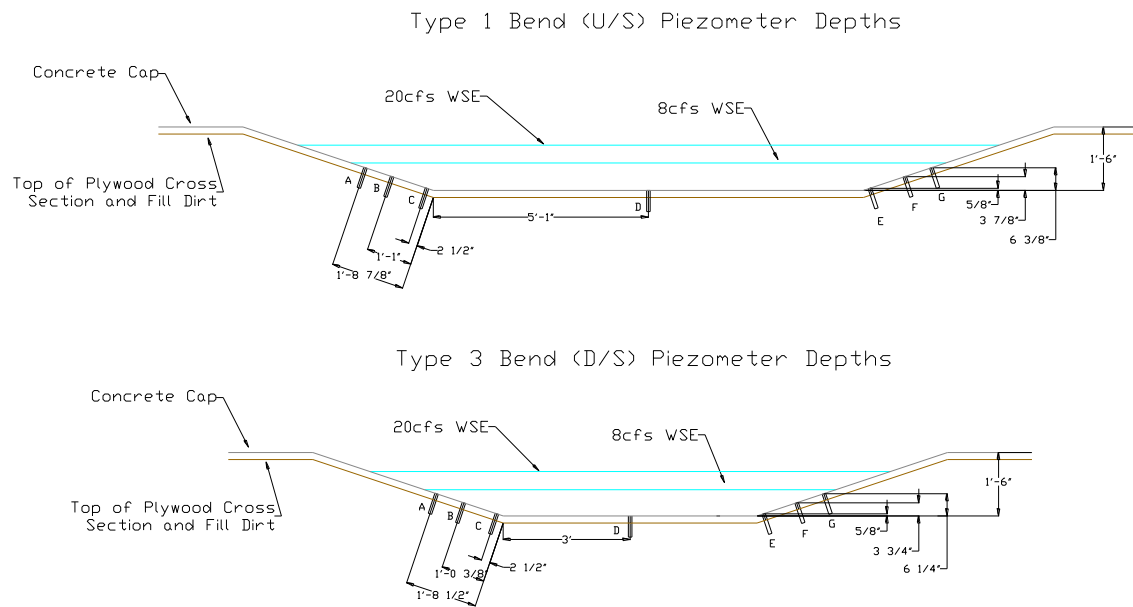


Figure A.13: Piezometer Locations for Type 1 and Type 3 Bends (from Heintz (2002))



Figure A.14: Installation of Piezometer for Flow Depth Measurement

The piezometers installed in the main channel are connected with a stilling well to allow data collection of water surface elevations (Heintz 2002). Two-inch diameter PVC pipe was placed in the stilling well. All piezometer taps were equipped with a back-flushing system to ensure that the stilling wells and the connecting tubing and taps would not clog (Heintz 2002). Figure A.15 shows the stilling wells used for water surface elevation measurements. The channel bed elevation was also measured using a point gauge. Flow depth was calculated by subtracting the bed elevation from the water surface elevation.

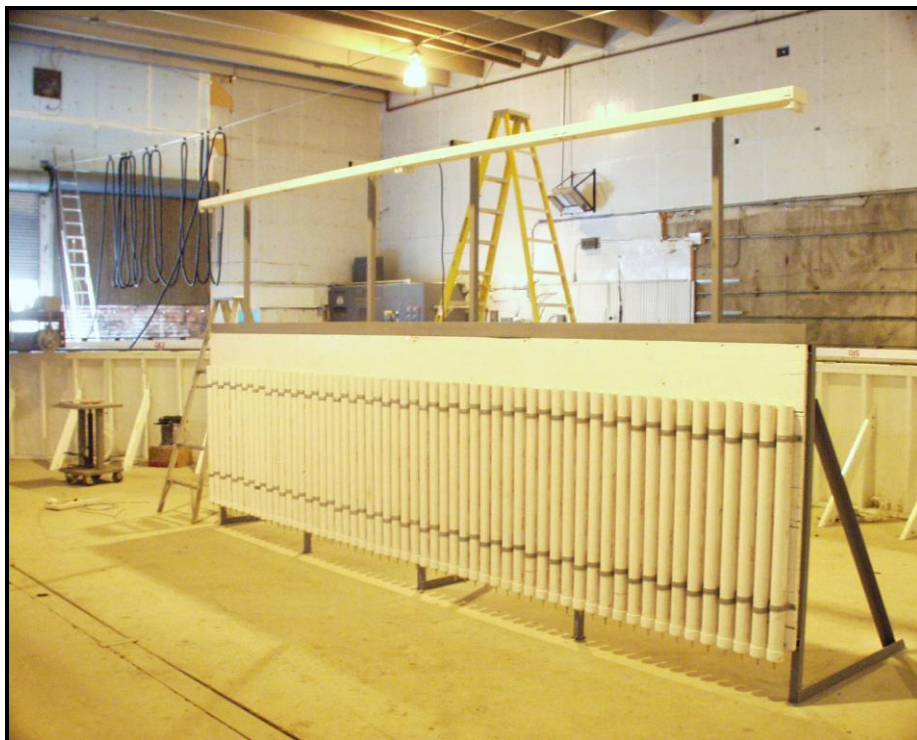


Figure A.15: Stilling Wells for Water Surface Elevation Measurements

A.3.3 Velocity Measurement

Since 1993, the ADV has been a popular instrument to collect flow data in field and laboratory studies (Wahl 2000). The installation of the ADV allowed for the measurement of flow velocity in each cross section of the model. ADV data were 3-D velocity measurements that were collected as V_x , V_y , and V_z . The ADV-measured velocity from a sampling volume was located 0.164 ft (5 cm) below the probe head. The sampling rate of the ADV is 10 Hz. The time period of data collection at each point is approximately 30 sec. At each point, 300 measurements

of flow velocity data were collected. The 3-D velocity measurements were collected with the ADV probe directed perpendicular to the cart, which was always oriented perpendicular to the direction of flow (Heintz 2002). Figure A.16 is a photograph of the ADV and Figure A.17 shows the ADV installation on the data-collection cart. After data collection by ADV, data processing is necessary because it is impossible to perform data analysis with the original ADV file. As a result, WinADV (computer software) was developed by Wahl (2000). The initial purpose of the software development was to facilitate displaying the velocity time series (Wahl 2000).



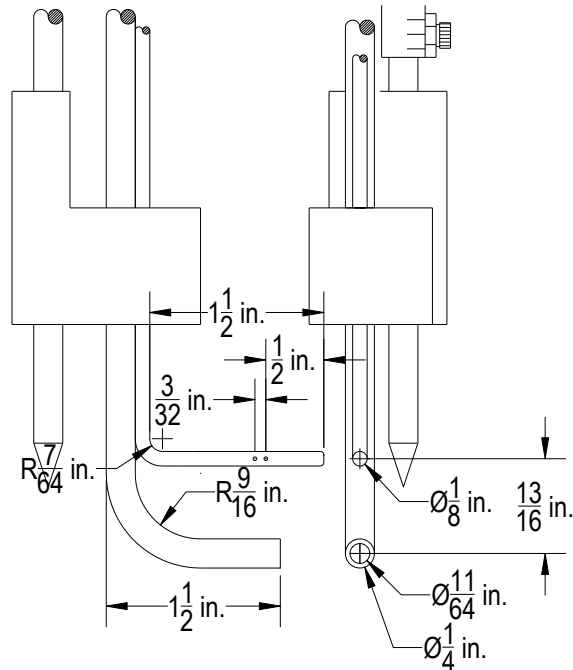
Figure A.16: Photograph of the ADV (from Heintz (2002))



Figure A.17: Installation of the ADV in the Data-collection Cart

A.3.4 Shear Stress Measurement

Sclafani (2008) performed shear stress data calibration in a 60-ft long, 4-ft wide, and 2.5-ft tall flume. A Preston tube was used to collect channel bed and bank data for the calculation of shear stress. The Preston tube presented in Figure A.18 allowed for measuring boundary shear stress of the model. The Preston tube technique utilizes a modified Pitot tube that was placed directly on the bed of the channel at the point of measurement. A pressure transducer was used for the collection of pressure data.



A.4 Model Calibration

There are two types of model calibration, the first is channel roughness calibration. Heintz (2002) conducted the model calibration by using the HEC-RAS program. As a result of the roughness calibration, the value of Manning's n (the roughness value) was determined to be 0.018. The second type of model calibration is backwater calibration by installing stop logs in the tailbox of the model. Using the computed roughness value of 0.018, the HEC-RAS model was then used to calculate a water surface profile for normal depth conditions at all flow rates (Heintz 2002). For the confirmation of the model calibration, Heintz (2002) compared the measured value of water surface profile and calculated value of water surface profile by using HEC-RAS. Figure A.19 and Figure A.20 show the installed stop logs of the model.



Figure A.19: Stop-log Installation of the Model



Figure A.20: Stop Log for Calibration of the Model

A.5 Test Program (Test Matrix)

There were four different discharges (8, 12, 16, and 20 cfs), eighteen cross sections (1 through 18), and seven piezometers (a through g) in each cross section. Figure A.21 shows a test matrix for the baseline test condition. For the discharges of 8 and 12 cfs, several piezometers were not submerged due to the low flow depth. Therefore, data of Piezo a, Piezo b, Piezo f, and Piezo g for 8 cfs, are excluded from the test matrix. Similar to the 8 cfs runs, Piezo a and Piezo g for 12 cfs could not collect data.

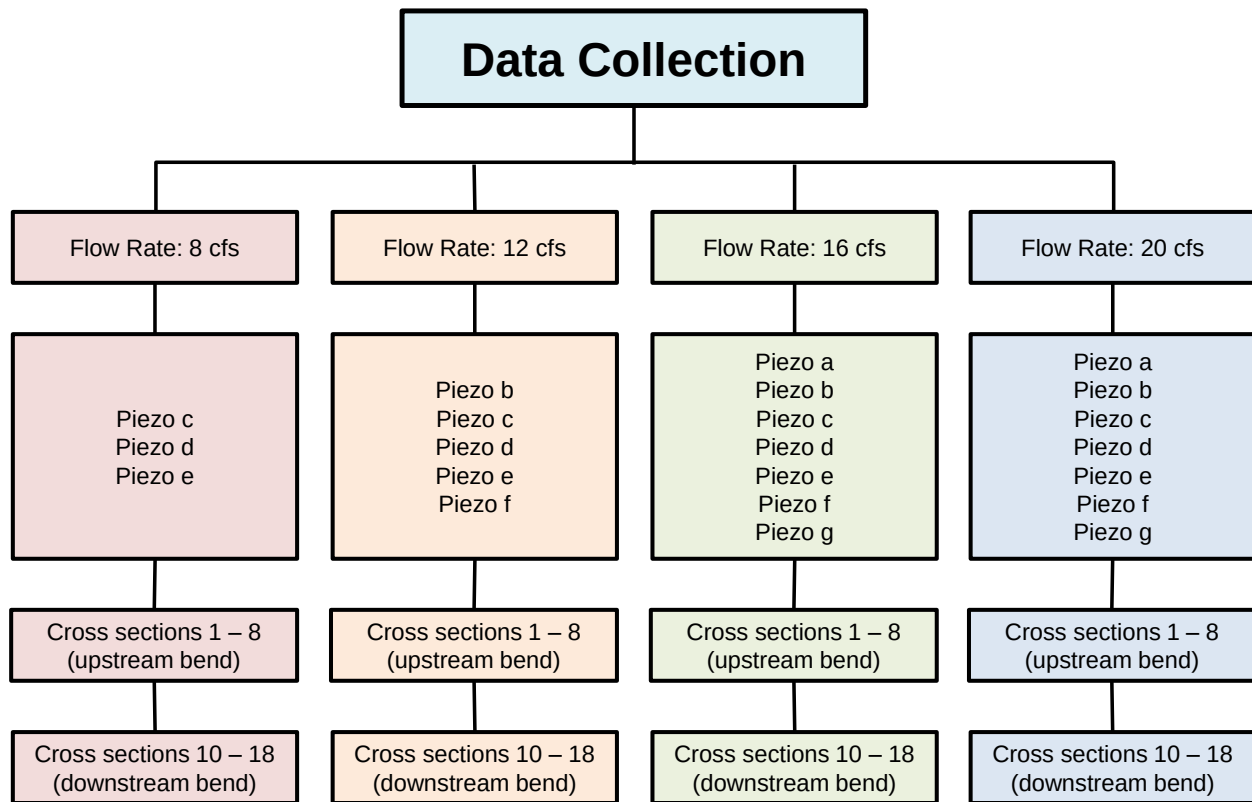


Figure A.21: Baseline Condition Test Matrix

APPENDIX B PRESTON TUBE CALIBRATION

Table B.1: Shear Stress from the Preston Tube Calibration for 8 cfs

Cross Section	Piezo	Shear Stress (psf)
4	c	0.0178
4	d	0.0166
4	e	0.0164
5	c	0.0183
5	d	0.0177
5	e	0.0000
6	c	0.0178
6	d	0.0189
6	e	0.0147
7	c	0.0172
7	d	0.0148
7	e	0.0125
8	c	0.0203
8	d	0.0160
8	e	0.0130
10	c	0.0438
10	d	0.0363
10	e	0.0322
11	c	0.0240
11	d	0.0384
11	e	0.0193
12	c	0.0342
12	d	0.0400
12	e	0.0289
13	c	0.0393
13	d	0.0361
13	e	0.0313
14	c	0.0263
14	d	0.0390
14	e	0.0346
15	c	0.0287
15	d	0.0484
15	e	0.0411
16	c	0.0286
16	d	0.0350
16	e	0.0376
17	c	0.0328
17	d	0.0388
17	e	0.0460

Table B.2: Shear Stress from the Preston Tube Calibration for 12 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	b	0.0239	14	b	0.0230
4	c	0.0211	14	c	0.0361
4	d	0.0223	14	d	0.0490
4	e	0.0248	14	e	0.0395
4	f	0.0175	14	f	0.0303
5	b	0.0273	15	b	0.0266
5	c	0.0267	15	c	0.0367
5	d	0.0244	15	d	0.0498
5	e	0.0246	15	e	0.0454
5	f	0.0175	15	f	0.0390
6	b	0.0227	16	b	0.0287
6	c	0.0238	16	c	0.0385
6	d	0.0209	16	d	0.0471
6	e	0.0210	16	e	0.0481
6	f	0.0172	16	f	0.0498
7	b	0.0250	17	b	0.0224
7	c	0.0205	17	c	0.0321
7	d	0.0185	17	d	0.0457
7	e	0.0152	17	e	0.0539
7	f	0.0148	17	f	0.0453
8	b	0.0274			
8	c	0.0241			
8	d	0.0170			
8	e	0.0186			
8	f	0.0143			
10	b	0.0556			
10	c	0.0555			
10	d	0.0396			
10	e	0.0320			
10	f	0.0323			
11	b	0.0352			
11	c	0.0343			
11	d	0.0452			
11	e	0.0264			
11	f	0.0192			
12	b	0.0337			
12	c	0.0496			
12	d	0.0389			
12	e	0.0334			
12	f	0.0287			
13	b	0.0307			
13	c	0.0457			
13	d	0.0430			
13	e	0.0338			
13	f	0.0313			

Table B.3: Shear Stress from the Preston Tube Calibration for 16 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	a	0.0299	11	d	0.0617
4	b	0.0266	11	e	0.0331
4	c	0.0258	11	f	0.0297
4	d	0.0260	11	g	0.0271
4	e	0.0285	12	a	0.0429
4	f	0.0234	12	b	0.0483
4	g	0.0165	12	c	0.0568
5	a	0.0335	12	d	0.0579
5	b	0.0343	12	e	0.0444
5	c	0.0261	12	f	0.0455
5	d	0.0271	12	g	0.0380
5	e	0.0289	13	a	0.0075
5	f	0.0251	13	b	0.0494
5	g	0.0185	13	c	0.0555
6	a	0.0311	13	d	0.0487
6	b	0.0346	13	e	0.0429
6	c	0.0270	13	f	0.0415
6	d	0.0283	13	g	0.0397
6	e	0.0280	14	a	0.0129
6	f	0.0238	14	b	0.0232
6	g	0.0198	14	c	0.0417
7	a	0.0321	14	d	0.0469
7	b	0.0290	14	e	0.0317
7	c	0.0259	14	f	0.0249
7	d	0.0285	14	g	0.0256
7	e	0.0251	15	a	0.0174
7	f	0.0219	15	b	0.0248
7	g	0.0162	15	c	0.0397
8	a	0.0243	15	d	0.0488
8	b	0.0278	15	e	0.0476
8	c	0.0293	15	f	0.0377
8	d	0.0280	15	g	0.0361
8	e	0.0259	16	a	0.0142
8	f	0.0188	16	b	0.0263
8	g	0.0158	16	c	0.0375
10	a	0.0678	16	d	0.0488
10	b	0.0630	16	e	0.0551
10	c	0.0700	16	f	0.0472
10	d	0.0445	16	g	0.0351
10	e	0.0428	17	a	0.0160
10	f	0.0408	17	b	0.0264
10	g	0.0395	17	c	0.0283
11	a	0.0454	17	d	0.0455
11	b	0.0543	17	e	0.0722
11	c	0.0691	17	f	0.0599
			17	g	0.0490

Table B.4: Shear Stress from the Preston Tube Calibration for 20 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	a	0.0240	11	d	0.0505
4	b	0.0213	11	e	0.0213
4	c	0.0170	11	f	0.0209
4	d	0.0210	11	g	0.0249
4	e	0.0268	12	a	0.0262
4	f	0.0198	12	b	0.0397
4	g	0.0151	12	c	0.0455
5	a	0.0299	12	d	0.0461
5	b	0.0265	12	e	0.0366
5	c	0.0194	12	f	0.0333
5	d	0.0192	12	g	0.0336
5	e	0.0252	13	a	0.0230
5	f	0.0197	13	b	0.0337
5	g	0.0150	13	c	0.0515
6	a	0.0265	13	d	0.0426
6	b	0.0273	13	e	0.0294
6	c	0.0196	13	f	0.0317
6	d	0.0195	13	g	0.0327
6	e	0.0219	14	a	0.0198
6	f	0.0199	14	b	0.0327
6	g	0.0123	14	c	0.0445
7	a	0.0193	14	d	0.0410
7	b	0.0242	14	e	0.0322
7	c	0.0212	14	f	0.0257
7	d	0.0219	14	g	0.0289
7	e	0.0186	15	a	0.0252
7	f	0.0210	15	b	0.0314
7	g	0.0119	15	c	0.0395
8	a	0.0194	15	d	0.0426
8	b	0.0260	15	e	0.0397
8	c	0.0392	15	f	0.0472
8	d	0.0186	15	g	0.0419
8	e	0.0177	16	a	0.0200
8	f	0.0099	16	b	0.0296
8	g	0.0080	16	c	0.0490
10	a	0.0630	16	d	0.0420
10	b	0.0514	16	e	0.0397
10	c	0.0674	16	f	0.0499
10	d	0.0493	16	g	0.0420
10	e	0.0381	17	a	0.0188
10	f	0.0349	17	b	0.0287
10	g	0.0339	17	c	0.0360
11	a	0.0399	17	d	0.0441
11	b	0.0459	17	e	0.0549
11	c	0.0562	17	f	0.0420
			17	g	0.0421

APPENDIX C LINEAR REGRESSION OF THE FLOW VELOCITY PROFILE

Table C.1: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 8 cfs

Cross Section	Piezo	Shear Stress (psf)
4	c	0.0030
4	d	0.0018
4	e	0.0011
5	c	0.0027
5	d	0.0038
5	e	0.0009
6	c	0.0034
6	d	0.0028
6	e	0.0006
7	c	0.0036
7	d	0.0045
7	e	0.0010
8	c	0.0037
8	d	0.0059
8	e	0.0000
10	c	0.0063
10	d	0.0077
10	e	0.0041
11	c	0.0115
11	d	0.0081
11	e	0.0067
12	c	0.0055
12	d	0.0097
12	e	0.0041
13	c	0.0044
13	d	0.0094
13	e	0.0070
14	c	0.0030
14	d	0.0043
14	e	0.0064
15	c	0.0006
15	d	0.0027
15	e	0.0049
16	c	0.0007
16	d	0.0074
16	e	0.0094
17	c	0.0007
17	d	0.0057
17	e	0.0061

Table C.2: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 12 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	b	0.0033	14	b	0.0034
4	c	0.0036	14	c	0.0046
4	d	0.0035	14	d	0.0163
4	e	0.0008	14	e	0.0089
4	f	0.0008	14	f	0.0071
5	b	0.0007	15	b	0.0014
5	c	0.0036	15	c	0.0022
5	d	0.0037	15	d	0.0137
5	e	0.0013	15	e	0.0140
5	f	0.0002	15	f	0.0051
6	b	0.0025	16	b	0.0003
6	c	0.0036	16	c	0.0005
6	d	0.0056	16	d	0.0223
6	e	0.0018	16	e	0.0111
6	f	0.0014	16	f	0.0047
7	b	0.0028	17	b	0.0003
7	c	0.0043	17	c	0.0021
7	d	0.0055	17	d	0.0121
7	e	0.0001	17	e	0.0101
7	f	0.0014	17	f	0.0118
8	b	0.0015			
8	c	0.0024			
8	d	0.0042			
8	e	0.0000			
8	f	0.0005			
10	b	0.0081			
10	c	0.0079			
10	d	0.0077			
10	e	0.0017			
10	f	0.0028			
11	b	0.0149			
11	c	0.0195			
11	d	0.0118			
11	e	0.0122			
11	f	0.0013			
12	b	0.0027			
12	c	0.0030			
12	d	0.0132			
12	e	0.0129			
12	f	0.0056			
13	b	0.0043			
13	c	0.0073			
13	d	0.0188			
13	e	0.0168			
13	f	0.0052			

Table C.3: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 16 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	a	0.0016	11	d	0.0115
4	b	0.0109	11	e	0.0116
4	c	0.0060	11	f	0.0131
4	d	0.0026	11	g	0.0059
4	e	0.0006	12	a	0.0036
4	f	0.0018	12	b	0.0086
4	g	0.0012	12	c	0.0241
5	a	0.0032	12	d	0.0193
5	b	0.0044	12	e	0.0119
5	c	0.0018	12	f	0.0107
5	d	0.0053	12	g	0.0070
5	e	0.0002	13	a	0.0019
5	f	0.0009	13	b	0.0045
5	g	0.0000	13	c	0.0119
6	a	0.0022	13	d	0.0267
6	b	0.0042	13	e	0.0093
6	c	0.0042	13	f	0.0107
6	d	0.0069	13	g	0.0079
6	e	0.0001	14	a	0.0021
6	f	0.0009	14	b	0.0044
6	g	0.0007	14	c	0.0093
7	a	0.0042	14	d	0.0278
7	b	0.0044	14	e	0.0171
7	c	0.0050	14	f	0.0086
7	d	0.0065	14	g	0.0054
7	e	0.0013	15	a	0.0000
7	f	0.0009	15	b	0.0022
7	g	0.0008	15	c	0.0033
8	a	0.0055	15	d	0.0163
8	b	0.0053	15	e	0.0173
8	c	0.0024	15	f	0.0087
8	d	0.0052	15	g	0.0072
8	e	0.0000	16	a	0.0012
8	f	0.0006	16	b	0.0011
8	g	0.0003	16	c	0.0060
10	a	0.0090	16	d	0.0290
10	b	0.0120	16	e	0.0156
10	c	0.0123	16	f	0.0086
10	d	0.0077	16	g	0.0055
10	e	0.0015	17	a	0.0004
10	f	0.0004	17	b	0.0016
10	g	0.0054	17	c	0.0048
11	a	0.0076	17	d	0.0177
11	b	0.0247	17	e	0.0174
11	c	0.0155	17	f	0.0054
			17	g	0.0006

Table C.4: Shear Stress (τ_x) from Linear Regression of the Flow Velocity Profile for 20 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	a	0.0116	11	d	0.0121
4	b	0.0030	11	e	0.0177
4	c	0.0038	11	f	0.0280
4	d	0.0030	11	g	0.0282
4	e	0.0000	12	a	0.0154
4	f	0.0001	12	b	0.0280
4	g	0.0027	12	c	0.0375
5	a	0.0054	12	d	0.0133
5	b	0.0036	12	e	0.0212
5	c	0.0024	12	f	0.0143
5	d	0.0118	12	g	0.0091
5	e	0.0005	13	a	0.0093
5	f	0.0003	13	b	0.0015
5	g	0.0004	13	c	0.0554
6	a	0.0042	13	d	0.0301
6	b	0.0083	13	e	0.0168
6	c	0.0024	13	f	0.0149
6	d	0.0094	13	g	0.0164
6	e	0.0016	14	a	0.0089
6	f	0.0019	14	b	0.0047
6	g	0.0031	14	c	0.0070
7	a	0.0099	14	d	0.0355
7	b	0.0064	14	e	0.0265
7	c	0.0013	14	f	0.0187
7	d	0.0066	14	g	0.0145
7	e	0.0001	15	a	0.0024
7	f	0.0017	15	b	0.0050
7	g	0.0004	15	c	0.0045
8	a	0.0040	15	d	0.0455
8	b	0.0060	15	e	0.0226
8	c	0.0026	15	f	0.0233
8	d	0.0019	15	g	0.0148
8	e	0.0005	16	a	0.0003
8	f	0.0002	16	b	0.0003
8	g	0.0002	16	c	0.0002
10	a	0.0461	16	d	0.0467
10	b	0.0044	16	e	0.0157
10	c	0.0075	16	f	0.0151
10	d	0.0077	16	g	0.0141
10	e	0.0004	17	a	0.0011
10	f	0.0010	17	b	0.0011
10	g	0.0027	17	c	0.0000
11	a	0.0756	17	d	0.0377
11	b	0.0079	17	e	0.0313
11	c	0.0313	17	f	0.0262
			17	g	0.0073

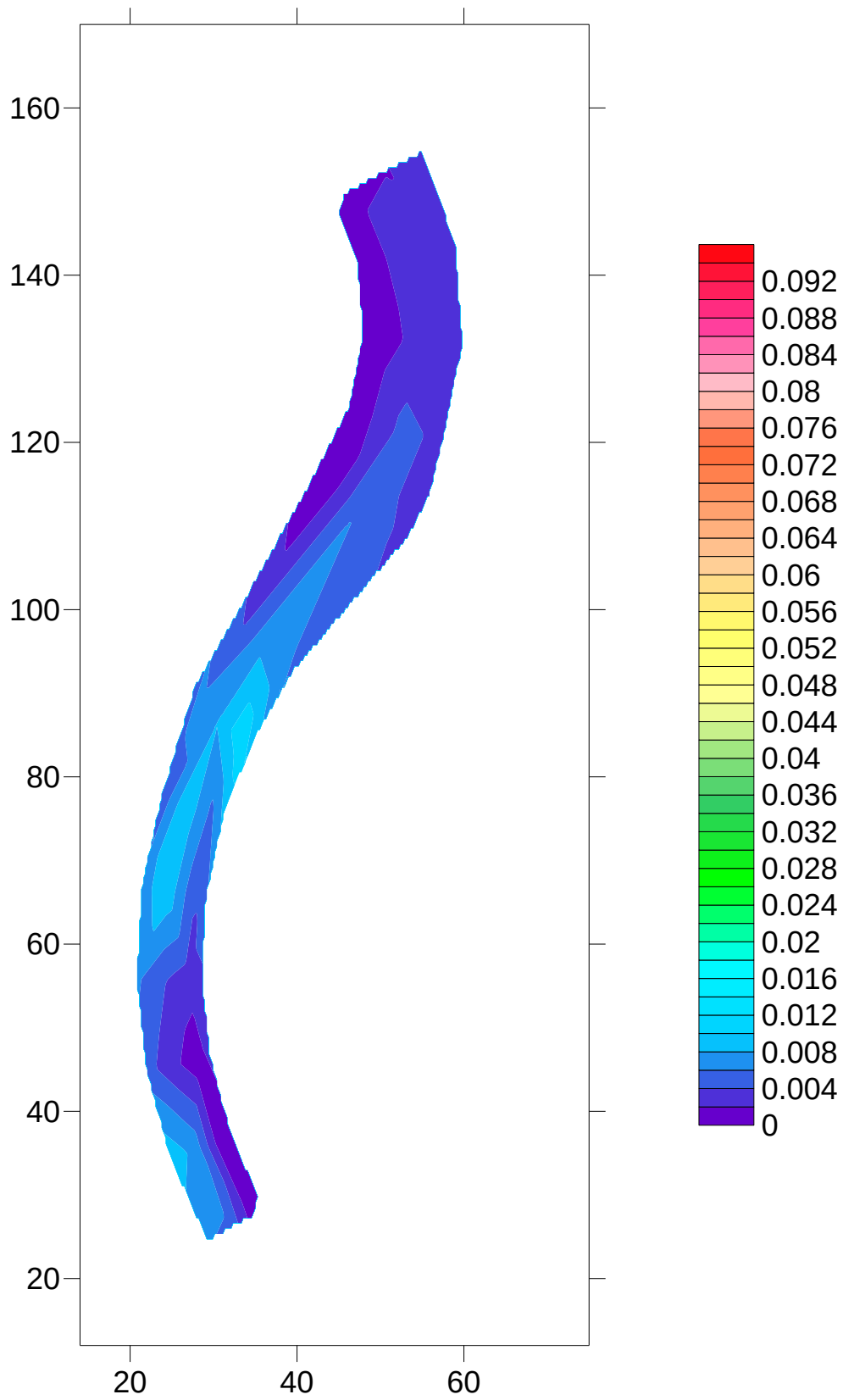


Figure C.1: Shear Stress Distribution (τ_x) by Linear Regression for 8 cfs (psf)

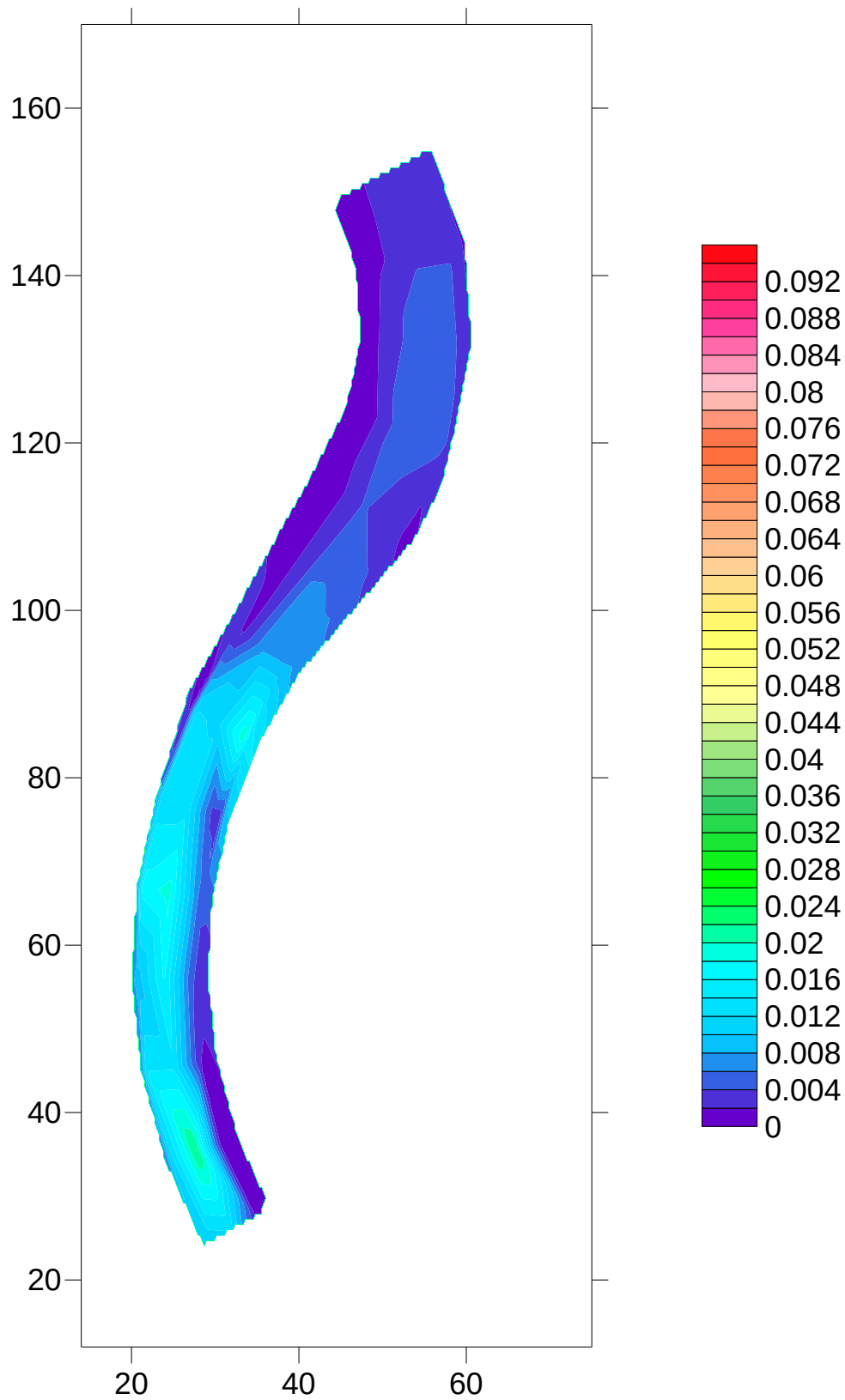


Figure C.2: Shear Stress Distribution (τ_x) by Linear Regression for 12 cfs (psf)

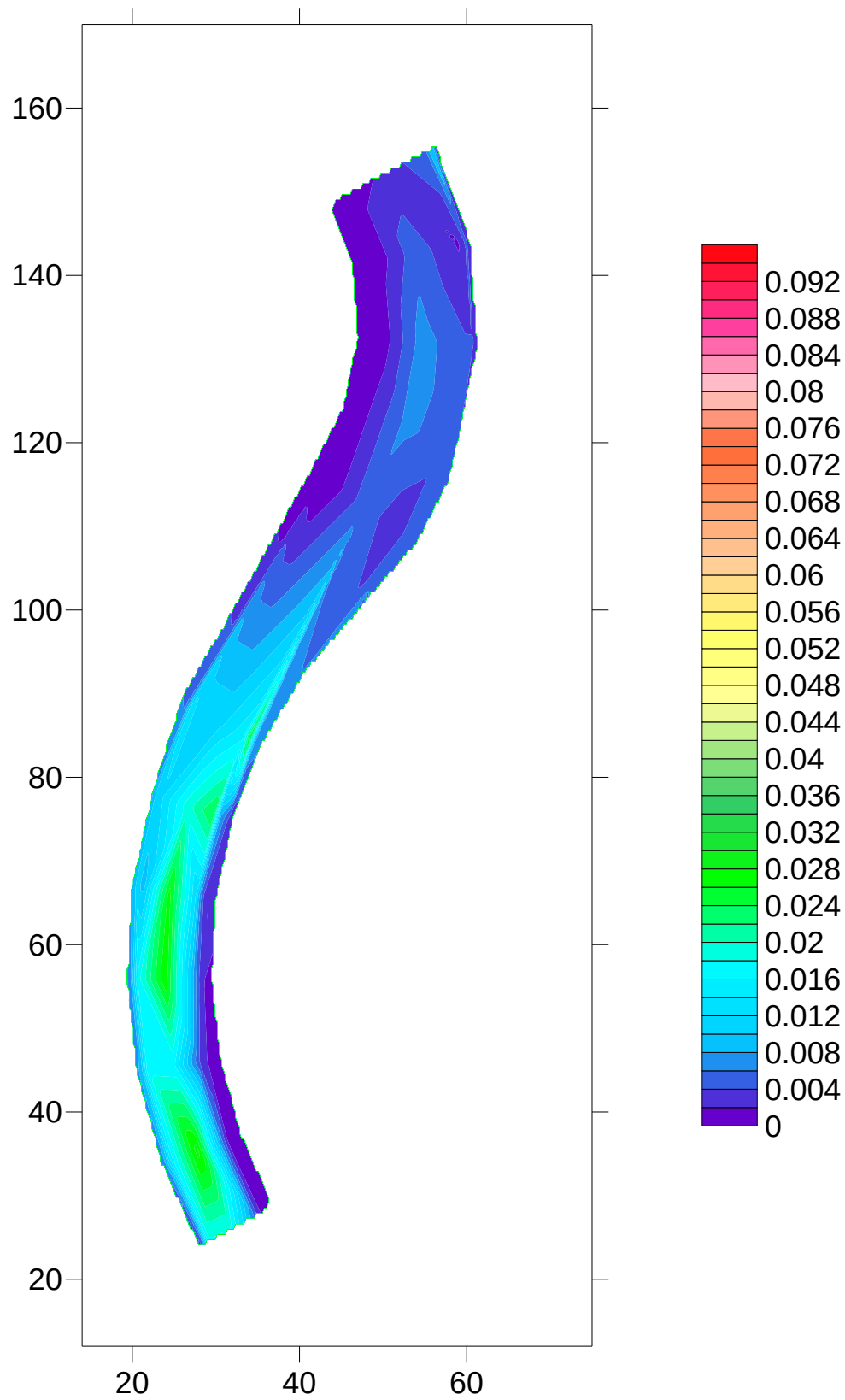


Figure C.3: Shear Stress Distribution (τ_v) by Linear Regression for 16 cfs (psf)

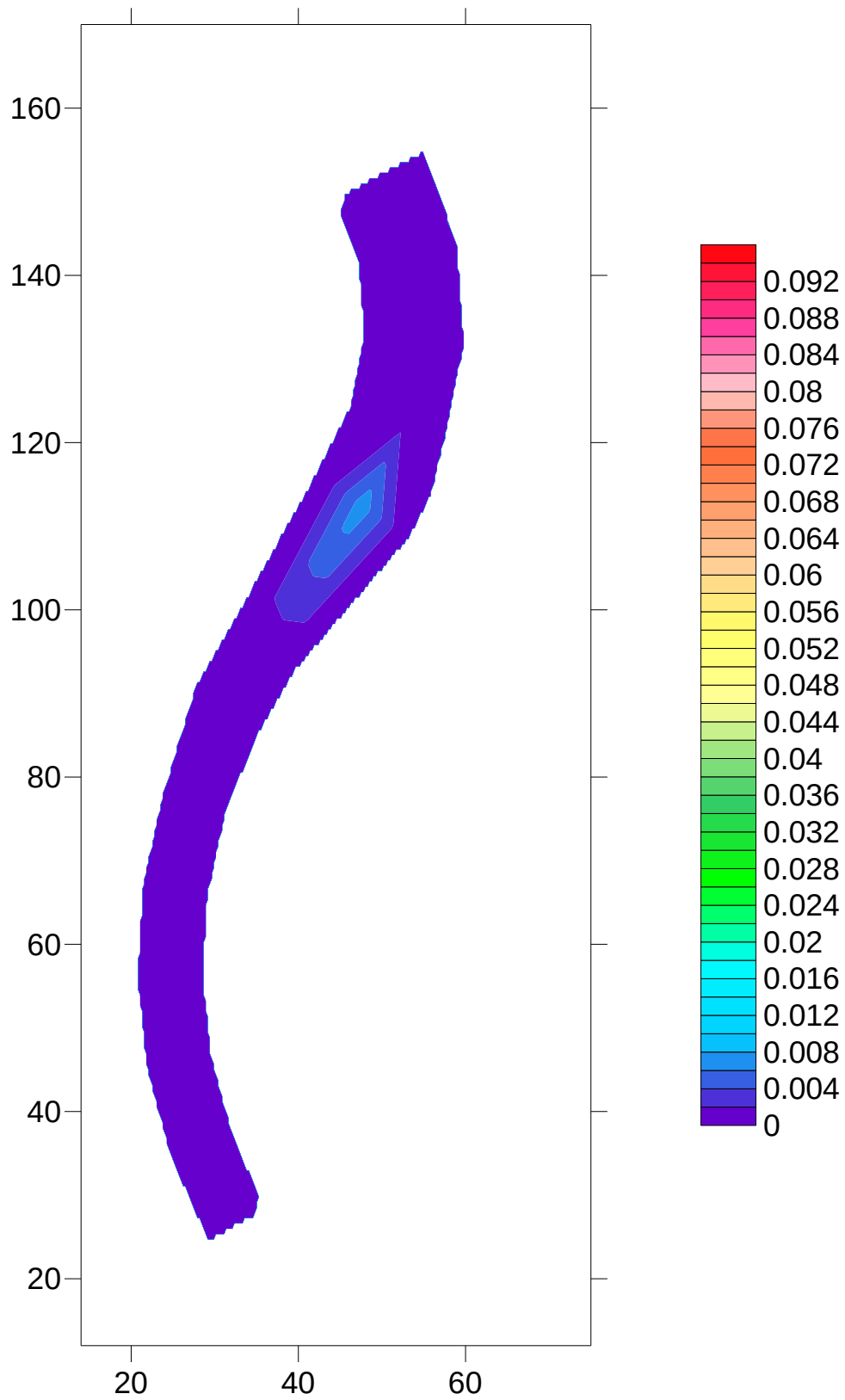


Figure C.4: Shear Stress Distribution (τ_y) by Linear Regression for 8 cfs (psf)

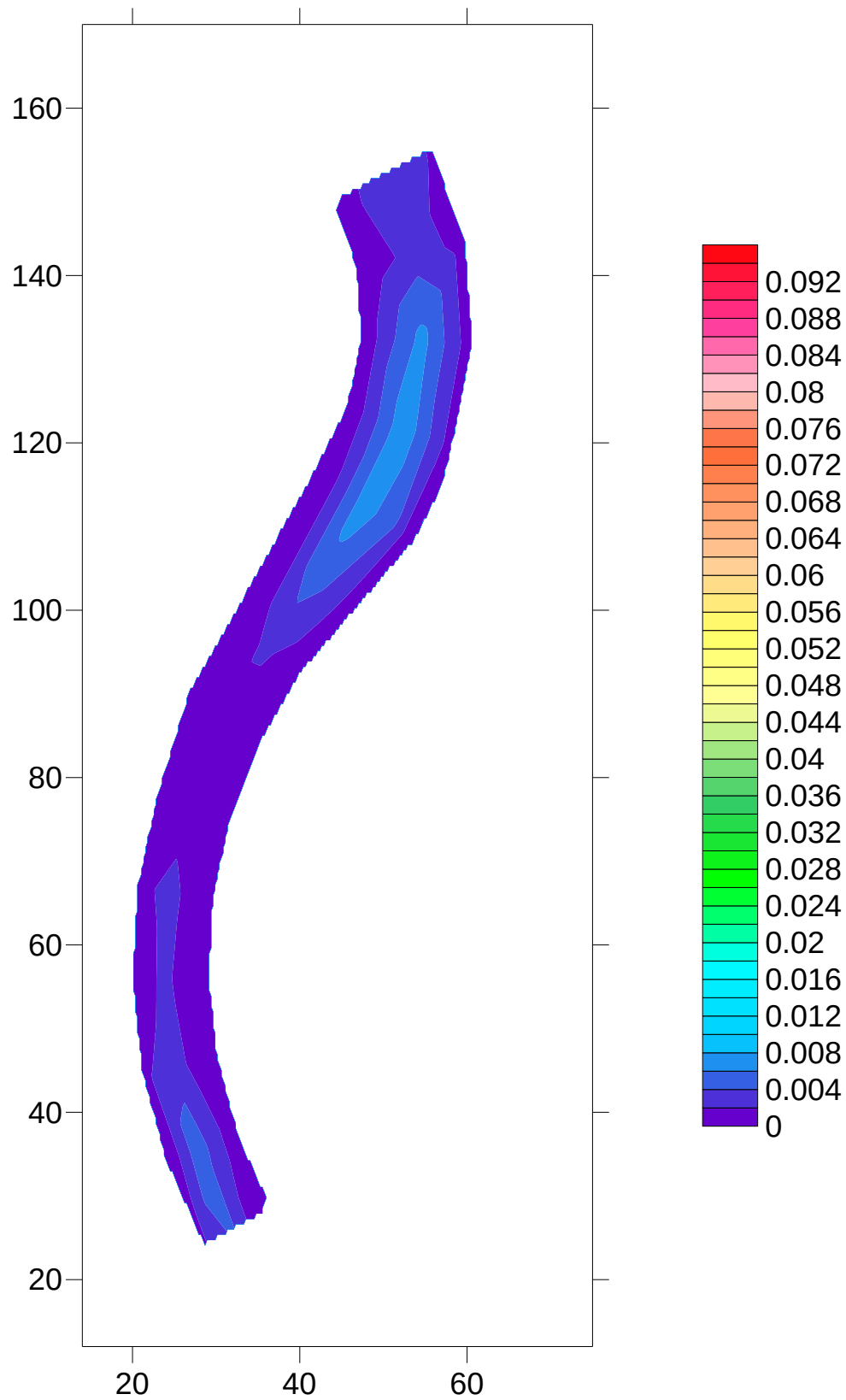


Figure C.5: Shear Stress Distribution (τ_v) by Linear Regression for 12 cfs (psf)

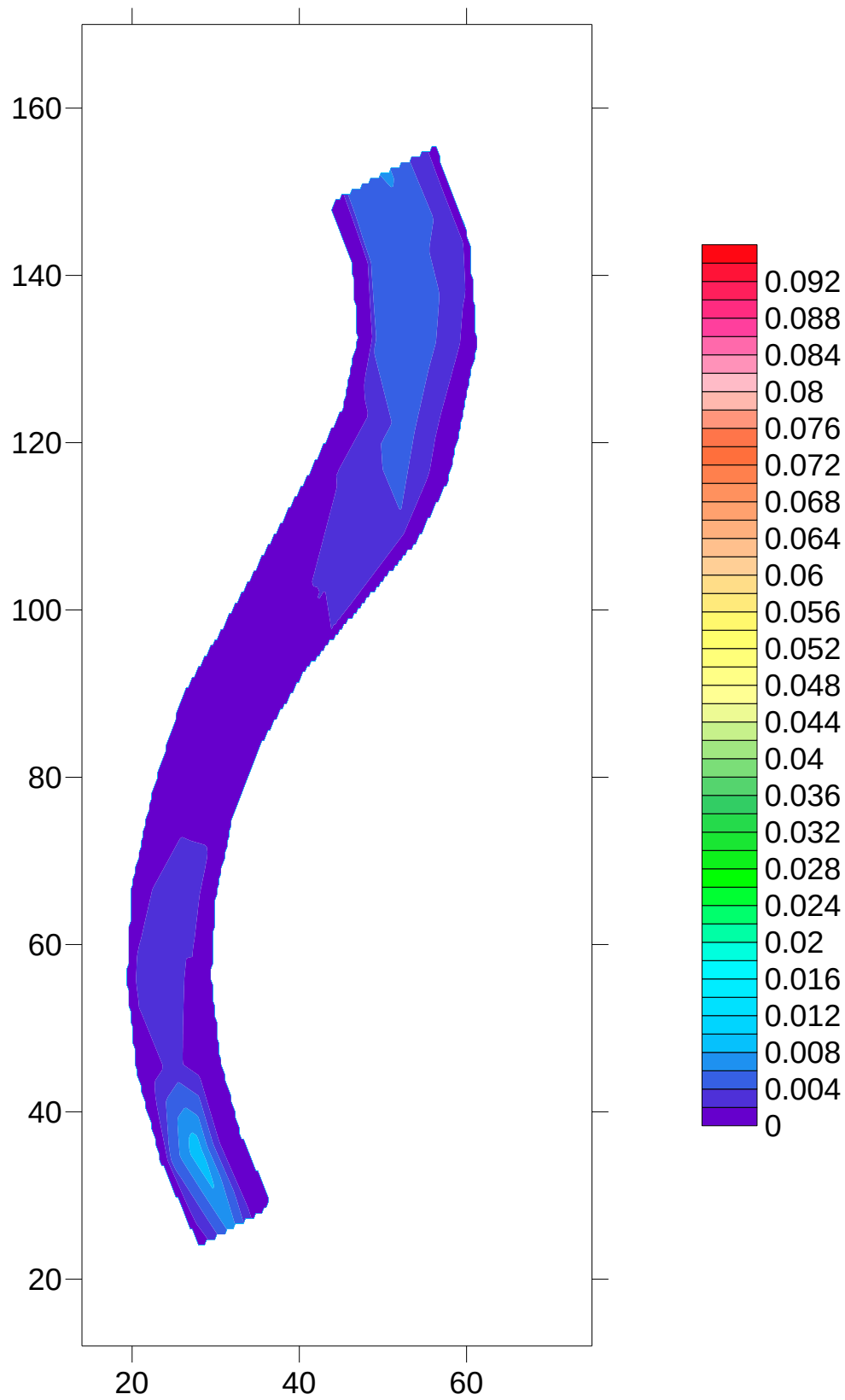


Figure C.6: Shear Stress Distribution (τ_y) by Linear Regression for 16 cfs (psf)

APPENDIX D SHEAR STRESS FROM ROZOVSKII (1961) METHOD

Table D.1: Shear stress (τ_y) from Rozovskii (1961) Method for 8 cfs

Cross Section	Piezo	Shear Stress (psf)
4	c	0.0607
4	d	0.0683
4	e	0.0559
5	c	0.0090
5	d	0.0100
5	e	0.0104
6	c	0.0250
6	d	0.0247
6	e	0.0269
7	c	-0.0218
7	d	-0.0239
7	e	-0.0135
8	c	-0.0290
8	d	-0.0314
8	e	-0.0134
10	c	-0.0478
10	d	-0.0426
10	e	-0.0344
11	c	0.0140
11	d	0.0145
11	e	0.0284
12	c	0.0347
12	d	0.0288
12	e	0.0367
13	c	0.0246
13	d	0.0200
13	e	0.0175
14	c	0.0054
14	d	-0.0145
14	e	-0.0093
15	c	0.0649
15	d	0.0547
15	e	0.0501
16	c	0.0383
16	d	0.0228
16	e	0.0188
17	c	0.0463
17	d	0.0280
17	e	0.0202

Table D.2: Shear stress (τ_y) from Rozovskii (1961) Method for 12 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	b	0.0518	14	b	0.0223
4	c	0.0765	14	c	0.0047
4	d	0.0803	14	d	-0.0240
4	e	0.0589	14	e	-0.0133
4	f	0.0564	14	f	0.0045
5	b	-0.0096	15	b	0.0379
5	c	-0.0222	15	c	0.0335
5	d	-0.0256	15	d	0.0006
5	e	-0.0339	15	e	0.0022
5	f	-0.0049	15	f	0.0166
6	b	0.0264	16	b	0.0684
6	c	0.0337	16	c	0.0815
6	d	0.0363	16	d	0.0497
6	e	0.0307	16	e	0.0443
6	f	0.0307	16	f	0.0391
7	b	-0.0021	17	b	0.1012
7	c	-0.0112	17	c	0.1328
7	d	-0.0153	17	d	0.1157
7	e	-0.0027	17	e	0.0941
7	f	0.0089	17	f	0.0727
8	b	0.0346			
8	c	0.0397			
8	d	0.0438			
8	e	0.0639			
8	f	0.0505			
10	b	-0.0074			
10	c	-0.0266			
10	d	-0.0068			
10	e	-0.0038			
10	f	0.0043			
11	b	0.0264			
11	c	0.0176			
11	d	0.0323			
11	e	0.0460			
11	f	0.0443			
12	b	0.0240			
12	c	0.0100			
12	d	0.0050			
12	e	0.0173			
12	f	0.0239			
13	b	0.0641			
13	c	0.0659			
13	d	0.0664			
13	e	0.0694			
13	f	0.0561			

Table D.3: Shear stress (τ_y) from Rozovskii (1961) Method for 16 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	a	0.0273	11	d	0.0004
4	b	0.0324	11	e	0.0199
4	c	0.0453	11	f	0.0269
4	d	0.0504	11	g	0.0238
4	e	0.0163	12	a	0.0106
4	f	0.0372	12	b	0.0162
4	g	0.0368	12	c	0.0231
5	a	0.0181	12	d	0.0247
5	b	0.0186	12	e	0.0234
5	c	0.0266	12	f	0.0167
5	d	0.0289	12	g	0.0112
5	e	0.0085	13	a	0.0526
5	f	0.0239	13	b	0.0626
5	g	0.0261	13	c	0.0384
6	a	0.0386	13	d	0.0417
6	b	0.0478	13	e	0.0516
6	c	0.0651	13	f	0.0495
6	d	0.0598	13	g	0.0409
6	e	0.0546	14	a	0.0316
6	f	0.0535	14	b	0.0316
6	g	0.0438	14	c	0.0070
7	a	-0.0084	14	d	-0.0208
7	b	-0.0203	14	e	0.0265
7	c	-0.0317	14	f	0.0070
7	d	-0.0396	14	g	0.0154
7	e	-0.0310	15	a	0.0261
7	f	-0.0049	15	b	0.0206
7	g	0.0061	15	c	0.0000
8	a	0.0208	15	d	-0.0425
8	b	0.0208	15	e	-0.0368
8	c	0.0229	15	f	-0.0144
8	d	0.0203	15	g	0.0039
8	e	0.0518	16	a	0.0709
8	f	0.0467	16	b	0.0972
8	g	0.0380	16	c	0.1124
10	a	-0.0258	16	d	0.0768
10	b	-0.0639	16	e	0.0652
10	c	-0.0958	16	f	0.0575
10	d	-0.0722	16	g	0.0485
10	e	-0.0590	17	a	0.0259
10	f	-0.0234	17	b	0.0246
10	g	-0.0067	17	c	0.0101
11	a	0.0587	17	d	-0.0374
11	b	-0.0001	17	e	-0.0476
11	c	-0.0197	17	f	-0.0152
			17	g	-0.0060

Table D.4: Shear stress (τ_y) from Rozovskii (1961) Method for 20 cfs

Cross Section	Piezo	Shear Stress (psf)	Cross Section	Piezo	Shear Stress (psf)
4	a	0.0305	11	d	-0.0551
4	b	0.0386	11	e	-0.0259
4	c	0.0578	11	f	-0.0070
4	d	0.0668	11	g	0.0050
4	e	0.0262	12	a	-0.0012
4	f	0.0434	12	b	-0.0406
4	g	0.0431	12	c	-0.1019
5	a	0.0326	12	d	-0.0745
5	b	0.0431	12	e	-0.0432
5	c	0.0956	12	f	-0.0247
5	d	0.0560	12	g	-0.0036
5	e	0.0344	13	a	0.0439
5	f	0.0373	13	b	0.0308
5	g	0.0450	13	c	-0.0192
6	a	0.0474	13	d	-0.0096
6	b	0.0383	13	e	0.0128
6	c	0.0522	13	f	0.0188
6	d	0.0302	13	g	0.0251
6	e	0.0369	14	a	0.0264
6	f	0.0420	14	b	0.0140
6	g	0.0435	14	c	-0.0321
7	a	0.0182	14	d	-0.0576
7	b	0.0193	14	e	-0.0334
7	c	0.0256	14	f	-0.0128
7	d	0.0094	14	g	-0.0019
7	e	0.0618	15	a	0.0464
7	f	0.0332	15	b	0.0470
7	g	0.0392	15	c	0.0271
8	a	-0.0388	15	d	-0.0134
8	b	-0.0593	15	e	-0.0051
8	c	-0.0866	15	f	0.0053
8	d	-0.0996	15	g	0.0156
8	e	-0.0622	16	a	0.0487
8	f	-0.0327	16	b	0.0466
8	g	-0.0119	16	c	0.0364
10	a	-0.0100	16	d	-0.0190
10	b	-0.0271	16	e	-0.0147
10	c	-0.0378	16	f	0.0034
10	d	-0.0127	16	g	0.0160
10	e	0.0049	17	a	0.0185
10	f	0.0082	17	b	0.0118
10	g	0.0157	17	c	-0.0177
11	a	-0.0169	17	d	-0.0774
11	b	-0.0096	17	e	-0.0859
11	c	-0.0736	17	f	-0.0522
			17	g	-0.0231

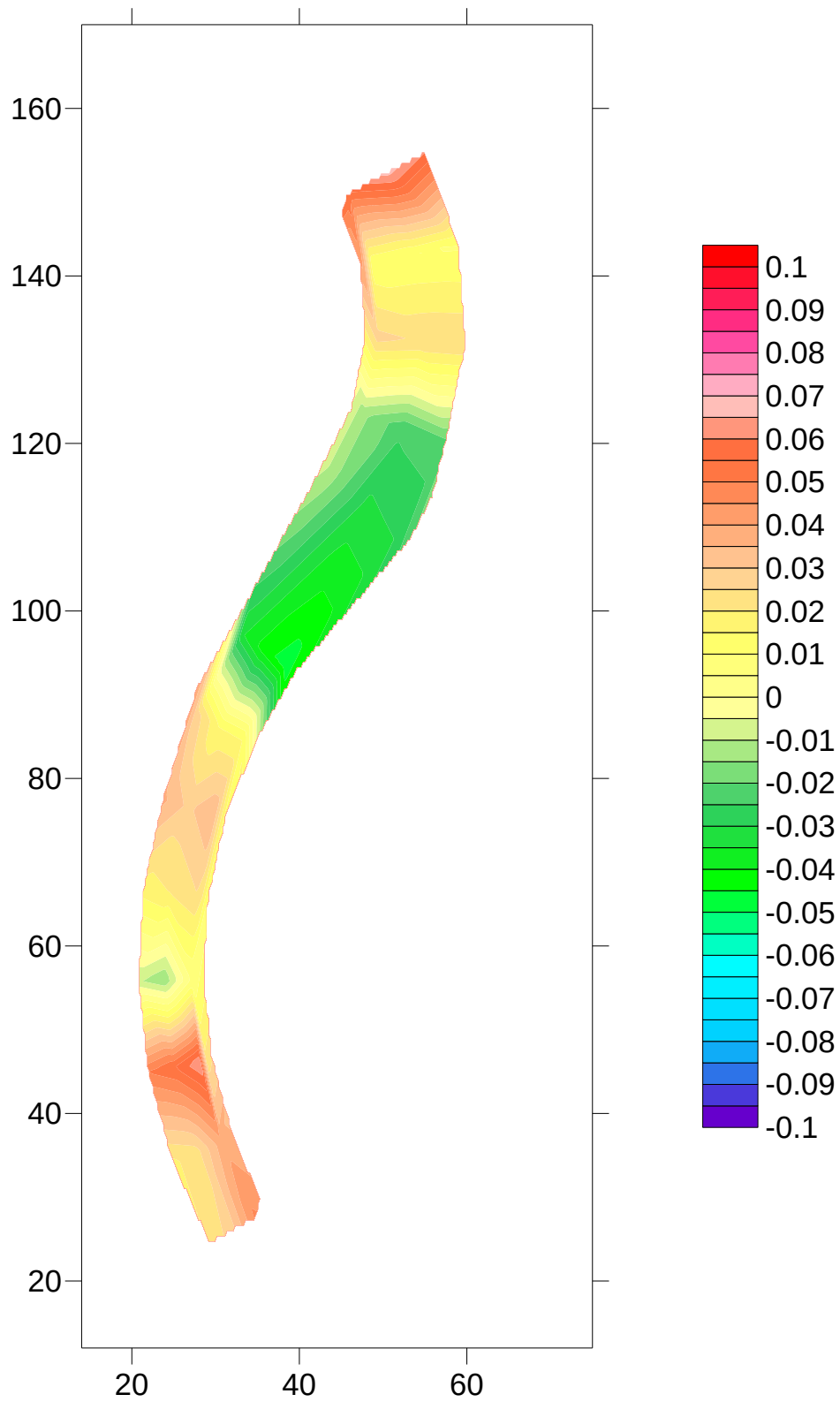


Figure D.1: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) Method for 8 cfs (psf)

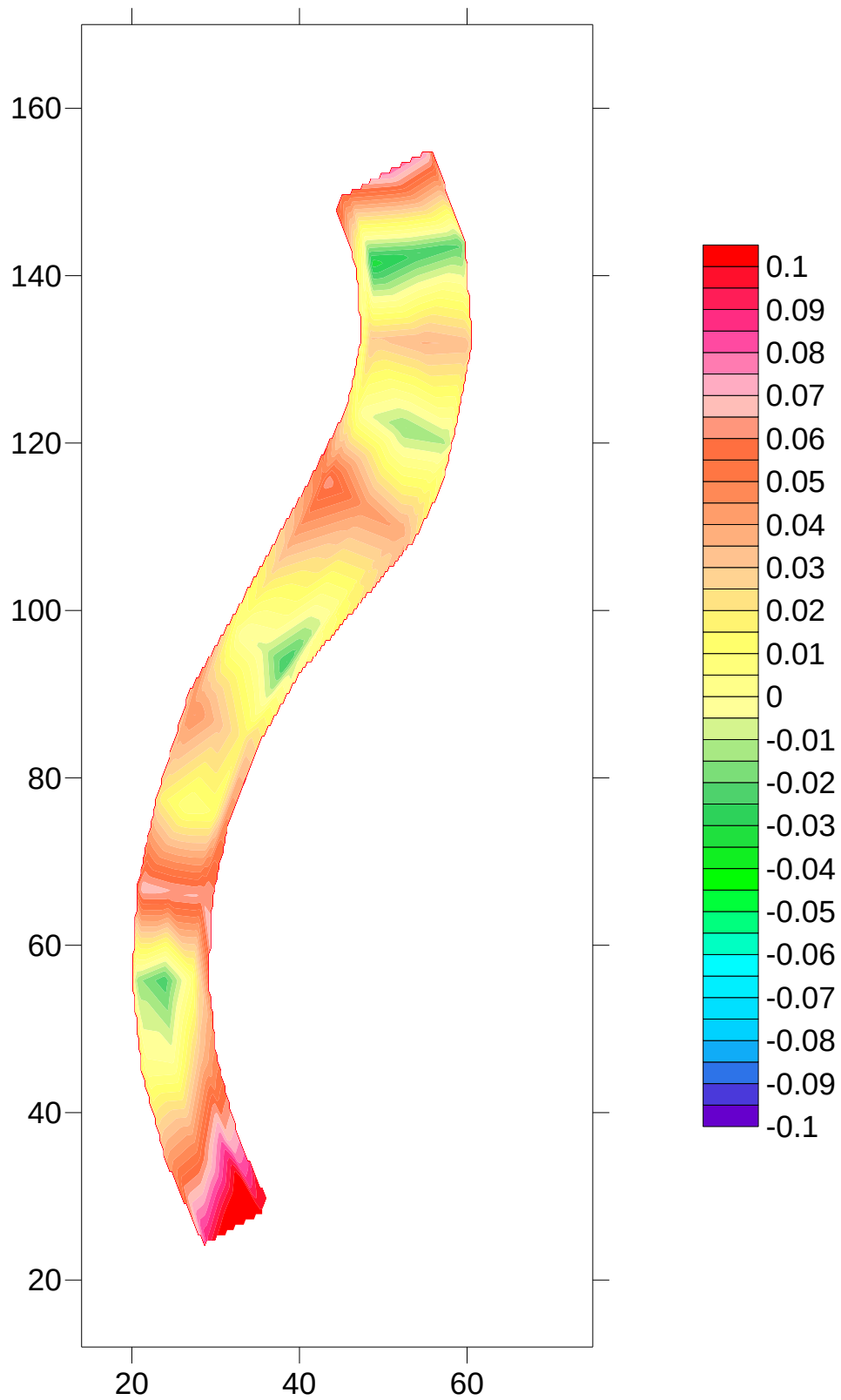


Figure D.2: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) Method for 12 cfs (psf)

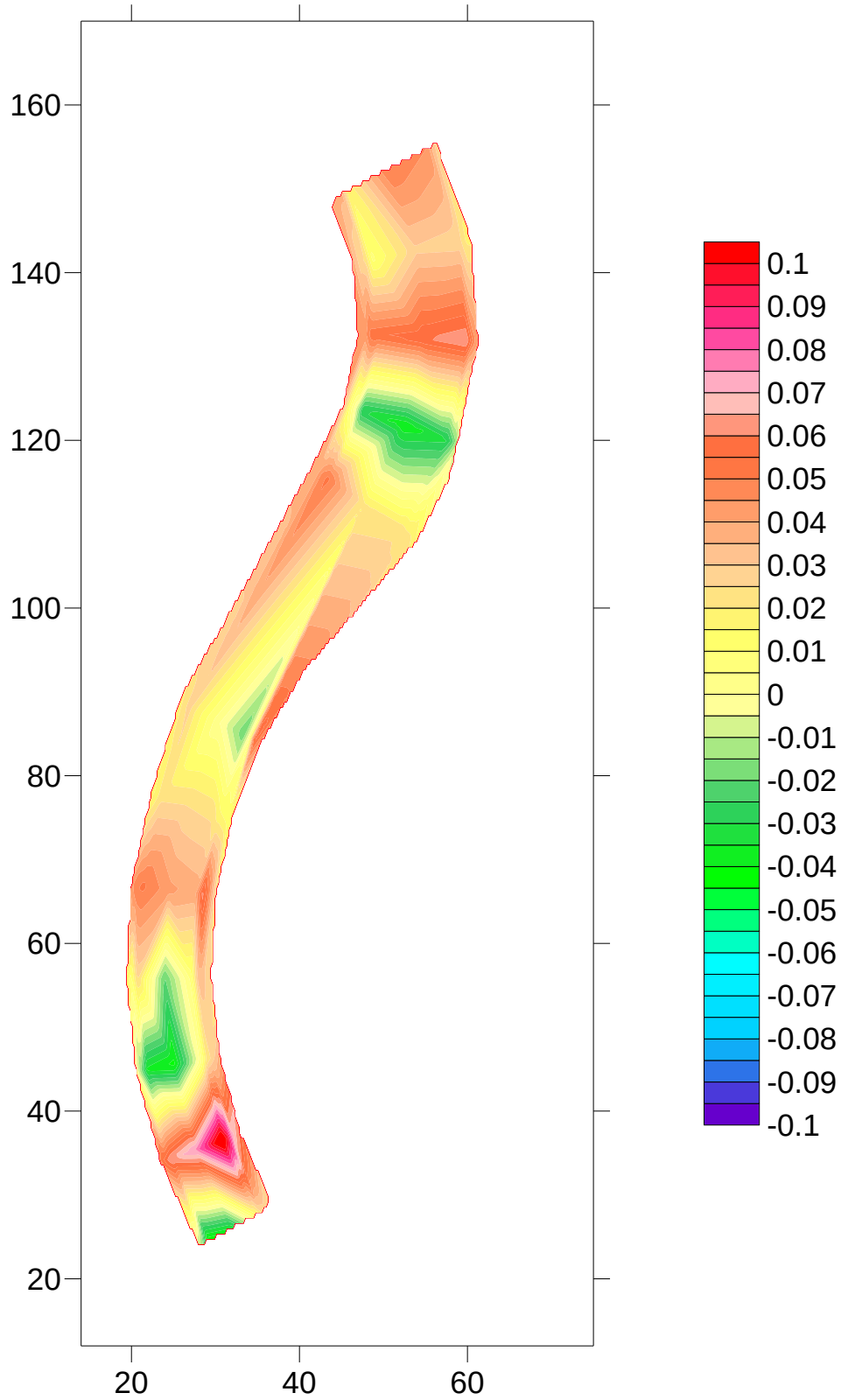


Figure D.3: Radial Shear Stress Distribution (τ_r) from Rozovskii (1961) Method for 16 cfs (psf)

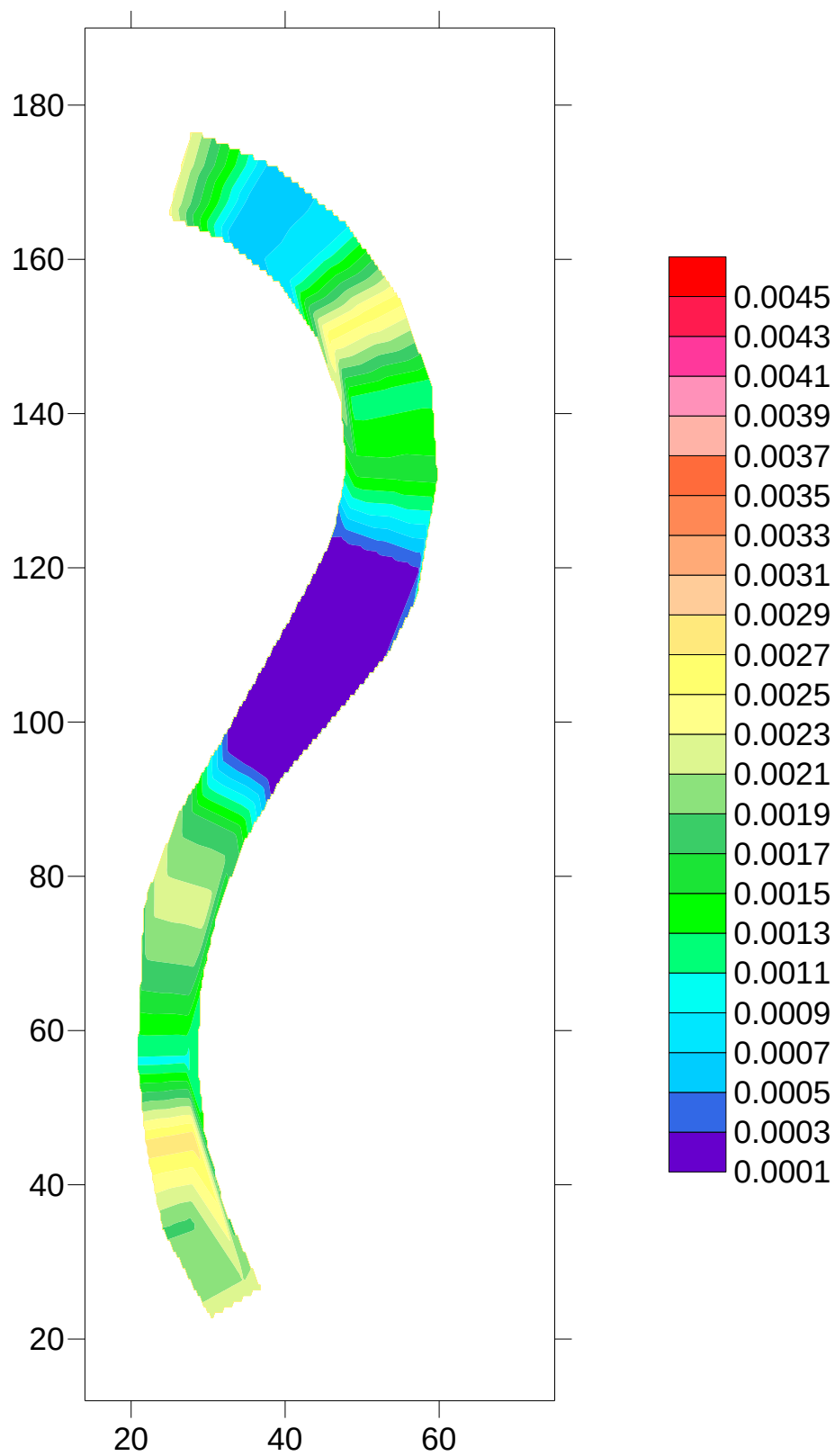


Figure D.4: Water Surface Slope Distribution for 8 cfs

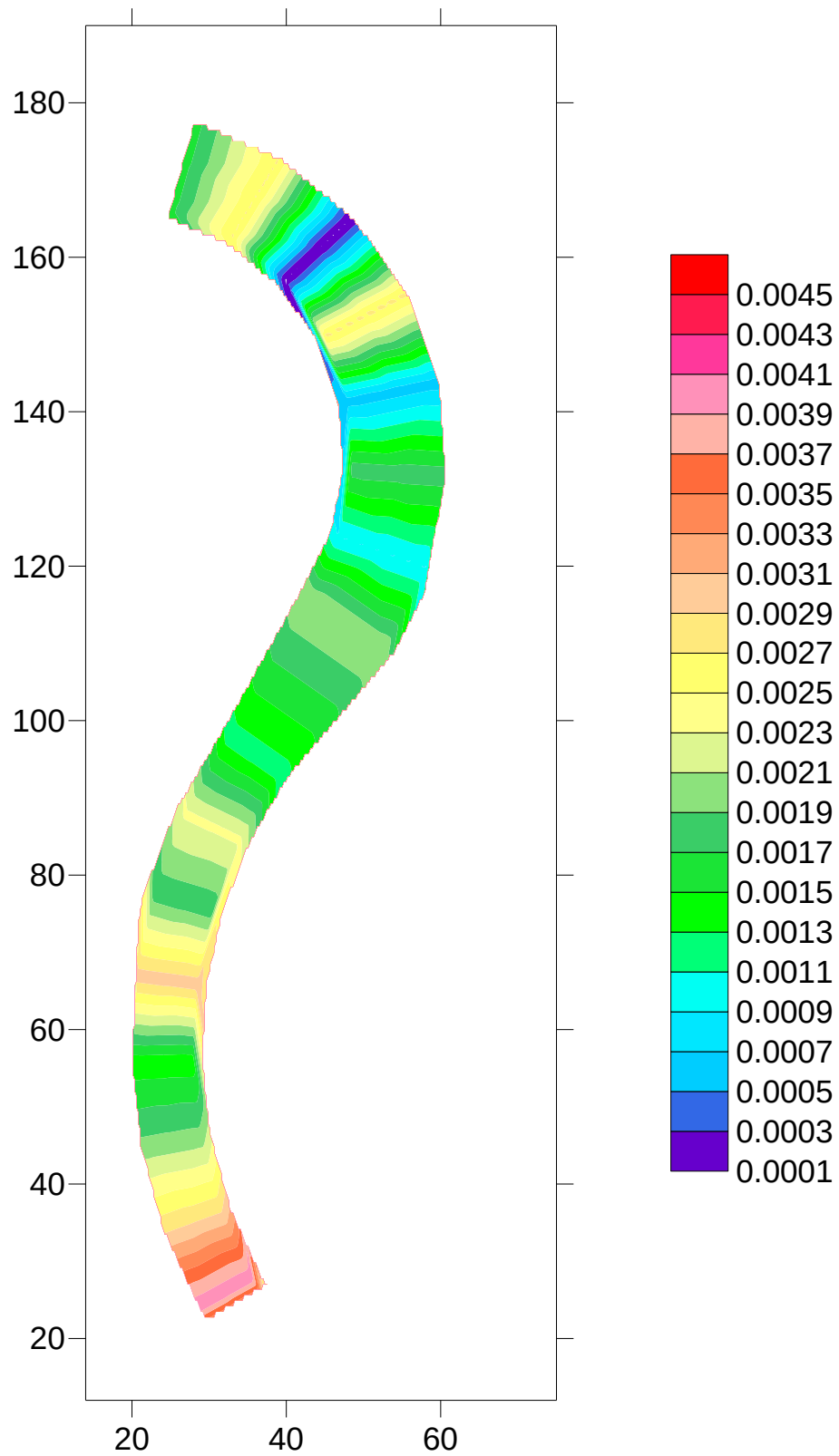


Figure D.5: Water Surface Slope Distribution for 12 cfs

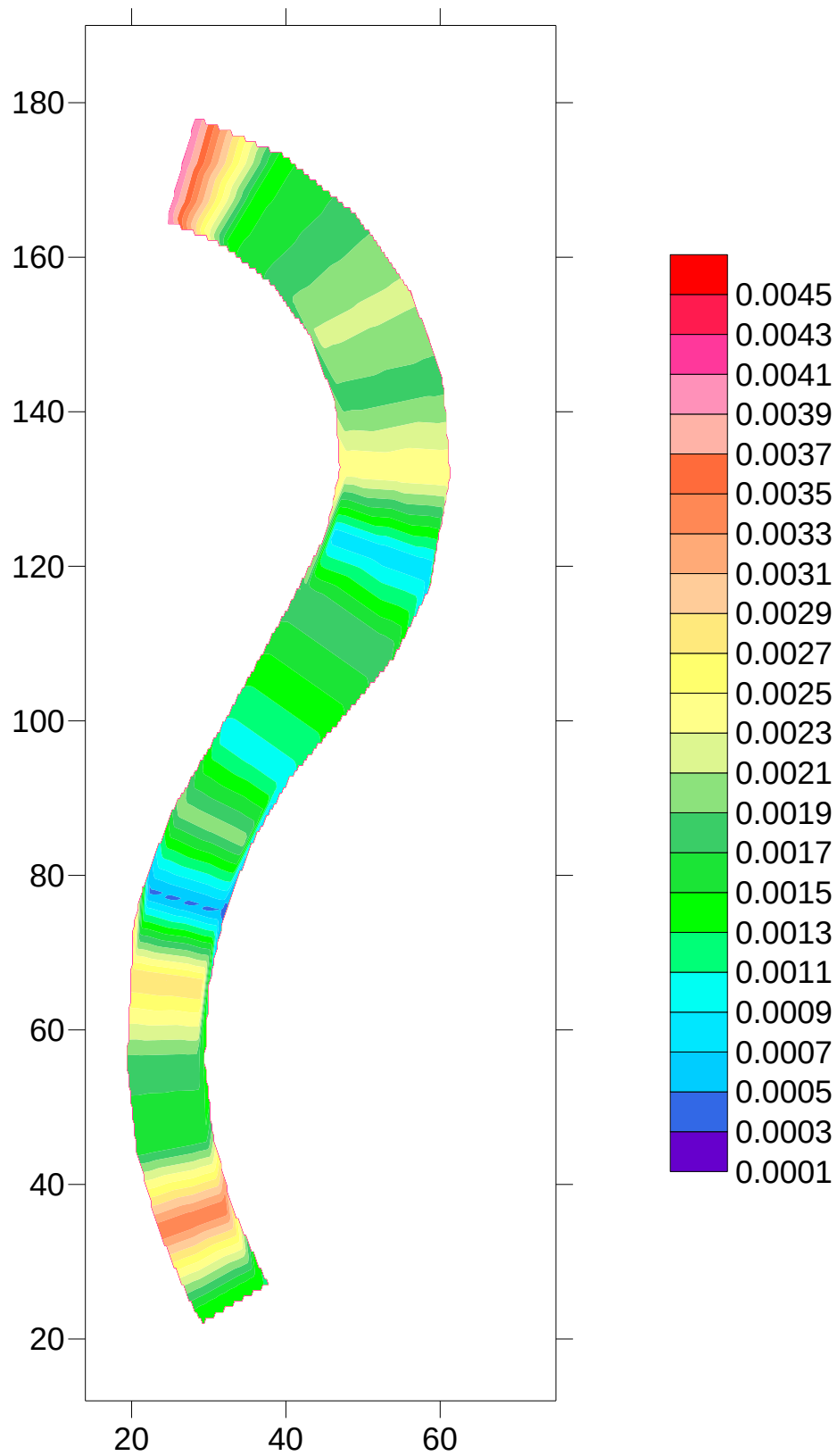


Figure D.6: Water Surface Slope Distribution for 16 cfs

APPENDIX E REYNOLDS SHEAR STRESS EXTRAPOLATION

This appendix is the summary of the calculation results from Reynolds shear stress extrapolation. Descriptions of seven of the columns in Table E.1 through Table E.9 are:

- Column 1 is the slope of linear regression equation of the Reynolds shear stress profile (τ_{xz}):
- Column 2 is the intercept with the y-axis of the linear regression equation;
- Column 3 is the Reynolds shear stress, obtained from linear regression;
- Columns 4 through 6 are the same parameters of the Reynolds shear stress; and
- Column 7 is bed shear stress computed from Equation 4.4.

If the Reynolds shear stress profile of τ_{xz} or τ_{zy} does not show similar trend with Figure 2.28, those data were presented as ‘out’ in Table E.1 through Table E.9.

Table E.1: Reynolds Shear Stress Calculation Summary for 8 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
4	c	-85.499	0.630	0.007	198.740	0.184	-0.001	0.007
4	d	out						
4	e	-74.965	0.508	0.007	-210.300	0.679	0.003	0.008
5	c	-159.730	0.668	0.004	980.910	0.295	0.000	0.004
5	d	out						
5	e	out						
6	c	-159.730	0.668	0.004	980.910	0.295	0.000	0.004
6	d	out						
6	e	out						
7	c	out						
7	d	out						
7	e	out						
8	c	out						
8	d	out						
8	e	-137.020	0.466	0.003	-1720.500	0.595	0.000	0.003
10	c	out						
10	d	-76.433	0.644	0.008	1017.000	-0.052	0.000	0.008
10	e	out						
11	c	-34.966	0.675	0.019	121.510	0.544	-0.004	0.020
11	d	out						
11	e	-56.983	0.607	0.011	-349.550	0.570	0.002	0.011
12	c	-62.656	0.723	0.012	835.890	2.459	-0.003	0.012
12	d	-60.051	0.756	0.013	-225.230	0.168	0.001	0.013
12	e	-59.604	0.717	0.012	-73.600	0.317	0.004	0.013
13	c	-45.212	0.588	0.013	113.030	0.669	-0.006	0.014
13	d	-51.017	0.872	0.017	-57.130	0.126	0.002	0.017
13	e	-67.681	0.779	0.012	-174.290	0.532	0.003	0.012
14	c	-203.800	1.299	0.006	-98.199	0.076	0.001	0.006
14	d	out						
14	e	-54.947	0.727	0.013	31.162	0.125	-0.004	0.014
15	c	-102.610	0.557	0.005	257.730	0.615	-0.002	0.006
15	d	-137.450	1.853	0.013	-150.880	0.060	0.000	0.013
15	e	-39.600	0.701	0.018	-97.857	0.285	0.003	0.018
16	c	out						
16	d	-32.438	0.732	0.023	237.930	0.655	-0.003	0.023
16	e	out						
17	c	out						
17	d	-43.757	0.693	0.016	-107.350	0.134	0.001	0.016
17	e	-47.680	0.741	0.016	-86.199	0.325	0.004	0.016

Table E.2: Reynolds Shear Stress Calculation Summary of Upstream Bend for 12 cfs (psf)

Cross Section	Column Piezo	1	2	3	4	5	6	7
		Slope	<i>b</i>	τ_{xz}	Slope	<i>b</i>	τ_{zy}	τ_{bed}
4	b	-19.205	0.432	0.022	158.380	0.382	-0.002	0.023
4	c	-39.107	0.517	0.013	-478.450	0.633	0.001	0.013
4	d	out						
4	e	-50.955	0.516	0.010	-98.052	0.584	0.006	0.012
4	f	out						
5	b	out						
5	c	-85.313	0.586	0.007	146.660	0.324	-0.002	0.007
5	d	-120.490	0.684	0.006	128.660	-0.016	0.000	0.006
5	e	-34.916	0.477	0.014	-144.470	0.672	0.005	0.014
5	f	-129.830	0.501	0.004	-188.650	0.607	0.003	0.005
6	b	-79.636	0.786	0.010	90.680	0.234	-0.003	0.010
6	c	out						
6	d	-137.420	0.942	0.007	95.307	0.125	-0.001	0.007
6	e	-76.112	0.631	0.008	-143.370	0.707	0.005	0.010
6	f	-111.540	0.448	0.004	241.340	0.089	0.000	0.004
7	b	-102.310	0.677	0.007	96.070	0.284	-0.003	0.007
7	c	out						
7	d	-66.081	0.695	0.011	-62.375	0.747	0.012	0.016
7	e	-61.544	0.597	0.010	-145.940	0.575	0.004	0.010
7	f	out						
8	b	-65.609	0.501	0.008	73.638	0.306	-0.004	0.009
8	c	-118.150	0.598	0.005	318.300	0.370	-0.001	0.005
8	d	-127.520	0.785	0.006	173.020	-0.142	0.001	0.006
8	e	-122.680	0.550	0.004	-247.340	0.631	0.003	0.005
8	f	-180.860	0.420	0.002	337.130	0.045	0.000	0.002

Table E.3: Reynolds Shear Stress Calculation Summary of Downstream Bend for 12 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
10	b	-53.036	0.636	0.012	160.080	0.477	-0.003	0.012
10	c	-43.527	0.422	0.010	179.770	0.405	-0.002	0.010
10	d	-90.334	0.663	0.007	230.060	0.036	0.000	0.007
10	e	out						
10	f	-69.233	0.514	0.007	139.520	0.029	0.000	0.007
11	b	out						
11	c	-36.108	0.612	0.017	170.530	0.563	-0.003	0.017
11	d	-60.498	0.652	0.011	69.190	0.332	-0.005	0.012
11	e	-86.993	0.718	0.008	-621.370	0.867	0.001	0.008
11	f	out						
12	b	out						
12	c	out						
12	d	-46.024	0.760	0.017	396.510	0.309	-0.001	0.017
12	e	-52.546	0.696	0.013	-59.134	0.345	0.006	0.014
12	f	out						
13	b	-46.268	0.459	0.010	-90.676	0.120	0.001	0.010
13	c	-30.808	0.640	0.021	72.360	0.658	-0.009	0.023
13	d	-60.780	1.010	0.017	192.070	0.667	-0.003	0.017
13	e	-46.912	0.731	0.016	-114.080	0.461	0.004	0.016
13	f	-32.038	0.489	0.015	-57.870	0.429	0.007	0.017
14	b	out						
14	c	-35.097	0.556	0.016	86.888	0.707	-0.008	0.018
14	d	-48.936	1.025	0.021	230.900	1.032	-0.004	0.021
14	e	-65.275	0.891	0.014	-74.848	0.373	0.005	0.015
14	f	out						
15	b	out						
15	c	out						
15	d	out						
15	e	-63.118	0.719	0.011	-93.491	0.488	0.005	0.013
15	f	out						
16	b	-37.227	0.414	0.011	-179.480	0.009	0.000	0.011
16	c	-32.074	0.477	0.015	179.940	0.795	-0.004	0.016
16	d	-27.345	0.862	0.032	92.003	0.884	-0.010	0.033
16	e	-42.445	0.846	0.020	-68.900	0.533	0.008	0.021
16	f	out						
17	b	-59.835	0.456	0.008	355.190	0.734	-0.002	0.008
17	c	out						
17	d	out						
17	e	-33.915	0.605	0.018	-98.617	0.680	0.007	0.019
17	f	out						

Table E.4: Reynolds Shear Stress Calculation Summary of Upstream Bend for 16 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
4	a	-41.251	0.597	0.014	15.382	0.298	-0.019	0.024
4	b	-23.399	0.376	0.016	135.440	0.323	-0.002	0.016
4	c	-13.906	0.263	0.019	28.790	0.201	-0.007	0.020
4	d	-39.013	0.409	0.010	-201.050	0.865	0.004	0.011
4	e	-33.259	0.419	0.013	-74.613	0.452	0.006	0.014
4	f	out						
4	g	out						
5	a	-110.960	1.055	0.010	43.892	0.342	-0.008	0.012
5	b	-41.011	0.480	0.012	96.719	0.312	-0.003	0.012
5	c	out						
5	d	out						
5	e	-64.555	0.566	0.009	-86.411	0.509	0.006	0.011
5	f	out						
5	g	out						
6	a	out						
6	b	-80.810	0.628	0.008	180.710	0.577	-0.003	0.008
6	c	-74.016	0.485	0.007	125.430	0.272	-0.002	0.007
6	d	out						
6	e	-43.589	0.539	0.012	-71.690	0.491	0.007	0.014
6	f	-99.020	0.574	0.006	-113.330	0.611	0.005	0.008
6	g	-171.370	0.475	0.003	407.430	-0.153	0.000	0.003
7	a	out						
7	b	out						
7	c	-47.538	0.398	0.008	78.854	0.170	-0.002	0.009
7	d	out						
7	e	-67.955	0.522	0.008	-189.170	0.679	0.004	0.008
7	f	out						
7	g	out						
8	a	-27.059	0.440	0.016	227.180	0.603	-0.003	0.016
8	b	-89.206	0.679	0.008	172.470	0.312	-0.002	0.008
8	c	out						
8	d	-66.188	0.546	0.008	80.191	0.055	-0.001	0.008
8	e	-66.734	0.470	0.007	-198.710	0.585	0.003	0.008
8	f	out						
8	g	out						

Table E.5: Reynolds Shear Stress Calculation Summary of Cross section 10 through Cross section 15 for 16 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
10	a	-49.208	0.695	0.014	-145.130	0.058	0.000	0.014
10	b	-21.206	0.404	0.019	202.930	0.522	-0.003	0.019
10	c	-15.326	0.323	0.021	212.810	0.433	-0.002	0.021
10	d	-56.462	0.696	0.012	-207.880	0.890	0.004	0.013
10	e	-70.256	0.529	0.008	-298.600	0.701	0.002	0.008
10	f	-42.268	0.399	0.009	151.360	0.093	-0.001	0.009
10	g	out						
11	a	-27.333	0.462	0.017	-100.770	-0.034	0.000	0.017
11	b	-19.721	0.669	0.034	93.748	0.621	-0.007	0.035
11	c	-19.130	0.468	0.024	87.194	0.414	-0.005	0.025
11	d	-53.413	0.694	0.013	175.230	0.350	-0.002	0.013
11	e	out						
11	f	-41.670	0.681	0.016	-155.000	0.514	0.003	0.017
11	g	out						
12	a	-347.220	2.035	0.006	-110.420	0.078	0.001	0.006
12	b	-25.429	0.542	0.021	-56.215	-0.068	-0.001	0.021
12	c	-31.288	0.658	0.021	98.299	0.610	-0.006	0.022
12	d	out						
12	e	-44.729	0.626	0.014	-60.891	0.271	0.004	0.015
12	f	-20.463	0.458	0.022	-78.113	0.378	0.005	0.023
12	g	out						
13	a	-94.605	0.631	0.007	260.210	0.570	-0.002	0.007
13	b	out						
13	c	-17.961	0.611	0.034	78.781	0.799	-0.010	0.035
13	d	-21.909	0.745	0.034	133.100	0.640	-0.005	0.034
13	e	-30.917	0.700	0.023	-103.550	0.350	0.003	0.023
13	f	out						
13	g	out						
14	a	-61.407	0.601	0.010	216.920	0.581	-0.003	0.010
14	b	-49.525	0.582	0.012	-98.922	-0.051	-0.001	0.012
14	c	-19.658	0.549	0.028	146.640	0.817	-0.006	0.028
14	d	-24.346	0.824	0.034	-65.408	0.159	0.002	0.034
14	e	-31.980	0.788	0.025	-86.134	0.370	0.004	0.025
14	f	out						
14	g	out						
15	a	out						
15	b	-47.069	0.432	0.009	60.839	0.368	-0.006	0.011
15	c	-39.447	0.548	0.014	145.200	0.712	-0.005	0.015
15	d	-27.061	0.833	0.031	-67.040	0.211	0.003	0.031
15	e	-26.746	0.666	0.025	-76.329	0.436	0.006	0.026
15	f	out						
15	g	out						

Table E.6: Reynolds Shear Stress Calculation Summary of Cross Section 16 and Cross Section 17 for 16 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
16	a	-47.288	0.458	0.010	-199.770	0.175	0.001	0.010
16	b	-46.247	0.504	0.011	179.810	0.834	-0.005	0.012
16	c	-29.265	0.577	0.020	176.080	0.710	-0.004	0.020
16	d	-35.927	1.099	0.031	48.576	0.864	-0.018	0.035
16	e	-36.451	0.896	0.025	-30.185	0.325	0.011	0.027
16	f	out						
16	g	out						
17	a	out						
17	b	-53.205	0.434	0.008	128.620	0.600	-0.005	0.009
17	c	-24.473	0.407	0.017	113.890	0.580	-0.005	0.017
17	d	-28.354	0.889	0.031	57.236	0.981	-0.017	0.036
17	e	out						
17	f	out						
17	g	out						

Table E.7: Reynolds Shear Stress Calculation Summary of Upstream Bend for 20 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
4	a	-37.102	0.449	0.012	15.536	0.253	-0.016	0.020
4	b	-85.760	0.415	0.005	-32.778	0.297	0.009	0.010
4	c	-61.512	0.478	0.008	118.180	0.217	-0.002	0.008
4	d	-68.330	0.666	0.010	-105.620	0.648	0.006	0.012
4	e	-29.445	0.534	0.018	-57.002	0.702	0.012	0.022
4	f	out						
4	g	out						
5	a	-72.948	0.603	0.008	7.410	0.306	-0.041	0.042
5	b	out						
5	c	-64.419	0.483	0.008	-193.940	0.401	0.002	0.008
5	d	-95.145	0.838	0.009	-103.370	1.020	0.010	0.013
5	e	-27.537	0.394	0.014	-59.209	0.578	0.010	0.017
5	f	out						
5	g	out						
6	a	-71.402	0.747	0.010	20.729	0.252	-0.012	0.016
6	b	-6.582	0.317	0.048	-60.666	0.314	0.005	0.048
6	c	-48.443	0.410	0.008	-395.840	0.541	0.001	0.009
6	d	out						
6	e	-49.203	0.551	0.011	-78.175	0.668	0.009	0.014
6	f	-60.304	0.536	0.009	-103.610	0.698	0.007	0.011
6	g	-95.615	0.404	0.004	107.930	0.018	0.000	0.004
7	a	-28.184	0.502	0.018	182.830	0.487	-0.003	0.018
7	b	-14.635	0.351	0.024	-268.680	0.398	0.001	0.024
7	c	-37.299	0.415	0.011	-696.570	0.714	0.001	0.011
7	d	-64.736	0.654	0.010	-54.499	0.918	0.017	0.020
7	e	-63.306	0.528	0.008	-91.264	0.687	0.008	0.011
7	f	-76.751	0.589	0.008	-120.930	0.714	0.006	0.010
7	g	out						
8	a	out						
8	b	-63.233	0.583	0.009	77.498	0.215	-0.003	0.010
8	c	-53.066	0.387	0.007	-366.460	0.839	0.002	0.008
8	d	-80.959	0.686	0.008	-53.388	0.827	0.015	0.018
8	e	-58.357	0.443	0.008	-133.030	0.700	0.005	0.009
8	f	-121.830	0.526	0.004	-152.670	0.659	0.004	0.006
8	g	out						

Table E.8: Reynolds Shear Stress Calculation Summary of Cross Section 10 through Cross Section 15 for 20 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
10	a	-34.832	0.713	0.020	-269.780	-0.042	0.000	0.020
10	b	-45.341	0.636	0.014	924.420	0.720	-0.001	0.014
10	c	-8.296	0.338	0.041	198.940	0.386	-0.002	0.041
10	d	-23.410	0.555	0.024	-264.620	1.107	0.004	0.024
10	e	-73.351	0.596	0.008	-181.650	0.681	0.004	0.009
10	f	-84.321	0.531	0.006	-316.250	0.904	0.003	0.007
10	g	-73.644	0.576	0.008	-388.540	1.040	0.003	0.008
11	a	out						
11	b	out						
11	c	-21.767	0.505	0.023	139.750	0.379	-0.003	0.023
11	d	out						
11	e	-49.223	0.692	0.014	-92.230	0.514	0.006	0.015
11	f	out						
11	g	out						
12	a	-15.267	0.522	0.034	-64.990	0.120	0.002	0.034
12	b	out						
12	c	-22.967	0.661	0.029	127.750	0.627	-0.005	0.029
12	d	-33.407	0.757	0.023	230.370	0.414	-0.002	0.023
12	e	out						
12	f	out						
12	g	-46.766	0.729	0.016	-41.799	0.353	0.008	0.018
13	a	-58.472	0.526	0.009	-90.486	0.049	0.001	0.009
13	b	out						
13	c	-15.042	0.671	0.045	70.757	0.694	-0.010	0.046
13	d	-22.680	0.684	0.030	99.242	0.598	-0.006	0.031
13	e	-47.751	0.780	0.016	289.290	0.465	-0.002	0.016
13	f	-37.787	0.769	0.020	-349.480	0.734	0.002	0.020
13	g	out						
14	a	-59.165	0.562	0.010	-105.600	0.014	0.000	0.010
14	b	-35.737	0.576	0.016	76.763	0.694	-0.009	0.018
14	c	-17.143	0.610	0.036	67.368	0.684	-0.010	0.037
14	d	-23.166	0.768	0.033	91.865	0.719	-0.008	0.034
14	e	-36.839	0.813	0.022	209.000	0.615	-0.003	0.022
14	f	out						
14	g	out						
15	a	-90.056	0.619	0.007	194.090	0.704	-0.004	0.008
15	b	-46.539	0.653	0.014	107.430	0.858	-0.008	0.016
15	c	-23.598	0.572	0.024	102.620	0.745	-0.007	0.025
15	d	-13.616	0.767	0.056	48.387	0.722	-0.015	0.058
15	e	-37.585	0.828	0.022	223.340	0.841	-0.004	0.022
15	f	-24.744	0.765	0.031	-104.440	0.505	0.005	0.031
15	g	out						

Table E.9: Reynolds Shear Stress Calculation Summary of Cross Section 16 and Cross Section 17 for 20 cfs (psf)

Cross Section	Column Piezo	1 Slope	2 b	3 τ_{xz}	4 Slope	5 b	6 τ_{zy}	7 τ_{bed}
16	a	-76.102	0.555	0.007	336.330	1.135	-0.003	0.008
16	b	-48.828	0.522	0.011	223.790	0.893	-0.004	0.011
16	c	-25.677	0.594	0.023	78.620	0.760	-0.010	0.025
16	d	out						
16	e	out						
16	f	-35.273	0.840	0.024	-34.175	0.443	0.013	0.027
16	g	out						
17	a	out						
17	b	-53.979	0.571	0.011	60.137	0.612	-0.010	0.015
17	c	-45.774	0.534	0.012	128.750	0.709	-0.006	0.013
17	d	out						
17	e	-23.885	0.678	0.028	-34.993	0.272	0.008	0.029
17	f	out						
17	g	out						

**APPENDIX F COMPARISON OF SHEAR STRESS FROM LINEAR
REGRESSION OF THE FLOW VELOCITY PROFILE FOR THE
LATERAL DIRECTION (τ_y) AND ROZOVSKII METHOD**

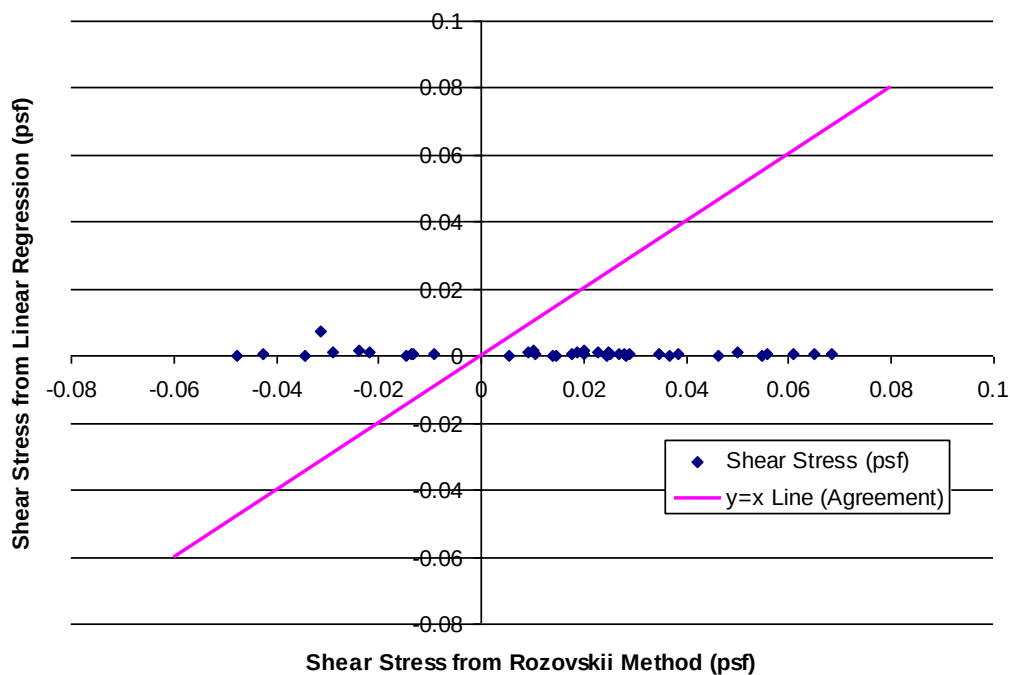


Figure F.1: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 8 cfs

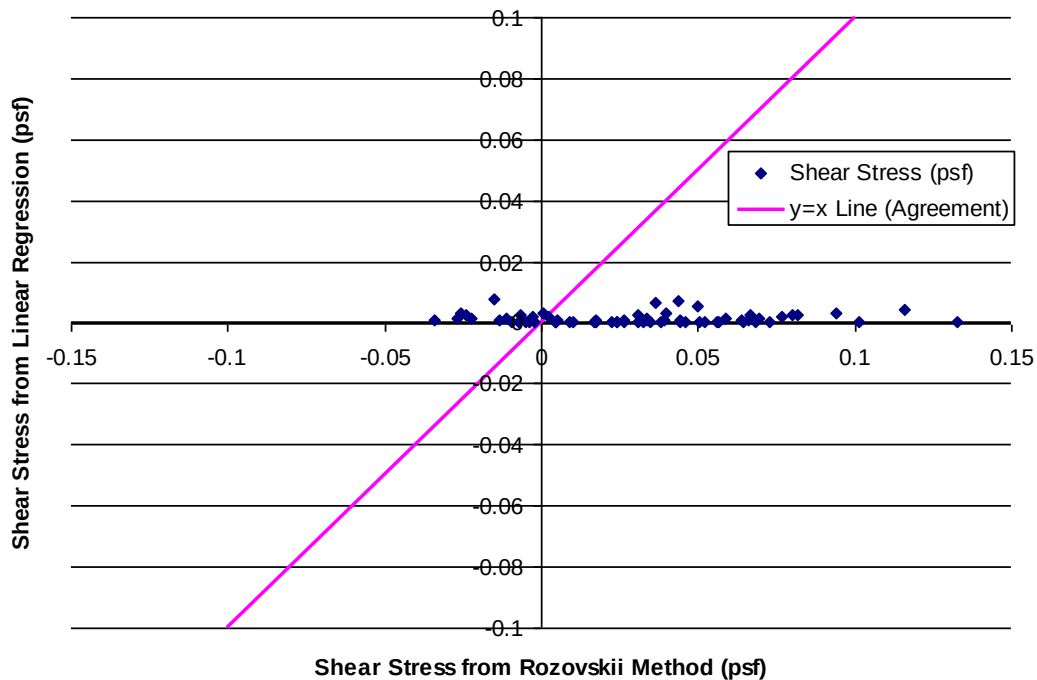


Figure F.2: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 12 cfs

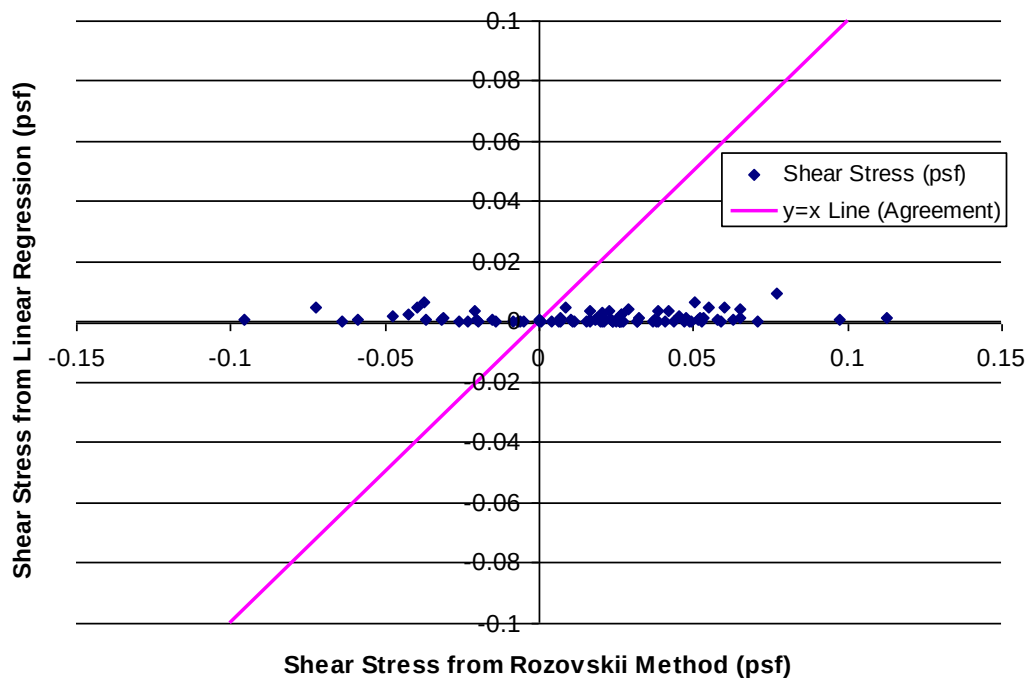


Figure F.3: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 16 cfs

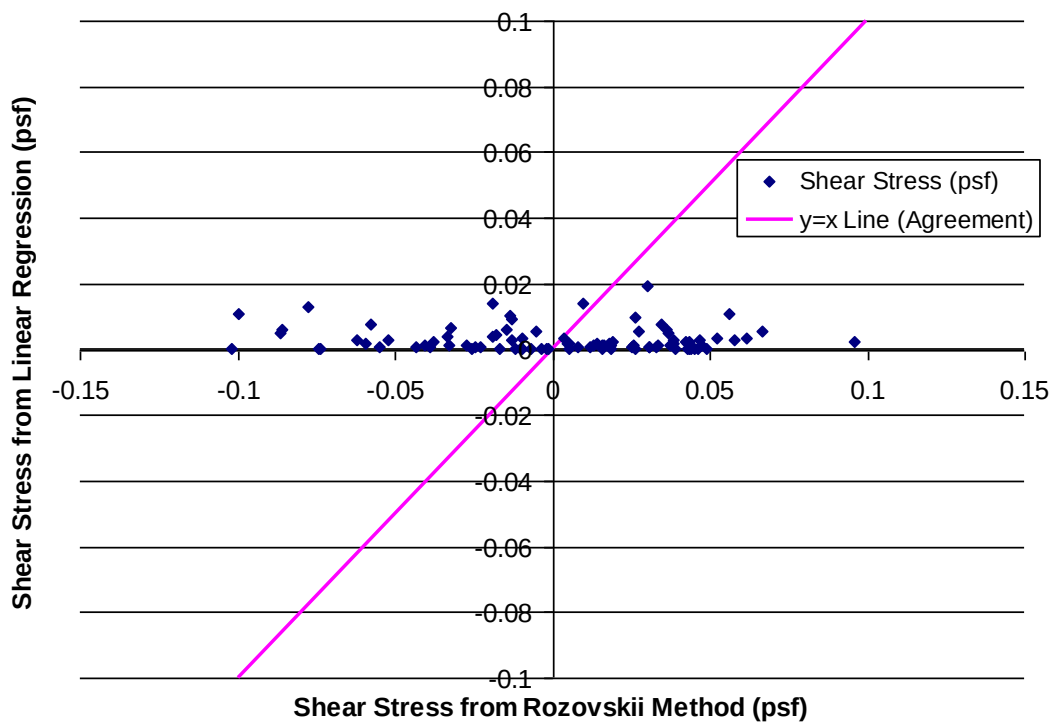


Figure F.4: Comparison of Shear Stress from the Preston Tube and Rozovskii Methods for 20 cfs

APPENDIX G COMPARISON OF SHEAR STRESS MAXIMUM VALUES

To compare shear stress calculation methods for different angles, maximum shear stress values from different methods are plotted in this appendix. The blue lines in the plots show the averaged shear stress value from HEC-RAS. In the upstream bend, shear stress from HEC-RAS in Cross section 1 and Cross section 2 are averaged. For the downstream bend, shear stresses from HEC-RAS in Cross section 10 and Cross section 11 are averaged. Theoretically, the order of maximum value of shear stress should be: shear stress from the Preston tube $\rightarrow$ shear stress from Reynolds shear stress extrapolation $\rightarrow$ shear stress from linear regression because of the measurement method. The plots and prediction of the order of maximum shear stress agreed well, except for the 20 cfs discharge in the upstream bend.

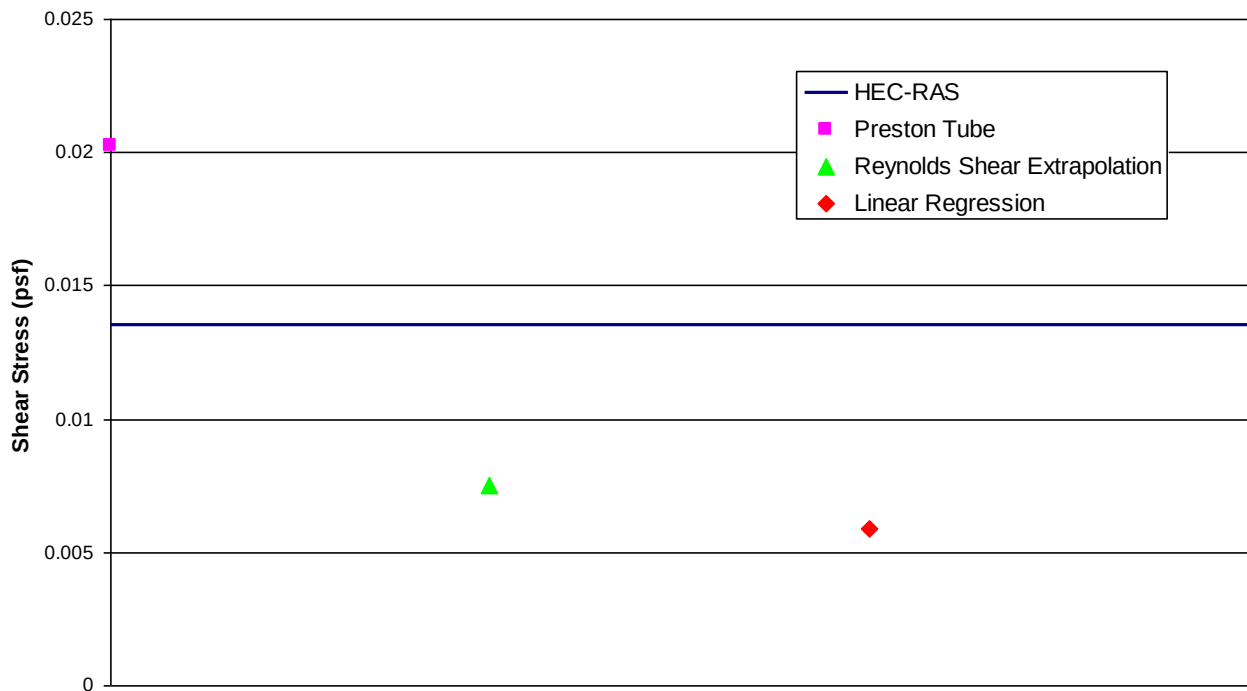


Figure G.1: Comparison of Shear Stress Maximum Values (8 cfs Upstream)

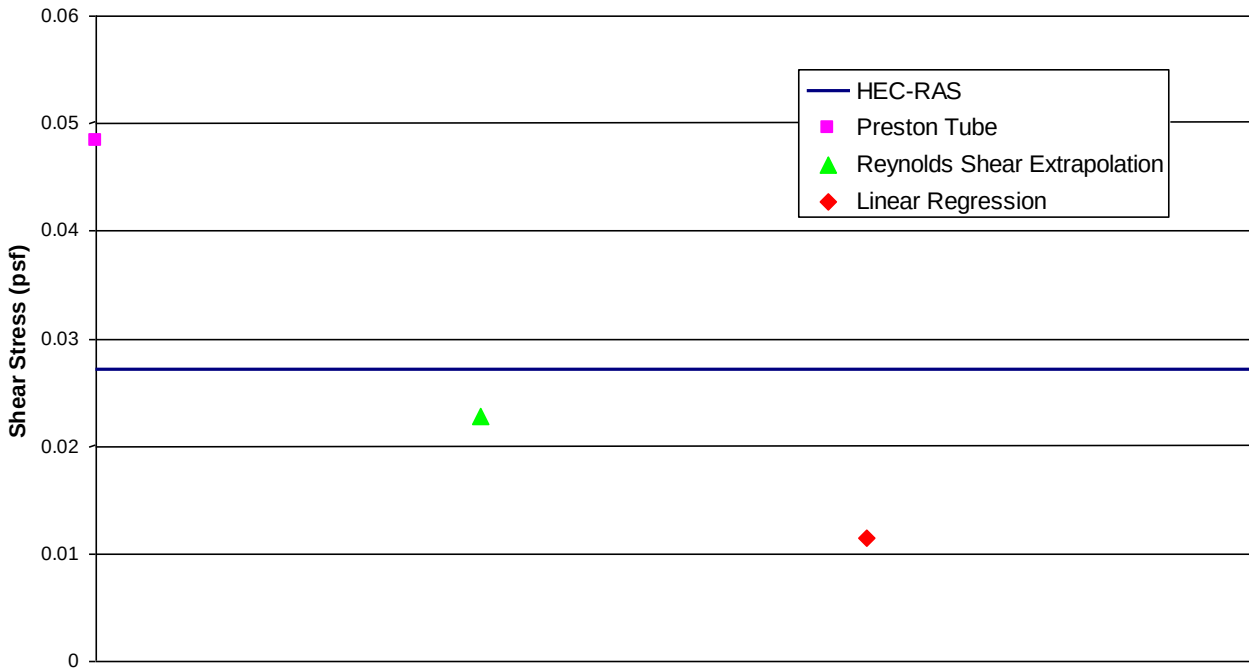


Figure G.2: Comparison of Shear Stress Maximum Values (8 cfs Downstream)

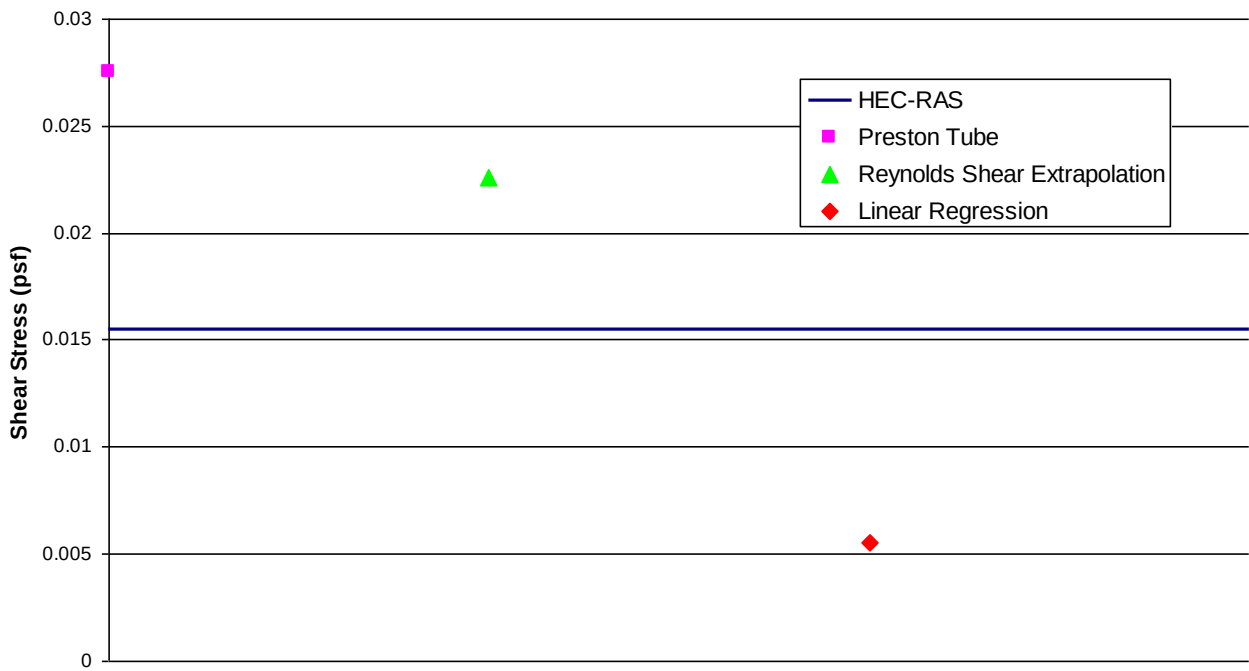


Figure G.3: Comparison of Shear Stress Maximum Values (12 cfs Upstream)

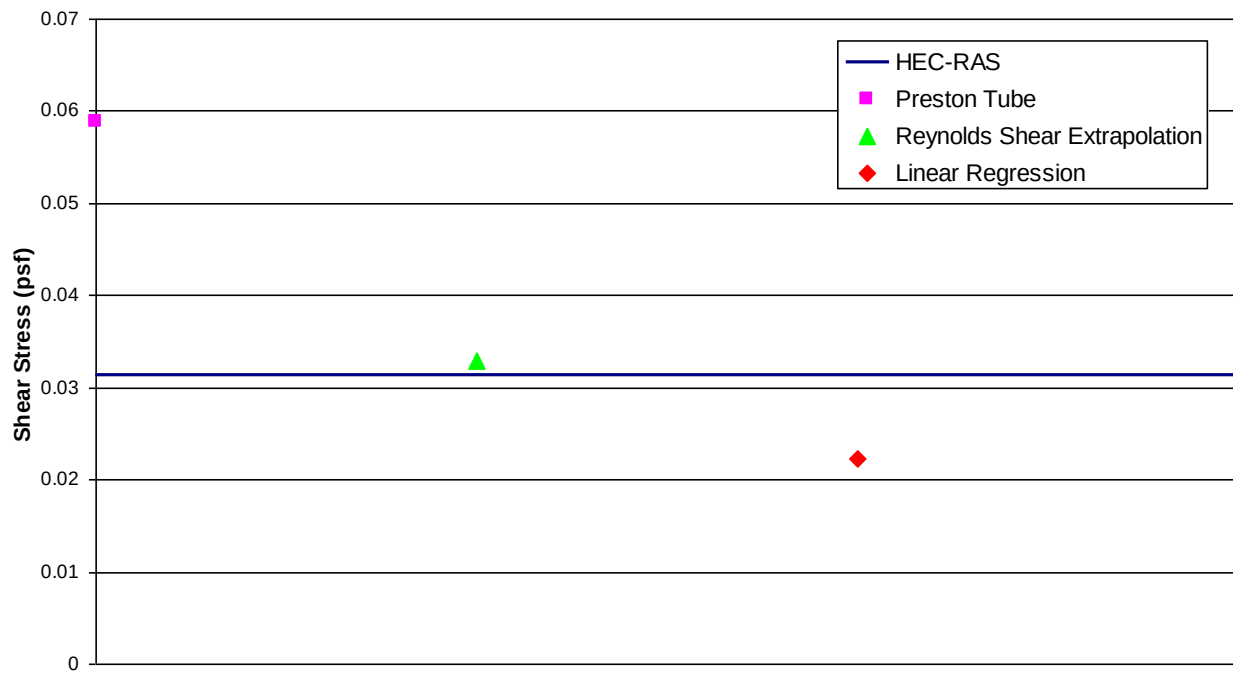


Figure G.4: Comparison of Shear Stress Maximum Values (12 cfs Downstream)

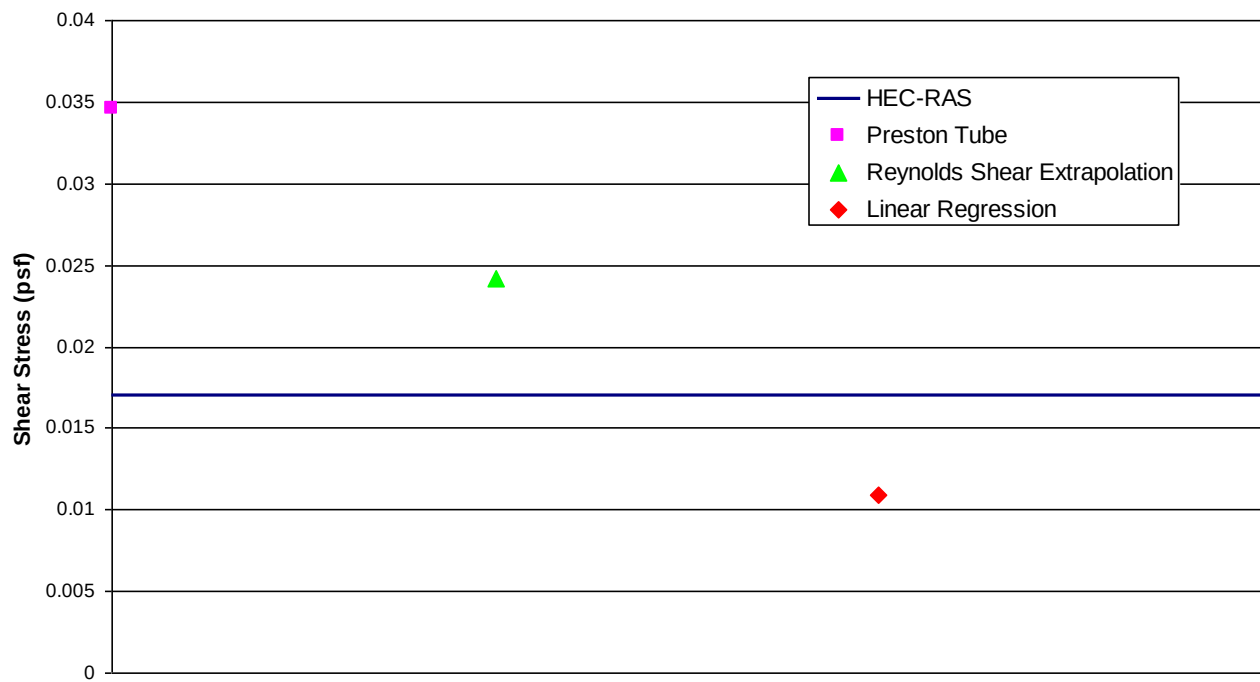


Figure G.5: Comparison of Shear Stress Maximum Values (16 cfs Upstream)

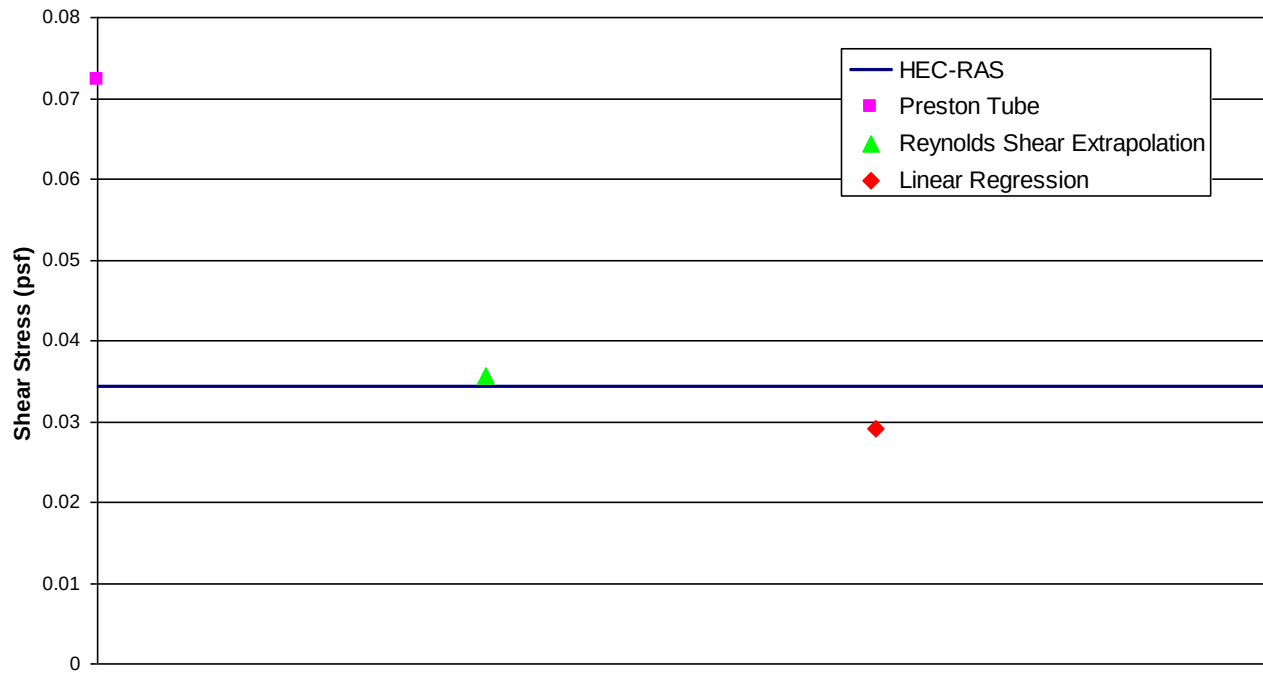


Figure G.6: Comparison of Shear Stress Maximum Values (16 cfs Downstream)

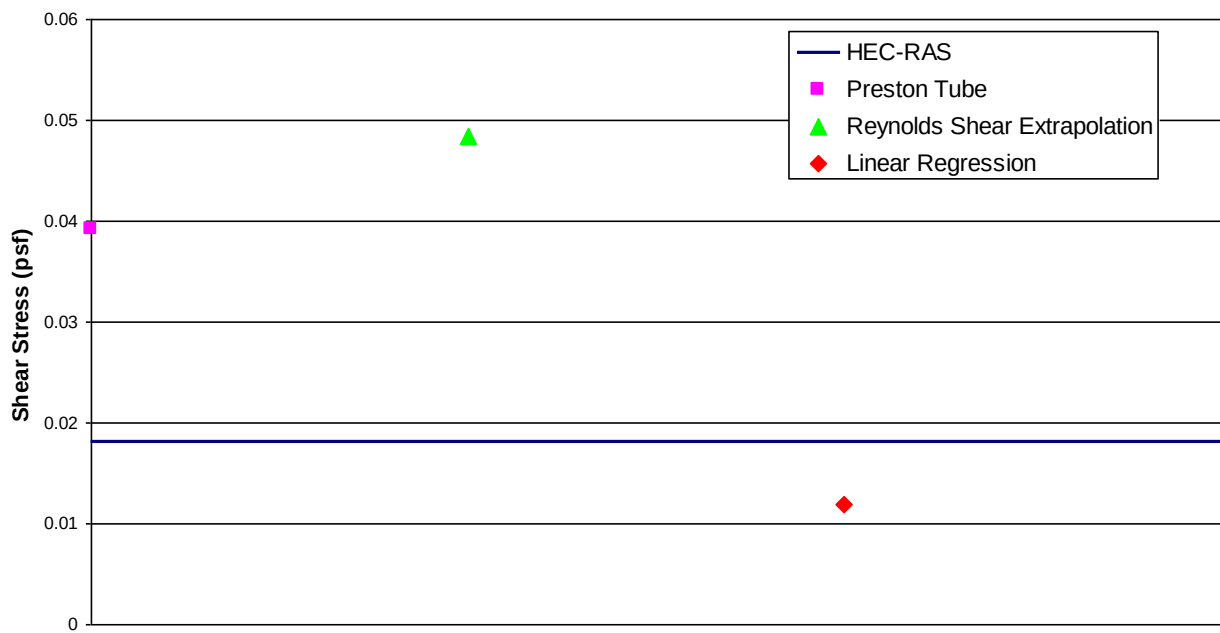


Figure G.7: Comparison of Shear Stress Maximum Values (20 cfs Upstream)

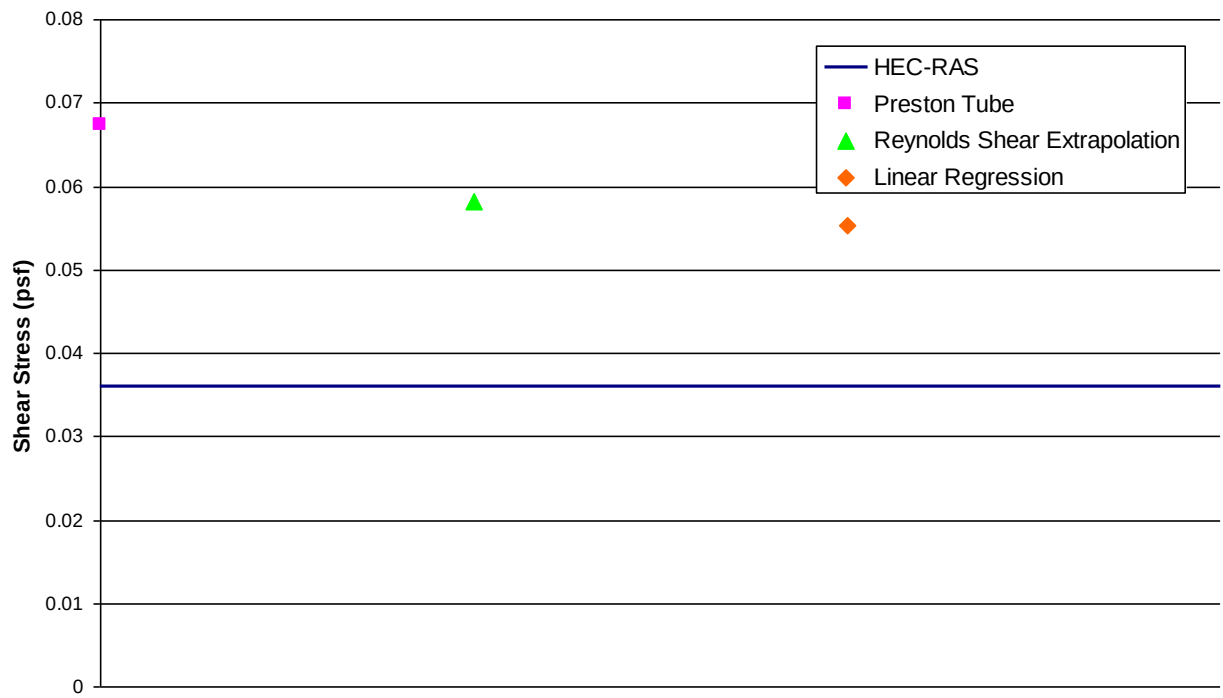


Figure G.8: Comparison of Shear Stress Maximum Values (20 cfs Downstream)