SAMPLE SIZES AND CONFIDENCE INTERVALS FOR WILDLIFE POPULATION RATIOS

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One of the most common types of data used in applied wildlife management is the population ratio. These data are used to gauge reproductive success and the effect of selective harvesting of wildlife.

A common problem facing wildlife managers is determining how many animals need to be classified in order to obtain an acceptable estimate of these population ratios. A closely related question is the determination of confidence intervals around the ratios obtained.

Given a population of animals there are 2 general sampling strategies for collecting data to estimate population ratios. One approach assumes that the area occupied by the population can be divided into quadrats (plots—perhaps belt transects) and the quadrats are randomly sampled within a given habitat type, i.e., the quadrat is the sampling unit. A survey (census) of the animals is made on each of the sampled quadrats and the numbers of (e.g.) fawns and does are recorded. In this case, the population ratio is estimated as usual while its precision is obtained according to "classical finite random sample theory," see for example, Cochran (1977). Sample size formulas exist for the necessary number of quadrats to survey. However, in practice, such a design is often impractical when faced with the reality of time, budgetary, and planning constraints. Also, the sample size formulas depend on estimates of the sample variances which often are not available until the data are at least partially collected. The computations necessary for determination of whether or not an adequate sample size has been obtained are rather tedious and do not lend themselves to quick review while in the field.

A second approach, which is the subject of this discussion, considers the individual animal as a unit. This procedure avoids most of the above-mentioned problems. However, the user must be willing to meet the following requirements which are listed in order of importance:

1. The population is clearly defined.
2. The individual animals are sampled in a random and unbiased manner. This is difficult to accomplish in applied management; the problem can only be addressed superficially in this paper as application will vary with species and habitat. Essentially the requirement is that each animal in the population has an equal and independent chance of being "sampled." Extreme effort should be made in design of the study to meet this condition. If the sampling is biased, e.g., for "too many" does in estimation of a buck : doe ratio, then naive application of the methodology presented will not help matters. The final ratios are still biased!
3. An upper limit on the population size is available, e.g., the manager must be willing to estimate that there are no more than $N = 5{,}000$ bucks and does in a population. Fortunately, sample size formulas are not sensitive to errors in this value. Also, if the

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estimated upper limit is set too high, then the indicated sample size is conservative (i.e., larger than necessary) for the desired precision and confidence. The upper limit is for the *part* of the population under consideration. For example, if a buck : doe ratio is desired, then an upper limit on the total number of bucks and does is required (total population minus the number of fawns).
4. In as far as possible, the sample consists of distinct animals, i.e., the sampling is conducted "without replacement." This is more efficient than allowing animals to be "resampled" and should be a goal of the study design. If the study is conducted so that a significant proportion of the animals could be classified more than once, then theoretically the population is infinite as one could continue sampling "with replacement" indefinitely. The figures and tables in this paper can be used when animals are resampled by taking the results corresponding to the largest population sizes reported (i.e., $N = 12{,}000$ is essentially infinite with respect to the formulas).

Justification of the procedure to be described is contained in the appendix where formulas are derived that will provide more exact answers than interpolation within the tables and curves presented. As a general guideline, all results are developed at the 90% confidence level (see the appendix for other levels of confidence).

ESTIMATING SAMPLE SIZE AND CONFIDENCE INTERVALS

Population ratios are often expressed in the form of young/100 females or males/100 females. This expression of a ratio will be denoted as "a:100." A ratio of 70 fawns/100 does would therefore be expressed as 70:100, where $a = 70$. Using this notation for expressing population ratios, the end points of the 90% confidence interval are denoted by:

$$(a \pm b):100$$

(e.g., [70 ± 8 fawns]/100 does).

1. *Example for calculating adequate sample sizes and confidence intervals.*

A wildlife manager needs an August fawn : doe ratio for a pronghorn antelope herd. The necessary number of does and fawns to be classified must be determined. There are approximately 5,000 animals in this herd and it is felt that at least 3,600 are does and fawns. Therefore, N equals 3,600 in the following. It is also hypothesized that there will be about 80 fawns/100 does. This will put the value of "a" at 80. The desired precision of the estimated ratio is $b = \pm 8$ fawns/100 does. Using Table 1, under the column headed 80:100, we find ±8.0:100 in the fifth row which corresponds to curve number 5 in Fig. 1. Using $N = 3{,}600$ in Fig. 1 and curve number 5, the required sample size is about 825. That is, a few more than 800 does and fawns should be classified to insure (with 90% confidence) that the fawn : doe ratio will have precision of ±8 fawns/100 does.

In a similar manner, Table 1 and Fig. 1 can be used in reverse to obtain an approximate confidence interval on the estimated ratio. In the above example, suppose the sample size actually obtained is $n = 1{,}200$ and the computed ratio is 70 fawns/100 does. The point $n = 1{,}200$ and $N = 3{,}600$ is close to curve number 6. Entering Table 1 with the ratio 70: 100 and curve (row) 6, the precision is ±5.7: 100. In conclusion, the fawn : doe ratio is within (70 ± 5.7 fawns)/100 does with approximately 90% confidence.

2. *Example for adequate sample sizes with more than 2 classes.*

Continuing example 1, suppose the buck : doe ratio is also of interest. With the hypothesized ratio of 80 fawns/100 does we expect 1,600 fawns, 2,000 does, and 1,400 bucks in

Table 1. Precision of confidence intervals at various population ratios (90% confidence) and sample size curves in Fig. 1.

Curve number on Fig. 1	Population ratio							
	10:100	20:100	30:100	40:100	50:100	60:100	70:100	80:100	100:100
1	±4.0:100	±6.2:100	±8.2:100	±10.3:100	±12.3:100	±14.4:100	±16.6:100	±18.8:100	±23.3:100
2	±2.8:100	±4.3:100	±5.8:100	±7.2:100	±8.6:100	±10.1:100	±11.6:100	±13.1:100	±16.3:100
3	±2.3:100	±3.5:100	±4.7:100	±5.8:100	±7.0:100	±8.1:100	±9.3:100	±10.6:100	±13.1:100
4	±2.0:100	±3.0:100	±4.0:100	±5.0:100	±6.0:100	±7.0:100	±8.0:100	±9.1:100	±11.3:100
5	±1.7:100	±2.7:100	±3.5:100	±4.4:100	±5.3:100	±6.2:100	±7.1:100	±8.0:100	±10.0:100
6	±1.4:100	±2.1:100	±2.8:100	±3.5:100	±4.2:100	±4.9:100	±5.7:100	±6.4:100	±8.0:100
7	±1.2:100	±1.8:100	±2.4:100	±3.0:100	±3.6:100	±4.2:100	±4.8:100	±5.4:100	±6.7:100
8	±0.9:100	±1.4:100	±1.9:100	±2.3:100	±2.8:100	±3.2:100	±3.7:100	±4.2:100	±5.2:100
9	±0.7:100	±1.1:100	±1.5:100	±1.9:100	±2.3:100	±2.7:100	±3.0:100	±3.4:100	±4.3:100

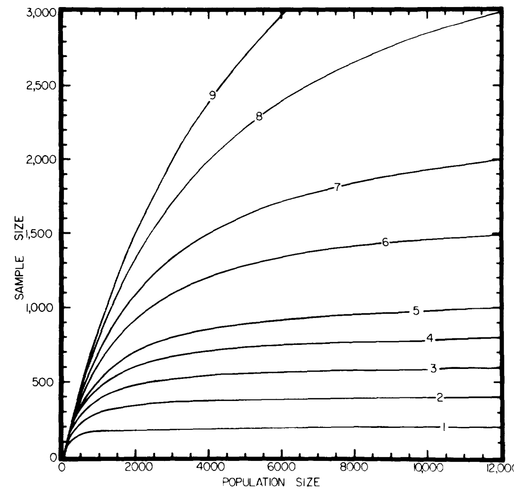

Fig. 1. Sample size (n) as a function of population size (N) of 2 age or sex classes and a given level of precision indicated in Table 1 (90% confidence).

the population of 5,000 animals. Also, about 367 fawns and 458 does should be in the sample of 825. The hypothesized buck : doe ratio is 1,400:2,000 or 70 bucks/100 does and suppose the desired precision is ±8 bucks/100 does. Using the column headed 70:100 in Table 1, ±8.0:100 is found to correspond to curve number 4 on Fig. 1. Also $N = 1{,}400 + 2{,}000 = 3{,}400$ in this case. From Fig. 1, about 690 bucks and does should be sampled, and we expect to obtain about 284 bucks and 406 does given the hypothesized ratio 70 bucks/100 does.

Taking the larger of the 2 values for the does (e.g., 458 does in example 1) the adequate size for the combined sample is:

$$n = 367 \text{ fawns} + 284 \text{ bucks} + 458 \text{ does} = 1{,}109 \text{ animals.}$$

About 1,100 total animals from the population of 5,000 should be sampled.

MINIMUM AND MAXIMUM SAMPLE SIZES

Fig. 2 contains a simplified version of Table 1 and Fig. 1. It is intended that Fig. 2 be used

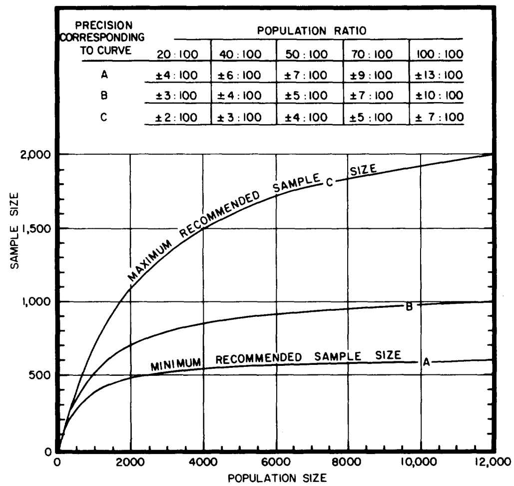

Fig. 2. Recommended sample size (n) as a function of population size (N) of 2 age or sex classes and a given level of precision indicated in the inserted table (90% confidence).

by field personnel as a general guideline for minimum sample size, maximum sample size, and the corresponding precision associated with a given combination of sample size and the estimated ratio at approximately 90% confidence.

3. *Example for calculating minimum recommended sample size.*

It is desired to estimate the preseason fawn : doe ratio in a mule deer herd. Current estimates suggest approximately 3,000 does and fawns are present in the population.

Using curve A from Fig. 2, the minimum number of does and fawns which we recommend for the sample is about 530. If the final fawn : doe ratio is approximately 70 fawns/100 does, the approximate 90% confidence interval is ±9:100, i.e., (70 ± 9 fawns)/100 does (see the table reproduced on Fig. 2).

Assuming the sampling procedure is random and unbiased, there is a 90% chance that the true fawn : doe ratio of the herd lies between 61 fawns/100 does and 79 fawns/100 does. If one desires a smaller confidence interval, then curve B or C from Fig. 2 should be selected. The sample size would be larger in both cases.

4. *Example for calculating maximum recommended sample size.*

We desire to estimate the postseason bull : cow ratio of an elk herd. We estimate approximately 5,000 bulls and cows are in the herd.

Using curve C from Fig. 1, a realistic maximum sample size is approximately 1,625. If 1,625 bulls and cows are classified and the bull : cow ratio approximates 20:100, then the 90% confidence interval from the top of Fig. 2 would be (20 ± 2):100. "Tighter" confidence intervals require larger sample sizes. However, increasing the sample size above 1,625 will not yield meaningful improvements in the precision. To see this, the sample size 1,625 corresponds to curve 7 in Table 1. Increasing the sample size to 2,750 (curve 9) yields (20 ± 1.1):100. The workload must be approximately doubled in order to improve the precision to ±1.1 bulls/100 cows.

SAMPLING "WITH REPLACEMENT"

If every animal has an equal chance of being classified (reclassified) on each sampling occasion, then theoretically the population is infinite ($N = \infty$). Figures 1 and 2 extend only to $N = 12{,}000$. However, the curves are almost flat in this region and $N = 12{,}000$ can safely be used in place of $N = \infty$ in all but the most precise studies corresponding to curve 9 in Fig. 1. In this extreme case (curve 9), sample sizes in the neighborhood of 4,000 to 5,000 are necessary to obtain the precision indicated in the bottom row of Table 1.

5. *Example for calculating recommended sample size when sampling with replacement.*

In a "quick and dirty" census, several different individuals are to cruise a network of roads over several weeks and classify (or reclassify) every antelope they see. Assuming that this can be done in a random and unbiased manner, how many does and fawns should be classified (reclassified) in order to achieve a precision of ±8.0 fawns/100 does? Also assume that under similar conditions this population has had a fawn : doe ratio of about 80:100. Using Table 1, curve 5 is selected and from Fig. 1 a sample size of $n = 1{,}000$ doe and fawn antelope corresponds to $N = 12{,}000$. Because curve 5 is almost flat in this region, a sample of about $n = 1{,}000$ is adequate for a precision of ±8.0 fawns/100 does even though animals are being reclassified. From Fig. 2, our minimum recommended sample size is about $n = 600$ and the maximum recommended sample size is about 2,000.

As in the other examples, the process can be reversed to obtain approximate 90% confidence intervals, i.e., use $N = 12{,}000$ in the figures.

SPECIAL CASES

6. *Example to be used for increases in the hypothesized ratio.*

Consider the fawn : doe sampling problem in Example 1, except that the hypothesized ratio is 100 fawns/100 does. Using the last column of Table 1 we must drop to row 6 to achieve a precision of ±8.0 fawns/100 does. Hence in Fig. 1 we must move up to curve 6 and the required sample size corresponding to $N = 3{,}600$ is approximately $n = 1{,}125$.

The sample size must increase from about 800 to about 1,100 in order to achieve the same precision (±8:100). This is a general result. As the hypothesized ratio increased (left to right in Table 1) the user must drop down in the table in order to achieve a fixed precision and hence up in Fig. 1 to a larger sample size.

7. *Example to be used for hypothesized ratios below 10:100.*

Consider a sample problem where the hypothesized ratio of males/females is 5:100. Also assume a maximum of $N = 2{,}000$ males and females and a desired precision of ±2 males/100 females. Because the necessary sample size decreases as the hypothesized ratio decreases (see Example 6), the column headed 10:100 in Table 1 is recommended and will give a conservative answer. Using curve 4 and $N = 2{,}000$ in Fig. 1, a sample size of 600 is larger than necessary for precision of ±2.0:100 if the "true" ratio is 5:100 (90% confidence).

We recommend that the left-hand column headed (10:100) in Table 1 be used for all hypothesized ratios below 10:100.

After data are collected and a computed ratio below 10:100 is obtained, say 8:100, the precision indicated in the first column of Table 1 is conservative. For example, with $n = 1{,}000$ animals sampled out of $N = 2{,}000$ total, $n = 1{,}000$ is about half-way between curves 6 and 7 in Fig. 1. Using the first column of Table 1 (10:100) the precision is better than ±1.3:100. The confidence interval (8 ± 1.3 males)/100 females can be reported with >90% confidence.

8. *Example to be used for hypothesized ratios above 100:100.*

If a ratio of 125 fawns/100 does has been obtained (for the given herd under similar conditions in previous years), we are again out of the range of Table 1. We recommend that the manager use the last column (100:100) when planning for sample sizes in this case. Theoretically, the role of does and fawns could be reversed to obtain 80 does/100 fawns and the column headed 80:100 used to determine sample sizes. However, most managers will express the ratio as fawns : doe and the larger sample sizes suggested by the last column are recommended. As an alternative, the more exact formulas in the appendix can be used. We hypothesized 125 fawns/100 does and suppose $N = 2{,}000$ is an upper bound on the number of does and fawns present. For precision of ±8:100 and using the column headed 100:100, curve 6 in Fig. 1 is suggested. This yields a recommended sample size of about 900. Suppose the data are collected with $n = 900$ and a ratio of 110 fawns/100 does is computed. As a trick to obtain a confidence interval, express 110 fawns/100 does as 91 does/100 fawns. Using row 6 and interpolating between 80:100 and 100:100, the precision is about ±7.3 does/100 fawns (90% confidence)—or (91 ± 7.3 does)/100 fawns which yields 83.7 does/100 fawns to 98.3 does/100 fawns. These last ratios convert back to 102 fawns/100 does to 119 fawns/100 does (90% confidence). The precision is almost (110 ± 8 fawns)/100 does.

CONCLUDING REMARKS

Table 1 and Fig. 1 are presented to allow interpolation when estimating the sample size necessary for a stated precision and hypothesized ratio (90% confidence). Also, they can be used to allow interpolation when obtaining approximate 90% confidence intervals for a computed population ratio.

Figure 2 contains our recommendation for minimum and maximum sample sizes and the corresponding level of precision. It is intended that Fig. 2 be used by field personnel during data collection.

Hypothesized ratios must be available to use the tables and figures. Such values may come from a preliminary sample, prior data collected in the same area under similar conditions, theoretical considerations (e.g., the ratio of male fawns to female fawns might be assumed to be 100:100 for planning purposes), the manager's intuition, etc. Clearly with more precise input information, adequate sample size is estimated with more precision.

The estimated sample sizes are not sensitive to errors in the estimated population size. The curves in Figs. 1 and 2 are relatively flat for populations exceeding 500 animals.

The requirement for a random and unbiased sampling procedure is difficult to meet. This is especially true when members of the classes occupy different segments of the habitat or when members of 1 class are more visible than members of the other due to plumage, feeding habits, tendency to run in herds, etc. It will test the manager's knowledge to overcome such problems and insure unbiased data collection techniques. A second problem, which is not quite so serious, is that hidden correlations may be introduced into the data when clusters (herds, all animals on a quadrat, etc.) are classified. These correlations will not cause biased ratios but the stated precision may be incorrect (too high or too low). No good guidelines exist in the literature for the solution of this problem (in as far as the authors are aware). If the precision is better than the value claimed by the theory behind this paper, then the manager is conservative and is using a sample size larger than necessary. In the other case, his sample size is not large enough. We recommend the methodology presented in this paper as a very general guideline for adequate sample size whenever the manager is confident that his sampling procedure is unbiased. For studies where the exact precision of the ratios must be known, a professional statistician, knowledgeable in finite sampling theory, should be consulted. A variation of quadrat sampling or cluster sampling will likely be recommended.

SUMMARY

Adequate sample sizes are given for the number of animals to be classified in a random and unbiased study for estimation of population age and sex ratios. The desired level of precision and confidence can be specified. Advantages of this procedure are: (1) the data and precision are summarized in standard forms (e.g., 40 ± 3 males/100 females), (2) the tables and curves are easy to use in the field, (3) minimum and maximum sample sizes can be recommended, and (4) the tables and curves can be used in reverse to obtain approximate confidence intervals on the estimated population ratio.

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APPENDIX

Consider a finite population of N animals where P is the proportion in the first category of the population and $Q = 1 - P$ is the proportion in the second category. Let n be the number of animals sampled without replacement and z_α be the "2-tailed" value from the normal distribution. For $nP \geq 5.0$ and $nQ \geq 5.0$ (roughly corresponding to 10:100, the minimum value considered in Table 1), a $100(1 - \alpha)\%$ probability interval on p is given by

$$P \pm z_\alpha \sqrt{\frac{PQ(N-n)}{n(N-1)}} \qquad (1)$$

where p is the observed proportion in the first category (Cochran 1977:51). To have precision of $\pm c$ for this interval we can solve

$$z_\alpha \sqrt{\frac{PQ(N-n)}{n(N-1)}} = c, \qquad (2)$$

for n

$$n = \frac{Nz^2PQ}{c^2(N-1) + z^2PQ}. \qquad (3)$$

The proportions can be transformed to the form of population ratios (a:100, $[A \pm B]$:100, etc.) by the formulas

$$A = \frac{100P}{(1-P)}, \quad P = \frac{A}{100 + A},$$
$$Q = \frac{100}{100 + A}, \quad a = \frac{100p}{(1-p)}. \qquad (4)$$

Given the probability interval, $p \pm c$, the resulting interval on a:100 would be from A_1:100 to A_2:100 where

$$A_1 = \frac{100(P-c)}{1-(P-c)} \quad \text{and}$$
$$A_2 = \frac{100(P+c)}{1-(P+c)}. \qquad (5)$$

Now

$$A - A_1 = \frac{100P}{(1-P)} - \frac{100(P-c)}{1-(P-c)}$$
$$= \frac{100c}{(1-P)^2 + c(1-P)} \qquad (6)$$

and

$$A_2 - A = \frac{100(P+c)}{1-(P+c)} - \frac{100P}{(1-P)}$$
$$= \frac{100c}{(1-P)^2 - c(1-P)}. \qquad (7)$$

For an approximate *symmetric* probability interval on a:100, i.e., $(A + B)$:100, take

$$B = \frac{100c}{(1-P)^2} \qquad (8)$$

an intermediate value between $(A - A_1)$ and $(A_2 - A)$ in equations (6) and (7). Solving equation (8) for c and substituting into (3) together with (4), we obtain

$$n = \frac{z_\alpha^2 AN}{\left(\dfrac{B^2 100(N-1)}{(100+A)^2} + z_\alpha^2 A\right)}. \qquad (9)$$

Given equation (9), the relationships in Table 1 and Figs. 1 and 2 were developed by a computer program.

Also, given a hypothetical value for A:100, the desired precision $\pm B$, the population size N and the desired level of confidence, $1 - \alpha$, (i.e., z_α) the adequate sample size can be computed by (9).

Finally, after the data are observed and A is estimated by a, equation (8) can be written in the form

$$b = \frac{z_\alpha(a + 100)}{10} \sqrt{\frac{a(N-n)}{n(N-1)}} \qquad (10)$$

to obtain an *approximate symmetric* $100(1 - \alpha)\%$ confidence interval on a:100.

If the sampling is conducted "with replacement" then equation (9) can be used with $N = \infty$. Dividing the numerator and denominator by N, it is easy to see that equation (9) will converge to

$$n = \frac{z_\alpha A(100 + A)^2}{100B^2}. \qquad (11)$$

Similarly, equation (10) converges to

$$b = \frac{z_\alpha(a + 100)}{10}\sqrt{a/n}.$$

9. *Example for the use of equation (9).*

Suppose the hypothesized buck : doe ratio for a post-season mule deer herd classification is 30 bucks/100 does. We estimate about $N = 2{,}000$ bucks and does in the herd. For 95% confidence, $z_{.05} = 1.96$ and from (9) the adequate sample size for precision of $\pm B = \pm 3$ bucks/100 does is

$$n = \frac{(1.96)^2(30)(2{,}000)}{\left(\frac{[\pm 3]^2 100[1{,}999]}{[100 + 30]^2} + (1.96)^2 30\right)}$$

$$= \frac{230{,}496}{[106.46 + 115.25]}$$

$$= 1{,}040 \text{ bucks and does.}$$

10. ***Example for the use of equation (10).***

There are 109 bucks and 402 does observed in an unbiased, randomized preseason antelope classification. The herd ratio is 109:402 or 27.1 bucks/100 does. Therefore, $a = 27.1$, $n = 109 + 402 = 511$, and we estimate that about $N = 2{,}000$ bucks and does are in the preseason herd. If 90% confidence is desired, $z_{.1} = 1.645$ and from equation (10)

$$b = \frac{1.645(27.1 + 100)}{10} \sqrt{\frac{27.1(2{,}000 - 511)}{511(1{,}999)}}$$

$$= 20.9(0.1988)$$

$$= 4.2.$$

Therefore, there is (approximately) a 90% chance that the true herd ratio falls in the interval

$$(27.1 \pm 4.2 \text{ bucks})/100 \text{ does.}$$

COYOTE PREDATION AVERSION WITH LITHIUM CHLORIDE: MANAGEMENT IMPLICATIONS AND COMMENTS

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Conditioned taste aversion has been proposed as a method to deter coyotes (*Canis latrans*) from preying on sheep. The method involves placing mutton baits laced with a strong emetic, lithium chloride (LiCl), on the range. Coyotes supposedly find and eat the baits, become ill, and subsequently avoid sheep because they associate the sheep with sickness (Gustavson et al. 1974, 1976, Ellins et al. 1977). Attempts also have been made to control coyote predation on turkeys (*Meleagris gallopavo*) using LiCl-laced turkey carcasses (Ellins and Catalano 1980).

Some investigations prior to 1978 cast doubt on the effectiveness of using LiCl baits to deter coyotes from killing (predation aversion) and, in a summary paper, Griffiths et al. (1978) concluded that no judgment could yet be made regarding the value of LiCl in preventing coyote predation. In this paper I synthesize the literature on the subject since 1978, discuss the most probable management implication (assuming LiCl produced predation aversion in coyotes), and point out some present information needs.

In defending an earlier publication, Conover et al. (1979) maintained that more research was needed on predation aversion with LiCl, whereas Gustavson (1979) felt that the existing studies demonstrated the success of the method. Cornell and Cornely (1979) believed that LiCl fed to coyotes in a variety of foods discouraged potentially dangerous coyotes from soliciting food at a campground. Burns (1980*b*) demonstrated that LiCl's salt flavor interfered with the ability of coyotes to form aversions to baits and prey killing. Burns and Connolly (1980) then produced a measurable black-tailed jack rabbit (*Lepus californicus*) bait aversion with low doses of LiCl that appeared not to provide flavor cues to coyotes. The bait aversion, however, was not transferred to predation aversion; coyotes continued to kill jack rabbits. Ellins and Martin (1981) later found that coyotes could detect LiCl and avoid baits at concentrations below