

Deploying initial attack resources for wildfire suppression: spatial coordination, budget constraints, and capacity constraints

Yohan Lee, Jeremy S. Fried, Heidi J. Albers, and Robert G. Haight

Abstract: We combine a scenario-based, standard-response optimization model with stochastic simulation to improve the efficiency of resource deployment for initial attack on wildland fires in three planning units in California. The optimization model minimizes the expected number of fires that do not receive a standard response — defined as the number of resources by type that must arrive at the fire within a specified time limit — subject to budget and station capacity constraints and uncertainty about the daily number and location of fires. We use the California Fire Economics Simulator to predict the number of fires not contained within initial attack modeling limits. Compared with the current deployment, the deployment obtained with optimization shifts resources from the planning unit with highest fire load to the planning unit with the highest standard response requirements but leaves simulated containment success unchanged. This result suggests that, under the current budget and capacity constraints, a range of deployments may perform equally well in terms of fire containment. Resource deployments that result from relaxing constraints on station capacity achieve greater containment success by encouraging consolidation of resources into stations with high dispatch frequency, thus increasing the probability of resource availability on high fire count days.

Résumé : Nous combinons un modèle d'optimisation de la réponse standard basée sur différents scénarios à une simulation stochastique pour améliorer l'efficacité du déploiement des ressources lors de l'attaque initiale des feux de forêt dans trois unités de gestion en Californie. Le modèle d'optimisation minimise le nombre attendu de feux qui ne reçoivent pas une réponse standard (définie comme la quantité de ressources de chaque type qui doit être déployée à l'intérieur d'une certaine limite de temps) à cause de contraintes de budget et de capacité des stations et de l'incertitude quant au nombre quotidien de feux et à leur localisation. Nous utilisons le simulateur de l'aspect économique des feux en Californie pour prédire le nombre de feux qui ne sont pas maîtrisés à l'intérieur des limites déterminées par la modélisation de l'attaque initiale. Comparativement au déploiement actuel, le déploiement obtenu par optimisation déplace des ressources de l'unité de gestion qui a le fardeau d'intervention le plus lourd vers l'unité de gestion qui a les exigences de réponse standard les plus élevées mais laisse le succès simulé de la maîtrise des feux inchangé. Ce résultat indique que dans les conditions actuelles de contraintes de budget et de capacité, différents déploiements peuvent avoir une aussi bonne performance en termes de maîtrise du feu. Les déploiements de ressources qui résultent de la réduction des contraintes de capacité des stations ont plus de succès dans la maîtrise des feux en favorisant la consolidation des ressources dans les stations qui ont une fréquence élevée de déploiement, augmentant par conséquent la probabilité que les ressources soient disponibles les jours où le nombre de feux est élevé. [Traduit par la Rédaction]

Introduction

Large wildfires in the United States pose significant challenges to fire management agencies charged with protecting human life, property, and natural resources. Since 1990, the area burned by large wildfires and suppression costs have increased significantly (Calkin et al. 2005; Littell et al. 2009). Further, the synchrony of large wildfires across broad geographic regions contributes to budget shortfalls when suppression costs exceed Congressional funds appropriated for suppression (Holmes et al. 2008).

As a strategy for preventing large and costly wildfires, fire managers prioritize aggressive initial attack of fire ignitions especially in places where high densities of people live in areas that have relatively high likelihood of high-intensity wildfire. It has long been understood that vigorous, rapid initial attack can contain a fire quickly before it becomes large and causes substantial damage (Parks 1964). Initial attack is generally defined as the first 1–8 h of fire suppression effort, during which the primary objective is containment of the fire at a small size in the shortest possible time. Examples of fire suppression resources used for initial attack include fire engines, bulldozers, hand crews, and water-dropping helicopters.

Fire planners and managers make two types of allocation decisions for initial attack resources (Martell 1982). First, they deploy resources to meet expected demand for fire suppression in coming days, weeks, or months. Then, as fires occur, they dispatch those resources to achieve the earliest possible containment while taking into account the contingency of synchronous fire ignitions. While most fire managers have a clearly defined goal of minimizing the number of escaped fires, they face substantial uncertainty about the number, location, and intensity of fires and they have limited funds to acquire suppression resources or construct operating bases. As a result, fire managers must efficiently deploy costly firefighting resources across dispersed locations with considerable uncertainty about where fires will occur and how difficult they will be to control.

Both simulation and optimization models aid resource deployment and dispatch decisions (see Martell 1982, 2007 for reviews). Location-specific deployment and dispatch rules are evaluated using simulation models that account for the stochastic properties of firefighting tactics, dispatch policies, fire behavior, and fireline production rates (Fried and Gilles 1999; Fried et al. 2006). While simulation

Received 6 October 2011. Accepted 12 November 2012.

Y. Lee. Department of Forest Ecosystems and Society, Oregon State University, 321 Richardson Hall, Corvallis, OR 97331, USA.

J.S. Fried. USDA Forest Service, Pacific Northwest Research Station, P.O. Box 3890, Portland, OR 97208, USA.

H.J. Albers. Applied Economics/FES, Oregon State University, 321 Richardson Hall, Corvallis, OR 97331, USA.

R.G. Haight. USDA Forest Service, Northern Research Station, 1992 Folwell Avenue, St. Paul, MN 55108, USA.

Corresponding author: Yohan Lee (e-mail: yohan.lee@oregonstate.edu).



models are excellent for exploring the impact of marginal changes to the system, or even for “test-driving” entirely new system designs, they generally are not suited for identifying optimal deployment and dispatch policies because their complex nonlinear and stochastic structures are difficult to include in optimization algorithms.

Optimization models determine deployment and dispatch rules that attain specific fire management objectives. For example, deployment models assign suppression resources to stations to minimize operating costs while meeting predefined resource requirements in surrounding areas (Hodgson and Newstead 1978; Greulich and O'Regan 1982). Models of dispatch determine the number and type of suppression resources to dispatch to a given fire to minimize suppression cost plus damage subject to resource availability constraints (Kourtz 1989; Mees et al. 1994; Donovan and Rideout 2003). A few models address deployment or dispatch problems that account for uncertainty in fire occurrence or behavior (MacLellan and Martell 1996; Hu and Ntairo 2009). Haight and Fried (2007) developed a model that optimizes both seasonal deployment and daily dispatch decisions while accounting for uncertainty in the number, location, and intensity of fires. In their model, ignition uncertainty is characterized with a set of fire scenarios, each listing the location and intensity of fires that could occur in a single day. Each potential fire also has an associated standard response — defined as the required number of resources that must reach the fire within a maximum response time (Marianov and ReVelle 1991) — based on fire location and intensity. Resources are deployed to fire stations before the number, location, and intensity of ignitions are known, and resources are dispatched to fires contingent on the standard response requirements of the fires that occur in each scenario. The objective is to minimize the expected number of fires that do not receive a standard response subject to a resource deployment budget.

While optimization models deploy resources based on assumptions about potential fire location and intensity, they do not model interactions among fire-fighting tactics, fireline production rates, fire intensity, and suppression. In contrast, stochastic simulation models of initial attack simulate fire intensity and containment based on fire-fighting tactics and fireline production (e.g., Fried and Gilles 1999). Therefore, combining optimization and simulation analyses of initial attack may improve the information available to decision makers (Haight and Fried 2007; Hu and Ntairo 2009).

We combine a scenario-based, standard-response optimization model with a stochastic simulation model of initial attack to assess and potentially improve the deployment of fire suppression resources among three central Sierra planning units administered by the California Department of Forestry and Fire Protection (CALFIRE). First, we use the optimization model to determine the joint deployment of resources among the three planning units. Then, we simulate the initial attack success of each deployment obtained from the optimization model using the California Fire Economics Simulator version 2 (CFES2), a stochastic simulation model of initial attack (Fried and Gilles 1999; Fried et al 2006). We use these models to assess how deployment and dispatch decisions obtained with the optimization model affect initial attack success relative to the performance of an existing resource deployment that is based on expert knowledge and experience. Then, we assess how changes in station capacity and budget constraints affect resource deployment decisions and initial attack success. Finally, we develop a simple deployment heuristic using CFES2 simulations and assess its performance relative to the deployments obtained with optimization.

Methods

Study area and simulation framework

The 1.2 million hectare study area consists of the central portions of three adjacent CALFIRE administrative units in the central Sierra region of California — Amador-El Dorado (AEU),

Nevada-Yuba-Placer (NEU), and Tuolumne-Calaveras (TCU) — where CALFIRE is responsible for wildfire suppression (Fig. 1). Topography includes rolling hills and steep, rugged river canyons with elevations rising 300–1200 m west to east and vegetation ranges from annual grasslands, shrublands, oak savannas, and open pine woodlands to mixed conifer and true fir forests along this elevation and precipitation gradient. Stratified by life form, dominant vegetation cover is 42% herbaceous, 39% shrub, and 19% forest (Franklin et al. 2000). Before European settlement, these vegetation types supported low-intensity fires with frequent return intervals (2–16 years) (Barbour et al. 2007). Since 1900, fuel loadings have increased owing to fire suppression, so wildfires burn with high intensity. Low fuel moisture and severe fire weather combine to create the greatest potential for large fires during high fire season (June–October). Rapid population growth during the 1990s greatly increased the value at risk in buildings and infrastructure. From 2005 through 2008, CALFIRE deployed engines and dozers in 45 stations, hand crews in four camps, and aircraft from six air bases to initial attack fires (Fig. 1).

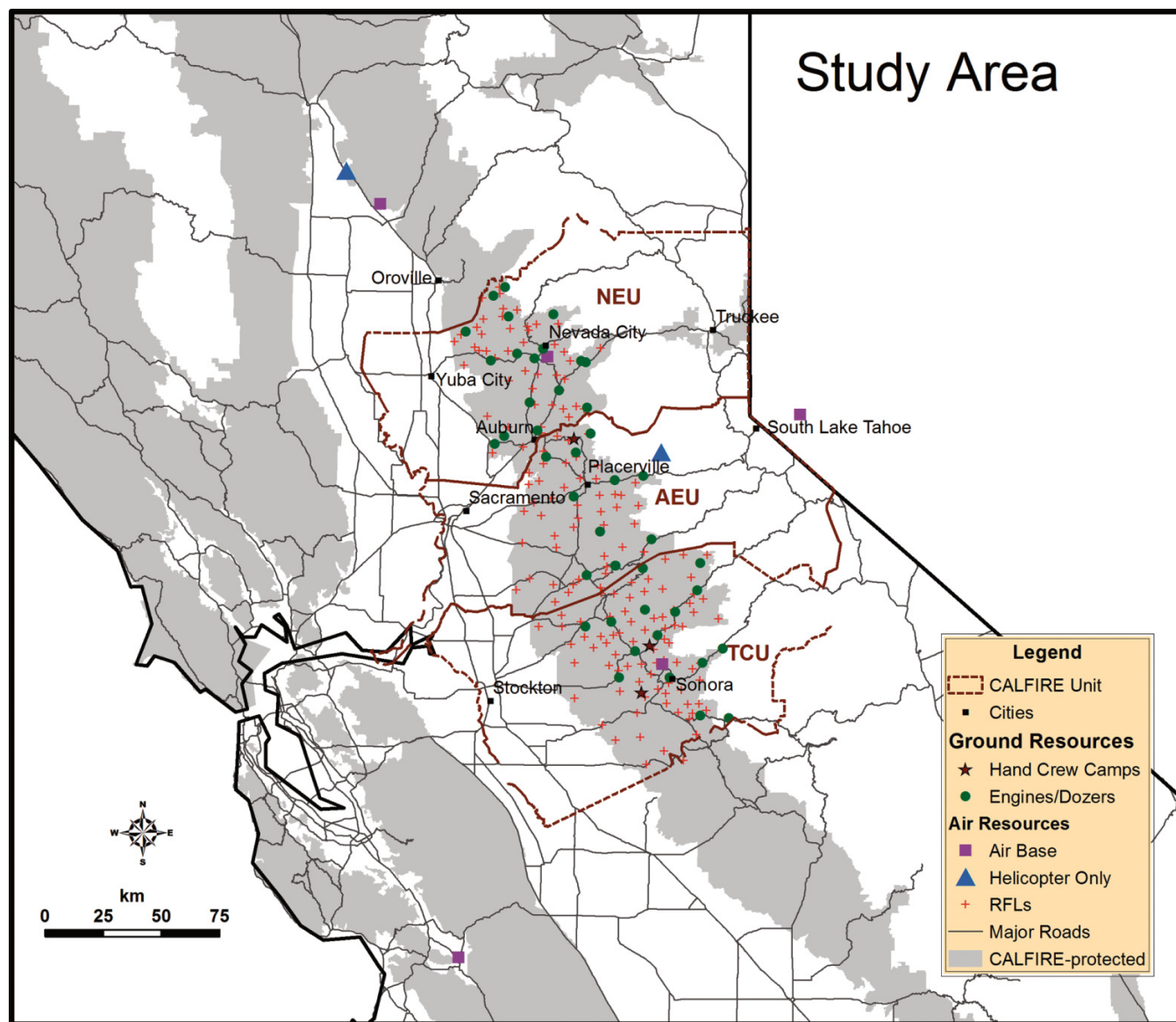
To conduct strategic planning using CFES2, CALFIRE stratified this area into 27 fire management analysis zones (FMAZ) described by broad fuel type and population density class. Within each FMAZ, fires are simulated at representative fire locations (RFLs), each characterized by a National Fire Danger Rating System fire behavior fuel model (narrative descriptions can be found in Deeming et al. 1977), slope class, representative fire weather station (Wolf Creek, White Cloud, Bald Mountain, Georgetown, Groveland, or Eliza Mountain), value at risk, and ease of access by firefighting resources, as represented by travel times. There are a total of 173 RFLs in the three units (Fig. 1), and each is assigned a weight consistent with the frequency of fires occurring at locations within the FMAZ that approximately match the RFL's characteristics.

Simulating initial attack

It is essential to understand that we use CFES2 at two points during this analysis, and for very different purposes: (1) to generate a set of fire scenarios and parameters that drive the optimization and (2) to simulate initial attack on fires in those scenarios to evaluate the performance of alternative resource deployments produced by the optimization. The CFES2 model uses stochastic simulation of fire occurrence and behavior and a mathematical model of perimeter containment (Fried and Fried 1996). It includes considerable operational detail designed to support decision making in wildland fire protection through quantitative analysis of the potential effects of marginal changes to the wildland fire management system. Examples of parameters that can be varied include availability and stationing of resources, rules for how many resources to dispatch, by kind, at each fire dispatch level, criteria for setting the fire dispatch level, schedules for when firefighting resources are staffed and available, and fireline-building tactics. The CFES2 model can be used to evaluate the contribution to initial attack effectiveness of several types of initial attack resources, alternate deployment of and dispatching rules for suppression resources, and multiunit and multiagency cooperation (Fried et al. 2006).

The occurrence model contains random variables for whether and how many fires occur on a given day, along with the location (RFL) and the ignition time for each fire (Fried and Gilles 1988). The behavior model contains random variables for fire spread rate and fire dispatch level depending on weather and time of day (Gilles and Fried 1999). To build a data set for the optimization model (described below), we used the CFES2 fire occurrence and behavior models, parameterized with data from 15 years of historical fire occurrences and fire weather observations between 1990 and 2010. Using this parameterization, CFES2 simulated 400 years of daily fire patterns with each day representing a particular com-

Fig. 1. CALFIRE administrative units in the central Sierra region of California: Amador-El Dorado (AEU), Nevada-Yuba-Placer (NEU), and Tuolumne-Calaveras (TCU). The study area is the CALFIRE-protected area (shaded) in the central portions of the three units. We excluded the CALFIRE-protected area in the eastern portion of NEU (shaded area surrounding Truckee) because it is too far away from other CALFIRE-protected areas to send or receive ground resources and depends primarily on US Forest Service and local suppression resources through mutual aid agreements.



bination of weather, fire count, fire locations, fire ignition times, fire behavior (intensity and rate of spread), and consequent demand for firefighting resources. On days where fire count is at least four in a unit, fires that occur later in the day frequently do not receive a full complement of resources, or some of the resources that they do receive have later arrival times, owing to close-by resources having already been committed to suppress earlier fires. We identified 5814 high fire season days with high fire counts (four or more in any one planning unit and no more than one fire at any RFL), representing 16% of the days on which any fire occurred in any of the three units and accounting for 42 835 simulated fires (7.35 fires per day on average), of which 43% were in NEU, 28% in AEU, and 29% in TCU. We use these 5814 high fire count and high fire season days as the fire scenarios to evaluate alternative deployment and dispatch decisions because these are the days when the initial attack resources will be challenged to meet demands for fire suppression and because escaped fires may

cause catastrophic damage and may be very expensive to extinguish. To reiterate the point made at the beginning of this section, we use the 5814 scenarios in two ways: (1) to generate a resource deployment using the standard response optimization model and (2) to simulate the performance of the resource deployment using CFES2.

Optimizing strategic resource deployment

We develop a scenario-based, standard-response optimization model to guide the annual, strategic deployment (or home-basing) of initial attack resources to stations and dispatch them to fires. The model minimizes the expected daily number of fires that do not receive a standard response subject to budget and station capacity constraints. The standard response is the number of resources by type that must arrive at a fire within a specified time limit. A standard response is defined for each of three fire dispatch levels by fire management experts in each unit and varies

among units due to differences in the number of resources by type stationed in the unit and the degree of reliance on air resources. To represent the uncertainty in fire ignition and behavior during a single day, the model includes multiple fire scenarios, each defining the number, location, and standard response requirements of fires that may occur. In aggregate, the multiple scenarios approximate the probability distribution of fire locations and intensities during a single day. The model deploys resources to stations at the beginning of the fire season or planning period before the number, location, and intensity of fire ignitions are known, and then resources are dispatched to fires contingent on the standard response requirements of the fires that occur in each scenario. The model addresses strategic deployment decisions within multiple fire management units and is described with the following notation.

Indices

- u, U = index and set of fire planning units (i.e., AEU, NEU, and TCU)
- i, I = index and set of suppression resource types
- j, J^u = index and set of fire stations in unit u
- k, K^u = index and set of potential fire locations in unit u
- s, S = index and set of fire scenarios

Parameters

- B = annual budget for total operating cost across all fire planning units
- c_i = annual cost of operating resource type i
- C_{iju} = upper limit on number of resources of type i at station j in unit u
- p_s = probability that fire scenario (fire day) s occurs
- r_{iksu} = number of resources type i required at location k in unit u during fire scenario s to satisfy requirements for a standard response
- $t_{iju'ku}$ = response time of resource type i from station j in unit u' to fire location k in unit u
- T_{iku} = maximum response time for resource type i to fire location k in unit u to satisfy a standard response requirement
- $N_{iku}^{u'}$ = set of stations j in unit u' from which resources of type i can reach location k in unit u within the maximum response time, i.e., $N_{iku}^{u'} = \{j \in J^{u'} \mid t_{iju'ku} < T_{iku}\}$

Decision variables

- x_{iju} = integer variable for number of resources of type i deployed at station j in unit u
 - $d_{iju'kus}$ = integer variable for number of resources of type i at station j in unit u' that are dispatched to fire location k in unit u during fire scenario s
 - z_{kus} = binary variable: 1 if fire location k in unit u receives a standard response during fire scenario s , 0 otherwise
- The model is formulated as follows:

$$[1] \quad \text{Minimize: } 0 = \sum_{u \in U} \left[\sum_{s \in S} p_s \sum_{k \in K^u} (1 - z_{kus}) \right]$$

Subject to

$$[2] \quad \sum_{u \in U} \sum_{i \in I} \sum_{j \in J^u} c_i x_{iju} \leq B$$

$$[3] \quad x_{iju} \leq C_{iju} \text{ for all } i \in I, j \in J^u, \text{ and } u \in U$$

$$[4] \quad \sum_{u' \in U} \sum_{k \in K^{u'}} d_{iju'kus} \leq x_{iju} \text{ for all } i \in I, j \in J^u, s \in S, \text{ and } u \in U$$

$$[5] \quad z_{kus} r_{iksu} \leq \sum_{u' \in U} \sum_{j \in N_{iku}^{u'}} d_{iju'kus} \text{ for all } i \in I, k \in K^u, s \in S, \text{ and } u \in U$$

$$[6] \quad z_{kus} \in \{0, 1\} \text{ for all } k \in K, s \in S, \text{ and } u \in U$$

Equation 1 minimizes the weighted sum of the expected number of fires that do not receive the standard response across all planning units, where the weight p_s represents the probability of the occurrence of fire scenario s . Inequality 2 requires that the total annual cost of operating suppression resources across the planning units is constrained to less than or equal to the budget limit. Inequality 3 represents the capacity of each station for each type of suppression resource. Inequality 4 requires that the number of each type of resource dispatched from each station during each fire scenario is less than or equal to the number of that type of resource deployed at the station. Inequality 5 represents the condition for whether a fire receives a standard response. A fire receives a standard response ($z_{kus} = 1$) only if, for each resource type i , the number of resources that are within the standard response time and dispatched to the fire from all available stations, $\sum_{u' \in U} \sum_{j \in N_{iku}^{u'}} d_{iju'kus}$, is greater than or equal to the number of resources required, r_{iksu} . The variable $d_{iju'ku}$ allows resources to be dispatched to locations in planning units other than their home unit. If $r_{iksu} = 0$ for all resource types i , there is no fire at location k in unit u during fire scenario s and z_{kus} may equal 1 with no resource commitment.

Our standard-response logic does rely on some simplification relative to the real world. The model will not send more resources to a fire than are defined in the standard response requirement because once the requirement is met, it is assumed that no further benefit is attainable. Further, the model will not send a partial response because benefit is contingent on the full standard response having been delivered. Annual operating costs, c_i , are fixed for each resource type without accounting for any incident-specific overtime pay or travel costs. Average values for these costs are reflected in the annual operating cost parameters. In the optimization model, an initial attack resource can be dispatched to at most one fire per day, while in the real world, and in CFES2 simulation, a resource may be used on multiple fires. Finally, while the standard response is a predefined number of resources arriving within a response time threshold for each fire, the dispatch decisions that comprise a standard response to a fire can vary: identical fires (location, severity, etc.) on different days may receive resources from different stations and planning units, depending on the other fires on those days.

Parameter values of the optimization model

Ideally, we would solve the optimization model with the complete set of 5814 fire scenarios generated with CFES2; however, due to computational limits, we use a set of 100 fire scenarios for the optimization model to approximate the probability distribution of days during the high fire season on which four or more fires occurred. Each of the 100 scenarios is randomly selected (without replacement) from the set of 5814 scenarios. Each scenario includes the location and dispatch level of each fire during a single day when there are at least four fires in any one of the three fire planning units and no more than one fire at any RFL. Mean daily number of fires for these 100 scenarios is 7.43, with a range of 4–12. Although we assume that the scenarios are equally likely (i.e., $p_s = 0.01$, $s = 1 \dots 100$) (MacLellan and Martell 1996; Haight and Fried 2007), their random selection from the larger set of 5814 implies that more likely fire scenarios are better represented in this sample of 100 than less likely fire scenarios. By assuming equal probability and aggregating the results, we can approximate the distribution of outcomes.

Table 1. Dispatch policy (number of resources by fire dispatch level) for initial attack in planning units Amador-El Dorado (AEU), Nevada-Yuba-Placer (NEU), and Tuolumne-Calaveras (TCU).

Fire dispatch level ^a	Resource type and planning unit											
	Engine			Dozer			Hand crew			Helicopter		
	AEU	NEU	TCU	AEU	NEU	TCU	AEU	NEU	TCU	AEU	NEU	TCU
1	3	4	2	0	0	1	0	0	1	0	0	1
2	4	6	4	1	1	2	1	1	3	0	1	2
3	5	8	6	1	2	3	2	2	5	1	2	3

^aFire dispatch level, derived from modeled fire behavior parameters, ranges from 1 (low) to 3 (high) and is designed to ensure a suppression response that is well matched to the challenge (e.g., growth rate or fire intensity) posed by a fire (Gilliss and Fried 1999).

The standard response to each fire depends on the fire's dispatch level, which ranges from 1 (low) to 3 (high) (Table 1). The fire dispatch levels are derived from the day's maximum burning index and scaled by a diurnal adjustment factor specific to the time of fire occurrence (Gilliss and Fried 1999). The dispatch levels assist CALFIRE personnel in determining how many resources of each type to dispatch for initial attack given the level of fire danger. In general, the higher the dispatch level, the more resources are required in the standard response. The standard response is zero for any location that does not have a fire.

The optimization model deploys engines, bulldozers, hand crews, and helicopters among stations owned and operated by CALFIRE. Response times for ground resources to travel between their home station and every RFL were estimated using Google Earth. Response times for helicopters were made using airspeed and distances between air bases and RFLs. In consultation with CALFIRE unit leaders, we established response time thresholds of 30 min for engines and 60 min for dozers, hand crews, and helicopters, beyond which a response would be considered unsatisfactory. Rapid response is critical because fast-spreading fires are likely to escape and cause considerable damage if concerted initial attack is not applied within the first 30 min (Arienti et al. 2006; Haight and Fried 2007). For application, we estimate the unit cost (i.e., annually operating costs) for each resource by type (Table 2).

Applications of the optimization model are solved on a Dell Pentium 4 desktop computer (CPU 2.4 GHz) with GAMS/CPLEX Solver. The termination criterion for the optimization runs is a combination of time limit and optimality: the solver is instructed to stop and report the solution after 16 h of runtime or after proven optimality is achieved, whichever happens first.

Computing bounds on the objective function value

Because the optimal solution of the problem (eqs. 1–6) with 100 fire scenarios is an approximation of the optimal solution of the problem with the complete set of 5814 scenarios, we compute bounds on the unknown optimal objective function value of the larger problem (Mak et al. 1999). The bounds are used to evaluate the quality of the approximation. For the upper bound ($\bar{U}$), we take the deployment for the optimal solution with 100 scenarios and compute the daily number of fires not receiving a standard response using all 5814 scenarios. For the lower bound ($\bar{L}$), we solve 20 replicates of the optimization problem with 20 independent sets of 100 scenarios. The lower bound is the mean of the objective function values from the 20 replicates. Finally, we compute the gap $(\bar{U} - \bar{L})/\bar{U}$, which represents the relative percentage difference between the upper and lower bounds on the unknown optimal objective function value for the problems (eqs. 1–6) with the complete set of 5814 fire scenarios.

Table 2. Crew size and operating costs of initial attack resources.

Attribute	Resource type			
	Engine	Dozer	Hand crew	Helicopter
Crew	3	1	17	6
Hourly cost (\$)	143	188	390	1051
Annual cost (\$) ^a	750 164	162 432	402 480	1 286 424

^aAnnual cost is based on hourly cost and estimated annual operating hours of each resource type obtained from consultation with CALFIRE personnel. Compared with engines, dozers have a higher hourly cost and lower annual cost because dozers are operated for fewer hours than engines.

Estimating the effects of employing a standard response objective

The first issue we address is how deployment and dispatch decisions obtained with an optimization model that minimizes the number of fires not receiving a standard response affect initial attack success. To address this issue, we formulate a base case using the CALFIRE resource deployment during the years 2005–2008 (Case A, Table 3). The deployment includes a total of 51 engines and seven dozers divided among 32 of the 45 stations in the study area (Case A, Fig. 2). In addition, 15 hand crew teams are deployed at four camps and eight helicopters are deployed at six air bases. The total annual operating cost of this deployment is \$55.7 million. We assume that resources may be dispatched between units to suppress fires. This CALFIRE resource deployment has remained relatively stable for many years despite changes in fire load, fire severity, values at risk, and access. Small changes in deployment do occur from year to year as CALFIRE adapts to changes in funding.

For comparison with this CALFIRE deployment case, we use the standard-response optimization model to deploy resources among the three planning units given the same budget level (\$55.7 million) and capacity constraints of the CALFIRE deployment, which we call the low capacity/current budget case (Case B, Table 3). The engine and dozer stations could each house up to two engines and two dozers. Conservation camps could each house up to five hand crews. Air bases could each house up to three helicopter crews. The optimization model also assumes that resources may be dispatched between planning units.

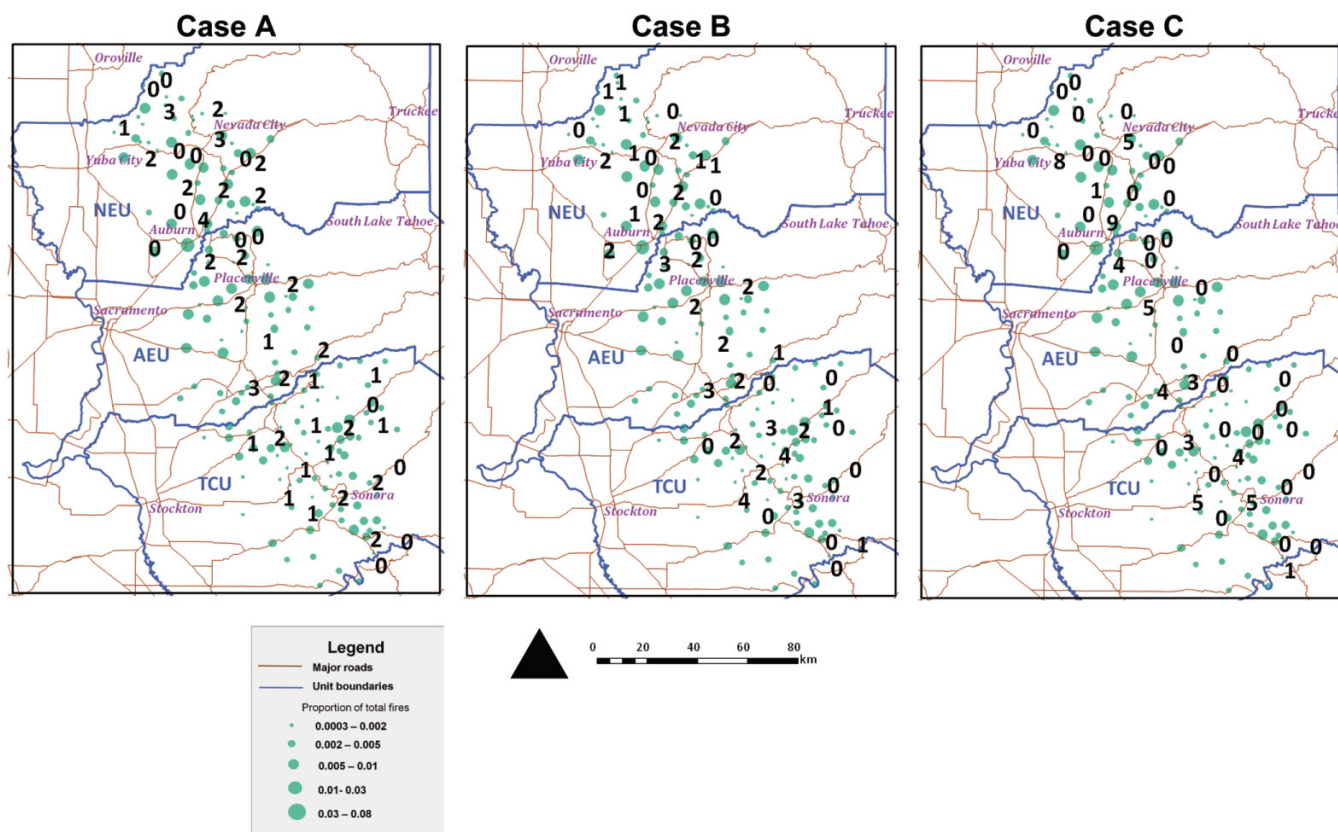
We measure the performance of the CALFIRE deployment and the deployment obtained with optimization by simulating initial attack using CFES2 and counting the number of fires that are not successfully contained. CFES2 is parameterized with the same inputs (e.g., for occurrence, fire behavior, fire locations, resource productivities, and response times) used in the simulations that generated the 5814 fire scenarios. Performance is measured by the number of fires that are not contained before they exceed simulation limits (ESL) on fire size or time, becoming “ESL” fires. The size limit is 50, 100, or 300 acres, depending on fuel and population density of the FMAZ where the fire occurs. The time limit is 2 h. These limits can be thought of as addressing both a goal (no

Table 3. Cases used for analysis.

Case ^a	Station capacity				Budget (\$million)
	Engine	Dozer	Hand crew	Helicopter	
A. Base (CALFIRE deployment)	2	2	5	3	55.7
B. Low capacity, current budget	2	2	5	3	55.7
C. High capacity, current budget	Unlimited	Unlimited	Unlimited	Unlimited	55.7
D. Low capacity, high budget	2	2	5	3	69.6
E. High capacity, high budget	Unlimited	Unlimited	Unlimited	Unlimited	69.6
F. Low capacity, low budget	2	2	5	3	41.8
G. High capacity, low budget	Unlimited	Unlimited	Unlimited	Unlimited	41.8
H. Heuristic, current budget	Unlimited	Unlimited	Unlimited	Unlimited	55.7

^aThe base case represents the current (2005–2008) deployment of resources in each planning unit with dispatch allowed between units. The other cases are resource deployments found by solving the scenario-based, standard-response optimization model with dispatch allowed between units and different budget and station capacity constraints.

Fig. 2. Deployment of engines and dozers in relation to representative fire locations in the current CALFIRE deployment (Case A), obtained with the optimization model with low station capacity and current budget (Case B), and obtained with the optimization model with high station capacity and current budget (Case C).



fires above a size limit or no fires with duration above a time limit) and a modeling constraint. A fire that exceeds either limit has likely transitioned from initial attack mode to extended attack mode, in which resources beyond the standard response are dispatched and the control strategy adjusted (e.g., pulling back to the next ridge or setting backfires rather than direct containment). The performances of resource deployments are estimated using the 5814 fire scenarios with high fire counts (defined as at least four fires in any unit in one day and no more than one fire at any RFL) and high fire season. The difference in performance between the CALFIRE deployment case and the optimization model's low capacity/current budget deployment case represents the effect of changing the number and type of resources in administrative units to minimize the number of fires not receiving the standard response requirements

for those units. In both the simulation model and the real world, a standard response does not guarantee that a fire will be contained (be prevented from becoming an ESL fire), although the vast majority of fires that receive a standard response are contained. Conversely, not receiving a standard response does not guarantee that a fire will become an ESL fire. From the perspective of fire managers and much of the public, however, ESL rates are far more germane than standard response achievement.

Note that CALFIRE relies on ESLs as a performance measure rather than a potentially more useful economic statistic like area burned in part because no initial attack model yet devised is capable of accurately predicting the size of fires that exceed initial attack, and these fires almost always account for nearly all of the area burned. Past attempts to assign average historic escaped fire

Table 4. Performance and cost of alternative initial attack resource deployments.

Case ^a	Number of resources deployed				Daily number of fires		Cost (\$million)			Gap ^d
	Engine	Dozer	Hand crew	Helicopter	ESL ^b	Not covered ^c	AEU	NEU	TCU	
A. Base (CALFIRE deployment)	51	7	15	8	0.522	2.86	16.9	20.7	18.1	
B. Low capacity, current budget	45	11	15	11	0.526	1.97	16.2	18.7	20.8	0.08
C. High capacity, current budget	46	11	13	11	0.478	1.92	15.4	23.9	16.4	0.10
D. Low capacity, high budget	57	16	22	12	0.488	1.80	21.5	21.7	25.3	0.05
E. High capacity, high budget	58	13	18	13	0.477	1.69	19.8	28.9	20.8	0.18
F. Low capacity, low budget	34	9	11	8	0.537	2.23	13.5	13.0	15.2	0.01
G. High capacity, low budget	35	10	9	8	0.531	2.22	13.7	18.9	9.2	0.02
H. Heuristic, current budget	51	7	15	8	0.490	2.50	17.4	22.6	15.7	

^aThe Base case (A) represents the current (2005–2008) deployment of resources in each planning unit with dispatch allowed between units. Cases B–G are resource deployments found by solving the scenario-based, standard-response optimization model with dispatch allowed between units and different budget and station capacity constraints. The optimization model is solved with a set of 100 randomly selected fire scenarios. Case H is the resource deployment found with the CFES2 simulation heuristic.

^bESL (exceed simulation limits) fires are computed using CFES2 and the complete set of 5814 fire scenarios.

^cDaily number of fires not covered (fires that do not receive a standard response) is computed over the complete set of 5814 fire scenarios.

^dThe gap is calculated as $(\bar{U} - \bar{L})/\bar{U}$ and represents the relative percentage difference between the upper bound ($\bar{U}$) and lower bound ($\bar{L}$) on the unknown optimal objective function value (daily number of fires not covered) for the problems (eqs. 1–6) with the complete set of 5814 fire scenarios.

sizes to ESLs (e.g., [USDA Forest Service 1985](#)) generated arbitrary results because such assignments are unavoidably an artifact of the period for which the average ESL size was computed. With increasing evidence that annual area burned is nonstationary, such assignments are also prone to bias.

Estimating the effects of station capacity constraints and annual operating budgets

The second issue we address is how changes in the station capacity constraints and budget affect resource deployment and initial attack success. To address this issue, we formulate and solve five additional optimization models with different combinations of constraints (Cases C–G, [Table 3](#)). In the first model (Case C), we remove the station capacity constraints while keeping the existing budget of \$55.7 million. The other four models (Cases D–G) are formulated with a different set of capacity and budget constraints in which budgets are increased or decreased by 25% and capacity constraints are included, or not. We use CFES2 to simulate the performance of the resource deployments obtained with each of these five models for comparison with the performance of the CALFIRE resource deployment. Performance is measured by the number of ESL fires per day based on the set of 5814 fire scenarios with high fire counts and high fire season.

Testing the performance of the optimization relative to a simulation-based heuristic

The standard response model does not attempt to minimize the number of ESL fires because optimization cannot be directly applied to that inordinately complex problem. Adopting the practical perspective of the analysts who routinely rely on CFES2 to conduct strategic fire planning, we developed a straightforward, iterative, simulation-based heuristic, well within the capabilities of a seasoned fire protection planning analyst, to seek ESL-reducing resource allocations that could be compared with allocations obtained with standard response optimization:

- (1) Use CFES2 to simulate the number of ESL fires for the current resource deployment.
- (2) Identify the least-used engine, dozer, and hand crew and re-deploy them to stations nearest the RFLs with high frequencies of ESL fires.
- (3) Return to step 1 and repeat.
- (4) Stop and produce a solution (termination condition: 1% gap of improvement).

We apply this heuristic using the current CALFIRE deployment as the starting point and assuming that the number of resources

by type remains fixed (Case H, [Table 3](#)). Thus, we consider trade-offs among resource deployments at different stations but not among resource types.

Results

Effects of employing a standard response objective

In the base case, the CALFIRE deployment of initial attack resources in 2005–2008 results in an average of 0.522 ESL fires per day for the days in which at least four fires occur in a single unit (Case A, [Table 4](#)). The average of 0.522 ESL fire per day represents 7.10% of the 42 835 fires included in the 5814 scenarios. The 51 engines and seven dozers are divided among 32 of 45 stations ([Fig. 2](#)) with the largest amount (40%) located in NEU, which has 43% of the fires.

The deployment obtained with the optimization model given the current budget and station capacity (low capacity and current budget, Case B in [Table 4](#)) uses more dozers and helicopters and fewer engines than does the CALFIRE deployment in Case A. More dozers and helicopters are deployed to meet the relatively high standard response requirements in TCU ([Table 1](#)). Engines and dozers are deployed in 29 of the 45 stations, and they are shifted from NEU, which has the highest fire load, to AEU and TCU to meet the standard response requirements in those units ([Fig. 2](#)). The optimal deployment averages 0.526 ESL fire per day ([Table 4](#)), which is not significantly different ($p < 0.05$) from the mean number of ESL fires per day for the Case A deployment. However, the optimal deployment reduces the expected number of fires per day that do not receive a standard response by 40% from 2.86 to 1.97 ([Table 4](#)), primarily because of the increased number of dozers and helicopters and the redeployment of engines and dozers from NEU to AEU and TCU.

To evaluate the quality of the deployment obtained in Case B, which is the optimal solution to the problem (eqs. 1–6) with 100 scenarios, we compute upper and lower bounds on the unknown optimal solution to the problem with the complete set of 5814 fire scenarios. The upper bound is the daily number of fires not covered (1.97), which is computed for the deployment obtained in Case B using the complete set of 5814 scenarios. The lower bound is the mean of the objective function values of 20 replicate solutions to the problem in Case B with 20 independent sets of 100 scenarios. The gap (relative percentage difference between upper and lower bounds, [Table 4](#)) is relatively small (0.08), suggesting that the optimal deployment obtained for the problem with 100 scenarios performs well relative to the unknown optimal solution to the problem with 5814 scenarios.

Table 5. Means of the objective function value (expected number of fires not receiving a standard response) for Case B (low station capacity and current budget) computed with sets of 20 replicates with increasing numbers of scenarios (N).

N	Lower bound ($\bar{L}$) 95% confidence interval ^a	Upper bound ($\bar{U}$) 95% confidence interval ^b	Optimal gap ($\bar{G}$) ^c	95% confidence interval ^c
30	1.72 ± 0.18	1.99 ± 0.04	0.14	[0, 0.49]
50	1.77 ± 0.10	2.01 ± 0.04	0.12	[0, 0.38]
100	1.82 ± 0.05	1.97 ± 0.04	0.08	[0, 0.24]
200	1.84 ± 0.03	1.97 ± 0.04	0.07	[0, 0.20]

^aThis average is obtained from the objective functions by solving eqs. 1–6 for 20 replicates with N scenarios.

^bFor each of the 20 optimal deployments obtained with N scenarios, we computed the associated expected number of fires not receiving a standard response using the complete set of 5814 scenarios.

^cThe optimal gap is calculated as $(\bar{U} - \bar{L})/\bar{U}$ and the confidence interval is calculated by using the method suggested by Mak et al. (1999).

To investigate how the number of scenarios used to compute the approximate solution affects the gap, we solve four sets of 20 replicates of the optimization problem for Case B. The 20 replicates in each set are constructed with N independent scenarios, with N equal to 30, 50, 100, and 200 scenarios to form the four sets. The lower bound estimate ($\bar{L}$) for each set is the mean of the objective function values over the 20 replicates (Table 5). The upper bound estimate for each set ($\bar{U}$) is the mean of the daily number of uncovered fires for the solutions to the 20 replicate problems with N scenarios, each computed using 5814 scenarios. Once we have the lower and upper bounds, we compute the gap ($\bar{G}$) and its confidence interval (Mak et al. 1999). The size of the gap is reduced 43% when the number of scenarios increases from 30 to 100. The size of the gap is reduced 12% when the number of scenarios increases from 100 to 200. This small reduction suggests that performance improvements will be small when increasing from 100 to 200 scenarios and beyond. An explanation is our 16 h limit on execution time: few of the problems with 100 or 200 scenarios are solved to a proven optimum within 16 h. Nevertheless, the gap we obtain using 100 scenarios is relatively small and the best bound we can obtain within resource limits.

Effects of station capacity constraints

When the current aggregate budget (\$55.7 million) is reallocated without station capacity constraints, the new deployment is designated as the high capacity and current budget Case C in Table 4. This deployment generates fewer ESL fires per day (0.478) and has fewer fires per day not receiving a standard response (1.92) compared with Case B. The 9% reduction in ESL fires is statistically significant ($p < 0.05$) and represents an improvement in the predicted performance of a deployment for which station capacity is not limiting.

Relaxing the station capacity constraints in Case C changes the location of initial attack resources while leaving the optimal mix of resources across the three planning units almost the same as in Case B, which has low station capacity and current budget (Table 4). Engines and dozers are concentrated in 13 of the 45 stations (Case C, Fig. 2). They are moved from TCU and AEU, which have the highest deployments in Case B, to NEU, which has the highest deployment in Case C. Further, with relaxed capacity constraints in Case C, seven of 11 helicopters can be deployed in a centrally located air base in NEU. The concentration of engines, dozers, and helicopters in stations in NEU is consistent with the relatively large fire load in NEU. These resources are well positioned to contribute towards a standard response for many fires; they are within 30 or 60 min, depending on resource type, of many possible fire locations. This deployment also contributes to the reduction in number of ESL fires and number of fires not receiving a standard response relative to Case B (Table 4). More-

over, the improvement in containment success may be understated because more concentrated basing may reduce costs of maintaining station infrastructure and free up funds for suppression resources, although some of these funds might be needed to cover the cost of adjusting capacity at stations hosting more resources than current rated capacity (e.g., for additional buildings to house equipment or staff).

The capacity constraints affect the optimal allocation of funding among the planning units. For the cases with high station capacity (Cases C, E, and G, Table 4), NEU has the highest budget allocation because NEU has the highest fire frequency. In these cases, more engines, dozers, and helicopters are deployed in NEU to cover fires in NEU and across the border in AEU. For the cases with low station capacity (Cases B, D, and F, Table 4), the optimal budget allocation favors TCU. In these cases, the upper limits on engines, dozers, and helicopters in NEU shift those resources to TCU where there are more stations.

Effects of the budget constraints

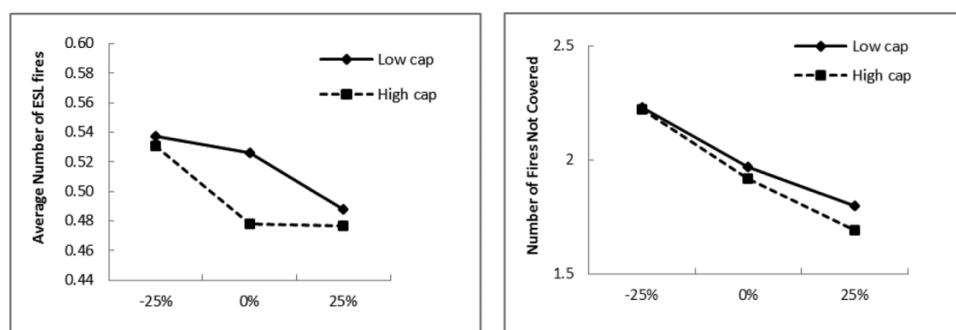
We found clear impacts of budget constraints on the daily number of ESL fires and the number of fires not receiving a standard response (Fig. 3). Increasing the budget level from 75% to 125% of the current level reduces the daily number of ESL fires from 0.537 to 0.488 in the low station capacity cases and from 0.531 to 0.477 in the high station capacity cases. Increasing the budget also reduces the daily number of fires not receiving a standard response.

The low budget cases provide guidance on how to reduce resources in the event of a budget reduction. The cases with lower budgets (Cases F and G, Table 4) have 11 fewer engines, one or two fewer dozers, four less hand crews, and three fewer helicopters than the cases with the current budget (Cases B and C, Table 4). The case with low station capacity has nine fewer stations when budget is reduced, while the case with high station capacity has four fewer stations, although of course this case had fewer stations before the budget reduction. Thus, a budget cut in the capacity constrained case is more likely to cause a complete shutdown of some stations by removing one or two deployed engines. In contrast, a budget cut in the case with unconstrained capacity reduces resources in most stations without closing them.

With a 25% higher budget, the optimization model increases all four types of resources and their deployment depends on the station capacity constraints. With capacity constraints, 17 engines and dozers, seven hand crews, and one helicopter are added to the three planning units (Cases B and D, Table 4). The new engines and dozers are added to eight new stations, mostly in TCU, which has 29% of the fires in the study area and the highest per fire standard response requirements for nonengine resources. Without station capacity constraints, 14 engines and dozers, five hand crews, and two helicopters are added to the planning units (Cases C and E, Table 4). The new deployment of engines and dozers is scattered among the 16 stations with no net gain in the number of stations.

Similar to Case B, we compute the gaps associated with the deployments obtained for Cases C–G, which range from 0.01 to 0.18 (Table 4). Each of these gaps represents the relative percentage difference between upper and lower bounds on the unknown optimal solution to the problem with the complete set of 5814 fire scenarios. While the gap for the deployment obtained with a high budget (Case E) is relatively high (0.18), the gaps for the deployments with low budgets (Cases F and G) are relatively low (0.01 and 0.02). With a high budget, more resources are allocated across stations, which produces high variation in the optimal deployment obtained for different sets of 100 fire scenarios. With a small budget, fewer resources are allocated across stations in a broad landscape, which produces relatively small variation in the optimal deployment for different sets of 100 scenarios. With the exception of Case E, the gaps are relatively small (≤ 0.10).

Fig. 3. Expected number of ESL fires (left) and number of fires not receiving a standard response (right) as the budget constraint is varied relative to the current budget (\$55.7 million).



Testing the performance of a simulation optimization heuristic

We applied the simulation optimization heuristic described above using the CALFIRE deployment (Case A) as the starting point. After eight iterations, the heuristic's solution involves shifting eight engines and two dozers from stations on the edges of the study area, where fire frequency is low, to centrally located stations near Auburn and Placerville, where fire frequency is high. The new deployment (Case H, Table 4) reduced the number of ESL fires by 6.6% relative to the performance of Case A. While the new deployment violated the capacity constraints at four stations, it allowed slightly more ESL fires than the optimal deployment in Case C, which had no capacity constraints and involved more complex changes to the mix of resources and deployment among the fire planning units. In addition to not finding a superior resource allocation for reducing ESL fires over the optimization model, the simulation optimization heuristic is time-consuming. The deployment obtained after eight iterations required about 19.5 h of execution time (2.4 h per simulation times eight simulations), slightly more than the execution time allowed for each application of the optimization model.

Discussion

We combine a scenario-based, standard-response optimization model with a stochastic fire simulation model (CFES2) to improve the efficiency of the deployment of suppression resources for initial attack on wildland fires. We expand the optimization model of Haight and Fried (2007), who focused on engine deployment in a single fire planning unit, by exploring opportunities for improved overall efficiency in multiple planning units where resources can be shared among units. We also add three types of resources to the mix — dozers, hand crews, and helicopters — that differ from engines and one another with respect to response time, fire line production, cost, and basing constraints, allowing us to investigate the optimal deployment of initial attack resources by type. We analyze the impacts of station capacity and budget constraints on resource deployment patterns. Finally, we evaluate the performance of the deployments obtained with optimization by predicting numbers of escaped fires with CFES2 and comparing the results with the performance of the current CALFIRE resource deployment.

The results provide insight into the performance of the optimization model, which deploys resources to minimize the expected number of fires that do not receive a standard response — defined as the number of resources by type that must arrive at the fire within a specified time limit — across multiple fire scenarios. Compared with the current CALFIRE deployment, the deployment obtained with optimization and the current budget and station capacities shifts resources from the planning unit with highest fire load to the planning unit with the highest standard response requirements. The new deployment is predicted to have

fewer fires that do not receive a standard response and no change in the number of ESL fires. We conclude that, in this case, an optimization model with a standard response objective provides resource deployments that perform at least as well as the predicted performance of an existing resource deployment based on expert knowledge and experience.

The results also provide insight into the performance of the existing resource deployment. Using the optimization model to deploy resources under existing budget and station capacity constraints, we did not find efficiency gains, in terms of reducing the predicted number of escaped fires. This result suggests that the current CALFIRE deployment is based more on fire loads and less on standard response requirements across the fire planning units, and the weight placed on fire load in determining resource deployment does not greatly affect simulated performance. However, we did find significant performance gains with increased budget and when station capacity was not bounded. While we expected that performance would scale with budget, the performance improvements associated with increasing station capacity were unexpected.

Budget and station capacity constraints not only limit the number of initial attack resources but also influence the appropriate mix of deployed resources due to the differences in cost and productivity across resource types. The change in mix of resources across management units and at particular stations as the budget and station capacity change depends on attributes like unit cost, productivity, response times, and abundance of each resource type. A reduction in budget or station capacity may decrease the use of some resources while increasing the use of others due to interplay among these attributes among resources types. For example, a budget cut that eliminates part of the funding for a helicopter may result in the rest of the helicopter funding being redirected into an increase in the number of dozers due to their lower cost. Considering the deployment of all initial attack resources simultaneously reveals complexities in the mix of resources because of differences in the usefulness and unit cost of each resource.

There are at least three important modeling assumptions that may affect results. In our models, an RFL is a map point that represents a proportion of the average annual fire load together with a particular mix of fuels, topography, and distances to fire stations. In practice, the mix of resources that are dispatched to fires, and the timing of their arrivals, will differ among fires represented by a given RFL. Some fires will be more, and others less, accessible to suppression resources than assumed by the RFL point, which may affect the accuracy of fire simulations. While it is conceptually possible to increase the number of RFLs without limit, it can be challenging to assign historical fires to a very large number of locations based on similarity across multiple attributes (e.g., geographic location, fuel, slope, resource arrival times, and complicating factors such as homes, fences, or unique terrain

features), and historical fire locations that are distant from the road network and lightning-prone ridges may not be useful predictors of future fire locations. Furthermore, simulation time increases at least linearly with RFL count. The second assumption involves edge effects. We assume that stations at the edge of our three-unit study area only serve fires within the study area and not outside. As a result, the optimization may deploy suppression resources in the interior of the study area where they have access to more fires. In practice, stations may serve fires in any adjacent fire planning unit and our results do not account for these edge effects. Third, this framework implicitly treats all fires that exceed initial attack size limits as equal; the deployments and dispatches described here do not reflect heterogeneity across space in the magnitude of the damage or the eventual size of the escaped fire. In practice, fires located near large populations or particularly valuable resources may receive a higher priority for initial attack than identical fires in other locations. In future work, we can weight the RFLs by a metric of importance in terms of avoiding ESL fires.

For comparison with results from the optimization model, we develop a simple deployment heuristic to apply with CFES2 simulations. Similar to the results of the optimization model without capacity constraints, the results of the heuristic suggest that we can improve the performance of initial attack resources by allocating them to stations with high fire loads, as these are also proximal to higher incidences of ESL fires. Considering the effect of budget and trade-offs among resources of different types requires developing more complex heuristics, which is beyond the scope of this study but is potentially a fruitful area for future study.

Our scenario-based, standard-response optimization model is static in the sense that it determines optimal deployment given an approximation of the probability distribution of fire locations and intensities during a single day with four or more fire ignitions during the high fire season. We solve for optimal deployment given uncertainty about the number and location of fires during a severe fire day because this is the type of day when the initial attack resources will be challenged to meet demands for fire suppression and because escaped fires may cause catastrophic damage and be very expensive to extinguish. Our model is not dynamic and does not account for a sequence of days during the fire season where what happens during one day influences what happens on the next. It may be possible to model fire days as a Markov process and use stochastic dynamic programming to determine optimal resource deployment.

Taken together, the results of our study emphasize the economic trade-offs among resources and across locations. The results also suggest that combining optimization and simulation models of initial attack can inform and supplement planners' intuition regarding the efficient deployment of suppression resources.

Acknowledgments

We are grateful to Jim Spero, Bill Holmes, and Douglas M. Ferro of the California Department of Forestry and Fire Protection and Lewis Ntamo of Texas A&M University for valuable discussions and suggestions. We thank the Associate Editor and three anonymous reviewers for constructive comments. Research funding was provided by the US Forest Service Northern Research Station and the National Fire Plan. We also gratefully acknowledge the

USFS-OSU 07-JV-11242300-086's funding for Y. Lee to pursue his doctoral degree.

References

- Arienti, M.C., Cumming, S.G., and Boutin, S. 2006. Empirical models of forest fire initial attack success probabilities: the effects of fuels, anthropogenic linear features, fire weather, and management. *Can. J. For. Res.* **36**(12): 3155–3166. doi:10.1139/x06-188.
- Barbour, M.G., Keeler-Wolf, T., and Schoenherr, A.A. 2007. *Terrestrial vegetation of California*. 3rd ed. University of California Press, Berkeley and Los Angeles, Calif.
- Calkin, D.E., Gebert, K.M., Jones, J.G., and Neilson, R.P. 2005. Forest Service large fire area burned and suppression expenditure trends, 1970–2002. *J. For.* **103**(4): 179–183.
- Deeming, J.E., Burgan, R.E., and Cohen, J.D. 1977. The National Fire-Danger Rating System — 1978. U.S. For. Serv. Gen. Tech. Rep. INT-39.
- Donovan, G.H., and Rideout, D.B. 2003. An integer programming model to optimize resource allocation for wildfire containment. *For. Sci.* **49**(2): 331–335.
- Franklin, J., Woodcock, C.E., and Warbington, R. 2000. Multi-attribute vegetation maps of Forest Service lands in California supporting resource management decisions. *Photogram. Eng. Remote Sens.* **66**(10): 1209–1217.
- Fried, J.S., and Fried, B.D. 1996. Simulating wildfire containment with realistic tactics. *For. Sci.* **42**: 267–281.
- Fried, J.S., and Gilless, J.K. 1988. Stochastic representation of fire occurrence in a wildland fire protection planning model for California. *For. Sci.* **34**(4): 948–955.
- Fried, J.S., and Gilless, J.K. 1999. CFES2: The California Fire Economics Simulator version 2 user's guide. Division of Agriculture and Natural Resources Publication 21580. University of California, Oakland, Calif.
- Fried, J.S., Gilless, J.K., and Spero, J. 2006. Analyzing initial attack on wildland fires using stochastic simulation. *Int. J. Wildl. Fire*, **15**: 137–146. doi:10.1071/WF05027.
- Gilless, J.K., and Fried, J.S. 1999. Stochastic representation of fire behavior in a wildland fire protection planning model for California. *For. Sci.* **45**(4): 492–499.
- Greulich, F.E., and O'Regan, W.G. 1982. Optimum use of air tankers in initial attack: selection, basing, and transfer rules. U.S. For. Serv. Res. Pap. PSW-163.
- Haight, R.G., and Fried, J.S. 2007. Deploying wildland fire suppression resources with a scenario-based standard response model. *INFOR J.* **45**(1): 31–39. doi:10.3138/infor.45.1.31.
- Hodgson, M.J., and Newstead, R.G. 1978. Location-allocation models for one-strike initial attack of forest fires by airtankers. *Can. J. For. Res.* **8**: 145–154. doi:10.1139/x78-024.
- Holmes, T.P., Huggett, R.J., Jr., and Westerling, A.L. 2008. Statistical analysis of large wildfires. In *The economics of forest disturbances: wildfires, storms, and invasive species*. Edited by T.P. Holmes, J.P. Prestemon, and K.L. Abt. Springer, New York. pp. 59–77.
- Hu, X.L., and Ntamo, L. 2009. Integrated simulation and optimization for wildfire containment. *ACM Trans. Model. Comput. Simul.* **19**(4): Article 19.
- Kourtz, P. 1989. Two dynamic programming algorithms for forest fire resource dispatching. *Can. J. For. Res.* **19**: 106–112. doi:10.1139/x89-014.
- Littell, J.S., McKenzie, D., Peterson, D.L., and Westerling, A.L. 2009. Climate and wildfire area burned in western U.S. *Ecol. Appl.* **19**(4): 1003–1021. doi:10.1890/07-1183.1.
- MacLellan, J.I., and Martell, D.L. 1996. Basing airtankers for forest fire control in Ontario. *Oper. Res.* **44**: 677–686. doi:10.1287/opre.44.5.677.
- Mak, W.K., Morton, D.P., and Wood, R.K. 1999. Monte Carlo bounding techniques for determining solution quality in stochastic programs. *Oper. Res. Lett.* **24**: 47–56. doi:10.1016/S0167-6377(98)00054-6.
- Marianov, V., and ReVelle, C. 1991. The standard response fire protection siting problem. *INFOR J.* **29**: 116–129.
- Martell, D.L. 1982. A review of operational research studies in forest fire management. *Can. J. For. Res.* **12**(2): 119–140. doi:10.1139/x82-020.
- Martell, D.L. 2007. Forest fire management: current practices and new challenges for operational researchers. In *Handbook of operations research in natural resources*. Springer Science plus Business Media, New York. Chap. 26. pp. 489–509.
- Mees, R., Strauss, D., and Chase, R. 1994. Minimizing the cost of wildland fire suppression: a model with uncertainty in predicted flame length and fire-line width produced. *Can. J. For. Res.* **24**: 1253–1259. doi:10.1139/x94-164.
- Parks, G.M. 1964. Development and application of a model for suppression of forest fires. *Manage. Sci.* **10**: 760–766. doi:10.1287/mnsc.10.4.760.
- USDA Forest Service. 1985. National fire management analysis system user's guide. Release of the initial action assessment model (FPLIAA2). Aviation and Fire Management, Washington, D.C.