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An Evaluation of a Proposed Revision of the ASTM D 1990 Grouping Procedure

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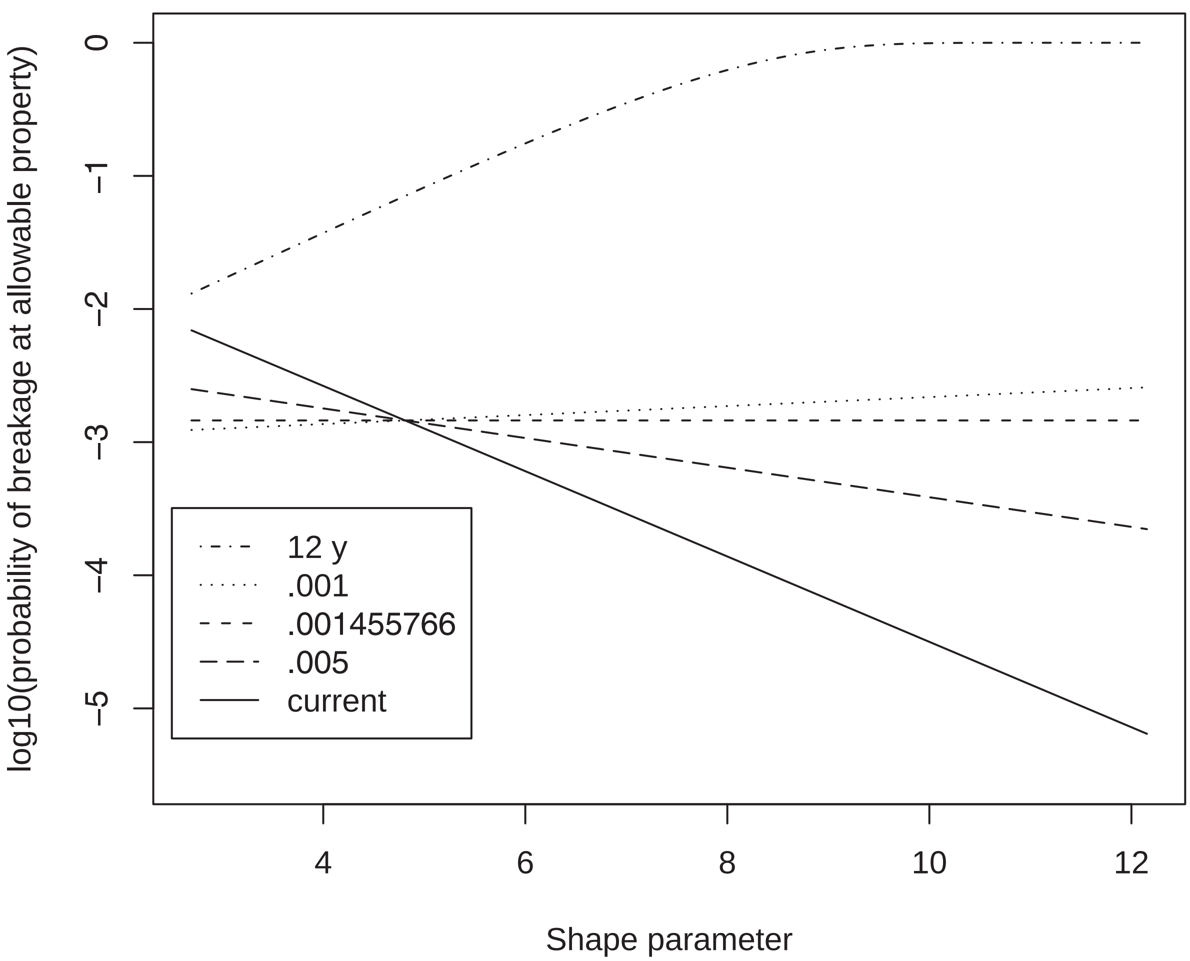

Abstract

Lum, Taylor, and Zidek have proposed a revised procedure for wood species grouping in ASTM standard D 1990. We applaud the authors' recognition of the importance of considering a strength distribution's variability as well as its fifth percentile. However, we have concerns about their proposed method of incorporating this information into a standard. We detail these concerns in this paper. We also provide both theoretical and empirical arguments that suggest that revisions of the standard should not be based on a two-parameter Weibull assumption.

Keywords: Reliability, modulus of rupture, modulus of elasticity, bivariate Gaussian–Weibull, pseudo-truncated Weibull, Weibull

Contents

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1 Introduction

In 2008, the American Lumber Standard Committee Board of Review formed a North American Lumber Properties Task Group to review the application and interpretation of ASTM structural lumber standards. The subjects under review included species grouping, withdrawal of species from existing species groups, the appropriateness of using parametric fits to censored tails of lumber data, and issues with sampling representativeness. The charge of one of the task subgroups was to investigate procedures that might be improvements over the currently accepted grouping procedure that is based on nonparametric estimates of fifth percentiles. In this paper, we evaluate a proposed procedure that is based on two-parameter Weibull fits to lumber strength data.

Two important wood properties are bending strength (modulus of rupture or MOR) and stiffness (modulus of elasticity or MOE). In the past, MOR populations have been modeled as lognormals or as two- or three-parameter Weibulls. MOE populations have generally been modeled as Gaussians. (See, for example, ASTM 2010a, Evans and Green 1988, and Green and Evans 1988.)

Design engineers must ensure that the loads to which wood systems are subjected rarely exceed the systems' strengths. To this end, ASTM D 2915 (ASTM 2010a), and ASTM D 245 or ASTM D 1990 (ASTM 2010b,c) describe the manner in which "allowable properties" are assigned to populations of structural lumber. In essence, an allowable strength property is calculated by estimating a fifth percentile of a population (actually a 95% content, one-sided lower 75% tolerance bound) and then dividing that value by duration of load and safety factors. The intent is that the population can only be used in applications in which the load does not exceed the allowable property.

It is often the case that lumber marketers want to assign a single allowable property to a grouping of multiple species. Currently, ASTM D 1990 details procedures for determining this shared allowable property. For strength properties, these procedures are currently solely focused on the fifth percentile of an individual species or group of species. Lum, Taylor, and Zidek (2010a,b,c) and others (see, for example, Verrill and Kretschmann 2009) have noted that under the current procedures, two strength distributions could share a fifth percentile and thus an allowable property, and yet, if the populations had different variabilities, the associated probabilities of breakage at the allowable property would differ. This is illustrated in Figure 1. The wide distribution has mean 1 and standard deviation .25. The narrow distribution has mean $1 - 1.645 \times .25 + 1.645 \times .05$ and standard deviation .05. The two distributions share a fifth percentile $(1 - 1.645 \times .25)$, and yet it is clear visually that a value drawn from the wide distribution is more likely to fall below the allowable property than a value drawn from the narrow distribution. (In fact, it can be shown that a value from the wide distribution falls below the allowable property with probability .002 while a value from the narrow distribution falls below the allowable property with probability 2.8E-15.)

Thus, an improved grouping procedure should take differing variabilities into account. In fact, ideally, an improved procedure would focus on grouping based on similar probabilities of failure for a given load distribution. Lum *et al.* (2010a,b,c) have proposed a grouping procedure that does take variability into account. (In their papers they emphasize that the proposal is a *draft* proposal.) However, it is not based on probabilities of failure. Apparently, the authors felt that for practical reasons, it was still necessary to base groupings on something like an allowable property based on a fifth percentile divided by a duration of load and safety factor. As we will detail below, they proposed to incorporate information about a population's variability by calculating an "adjusted fifth percentile." For populations with a coefficient of variation (CV) above some standard value (more variable populations), the adjusted fifth percentile would lie below the actual fifth percentile. For populations with a coefficient of variation below the standard value (less variable populations), the adjusted fifth percentile would lie above the actual fifth percentile.

We applaud the authors for their insight, but believe that their current proposal needs further work (as expected given its draft status). We believe that their proposal is unsatisfactory for four reasons:

1. Their current proposal still permits probabilities of breakage at allowable properties to differ among species and among groups. In Section 4.1, we do describe a particular implementation of their proposal that would address this problem. (This particular implementation is simpler in conception and execution than their more general proposal.) However, in Section 4.2, we note that equal probabilities of breakage cannot be guaranteed under all duration of load conditions.

2. In Section 4.3, we discuss the fact that their proposal needs to be investigated for safety consequences.

3. In Section 4.4, we note that the proposed procedure can still lead to unexpected results (the probability of breakage of the "controlling" species at the allowable property can be less than that of a non-controlling species).

4. The current proposal is based on an assumption of a two-parameter Weibull strength distribution. Verrill, Evans, Kretschmann, and Hatfield (2012) demonstrate that if the full MOE, MOR distribution is bivariate Gaussian–Weibull, then a subcategory based on machine stress rated (MSR) values (or a normally distributed quality characteristic that is implicit in visually graded categories) will not be distributed as a Weibull. Instead, it will be distributed as a pseudo-truncated Weibull (PTW). In Sections 5.2 and 5.3 we discuss empirical evidence that actual MOR distributions are indeed PTW rather than Weibull. In Section 5.4 we discuss simulations that we have conducted that demonstrate that breakage rate calculations that are based on Weibull fits are susceptible to significant error if the strength distribution is actually a pseudo-truncated Weibull.

Before proceeding with our analysis, we would like to clarify one point. We have spoken about "probabilities of breakage at the allowable property" and about "similar probabilities of failure for a given load distribution." Ultimately, we would like to see (if possible) standards that assure us that if a wood assembly's environment (heat, moisture, biological ...) is drawn from a given "distribution" and its load history is drawn from a given "distribution," then if the members of the assembly are drawn from a given species grouping, the probability that the assembly "fails" (in terms of safety or, alternatively, serviceability) prior to a given time is less than a given value. However, in the current paper, we are focusing on the probability that a piece of lumber breaks when it is subjected to a load equal to the allowable strength property of the population from which the piece is drawn. We will denote this probability by p_{Br} and refer to it as "the probability of breakage."

We recognize that our "probabilities" cannot be taken entirely literally. Our probability calculations are mathematically correct, but they rely on simplified models that may not adequately reflect the real world. Our paper is primarily an attempt to evaluate Lum *et al.*'s (2010a,b,c) proposed fifth percentile adjustment based on currently available data and models. As our data and models improve, our probability estimates will also improve.

2 Aside

In connection to Figure 1, a reviewer of this paper noted 1) that the load is, of course, variable, 2) that strength populations are unlikely to have Gaussian distributions (the reviewer suggested doing some modeling with lognormal strength and load distributions), and 3) that when variable loads are included, the "unfair" advantage that a wide distribution is given when we group solely on the basis of fifth percentiles is lessened. The reviewer felt that if we did some calculations with lognormal loads and strengths (which are mathematically tractable), "you will conclude that 'messing' with the fifth/2.1 can't be justified in the context of structural reliability or safety."

One of our responses was that we were simply using Figure 1 to illustrate the obvious, but sometimes overlooked point that a fifth percentile is not sufficient to characterize the behavior of a population, and that if we group based solely on fifth percentiles, we are grouping on a basis that is not justified from a reliability perspective. A second response was that we were evaluating a proposal that made a Weibull strength assumption so, indeed, we would not be making a Gaussian strength assumption in the remainder of the paper. A third response was that although full reliability calculations must include a load distribution, we could gain insights into the proposed method by evaluating the probability that a Weibull strength distribution would lie below an allowable property. This makes the Weibull math tractable and leads to some elegant simplifications (see, for example, Section 4) that permit us to gain a better understanding of the proposed method. A fourth response is that the focus of this report is the proposed method for adjusting fifth percentiles prior to grouping. Our purpose here is not to present a full-throated defense of the argument that grouping should be based on similar probabilities of failure given similar load distributions rather than on similar 5th percentiles. However, in response to the reviewer, we did perform some calculations based on the lognormal distribution. The details of the calculations are provided in the Appendix. We wanted to check whether significant "unfairness" (significant differences in reliabilities) continued to exist for "narrow" and "wide" distributions that shared the same fifth percentile when we calculate probabilities of failure based on a load distribution rather than at a fixed point (treating the allowable property as a "worst case"). The results from these calculations are listed in Table 1, and the second row of Table 1 is illustrated in Figure 2. In line with the suggestions of the reviewer, we assumed lognormal strength and load distributions. The reviewer also suggested that in our calculations, we let the load distribution have a CV approximately equal to 0.3, and let the probability that the load exceeds the allowable property be approximately 0.02. We considered 15 cases. In each case, the CV of the load distribution was taken to be 0.3. We let the probability that the load exceeded the allowable property equal 0.01, 0.02, 0.05, 0.10, or 0.20. We let the CV's of the narrow strength populations be 0.05, 0.1, or 0.2 and the CV's of the corresponding wide strength populations be 0.25, 0.3, or 0.4. From Table 1, we can see that for the conditions considered, the reviewer's intuition is at least partially validated. That is, the p_{F} narr and p_{F} wide values for the cases considered in Table 1 are closer than the 2.8E-15 and .002 associated with Figure 1. However, the results continue to suggest that equal fifth percentiles do not in general lead to probabilities of failure that are so close that differences can be neglected. Of course, this is hardly a comprehensive study of the issue. It could be argued, for example, that strength distributions are not lognormal or Weibull but pseudo-truncated versions of standard strength distributions (see Section 5). This might lead to small probabilities of failure for both "narrow" and "wide" strength distributions and differences in these probabilities might not be of practical importance. Also, we have compared distributions with highly diverging CV's (for example, 0.05 and 0.25). In practice, differences in strength distribution CV's might be slight enough so that differences in probabilities of failure will again not be of practical importance. We leave further exploration of these issues to future papers.

3 The Proposal of Lum, Taylor, and Zidek (2010)

In essence, their proposal was:

1. Strength distributions would be modeled as two-parameter Weibulls.
2. The controlling species in a group of species would be the one with the lowest pth quantile for some pre-specified p.
3. The allowable strength property for a group would be $x_{.05,\text{adj}}/2.1$ where $x_{.05,\text{adj}}$ is the adjusted fifth percentile of the controlling species. This adjusted fifth percentile would be a function of the controlling population's coefficient of variation and some standard coefficient of variation. If a controlling population's coefficient of variation were smaller than the standard coefficient of variation, then the adjusted fifth percentile would be boosted up. If a controlling population's coefficient of variation were larger than the standard coefficient of variation, then its fifth percentile would be adjusted down.

Here is a detailed description of their algorithm for calculating $x_{.05,\text{adj}}$.

Let β be the shape and $1/\gamma$ be the scale of the assumed Weibull strength distribution of the controlling species. The pth quantile, x_p, of this distribution is given by

$$p = 1 - \exp\left(-(\gamma \times x_p)^\beta\right)$$

or

$$x_p = (-\ln(1-p))^{1/\beta}/\gamma \tag{1}$$

Let β_0 be the shape parameter corresponding to the standard coefficient of variation. (For two-parameter Weibull distributions, the coefficient of variation is solely a function of the shape parameter.)

Find the γ_0 that satisfies

$$p = 1 - \exp\left(-(\gamma_0 \times x_p)^{\beta_0}\right)$$

or

$$\gamma_0 = (-\ln(1-p))^{1/\beta_0}/x_p \tag{2}$$

The adjusted fifth percentile is then given by

$$.05 = 1 - \exp\left(-(\gamma_0 \times x_{.05,\text{adj}})^{\beta_0}\right)$$

or, using (2) and (1),

$$\begin{aligned} x_{.05,\mathrm{adj}} &= (-\ln(1-.05))^{1/\beta_0}/\gamma_0 \\ &= \left(\frac{\ln(1-.05)}{\ln(1-p)}\right)^{1/\beta_0} \times x_p \\ &= \left(\frac{\ln(1-.05)}{\ln(1-p)}\right)^{1/\beta_0} \times \frac{(-\ln(1-p))^{1/\beta}}{\gamma} \qquad (3) \end{aligned}$$

and the corresponding adjusted allowable property is $x_{.05,\mathrm{adj}}/2.1$. (Note that the unadjusted fifth percentile, $x_{.05}$, is given by replacing p in (3) by .05.)

4 Performance of the Proposed Procedure

To evaluate the performance of this procedure, we calculate the probability that a piece of lumber drawn from the controlling species population breaks when it is subjected to a load equal to the allowable strength property of the species.

From (3) we have

$$\begin{aligned} p_{\mathrm{Br}} &= 1 - \exp\left(-(\gamma \times x_{.05,\mathrm{adj}}/2.1)^{\beta}\right) \qquad (4) \\ &= 1 - \exp\left(-(\mathrm{arg})^{\beta}\right) \end{aligned}$$

where

$$\mathrm{arg}^{\beta} = \left[\frac{1}{2.1} \times \left(\frac{\ln(1-.05)}{\ln(1-p)}\right)^{1/\beta_0}\right]^{\beta} \times (-\ln(1-p)) \qquad (5)$$

In the next subsection, we will discuss a figure in which we plot $\log(p_{\mathrm{Br}})$ versus β for a variety of p's. Somewhat surprisingly, these plots will appear to be linear. This is due to the fact that, for the p's and β's that we consider, arg^{β} is "small" so

$$\exp\left(-(\mathrm{arg})^{\beta}\right) \approx 1 - (\mathrm{arg})^{\beta}$$

or

$$1 - \exp\left(-(\mathrm{arg})^{\beta}\right) \approx (\mathrm{arg})^{\beta}$$

and from (5),

$$\begin{aligned} \ln(p_{\mathrm{Br}}) &= \ln\left(1 - \exp\left(-(\mathrm{arg})^{\beta}\right)\right) \\ &\approx \beta \times \ln\left[\frac{1}{2.1} \times \left(\frac{\ln(1-.05)}{\ln(1-p)}\right)^{1/\beta_0}\right] \\ &\quad + \ln(-\ln(1-p)) \qquad (6) \end{aligned}$$

which is a linear function of β. (For unadjusted fifth percentiles, (6) becomes $\ln(p_{\mathrm{Br}}) \approx -\ln(2.1) \times \beta + \ln(-\ln(1-.05))$.)

4.1 Is the proposed procedure "fairer" than one based on unadjusted fifth percentiles?

In Figure 3, we plot the log base 10 of the probability of breakage of a (controlling) species at its allowable load versus shape. The solid line represents the "current situation"—that is, the allowable property is taken to be an unadjusted fifth percentile of a two-parameter Weibull divided by 2.1. The long-dashed line represents the situation under the Canadian proposal when the quantile p being considered is .005 and the baseline shape is 4.8. The short-dashed line represents the situation under the Canadian proposal when the quantile p being considered is .001455766 and the baseline shape is 4.8. The dotted line represents the situation under the Canadian proposal when the quantile p being considered is .001 and the baseline shape is 4.8.

From Figure 3, we can see that a procedure based on an unadjusted fifth is unfair in that, under it, populations with lower shapes (higher coefficients of variation) are permitted higher probabilities of breakage at their allowable properties. The Canadian procedure is fairer as it jacks down the permissible probabilities of breakage for shapes below 4.8 and increases them for shapes above 4.8. As the quantile used in the procedure decreases, the slope of the line increases.

From Figure 3, we can also see that, under the proposed approach, it is possible to develop a "fair procedure" that yields the same probability of breakage at the allowable property for all controlling species. That is, given any baseline shape, β_0, we can specify a corresponding p (0.001455766 for $\beta_0 = 4.8$) for which the p_{Br} versus β curve is flat (the short-dashed line in Figure 3).

This can be demonstrated as follows. From results (4) and (5), we know that p_{Br} will be a constant function of β if

$$\left(\frac{1}{2.1}\right)^{\beta_0} \times \left(\frac{\ln(1-.05)}{\ln(1-p)}\right) = 1 \tag{7}$$

or

$$\ln(1-p) = \ln(1-.05)/2.1^{\beta_0}$$

or

$$1 - p = \exp\left(\ln(1-.05)/2.1^{\beta_0}\right)$$

or

$$p = 1 - \exp\left(\ln(1-.05)/2.1^{\beta_0}\right) \tag{8}$$

(As noted above, for baseline $\beta_0 = 4.8$, this p equals .001455766.)

In this constant p_{Br} case, from (4), (5), and (7), we have

$$\begin{aligned} p_{\text{Br}} &= 1 - \exp\left(-(\text{arg})^{\beta}\right) \\ &= 1 - \exp(\ln(1-p)) = p \end{aligned} \tag{9}$$

That is, the probability of breakage in this "fair case" is just the p value that was used in the proposed procedure. This tells us that in the "fair case," the proposed procedure simply amounts to choosing a p value and then stating that the allowable property associated with any controlling species will be its pth quantile (so that the probability that any controlling species will break when subjected to a load equal to its allowable property will be p).

4.2 Duration of load and inherent unfairness

Any procedure is *inherently* unfair. To understand this central point, consider the current situation. Why is it "unfair" to narrower distributions? As illustrated in Figure 1, in a probability sense a .05 quantile (fifth percentile) divided by 2.1 is farther down in the tail ("more standard deviations below the mean" for a normal distribution) for a narrow distribution than for a broad distribution. Thus the p_{Br} (probability of breakage at the allowable property) for the narrower distribution at its fifth/2.1 is smaller than the p_{Br} for the broader distribution at its fifth/2.1. If instead, we use the pth quantile of a distribution as its allowable property, then p_{Br} would be the same (p) for all distributions.

However, as soon as specimens are subjected to duration of load and their strengths are reduced, this "fairness" is likely gone. Now *narrower* distributions are likely to be favored. Gerhards (2000) found that after 12.27 years under load, 16 of 50 No. 2 two by fours failed. The load was intended to be the "10-year design load" of the distribution from which the two by fours were drawn, that is $x_{.05}/1.62$. (The 1.3 factor for safety was not included, but this does not affect our conclusions. See below.) Gerhards later concluded that the load was actually $x_{.08}/1.62$. (Again, this does not affect our conclusions. See below.) He also calculated (by assuming that the original strength of the strongest piece of lumber to fail was equal to the expected strength of the 16th order statistic of 50 from the strength population) that the approximate load to strength value for the strongest piece of lumber to fail was 0.442. If we assume that all pieces with load to strength ratios greater than 0.442 will fail by 12.27 years, then we can calculate the probability of breakage after 12.27 years for loads set to "fair" adjusted allowable properties at day 0:

$$\begin{aligned} p_{\mathrm{Br}} &= \mathrm{Prob}\left(\text{Initial Strength} < \frac{x_{.05,\mathrm{adj}}}{2.1} \times \frac{1}{.442}\right) \\ &= 1 - \exp\left(-\left(\gamma \times \frac{x_{.05,\mathrm{adj}}}{2.1} \times \frac{1}{.442}\right)^{\beta}\right) \\ &= 1 - \exp\left(-(\mathrm{arg})^{\beta}\right) \end{aligned} \qquad (10)$$

where

$$\mathrm{arg}^{\beta} = \left[\frac{1}{(2.1)(.442)} \times \left(\frac{\ln(1-.05)}{\ln(1-p)}\right)^{1/\beta_0}\right]^{\beta} \times (-\ln(1-p))$$

and, for $\beta_0 = 4.8$ and a flat initial p_{Br} versus β curve,

$$p = 0.001455766$$

In Figure 4, we replicate Figure 3 but also add the p_{Br} versus β curve where p_{Br} is calculated via (10). Figure 4 demonstrates that after 12 years of load, the probability of breakage is no longer "fair" — P_{Br} is no longer a flat function of the CV. Instead, it increases as CV decreases (as β increases).

4.3 Is the proposed procedure "as safe" as one based on unadjusted fifth percentiles?

We can see from Figure 3 that implementing the proposed approach would (for at least some p values) decrease the probability of breakage at the allowable load for species with shape values less than the baseline shape. However, it would *increase* the probability of breakage at the allowable load for species with shape values greater than the baseline shape. It is not immediately clear that this would translate into a situation that is as safe as the current situation. (The species associated with the highest probabilities of breakage at their allowable properties under the current system would have these probabilities reduced under the proposal. However, are these species as common as those whose probabilities of breakage would increase under the proposal?)

In Figure 5 we plot log base 10 of the ratio of the probability of breakage under the proposed procedure to the probability of breakage under the current procedure versus controlling species shape for a baseline of 4.8. As a reference, we also plot the $\log 10(\text{ratio}) = 0$ line. (p_{Br} under the current procedure is just $1 - \exp(\ln(1 - .05)/2.1^{\beta})$ and p_{Br} under the proposed procedure is given by (4).)

Obviously, the ratios begin below 1, increase to 1 at the baseline shape, and then continue to increase. The increases can be quite large. (The minimum ratio in Figure 5 is .210 at controlling species $\beta = 2.6956$ (corresponding to a CV of 40%). The maximum ratio in Figure 5 is 226 at controlling species $\beta = 12.154$ (corresponding to a CV of 10%).) Thus, again, it is not immediately clear that we would not be degrading our reliability by adopting the proposed procedure.

4.4 Anomalous behavior of the proposed procedure

Suppose that there are two species in the group, species A and species B. Further suppose that it has been decided to base the procedure on the pth quantile, and that the pth quantile of species A, $x_{p,\text{A}}$, lies below the pth quantile, $x_{p,\text{B}}$, of species B. Then species A is the controlling species.

Now suppose that p and the baseline shape, β_0, are chosen so that the p_{Br} versus β curve is flat. In this case, as demonstrated in Subsection 4.1 (see equation (9)), $p_{\text{Br,A}} = p$ and the allowable property is just $x_{p,\text{A}}$. Since

$$x_{p,\text{B}} > x_{p,\text{A}} = \text{the allowable property}$$

$$\begin{aligned} p_{\text{Br,B}} &= \text{prob}(X_{\text{B}} < \text{the allowable property}) \\ &= \text{prob}(X_{\text{B}} < x_{p,\text{A}}) \\ &\leq \text{prob}(X_{\text{B}} < x_{p,\text{B}}) = p = p_{\text{Br,A}} \end{aligned}$$

Thus, species B is at least as safe at the allowable property as the controlling species A.

However, suppose, instead, that p and the baseline shape, β_0, are not chosen so that the p_{Br} versus β curve is flat. In this case, it is possible for the allowable property to lie below or above $x_{p,\text{A}}$, and undesirable behavior can result.

If the allowable property lies below $x_{p,\text{A}}$ and species B has a broader distribution than species A, it will be possible for $p_{\text{Br,B}}$ to exceed $p_{\text{Br,A}}$ even though species A is the "controlling species." For example, for $\beta_0 = 4.8$, $p = 0.01$, $\beta_{\text{A}} = 6$, $\gamma_{\text{A}} = 0.00941$, $\beta_{\text{B}} = 3$, $\gamma_{\text{B}} = 0.004$, we have allowable property = 33.0, $x_{p,\text{A}} = 49.4$, $x_{p,\text{B}} = 54.0$, $p_{\text{Br,A}} = 0.000899$, and $p_{\text{Br,B}} = 0.00230$. That is, $p_{\text{Br,B}} > p_{\text{Br,A}}$ even though $x_{p,\text{A}} < x_{p,\text{B}}$. The situation is depicted in Figure 6.

Similarly, if the allowable property lies above $x_{p,\text{A}}$ and species B has a narrower distribution than species A, it will be possible for $p_{\text{Br,B}}$ to exceed $p_{\text{Br,A}}$ even though species A is the "controlling species." For example, for $\beta_0 = 4.8$, $p = 0.0005$, $\beta_{\text{A}} = 5$, $\gamma_{\text{A}} = 0.00929$, $\beta_{\text{B}} = 9$, $\gamma_{\text{B}} = 0.017$, we have allowable property = 29.4, $x_{p,\text{A}} = 23.5$, $x_{p,\text{B}} = 25.3$, $p_{\text{Br,A}} = 0.00152$, and $p_{\text{Br,B}} = 0.00195$. That is, $p_{\text{Br,B}} > p_{\text{Br,A}}$ even though $x_{p,\text{A}} < x_{p,\text{B}}$. The situation is depicted in Figure 7.

5 Are Strength Distributions of Grades of Lumber Two-parameter Weibull?

The answer, both theoretically and empirically, is no.

5.1 The theoretical argument

This argument is based upon the process by which "bins" of lumber are produced. For example, in the United States, construction grade two by fours are often classified into visual categories—select structural, number 1, number 2—or into machine stress-rated (MSR) grades. In the case of MSR grades, modulus of elasticity (MOE) boundaries are selected, the MOE of a piece of lumber is measured non-destructively, and pieces are placed into categories based upon the MOE bins into which the pieces fall. Because MOE and MOR are correlated, bins with higher MOE boundaries also tend to contain lumber populations with higher MOR values. However, because the correlation between MOE and MOR is not 1, the MOR population corresponding to a MOE bin is not a truncated Weibull. In Verrill *et al.* (2012), we show that if the joint distribution of the full MOE and MOR populations is bivariate Gaussian–Weibull, then the strength distribution of the specimens in a MSR grade is a pseudo-truncated Weibull (PTW), not a Weibull. In that paper, we obtain the density function of the PTW and develop asymptotically efficient methods for fitting the underlying bivariate Gaussian–Weibull. We note that the arguments that apply to MSR lumber also apply to visually graded lumber assuming that there is an implicit quality measurement in the visual grading process that is analogous to the MOE measurement in the MSR process.

5.2 The empirical argument

Lum *et al.* (2010a,b,c) reported that for species A (not revealed), number 2 lumber, the estimated shape parameter increased from 3.1 to 4.1 to 4.9 as the data fitted went from full to bottom 20% to bottom 10%. For species B (not revealed), number 2 lumber, the estimated shape parameter increased from 2.9 to 3.7 to 4.5. For species C (not revealed), number 2 lumber, the estimated shape parameter increased from 2.4 to 4.1 to 4.9. That is, in all three cases the estimated shape parameter increased as the censoring increased. Similarly, we fit 19 cells of select structural and number 2 data that were obtained in the Ingrade Program (Evans and Green (1988), Green and Evans (1988)). See Table 2. In 17 of the 19 cases considered, the estimated shape parameter increased as the censoring increased (and the estimated scale parameter decreased as the censoring increased). (We note that we were using correct censored data estimation techniques in the censored data cases. See Cohen (1965) or ASTM D 5457 (ASTM 2011).)

We claim that these increases are evidence that these real strength distributions are not two-parameter Weibull. Given the increase in slope estimates with increased censoring, statisticians should immediately question the two-parameter Weibull assumption because if this assumption were correct, maximum likelihood estimates of the shape parameter would be asymptotically unbiased regardless of whether we were using censored estimates or full sample estimates. Thus, there should be no systematic differences in the means of the shape estimates. We checked this intuition via simulation. We generated 20 samples of size 1200 from a two-parameter Weibull with shape parameter 3.701 (CV approximately equal to 0.30). We then generated maximum likelihood estimates of the shape parameter for the full data, the bottom 20% of the data, and the bottom 10% of the data for each of the 20 samples. The 20 triples of estimates are reported in Table 3. The means of the three estimates (full, bottom 20%, bottom 10%) were 3.72, 3.70, and 3.76. Their standard deviations were .09, .19, and .27. This simulation does not suggest that we would consistently see shape values increasing as censoring increases if we were truly fitting a two-parameter Weibull. A listing of the program that produced Table 3 can be found at `http://www1.fpl.fs.fed.us/bivar3.html`. See the testcens.f link.

Two additional checks of the two-parameter Weibull assumption can be made via probability plots and goodness-of-fit tests. (See, for example, D'Agostino and Stephens, 1986.) In the upper halves of Figures 8 and 9, we provide two-parameter Weibull probability plots of two of the 19 data sets described in Table 2. In the lower halves of Figures 8 and 9, we provide Weibull probability plots of generated two-parameter Weibull data. The generating scales and shapes were the fitted values for the data sets plotted in the upper halves of the figures.

Notice the differences between the paired upper and lower plots. In each upper plot, the measured data set has a left-hand tail that is "short" compared to what one would expect if the population truly were Weibull (hence the points lying above the line at the left side of the plot). This left tail shortness is what one would expect from a PTW. This left side shortness does not appear in the generated two-parameter Weibull data displayed in the lower plots. We produced similar plots for the other 17 data sets listed in Table 2. Fourteen of the 19 data sets displayed the left tail shortness for the observed data. Only two of the 19 generated data sets displayed any left tail shortness.

There was no corresponding right-tail shortness in the observed strength data. At first, this might suggest pseudo-truncation at the left and none at the right. (This is correct for the SS data.) However, while a two-parameter Weibull must have support that extends down to 0, and thus below a PTW, there is no required upper extension, and it is possible that a best fit of a Weibull model to PTW data will involve a pulled-in Weibull right tail which can lead to data points at or above the $y = x$ line at the right side of the probability plot.

We also performed Cramér–Von Mises and Anderson–Darling goodness-of-fit tests of both the observed and generated strength data. The results from these tests are displayed in Table 4. "NS" denotes not statistically significant at a .25 level. ".25", ".10", ".05", ".025", and ".01" indicate statistically significant at the listed levels. From these results, it is clear that the observed strength data is not distributed as a two-parameter Weibull.

A listing of the program that produced the probability plots and performed the goodness-of-fit tests can be found at `http://www1.fpl.fs.fed.us/bivar3.html`. See the weibpp.f link.

5.3 Empirical evidence of pseudo-truncated Weibull behavior

In Section 5.1 we noted that, provided that the full MOE/MOR population has a particular form of a bivariate Gaussian–Weibull distribution, we can mathematically *prove* that the marginal MOR distribution for MSR lumber is not Weibull. Instead, the marginal distribution is a pseudo-truncated Weibull (PTW) where the distribution function of a PTW is derived in Verrill *et al.* (2012). We also noted that this theoretical argument also applies to visually graded lumber.

In Section 5.2, we demonstrated that there is empirical evidence that the MOR distributions of visual grades are not Weibull.

In this section, we establish that there is empirical evidence that the distributions of visual grades of lumber share at least one characteristic of PTW distributions. In particular, here we report results from simulations of PTW distributions that demonstrate that fits of two-parameter Weibulls to PTW data yield the same pattern of increasing shape estimates (and decreasing scale estimates) with increasing censoring that was observed (see Table 2) with actual visually graded lumber.

We created simulated number 2 populations as follows:

1. Randomly generate 1,000,000 N(0,1) x's (MOE's).

2. Randomly generate 1,000,000 correlated normally distributed y's via

$$y_i = \rho \times x_i + \sqrt{1-\rho^2} \times \mathrm{N}(0,1)$$

   where the N(0,1)'s in the equation are statistically independent of the x_i's.

3. Sort the x's and bring along the corresponding y's. (This models machine stress rating of lumber.)

4. Treat the y's that correspond to the top 600,000 x's as number 2 and betters. Treat the bottom 400,000 of this 600,000 as number 2's.

5. Transform these normals to Weibulls by first transforming them to U(0,1)'s and then to Weibulls. (We have done simulation studies that indicate that the correlation value between the MOEs and the Weibulls remains essentially the same as the correlation between the two sets of normals.) (Shapes were determined by CV's. Scales were chosen so that median values were 100.)

Listings of the simulation programs are available at `http://www1.fpl.fs.fed.us/bivar3.html`. See the num2sim.1m.2.f link.

We generated number 2 populations for four correlations between the underlying normals—.50, .60, .70, .80—and three coefficient of variation (CV) values—20, 30, and 40. From each generated population, we randomly drew (with replacement) 100 samples of size 400. For each of these samples, we used linear regression and maximum likelihood techniques to fit two-parameter Weibulls to the full sample, and to the bottom 20 and 10 percent of the sample. (Correct censored data estimation techniques were used to fit the censored data. See Cohen (1965) or ASTM D 5457 (ASTM 2011).) We averaged the parameter estimates over the 100 fits. These averages are reported in Table 5 and show the same pattern of increasing shape estimates and decreasing scale estimates with increasing censoring that was reported for the MOR data discussed in Subsection 5.2.

5.4 If the strength populations are not really two-parameter Weibull, does it matter?

In the subsections above, we have provided theoretical and empirical evidence that MOR subpopulations generated via MOE or visual-grading binning do not have a two-parameter Weibull distribution. Suppose, however, that we ignore this fact and proceed to fit such data with Weibull distributions. Does this hurt us? After all, ultimately we are not interested in estimates of shape and scale. Instead, we are interested in estimates of probabilities of breakage. If we fit a Weibull distribution to PTW data, are we seriously misled about probabilities of breakage? In Tables 6–9 we report results from simulations performed to investigate this question. (A listing of the program that produced these tables can be found at `http://www1.fpl.fs.fed.us/bivar3.html`. See the pfsim4.f link.)

For four generating correlations—.50, .60, .70, .80—and three coefficient of variation (CV) values—20, 30, and 40, we generated 10,000 bivariate Gaussian–Weibull data sets. These data data sets were generated so that each yielded 400 specimens with a Gaussian value that lay between the 40th and 80th percentiles of the Gaussian distribution. This required approximately 1000 bivariate Gaussian–Weibull pairs in each data set. For each of these 10,000 data sets we calculated:

1. (W/Reg/All) A (theoretically incorrect) Weibull regression fit based on all 400 simulated pseudo-truncated MOR data values.

2. (W/Reg/20) A (theoretically incorrect) Weibull regression fit based on the bottom 20% of the 400 simulated pseudo-truncated MOR data values.

3. (W/Reg/10) A (theoretically incorrect) Weibull regression fit based on the bottom 10% of the 400 simulated pseudo-truncated MOR data values.

4. (W/MLE/All) A (theoretically incorrect) Weibull maximum likelihood (ML) fit based on all 400 simulated pseudo-truncated MOR data values.

5. (W/MLE/20) A (theoretically incorrect) Weibull ML fit based on the bottom 20% of the 400 simulated pseudo-truncated MOR data values.

6. (W/MLE/10) A (theoretically incorrect) Weibull ML fit based on the bottom 10% of the 400 simulated pseudo-truncated MOR data values.

7. (PTW trunc) A (theoretically correct) fit of the 400 bivariate Gaussian–Weibull data values binned by the Gaussian value (the Gaussian member of the pair lies between the 40th and 80th percentiles of the Gaussian distribution). (Bivariate data that has been truncated based on one of its components is used to fit a bivariate Gaussian–Weibull which permits one to characterize the corresponding univariate PTW.)

8. (PTW 400) A (theoretically correct) fit of the first (not the lowest) 400 of the approximately 1000 unbinned Gaussian-Weibull values. (Bivariate data that has *not* been truncated based on one of its components is used to fit a bivariate Gaussian–Weibull which permits one to characterize the corresponding univariate PTW.)

9. (PTW 1000) A (theoretically correct) fit of all of the approximately 1000 unbinned Gaussian-Weibull values. (Bivariate data that has *not* been truncated based on one of its components is used to fit a bivariate Gaussian–Weibull which permits one to characterize the corresponding univariate PTW.)

(The first six fits use correct *methods*. That is, they use censored data methods for censored data and full sample methods for full sample data. They are "theoretically incorrect" fits because they are based on the incorrect assumption that the data come from a Weibull population.) Each of these fits yielded an allowable property (estimated fifth percentile divided by 2.1). In each case we calculated the predicted probability, $p_{\mathrm{Br,est}}$, that an MOR would lie beneath the allowable property. This value was calculated from the fitted distribution. In each case we also calculated the true probability, $p_{\mathrm{Br,true}}$, that an MOR would lie beneath the allowable property. This value was calculated from the (known) generating bivariate Gaussian–Weibull. Information about the ratios of the true "probabilities of breakage" (probability that a specimen has strength lower than the allowable property) to the estimated probabilities of breakage is presented in Tables 6–9.

In column 1 of these tables, we provide the generating coefficient of variation. In column 2 we describe the type of fit. (These correspond to the nine fitting techniques listed above.) In column 3 we provide the median of the 10,000 ratios of true to estimated probabilities of breakage. A value below 1 indicates an approach that is conservative at the median. However, as can be seen from the remainder of the table, the median represents an insufficient summary. The remaining columns list the fraction of the time that a particular technique had ratios of true to estimated probabilities of breakage that lay in the intervals $[0, .01]$, $(.01, .02]$, $(.02, .1]$, $(.1, .2]$, $(.2, .5]$, $(.5, 1)$, $(1, 2)$, $[2, 5)$, $[5, 10)$, $[10, 50)$, $[50, 100)$, $[100, \infty)$. Obviously, we could have chosen other intervals. However, our main points are clear from the current tables:

- The Weibull fits (regression and ML) tend to be overly conservative. That is, true probabilities of breakage can be much less than estimated probabilities of breakage.

- At the same time, the censored (20% and 10%) Weibull fits can be much more variable than the correct bivariate Gaussian–Weibull fits with the result that they can occasionally yield highly non-conservative fits. That is, the actual probabilities of breakage can be much greater than estimated probabilities of breakage.

- If the joint MOE/MOR distribution is truly bivariate Gaussian–Weibull, we can obtain better estimates of the probability of breakage by taking the same number of specimens from the full distribution than by restricting ourselves to binned values. (Compare the PTW 400 and PTW trunc results.)

6 Conclusions

We have evaluated a proposed method for combining wood species into groups with similar strength properties. We commend the authors of the proposal for recognizing the importance of considering a strength distribution's variability as well as its fifth percentile. However, we have concerns about the proposed implementation:

- The proposed procedure will reduce the allowable properties of more variable populations and increase the allowable properties of less variable populations. However, unless we choose the p and β_0 tuning parameters to yield a flat probability of breakage versus β curve, there will still be "unfairness" in the allowable property calculations. That is, the probability of breakage at the allowable property will depend on a population's CV.

- If we do tune the procedure to ensure a flat probability of breakage versus β curve, then the procedure simply amounts to choosing a p value (e.g., .01 or .001) and taking all controlling species' allowable properties to be their pth quantiles.

- However, as we demonstrated in Section 4.2, if we achieve "fairness" via this method at one time, the "fairness" will disappear as the lumber is subjected to duration of load.

- We also noted that although the proposed procedure could reduce the probabilities of breakage associated with more variable species, it will increase the probabilities of breakage associated with less variable species. Thus, we are not assured (without further work) that an implementation of the procedure would preserve current safety levels.

- Currently, the proposed procedure makes use of the Weibull distribution to obtain estimates of allowable properties. We have demonstrated that MOR grade populations are not Weibull distributed and that we can obtain poor estimates of probabilities of breakage at allowable properties if we make a Weibull assumption.

Given these issues, a great deal of additional work would need to be done before the proposed procedure could be accepted for implementation as part of an ASTM standard. Having said this, we do support continued work by Lum, Taylor, and Zidek (and others) in this area. Ultimately, if we want to take a reliability based approach to species grouping, the grouping cannot be based on a single parameter of a strength distribution (the fifth percentile) given the simple fact that strength distributions cannot generally be characterized by single parameters.

We note that our work also suggests that it would be worthwhile to investigate whether full joint MOE/MOR populations truly have a bivariate Gaussian–Weibull distribution. If so, we could exploit this fact to obtain more accurate and precise estimates of probabilities of failure. (However, we fear that given the mixture nature of lumber populations, it is likely that no "simple" joint distribution is appropriate for modeling a full population of MOE/MOR pairs.)

7 References

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8 Appendix

In this Appendix, we describe the manner in which we created Table 1 and Figure 2. None of the mathematics is novel. We describe it in some detail simply because this permits easy replication of our work, and because this highlights the facts that we need two constraints to characterize a two parameter strength or load distribution, that specifying a fifth percentile of a strength distribution does not permit us to calculate a probability of failure, and thus that equality or similarity of fifth percentiles alone should not be a basis for grouping.

8.1 Characterizing a lognormal distribution from its CV and its fifth percentile

By definition, a random variable X is distributed as a lognormal(μ, σ^2) if $\ln(X)$ is distributed as a normal(μ, σ^2). Thus, characterizing the lognormal is equivalent to determining the two parameters μ and σ. For a lognormal distribution, it can be shown that

$$\mathrm{CV} = \sqrt{\exp(\sigma^2) - 1}$$

or

$$\ln\left(1 + \mathrm{CV}^2\right) = \sigma^2 \tag{11}$$

Thus, for lognormals, the parameter σ can be calculated from a knowledge of the (alternative parameter) CV.

Now let $x_{.05}$ denote the fifth percentile of the lognormal. Then

$$\mathrm{Prob}\left(\mathrm{LN}(\mu, \sigma^2) \leq x_{.05}\right) = 0.05$$

or

$$\mathrm{Prob}\left(\mathrm{N}(\mu, \sigma^2) \leq \ln(x_{.05})\right) = 0.05$$

or

$$\mathrm{Prob}\left(\mathrm{N}(0, 1) \leq \frac{\ln(x_{.05}) - \mu}{\sigma}\right) = 0.05$$

or

$$\frac{\ln(x_{.05}) - \mu}{\sigma} = \Phi^{-1}(.05)$$

where Φ denotes the N(0,1) cumulative distribution function. Thus,

$$\mu = \ln(x_{.05}) - \sigma \times \Phi^{-1}(.05) \tag{12}$$

From results (11) and (12), we see that we can calculate the two parameters μ and σ of a lognormal distribution from a specification of its fifth percentile and its coefficient of variation. Thus, in our calculations for Table 1, we were able to fully characterize the "narrow" and "wide" lognormal strength distributions by specifying a shared fifth percentile and the CV's of the narrow and wide strength distributions. (For the shared fifth percentile, we arbitrarily choose $x_{.05} = 50$. The specific shared value does not matter because different values simply amount to changes in scale.)

8.2 Characterizing a lognormal distribution from its CV and the probability that a value randomly drawn from the distribution exceeds a particular value

Next, we claim that we can characterize a lognormal load distribution by specifying its CV and the probability, q, that it lies above the allowable property (given that we have previously specified $x_{.05}$).

From (11) we know that we can obtain a value for its σ parameter from its CV. We calculate its μ parameter as follows.

$$\text{Prob}\left(\text{LN}(\mu,\sigma^2) \le x_{.05}/2.1\right) = 1-q$$

or

$$\text{Prob}\left(\text{N}(\mu,\sigma^2) \le \ln(x_{.05}/2.1)\right) = 1-q$$

or

$$\text{Prob}\left(\text{N}(0,1) \le \frac{\ln(x_{.05}/2.1)-\mu}{\sigma}\right) = 1-q$$

or

$$\frac{\ln(x_{.05}/2.1)-\mu}{\sigma} = \Phi^{-1}(1-q)$$

Thus,

$$\mu = \ln(x_{.05}/2.1) - \sigma \times \Phi^{-1}(1-q) \tag{13}$$

So, assuming lognormality of both the load and strength distributions, specifying the shared (in our case) $x_{.05}$, the CV's of the narrow and wide strength distributions and the load distribution, and the probability that the load distribution lies above the allowable property permits us to fully characterize the load and strength distributions (that is, we can calculate their parameters μ and σ).

Finally, we note that if X_1 is distributed as a $\text{LN}(\mu_1,\sigma_1^2)$ and X_2 is distributed as an independent $\text{LN}(\mu_2,\sigma_2^2)$, then

$$\begin{aligned}
\text{Prob}(X_2 < X_1) &= \text{Prob}\left(\text{LN}(\mu_2,\sigma_2^2) < \text{LN}(\mu_1,\sigma_1^2)\right)\\
&= \text{Prob}\left(\text{N}(\mu_2,\sigma_2^2) < \text{N}(\mu_1,\sigma_1^2)\right)\\
&= \text{Prob}\left(\text{N}(\mu_2,\sigma_2^2) - \text{N}(\mu_1,\sigma_1^2) < 0\right)\\
&= \text{Prob}\left(\text{N}(\mu_2-\mu_1,\sigma_1^2+\sigma_2^2) < 0\right)\\
&= \Phi\left(\frac{\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right) \qquad (14)
\end{aligned}$$

Result (14) permits us to calculate, for example, the probability that a random value drawn from a wide lognormal strength distribution falls below a random value drawn from the load distribution, that is, the probability of failure for pieces of lumber drawn from the wide distribution when they are subjected to the load distribution.

At `http://www1.fpl.fs.fed.us/bivar3.html`, we provide listings of FORTRAN and S versions of a program that we developed to utilize Equations (11) – (14) to produce Table 1 and Figure 2. See the narr.wide.f and narr.wide.s links.

CV's of narrow and wide	q	p_{F} narr	p_{F} wide	p_{F} ratio
.05, .25	.01	.21E-6	.89E-6	.23
	.02	.82E-6	.25E-5	.33
	.05	.57E-5	.11E-4	.54
	.10	.28E-4	.35E-4	.79
	.20	.16E-3	.14E-3	1.17
.10, .30	.01	.15E-6	.22E-5	.07
	.02	.57E-6	.54E-5	.11
	.05	.37E-5	.20E-4	.19
	.10	.18E-4	.58E-4	.31
	.20	.10E-3	.20E-3	.51
.20, .40	.01	.38E-6	.11E-4	.036
	.02	.12E-5	.22E-4	.054
	.05	.60E-5	.62E-4	.096
	.10	.23E-4	.15E-3	.153
	.20	.10E-3	.40E-3	.254

Table 1: q is the probability that the load lies above the allowable property. p_{F} narr is the probability that a value drawn from the narrow strength distribution falls below a value drawn independently from the load distribution. p_{F} wide is the probability that a value drawn from the wide strength distribution falls below a value drawn independently from the load distribution. p_{F} ratio $= (p_{\mathrm{F}}$ narr$)/(p_{\mathrm{F}}$ wide$)$.

Species	Lumber size	Grade	Sample size	Censored size	Shape estimate		Scale estimate	
					Censored	Full	Censored	Full
DF	2×4	SS	414	60	5.664	4.618	10.474	11.074
DF	2×8	SS	493	60	4.066	3.747	8.051	8.446
DF	2×10	SS	414	60	4.980	3.985	7.083	7.866
DF	2×4	2	386	60	5.121	3.017	6.612	8.728
DF	2×8	2	1964	196	4.462	2.597	4.530	6.384
DF	2×10	2	388	60	5.188	2.436	3.712	5.682
HF	2×4	SS	428	60	5.902	4.571	8.908	9.420
HF	2×8	SS	375	60	5.495	4.242	6.715	7.508
HF	2×10	SS	368	60	4.858	4.251	6.295	6.425
HF	2×4	2	406	60	5.076	2.897	5.580	7.336
HF	2×8	2	372	60	4.229	2.717	4.630	5.859
HF	2×10	2	361	60	4.944	2.796	3.717	4.913
SP	2×4	SS	413	60	6.280	4.606	10.926	11.989
SP	2×8	SS	626	62	4.339	4.625	9.870	9.262
SP	2×10	SS	413	60	6.172	6.321	8.279	7.864
SP	2×4	2	413	60	4.722	2.783	7.135	8.888
SP	2×6	2	413	60	4.541	2.647	5.650	7.866
SP	2×8	2	1367	136	4.552	2.764	4.955	6.967
SP	2×10	2	412	60	4.256	3.469	5.630	6.441

Table 2: Maximum likelihood estimates of the shape and scale parameters for both censored and full data sets.

Full	Bottom 20%	Bottom 10%
3.70	4.05	4.14
3.68	3.68	4.13
3.74	3.67	3.76
3.58	3.61	3.73
3.91	3.91	3.88
3.59	3.81	4.09
3.81	3.97	4.07
3.82	3.95	3.96
3.67	3.61	3.45
3.81	3.89	3.55
3.64	3.68	3.82
3.65	3.77	3.76
3.84	3.78	3.93
3.70	3.60	3.41
3.57	3.65	4.11
3.75	3.55	3.36
3.74	3.47	3.65
3.69	3.46	3.51
3.70	3.51	3.41
3.75	3.33	3.50

Table 3: Maximum likelihood estimates of the shape parameter from simulated two-parameter Weibull data with shape parameter 3.701.

Species	Lumber size	Grade	Sample size	Observed data		Generated data	
				CVM	AD	CVM	AD
DF	2×4	SS	414	.10	.05	NS	NS
DF	2×8	SS	493	NS	NS	NS	NS
DF	2×10	SS	414	NS	NS	NS	NS
DF	2×4	2	386	.25	.10	NS	NS
DF	2×8	2	1964	.01	.01	NS	NS
DF	2×10	2	388	.01	.01	NS	NS
HF	2×4	SS	428	.05	.01	NS	NS
HF	2×8	SS	375	.05	.05	NS	NS
HF	2×10	SS	368	.10	.05	NS	NS
HF	2×4	2	406	.01	.01	NS	NS
HF	2×8	2	372	.01	.01	NS	NS
HF	2×10	2	361	.01	.01	.25	.25
SP	2×4	SS	413	.05	.025	.25	.05
SP	2×8	SS	626	.05	.025	.25	NS
SP	2×10	SS	413	.01	.01	NS	NS
SP	2×4	2	413	.01	.01	NS	NS
SP	2×6	2	413	.01	.01	NS	NS
SP	2×8	2	1367	.01	.01	.10	.25
SP	2×10	2	412	.10	.25	NS	NS

Table 4: Cramér–Von Mises and Anderson–Darling goodness-of-fit tests of both observed and generated data.

			Shape estimate		Scale estimate	
ρ	CV	Data	Reg	MLE	Reg	MLE
.50	20	all	7.040	6.861	108.5	108.6
		20%	7.368	7.556	107.6	106.8
		10%	7.414	7.691	108.2	106.5
	30	all	4.500	4.382	113.6	113.7
		20%	4.756	4.826	112.3	111.1
		10%	4.887	5.001	112.9	109.7
	40	all	3.283	3.209	119.4	119.7
		20%	3.505	3.504	116.4	115.9
		10%	3.602	3.698	117.4	112.3
.60	20	all	7.759	7.403	108.8	109.0
		20%	8.601	8.635	106.4	106.2
		10%	8.834	9.065	106.4	105.0
	30	all	4.930	4.726	113.8	114.1
		20%	5.400	5.415	110.3	110.0
		10%	5.552	5.662	110.4	107.9
	40	all	3.610	3.448	119.7	120.2
		20%	3.977	3.979	114.5	114.4
		10%	4.099	4.164	113.7	111.6
.70	20	all	8.700	8.170	108.7	109.0
		20%	9.820	9.772	106.2	106.2
		10%	10.137	10.309	106.1	105.0
	30	all	5.597	5.255	114.1	114.5
		20%	6.452	6.341	109.3	109.7
		10%	6.723	6.749	108.8	107.4
	40	all	4.063	3.820	120.2	120.7
		20%	4.651	4.565	113.3	114.2
		10%	4.818	4.911	112.3	110.2
.80	20	all	10.303	9.430	108.7	109.0
		20%	12.392	12.222	105.2	105.5
		10%	12.918	12.977	104.7	104.3
	30	all	6.607	6.014	114.1	114.6
		20%	8.139	7.935	107.5	108.3
		10%	8.490	8.533	106.3	106.0
	40	all	4.761	4.359	119.8	120.5
		20%	5.639	5.610	112.4	112.7
		10%	5.877	5.897	110.9	110.5

Table 5. Maximum likelihood and regression Weibull parameter estimates for pseudo-truncated Weibull data as a function of correlation, coefficient of variation, and censoring.

COV	Fit	Median p_{Br} ratio	p_{Br} ratio frequencies [0, .01]	(.01, .02]	(.02, .1]	(.1, .2]	(.2, .5]	(.5, 1)	(1, 2)	[2, 5)	[5, 10)	[10, 50)	[50, 100)	[100, ∞)
0.20	W/Reg/All	.62	.0000	.0000	.0002*†	.0028*†	.2983*†	.5661	.1298	.0028	.0000	.0000	.0000	.0000
	W/Reg/20	.84	.0000	.0000	.0041*†	.0444*†	.2409*†	.2880	.2344	.1418 †	.0348*†	.0111*†	.0004*†	.0001*†
	W/Reg/10	.86	.0000	.0001*†	.0123*†	.0679*†	.2374*†	.2344	.1952	.1495 †	.0578*†	.0378*†	.0047*†	.0029*†
	W/MLE/All	.51	.0000	.0000	.0002*†	.0021*†	.4662*†	.5028	.0287	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.93	.0000	.0000	.0006*†	.0150*†	.1840*†	.3378	.2903	.1453 †	.0236 †	.0034 †	.0000	.0000
	W/MLE/10	1.10	.0000	.0000	.0018*†	.0232*†	.1770*†	.2615	.2440	.1943*†	.0615*†	.0338*†	.0022*†	.0007*†
	PTW trunc	1.06	.0000	.0000	.0000	.0000	.0647 †	.3941	.3529	.1510 †	.0285 †	.0088 †	.0000	.0000
	PTW 400	1.04	.0000	.0000	.0000	.0000	.0366	.4307	.4694	.0630	.0003	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0025	.4773	.5154	.0048	.0000	.0000	.0000	.0000
0.30	W/Reg/All	.74	.0000	.0001*†	.0002*†	.0002*†	.1213*†	.7013	.1758	.0011	.0000	.0000	.0000	.0000
	W/Reg/20	.90	.0000	.0000	.0001*†	.0053*†	.1716*†	.3861	.3091	.1178*†	.0092*†	.0008*†	.0000	.0000
	W/Reg/10	.91	.0000	.0000	.0000	.0097*†	.1989*†	.3351	.2663	.1483*†	.0323*†	.0090*†	.0003*†	.0001*†
	W/MLE/All	.64	.0000	.0000	.0000	.0000	.1888*†	.7678	.0432	.0002	.0000	.0000	.0000	.0000
	W/MLE/20	.99	.0001*†	.0000	.0001*†	.0004*†	.1025*†	.4075	.3795	.1066*†	.0030 †	.0003 †	.0000	.0000
	W/MLE/10	1.09	.0001*†	.0000	.0002*†	.0019*†	.1137*†	.3304	.3474	.1769*†	.0248*†	.0045*†	.0001*†	.0000
	PTW trunc	1.05	.0000	.0000	.0000	.0000	.0199 †	.4341	.4369	.1029 †	.0058 †	.0004 †	.0000	.0000
	PTW 400	1.03	.0000	.0000	.0000	.0000	.0092	.4558	.5143	.0207	.0000	.0000	.0000	.0000
	PTW 1000	1.02	.0000	.0000	.0000	.0000	.0001	.4692	.5305	.0002	.0000	.0000	.0000	.0000
0.40	W/Reg/All	.80	.0000	.0000	.0001*†	.0000	.0548*†	.7435	.2005	.0011	.0000	.0000	.0000	.0000
	W/Reg/20	.92	.0000	.0000	.0000	.0007*†	.1085*†	.4504	.3571	.0806*†	.0027*†	.0000	.0000	.0000
	W/Reg/10	.94	.0000	.0000	.0000	.0006*†	.1243*†	.4121	.3309	.1183*†	.0115*†	.0023*†	.0000	.0000
	W/MLE/All	.70	.0000	.0000	.0000	.0000	.0827*†	.8478	.0694	.0001	.0000	.0000	.0000	.0000
	W/MLE/20	.99	.0000	.0000	.0001*†	.0001*†	.0547*†	.4513	.4293	.0636*†	.0009*†	.0000	.0000	.0000
	W/MLE/10	1.07	.0000	.0001*†	.0000	.0002*†	.0627*†	.3892	.4032	.1357*†	.0078*†	.0011*†	.0000	.0000
	PTW trunc	1.04	.0000	.0000	.0000	.0000	.0059 †	.4473	.4919	.0546 †	.0003 †	.0000	.0000	.0000
	PTW 400	1.02	.0000	.0000	.0000	.0000	.0037	.4646	.5246	.0071	.0000	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0001	.4818	.5180	.0001	.0000	.0000	.0000	.0000

Table 6: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for a bivariate Gaussian–Weibull with generating correlation .50. *'s denote "worse behavior" than PTW trunc behavior. †'s denote "worse behavior" than PTW 400 behavior.

COV	Fit	Median p_{Br} ratio	p_{Br} ratio frequencies [0, .01]	(.01, .02]	(.02, .1]	(.1, .2]	(.2, .5]	(.5, 1)	(1, 2)	[2, 5)	[5, 10)	[10, 50)	[50, 100)	[100, ∞)
0.20	W/Reg/All	.44	.0000	.0000	.0002*†	.0348*†	.5761*†	.3561	.0326	.0002	.0000	.0000	.0000	.0000
	W/Reg/20	.73	.0000	.0000	.0152*†	.0688*†	.2745*†	.2605	.2071	.1308 †	.0297 †	.0131 †	.0002 †	.0001 †
	W/Reg/10	.78	.0000	.0000	.0327*†	.0900*†	.2431*†	.2084	.1720	.1473 †	.0548*†	.0447*†	.0032*†	.0038*†
	W/MLE/All	.31	.0000	.0000	.0012*†	.1025*†	.7927*†	.1027	.0009	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.79	.0000	.0000	.0024*†	.0336*†	.2442*†	.3272	.2512	.1202 †	.0178 †	.0032 †	.0002 †	.0000
	W/MLE/10	.99	.0000	.0000	.0071*†	.0438*†	.2094*†	.2427	.2194	.1785*†	.0602*†	.0359*†	.0019*†	.0011*†
	PTW trunc	1.05	.0000	.0000	.0000	.0011 †	.1271 †	.3445	.2975	.1679 †	.0396 †	.0214 †	.0007 †	.0002 †
	PTW 400	1.03	.0000	.0000	.0000	.0002	.0534	.4197	.4416	.0846	.0005	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0067	.4819	.4988	.0126	.0000	.0000	.0000	.0000
0.30	W/Reg/All	.60	.0000	.0000	.0002*†	.0006*†	.2987*†	.6315	.0690	.0000	.0000	.0000	.0000	.0000
	W/Reg/20	.85	.0000	.0000	.0001*†	.0125*†	.2095*†	.3670	.2847	.1129 †	.0113 †	.0020*†	.0000	.0000
	W/Reg/10	.88	.0000	.0000	.0006*†	.0200*†	.2292*†	.3073	.2448	.1485*†	.0352*†	.0137*†	.0004*†	.0003*†
	W/MLE/All	.46	.0000	.0000	.0001*†	.0016*†	.6046*†	.3895	.0042	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.91	.0000	.0000	.0001*†	.0031*†	.1478*†	.4117	.3411	.0915 †	.0043 †	.0004 †	.0000	.0000
	W/MLE/10	1.05	.0000	.0000	.0002*†	.0055*†	.1388*†	.3329	.3103	.1737*†	.0309*†	.0076*†	.0001*†	.0000
	PTW trunc	1.05	.0000	.0000	.0000	.0000	.0496 †	.4146	.3919	.1304 †	.0124 †	.0011 †	.0000	.0000
	PTW 400	1.03	.0000	.0000	.0000	.0000	.0164	.4532	.4993	.0311	.0000	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0009	.4814	.5169	.0008	.0000	.0000	.0000	.0000
0.40	W/Reg/All	.69	.0000	.0000	.0000	.0005*†	.1442*†	.7471	.1080	.0002	.0000	.0000	.0000	.0000
	W/Reg/20	.90	.0000	.0000	.0001*†	.0020*†	.1398*†	.4320	.3336	.0882*†	.0040*†	.0003*†	.0000	.0000
	W/Reg/10	.92	.0000	.0000	.0001*†	.0025*†	.1571*†	.3848	.3043	.1298*†	.0168*†	.0046*†	.0000	.0000
	W/MLE/All	.55	.0000	.0000	.0001*†	.0001*†	.3519*†	.6396	.0083	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.94	.0000	.0000	.0002*†	.0006*†	.0854*†	.4646	.3810	.0672 †	.0010 †	.0000	.0000	.0000
	W/MLE/10	1.05	.0000	.0000	.0001*†	.0003*†	.0883*†	.3797	.3731	.1427*†	.0134*†	.0024*†	.0000	.0000
	PTW trunc	1.04	.0000	.0000	.0000	.0000	.0188 †	.4482	.4433	.0875 †	.0022 †	.0000	.0000	.0000
	PTW 400	1.02	.0000	.0000	.0000	.0000	.0081	.4682	.5122	.0115	.0000	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0001	.4812	.5186	.0001	.0000	.0000	.0000	.0000

Table 7: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for a bivariate Gaussian–Weibull with generating correlation .60. *'s denote "worse behavior" than PTW trunc behavior. †'s denote "worse behavior" than PTW 400 behavior.

COV	Fit	Median p_{Br} ratio	p_{Br} ratio frequencies											
			$[0, .01]$	$(.01, .02]$	$(.02, .1]$	$(.1, .2]$	$(.2, .5]$	$(.5, 1)$	$(1, 2)$	$[2, 5)$	$[5, 10)$	$[10, 50)$	$[50, 100)$	$[100, \infty)$
0.20	W/Reg/All	.24	.0000	.0000	.0355*†	.3298*†	.5773*†	.0565	.0009	.0000	.0000	.0000	.0000	.0000
	W/Reg/20	.56	.0000	.0002*†	.0503*†	.1184*†	.2899*†	.2254	.1615	.1067	.0326 †	.0140 †	.0009 †	.0001 †
	W/Reg/10	.64	.0000	.0013*†	.0757*†	.1251*†	.2288*†	.1791	.1420	.1265 †	.0535 †	.0556*†	.0063*†	.0061*†
	W/MLE/All	.13	.0000	.0001*†	.2359*†	.6097*†	.1528 †	.0015	.0000	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.55	.0000	.0000	.0204*†	.0989*†	.3320*†	.2786	.1688	.0818	.0144 †	.0051 †	.0000	.0000
	W/MLE/10	.81	.0000	.0003*†	.0284*†	.0878*†	.2333*†	.2156	.1810	.1490 †	.0578*†	.0403 †	.0047*†	.0018 †
	PTW trunc	1.07	.0000	.0000	.0004 †	.0158 †	.1889 †	.2694	.2353	.1780 †	.0566 †	.0481 †	.0040 †	.0035 †
	PTW 400	1.05	.0000	.0000	.0000	.0008	.0868	.3791	.4106	.1174	.0048	.0005	.0000	.0000
	PTW 1000	1.02	.0000	.0000	.0000	.0000	.0183	.4578	.4950	.0289	.0000	.0000	.0000	.0000
0.30	W/Reg/All	.41	.0000	.0000	.0002*†	.0259*†	.6829*†	.2818	.0092	.0000	.0000	.0000	.0000	.0000
	W/Reg/20	.73	.0000	.0000	.0019*†	.0345*†	.2766*†	.3376	.2294	.1018 †	.0149 †	.0033 †	.0000	.0000
	W/Reg/10	.79	.0000	.0000	.0041*†	.0507*†	.2664*†	.2691	.2080	.1383 †	.0404*†	.0213*†	.0013*†	.0004*†
	W/MLE/All	.27	.0000	.0000	.0008*†	.1846*†	.7907*†	.0239	.0000	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.74	.0000	.0000	.0002*†	.0142*†	.2540*†	.4076	.2484	.0697 †	.0055 †	.0004 †	.0000	.0000
	W/MLE/10	.92	.0001*†	.0000	.0005*†	.0182*†	.2069*†	.3132	.2577	.1549*†	.0340*†	.0139*†	.0006*†	.0000
	PTW trunc	1.04	.0000	.0000	.0000	.0005 †	.1047 †	.3710	.3341	.1536 †	.0283 †	.0077 †	.0000	.0001 †
	PTW 400	1.03	.0000	.0000	.0000	.0000	.0343	.4380	.4710	.0567	.0000	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0023	.4807	.5133	.0037	.0000	.0000	.0000	.0000
0.40	W/Reg/All	.53	.0000	.0000	.0000	.0008*†	.4249*†	.5530	.0213	.0000	.0000	.0000	.0000	.0000
	W/Reg/20	.83	.0000	.0001*†	.0000	.0065*†	.1969*†	.4150	.2852	.0896 †	.0064 †	.0003 †	.0000	.0000
	W/Reg/10	.87	.0000	.0000	.0000	.0090*†	.2053*†	.3580	.2648	.1311*†	.0250*†	.0067*†	.0001*†	.0000
	W/MLE/All	.37	.0000	.0000	.0001*†	.0115*†	.8589*†	.1295	.0000	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.83	.0000	.0000	.0001*†	.0016*†	.1582*†	.4729	.3055	.0601 †	.0015 †	.0001 †	.0000	.0000
	W/MLE/10	1.00	.0000	.0000	.0001*†	.0027*†	.1386*†	.3576	.3421	.1398*†	.0160*†	.0031*†	.0000	.0000
	PTW trunc	1.03	.0000	.0000	.0000	.0000	.0480 †	.4284	.4021	.1133 †	.0077 †	.0005 †	.0000	.0000
	PTW 400	1.02	.0000	.0000	.0000	.0000	.0102	.4635	.5032	.0231	.0000	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0004	.4821	.5164	.0011	.0000	.0000	.0000	.0000

Table 8: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for a bivariate Gaussian–Weibull with generating correlation .70. *'s denote "worse behavior" than PTW trunc behavior. †'s denote "worse behavior" than PTW 400 behavior.

COV	Fit	Median p_{Br} ratio	p_{Br} ratio frequencies $[0, .01]$	$(.01, .02]$	$(.02, .1]$	$(.1, .2]$	$(.2, .5]$	$(.5, 1)$	$(1, 2)$	$[2, 5)$	$[5, 10)$	$[10, 50)$	$[50, 100)$	$[100, \infty)$
0.20	W/Reg/All	.06	.0006*†	.0223*†	.8303*†	.1364*†	.0097	.0007	.0000	.0000	.0000	.0000	.0000	.0000
	W/Reg/20	.27	.0026*†	.0126*†	.2162*†	.1836*†	.2536*†	.1482	.0951	.0577	.0183	.0110 †	.0009 †	.0002 †
	W/Reg/10	.35	.0081*†	.0235*†	.2039*†	.1426*†	.1993 †	.1256	.0995	.0934	.0387 †	.0461 †	.0102 †	.0091 †
	W/MLE/All	.02	.0444*†	.3862*†	.5678*†	.0006	.0006	.0004	.0000	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.23	.0004*†	.0070*†	.2127*†	.2368*†	.2986*†	.1381	.0701	.0297	.0047	.0019	.0000	.0000
	W/MLE/10	.41	.0015*†	.0081*†	.1586*†	.1527*†	.2263*†	.1622	.1161	.0942	.0392 †	.0340 †	.0037 †	.0034 †
	PTW trunc	1.09	.0000	.0000	.0212 †	.0720 †	.1996 †	.1868	.1691	.1555	.0780 †	.0849 †	.0145 †	.0184 †
	PTW 400	1.04	.0000	.0000	.0002	.0066	.1436	.3295	.3300	.1678	.0199	.0024	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0001	.0574	.4323	.4337	.0753	.0012	.0000	.0000	.0000
0.30	W/Reg/All	.18	.0000	.0000	.0695*†	.5183*†	.4053*†	.0069	.0000	.0000	.0000	.0000	.0000	.0000
	W/Reg/20	.51	.0000	.0000	.0308*†	.1140*†	.3424*†	.2677	.1529	.0748	.0140 †	.0032 †	.0002 †	.0000
	W/Reg/10	.60	.0000	.0001*†	.0438*†	.1172*†	.2794*†	.2157	.1566	.1146 †	.0377 †	.0306*†	.0033*†	.0010*†
	W/MLE/All	.09	.0001*†	.0000	.6420*†	.3438*†	.0137	.0004	.0000	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.46	.0001*†	.0000	.0153*†	.1083*†	.4175*†	.2868	.1324	.0368	.0026 †	.0002 †	.0000	.0000
	W/MLE/10	.68	.0000	.0001*†	.0185*†	.0777*†	.2861*†	.2663	.1849	.1151 †	.0332 †	.0168 †	.0011*†	.0002 †
	PTW trunc	1.04	.0000	.0000	.0004 †	.0111 †	.1754 †	.2972	.2547	.1791 †	.0520 †	.0290 †	.0009 †	.0002 †
	PTW 400	1.03	.0000	.0000	.0000	.0003	.0652	.4134	.4284	.0911	.0016	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0086	.4763	.4985	.0166	.0000	.0000	.0000	.0000
0.40	W/Reg/All	.31	.0000	.0000	.0003*†	.0997*†	.8158*†	.0836	.0006	.0000	.0000	.0000	.0000	.0000
	W/Reg/20	.69	.0000	.0000	.0015*†	.0317*†	.3013*†	.3616	.2185	.0747 †	.0092 †	.0014 †	.0001*†	.0000
	W/Reg/10	.76	.0000	.0000	.0022*†	.0420*†	.2757*†	.2921	.2170	.1274 †	.0290*†	.0134*†	.0008*†	.0004*†
	W/MLE/All	.17	.0001*†	.0000	.0494*†	.6682*†	.2817*†	.0006	.0000	.0000	.0000	.0000	.0000	.0000
	W/MLE/20	.64	.0000	.0000	.0011*†	.0199*†	.3230*†	.4219	.1902	.0422	.0015 †	.0002 †	.0000	.0000
	W/MLE/10	.84	.0000	.0000	.0006*†	.0176*†	.2280*†	.3391	.2583	.1265 †	.0229 †	.0067*†	.0003*†	.0000
	PTW trunc	1.03	.0000	.0000	.0000	.0010 †	.1116 †	.3733	.3258	.1599 †	.0229 †	.0055 †	.0000	.0000
	PTW 400	1.04	.0000	.0000	.0000	.0000	.0244	.4413	.4863	.0477	.0003	.0000	.0000	.0000
	PTW 1000	1.01	.0000	.0000	.0000	.0000	.0007	.4837	.5122	.0034	.0000	.0000	.0000	.0000

Table 9: Frequencies for $p_{\mathrm{Br,true}}/p_{\mathrm{Br,est}}$ for a bivariate Gaussian–Weibull with generating correlation .80. *'s denote "worse behavior" than PTW trunc behavior. †'s denote "worse behavior" than PTW 400 behavior.

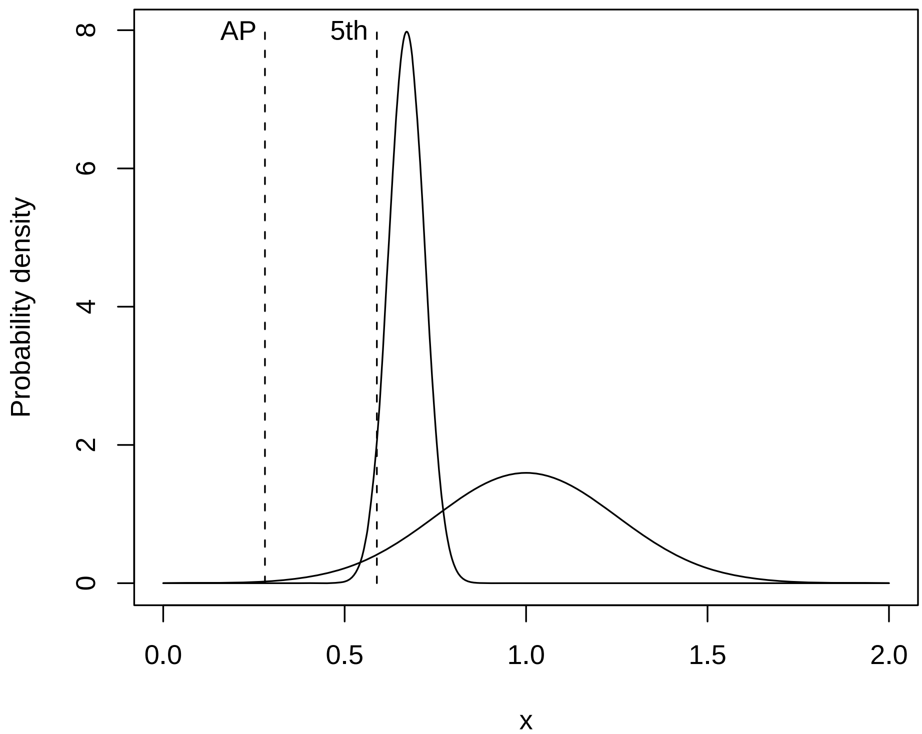

Figure 1: The two normal distributions have the same 5th percentile, but different variabilities. Thus, under the current system, the two distributions would share an allowable property (AP), but the probabilities associated with the region to the left of the allowable property differ between the two distributions.

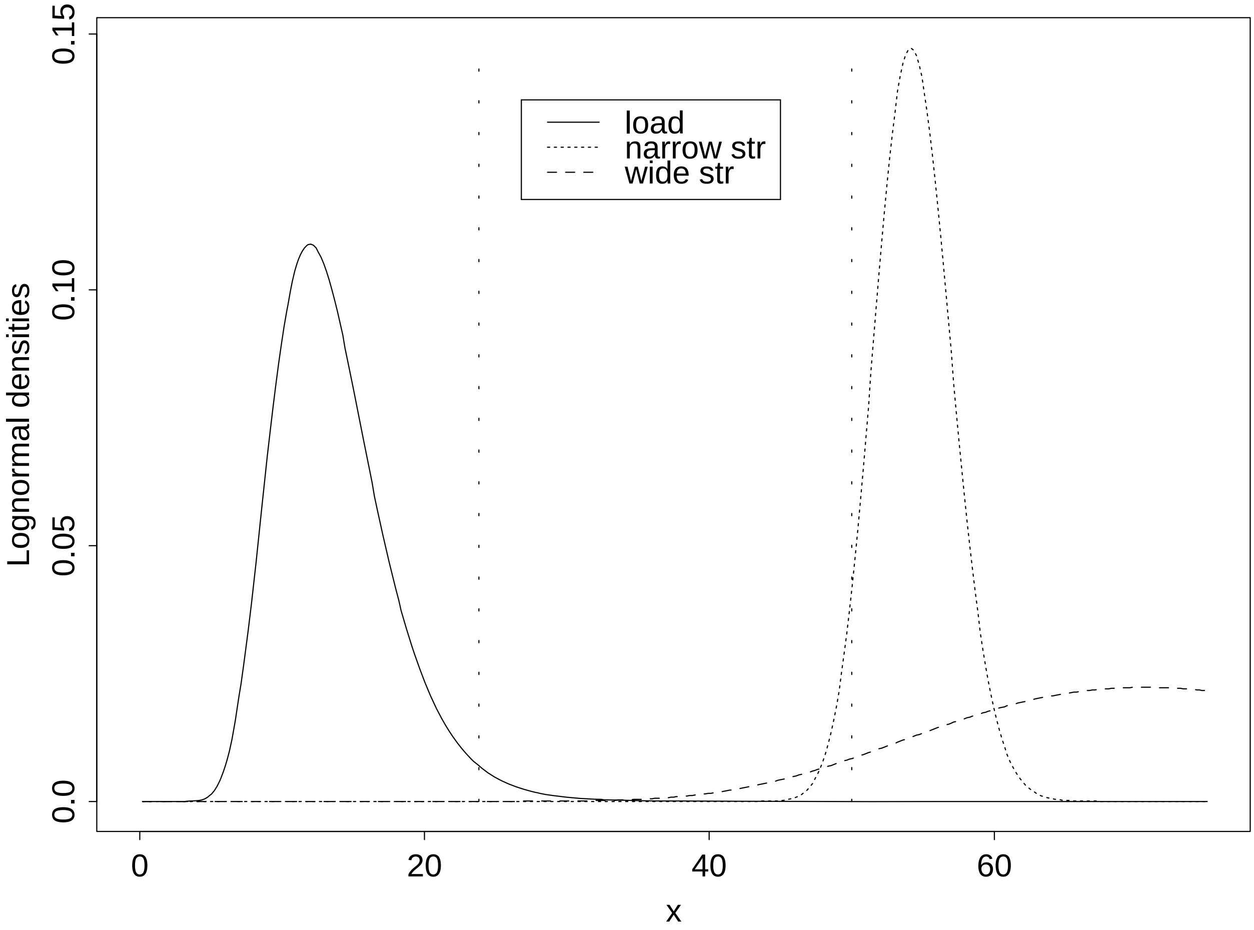

Figure 2: The two lognormal strength distributions have the same 5th percentile, but CV's 0.05 and 0.25. The lognormal load distribution has CV 0.30 and it exceeds the allowable property with probability 0.02. The vertical lines are at the shared allowable property and shared fifth percentile of the strength distributions.

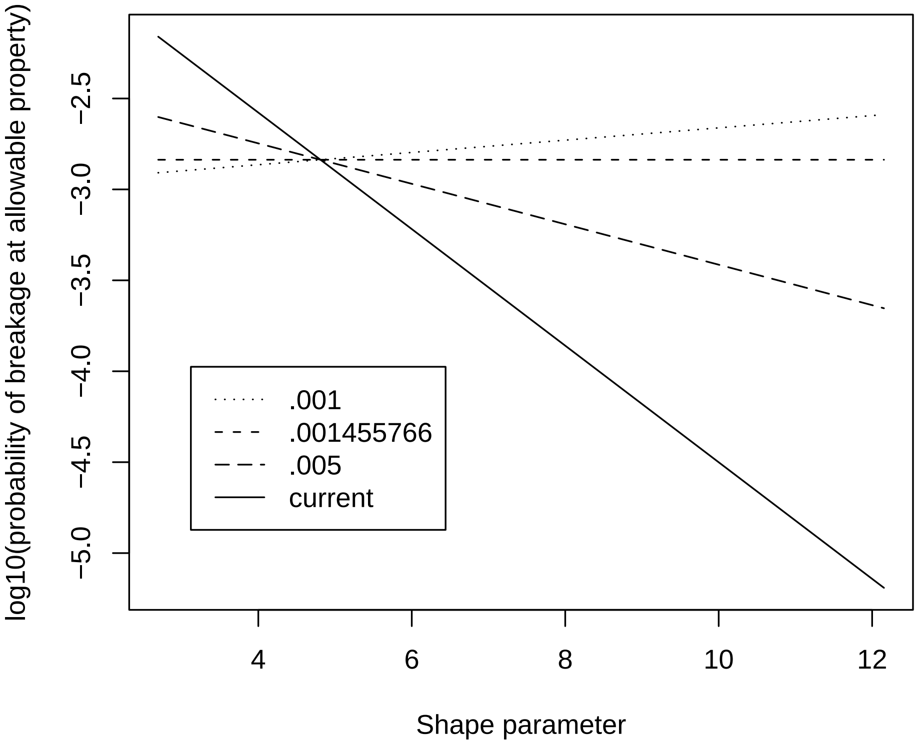

Figure 3: For a given baseline shape, the Lum–Taylor–Zidek procedure permits one to choose a quantile probability p so that the probability of breakage at the allowable load does not change with the shape of the controlling species. For a baseline shape of 4.8, this p value is .001455766.

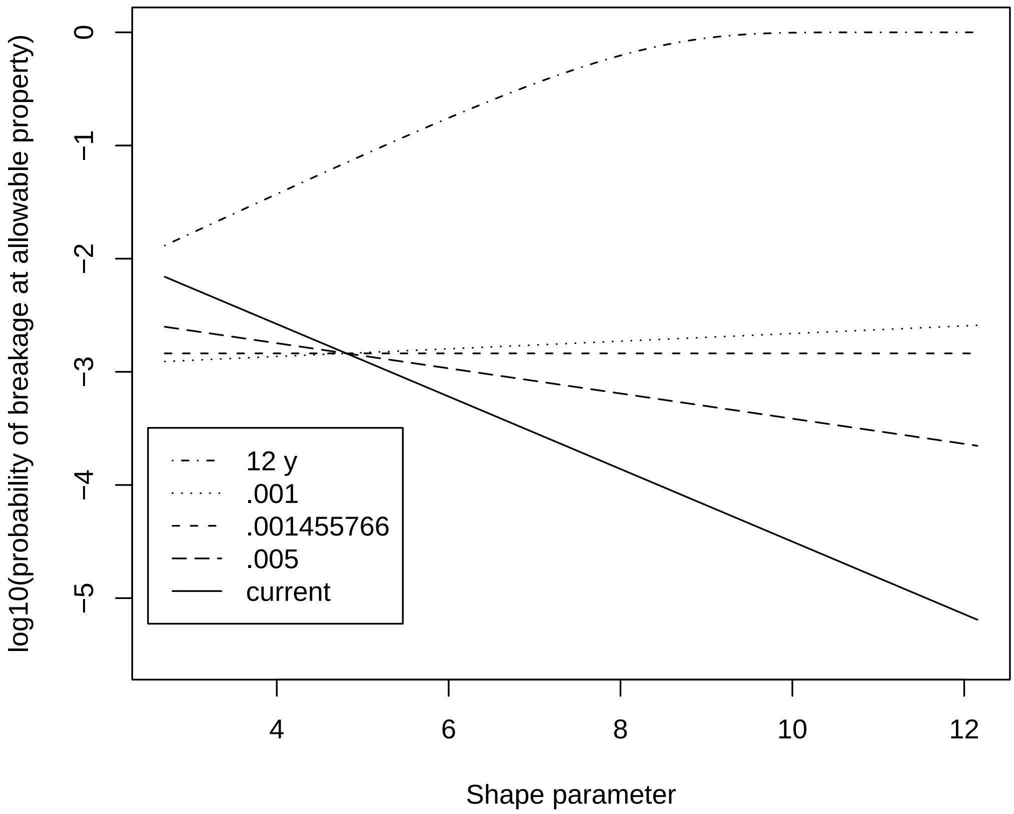

Figure 4: In this plot, we have reproduced Figure 3 and have added a line that represents the probability of breakage when we begin with the flat "fair" case and then subject the pieces of lumber to load $x_{.05,\mathrm{adj}}/2.1$ for 12.3 years. (Based on work of Gerhards (2000).)

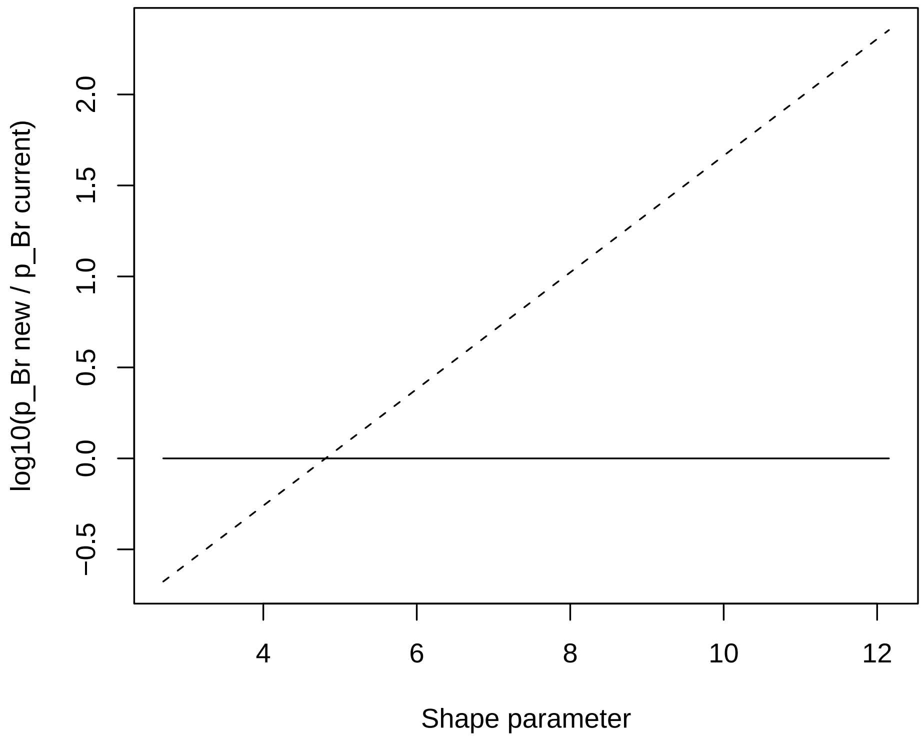

Figure 5: Dashed line — the log base 10 of the ratio between the probability of breakage at the allowable property under the proposed procedure to the probability of breakage at the allowable property under the current procedure when the baseline shape (β_0) is 4.8 and the quantile p value is 0.001455766. Note that the new procedure is conservative compared to the current procedure for $\beta < \beta_0$. However, it is non-conservative compared to the current procedure for $\beta > \beta_0$.

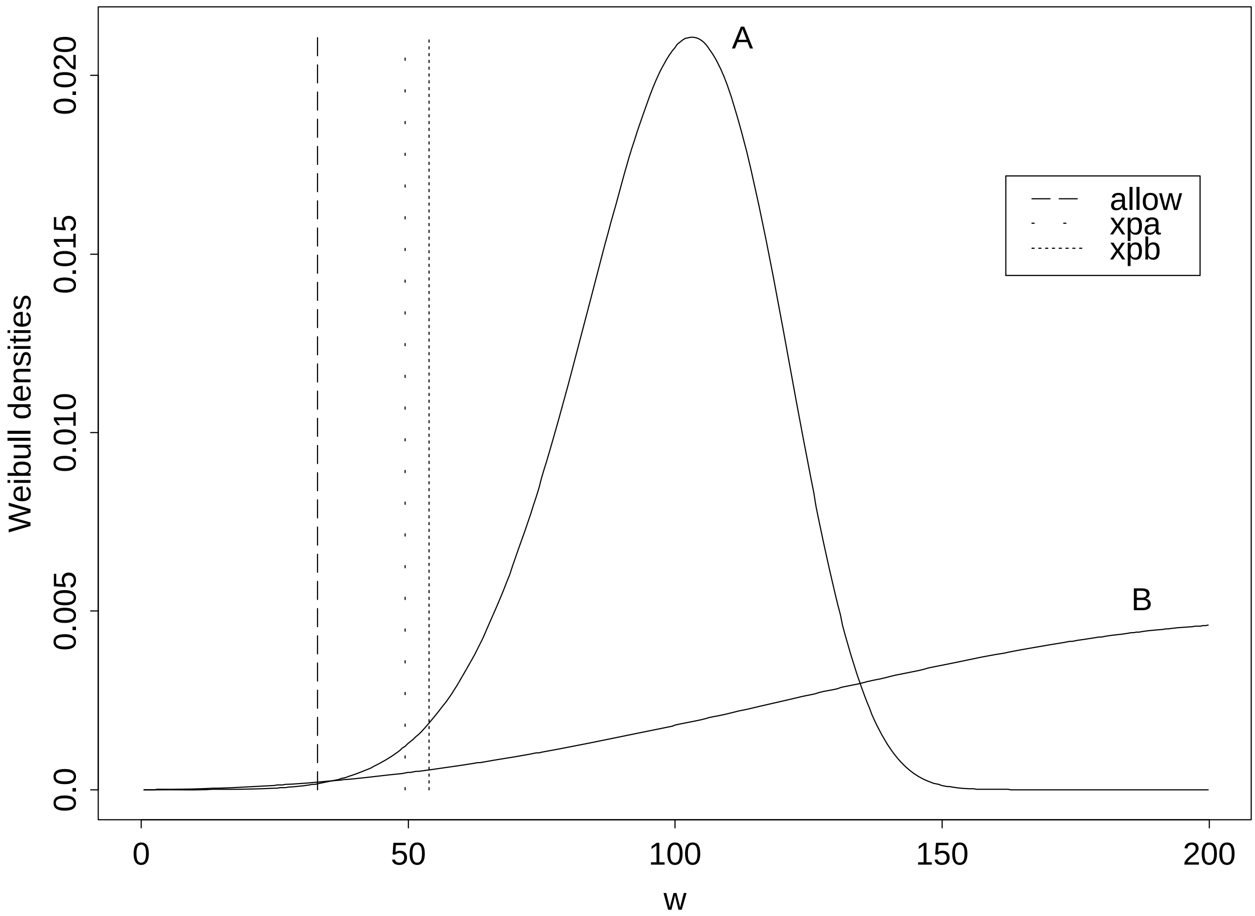

Figure 6: This depicts a case in which we have $x_{p,\mathrm{A}} < x_{p,\mathrm{B}}$ and yet $p_{Br,\mathrm{B}} > p_{Br,\mathrm{A}}$. Here, population A is *less* variable than population B. See the text in Section 4.4 for details.

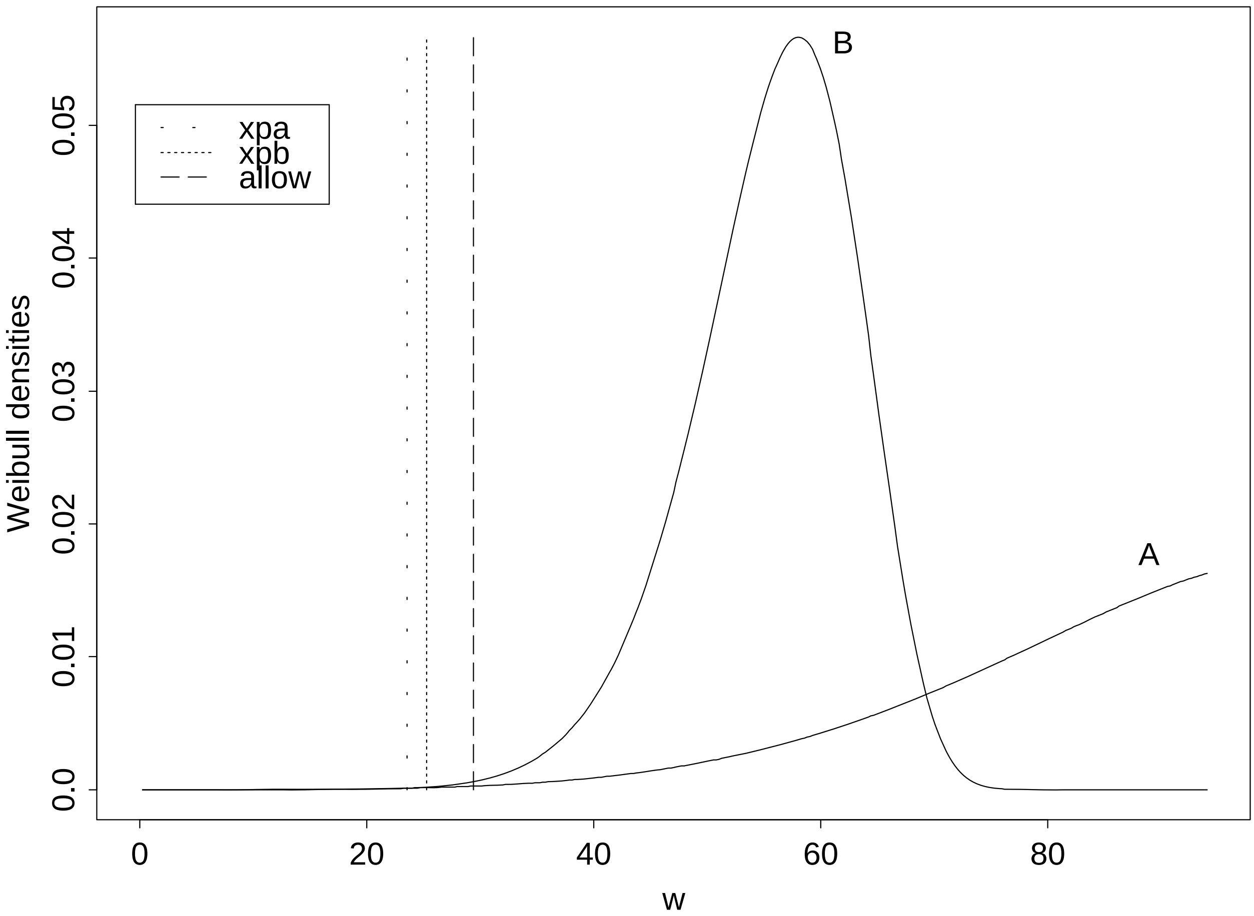

Figure 7: This depicts a case in which we have $x_{p,\mathrm{A}} < x_{p,\mathrm{B}}$ and yet $p_{Br,\mathrm{B}} > p_{Br,\mathrm{A}}$. Here, population A is *more* variable than population B. See the text in Section 4.4 for details.

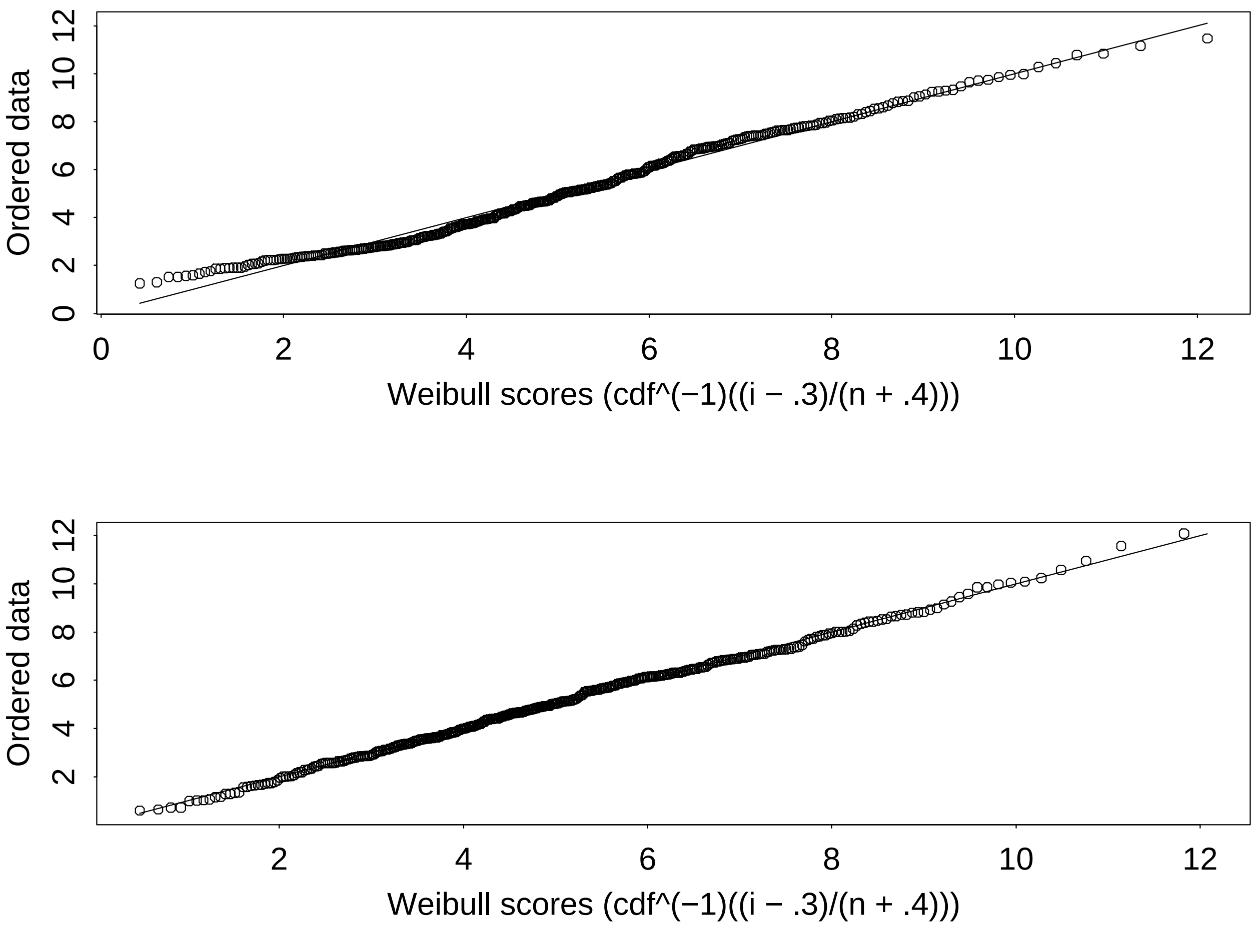

Figure 8: Upper plot: observed Ingrade MOR data (Douglas Fir, 2×10, Number 2). Lower plot: generated 2-parameter Weibull data.

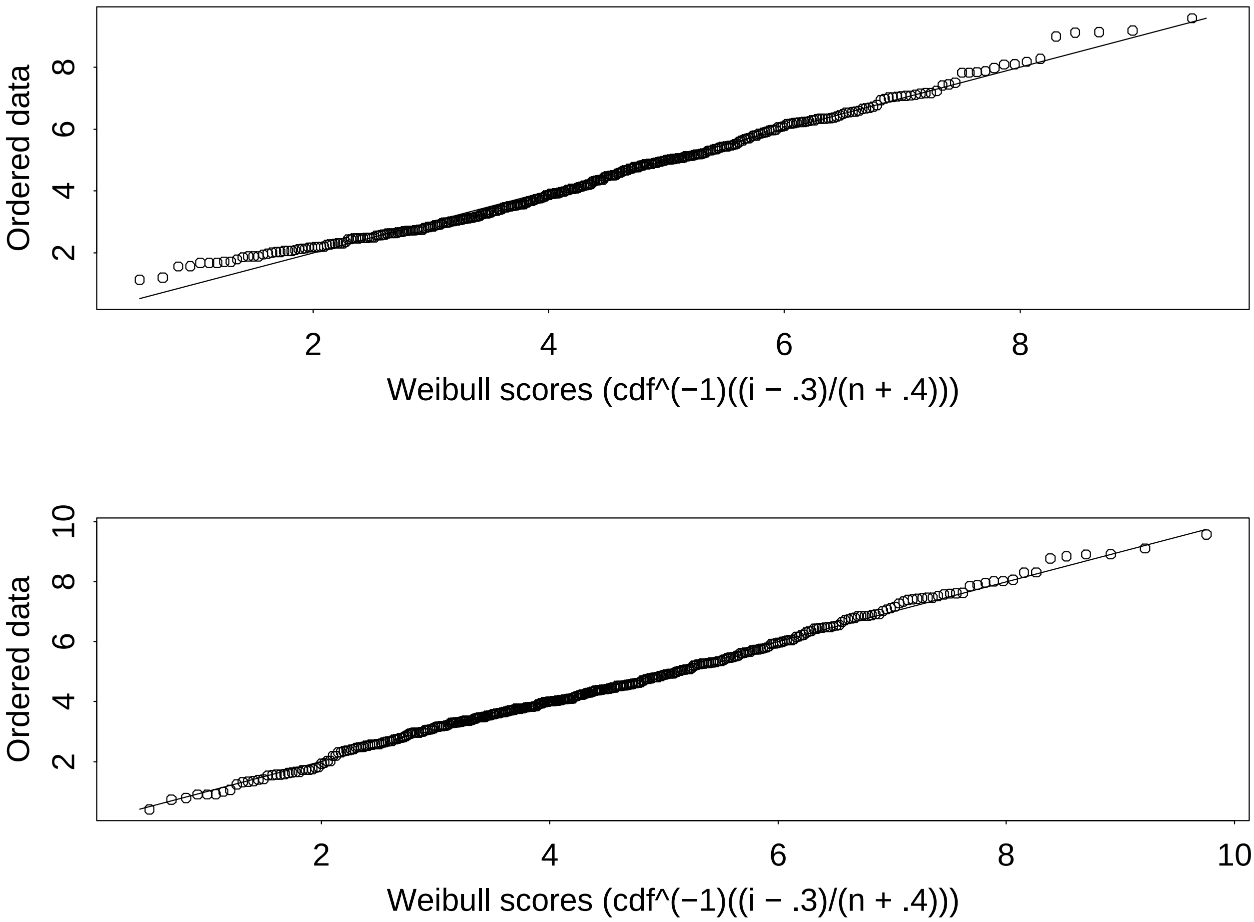

Figure 9: Upper plot: observed Ingrade MOR data (Hem Fir, 2×10, Number 2). Lower plot: generated 2-parameter Weibull data.